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# THE STEAM ENGINE

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# GAS AND OIL ENGINES

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# HE STEAM ENGINE

AND

# GAS AND OIL ENGINES

A BOOK FOR THE USE OF STUDENTS
WHO HAVE TIME TO MAKE
EXPERIMENTS AND
CALCULATIONS


BY

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# PREFACE

THIS is a book for students who have time to work many exercises. Almost every table of numbers is supposed to be worked out by the reader himself, or if he is supposed to verify only some of the numbers he must use the table in working other exercises. As an example of what I mean, consider Chap. III., in which a student is supposed to work out every number. If he is a beginner who knows but little mathematics he will work on squared paper, and he is led gradually through his own work to see, not only the value of expansion but the limit to its value because of back pressure and condensation; he sees for himself also the nature of the Willans Law. But the very same work ought to be done by an advanced student, only he will probably use formulæ which he can prove to be correct, instead of squared paper. Now the knowledge conveyed in this simple manner is of the very greatest importance, but it is usually assumed that no beginner can take it in. Indeed I may say that advanced students have usually only a very vague comprehension of this kind of knowledge. There is all the difference in the world between an attempt to study by mere reading and a real study through the actual doing of work.

Readers have great faith. Tell them that some philosopher obtained a certain law of adiabatic expansion of steam and they use that law, never testing it for themselves, although the test may only need half an hour's work. Tell them that there is a method used by everybody for showing the wetness of the steam in a cylinder, on the indicator diagram, and they use that method, although the exercise of a little common-sense would show them that the method is based on a fallacious assumption. There has been far too much of this

taking things for granted; there may have been some excuse f
doing it in the past, but there is no excuse now, for through M
McFarlane Gray and others we have very easy means of testin
things for ourselves. I am sorry to say that since Rankine's tim
no man with a good knowledge of physics and mathematics seems t
have devoted himself to a study of the steam engine. There a
men who have done very useful work; the text books are filled wit
the names of men who have done useful small things, but unfort
nately the text books give as great weight to some of the result
arrived at logically from wrong data as if Rankine himself had work
them out. There is a man better equipped than even Rankine wa
for the solution of steam engine problems, but unfortunately h
devotes himself to isolated problems having only an indirect bearin
upon steam-engine practice.

If I am looked upon as a person who wishes to give results to b
used in faith by my pupils, it will be very easy to find many faults i
this book. But I beg to say that I occupy a very different positio
I aim, throughout, at showing a student how he, himself, may attac
problems which are, as yet, only partially solved, and if I give som
of my own speculations, it is only when they are suggestive an
likely to incite a student to go on with the study through experime
and calculation along lines which seem to me good ones.

JOHN PERRY.

ROYAL COLLEGE OF SCIENCE,
*22nd February*, 1899.

*January*, 1902.—I beg to thank those readers who have sent m
corrections, and especially Mr. A. Hall, who has carefully gon
through the proofs of this third edition. Several of the illustration
of details of engines have been replaced by others, I hope to th
very considerable improvement of the book.

# CONTENTS

# THE STEAM ENGINE.

## CHAPTER I.

### INTRODUCTORY.

1. EVERYBODY thinks that the books he read in his boyhood were far more interesting than boys' books now. In one of my school books there was a story about people cast away on a desert island, who discovered and made friends with three delightful giants, who actually loved to do work, and only wanted to be superintended. Their names were "Flowing Water," "Wind," and "Vapour." **Nature's stores of energy** are indeed like helpful giants to us but they need superintendence, and we need to study their ways.

In that old story the men who discovered and utilized the services of the giants were men who had reverence and wonder and an eye for beauty of all kinds, for without these can no man invent; and because they had these fine qualities they also had that uncommon gift called common sense, and so they knew that two and three make five, and not six or merely four. That is, these men could calculate; they had a quantitative experimental knowledge of mechanics and physics. Without these kinds of knowledge you cannot understand the steam engine, although it is possible that you may get to be called engineers, for there are many children of Gibeon who can get people to call them engineers.

That a student may be aware of the kind of knowledge which ought to be familiar to engineers, I give many numerical exercises, and these ought to be worked. I must assume then that my readers know something of applied mechanics, and how to calculate the work necessary to be done in many common operations, and also how to make calculations concerning stores of mechanical energy. Mechanical energy is convertible into heat by friction, and everybody

knows this, but I assume that my readers have a **quantitative knowledge** of the fact, based upon their own experience; in fact that they have measured Joule's equivalent for themselves, and worked many numerical exercises on the conversion of one form of energy into another. Again, they are supposed to know something of chemistry, sufficient to let them grasp the idea that by letting certain chemical substances combine we can obtain energy: in the case of coal and the oxygen of the air, we usually get the energy in the form of heat, but in the case of some other substances, we get the energy in the much more manageable form of electrical energy. Mere reading and numerical work and listening to lectures are of themselves of no use; laboratory work of itself is of no use; a wide and exact knowledge of this great subject comes to us only gradually, and it never comes to a man who does not combine these various methods of study.

We have first to recognize Nature's great stores of energy, and to estimate their magnitude; in the second place we must learn how to make them available for our purposes. In the following pages I shall sometimes assume that my readers already know a great deal about the subject, and at other places I shall assume that they do not yet really know some of the most elementary facts of heat and mechanics. It is easy to make use of water-power, and my readers know how to make all sorts of calculations about it. It is heat from the sun which causes evaporation from seas to form rain and waterfalls. Wind power was utilized by our ancestors before they knew the use of metals. When we utilize this gift of Nature we steal not from the energy of rotation of the earth on its axis, but from the sun's heat. When we use fuel we utilize the energy radiated in past times to the earth, as heat and light from the sun, and perhaps it is only when we convert the mechanical energy given out by our bodies into heat by friction that we learn how intense is the storage of energy in a pound of fuel.

**2.** Nature's stores of energy are enormous when we compare them with, say, the work that a strong labourer will do in a day. When a labourer lifts 50 lbs., 60 feet high, he does 3,000 foot pounds of work. When I carelessly run off a bath full of hot water, say 20 cubic feet or 1,250 lbs. of water at 38° C. (or 100° F.), on a winter day, when the supply water is, say at 2° C. (or 35° F.), the energy that escapes is equivalent very closely to the work done by the labourer in 20,000 of his journeys, or 62½ millions of foot pounds. The energy obtainable from the burning of a pound of coal is about 12 million foot pounds, and from a pound of kerosene 17 millions. Now consider

that we may take **the sun** to have a surface as great as 12,000 times that of the earth, and we may imagine that more than half a ton (1,200 lbs.) of coal is burnt completely on every square foot of that surface every hour (about 60 times the intensity of firing in the best factory boiler furnace) [this is really about 7,250 horse-power developed as heat on every square foot]; this will give us a fair idea of the rate at which the sun is losing heat; now imagine that this enormous waste has been going on for 1,000 million years, and you have some idea of the waste of energy that has gone on in our corner of the universe. Or rather, you begin to see how hopeless it is to imagine the greatness of Nature's waste of energy. It is probable that the store of energy in any small portion of the universe in another form than that known to mechanical, or heat, or chemical engineers, might lead us to figures very much greater still; but it is not necessary here to refer to this quite different matter. Nature's greatest stores of energy are not available, possibly through our present want of knowledge; but I am inclined to think that they are really not available at all. Of the available stores the most important is that of coal, and it is necessary at once for us to become possessed of a definite knowledge of the value of coal.

When a **pound of average coal** is carefully burnt and all the available heat is measured, we find that it gives out about 8,500 centigrade or 11,700 Fahrenheit heat units, and this is equivalent to 12 million foot pounds. This 12 million foot pounds is a good figure to keep in one's memory as the calorific value of one pound of average coal (see Art. 256). Other good numbers to remember are 17 million for a pound of kerosene and 530,000 foot pounds as the calorific value of one cubic foot of average coal gas at atmospheric pressure and 0° C. Now if it is remembered that the engineer's unit of power is

$$1 \text{ horse-power} = 33{,}000 \text{ foot pounds per minute},$$

it is quite easy to make certain calculations which engineers require to do nearly every day of their lives. Thus a supply of 1 lb. of coal per hour means a supply of 12 million foot pounds of energy per hour, or 200,000 foot pounds per minute, or 6 horse-power.

It is only a **large and good steam engine** which gives out actually one useful horse-power for every 2 lbs. of coal per hour burnt in the furnace; hence a very good steam engine takes 12 horse-power as heat and gives out only 1 horse-power usefully mechanically. Even a very good engine (including the boiler) therefore takes a shilling, returns a penny usefully, and wastes elevenpence. In any

sometimes we usually mean by *efficiency* the useful power given out divided by the total power supplied: we see that even a very good steam engine and boiler has an efficiency of only $\frac{1}{10}$. It will be found later that this inefficiency is incidental to all engines which take energy in heat and give out mechanical energy.

Steam engines are approaching perfection, and for reasons to be given in Chap. XVI. we cannot expect much better results than the above. There is much better promise in gas engines. Well-made large engines are always more efficient than small ones. It is only a large steam engine of 200 horse-power or more that will give the above result. Even a large engine, if it works on a varying load like the engines of an electric or hydraulic company, will give results only one-third as good as the above; whereas many small common engines give out on the average only 1 per cent. of the whole energy supplied to them, wasting the other 99 per cent.

Now even a **small gas engine** using Dowson gas, made from anthracite, has been known to give out one useful horse-power for 1 lb. of coal. This means an efficiency which is twice as great as that of many large factory steam engines, of whose performance their makers are proud.

If coal could be burnt as zinc is burnt in an electric battery, and used in an electric engine instead of a heat engine, we might expect to convert more than 90 per cent. of the total energy into mechanical work instead of less than 8 per cent. The **fuel consumed by animals** is converted so largely into useful work that we are perfectly certain that the engine of animals is not a heat engine, but rather an electric engine. We are gradually getting some knowledge of the animal mechanism, and when we are able to imitate Nature's methods our steam and other heat engines will be looked upon as barbarous. In the meantime we are improving the steam engine. It is inherently wasteful, but it gives us great power with comparatively small weight and size. Every traveller by land or water knows how easily the power of many hundreds or thousands of horses is given out by a compact machine under easy control, and how the civilization of the world rests mainly upon the much maligned steam engine.

6. If a student can easily put his hand upon a few price lists of the best engineering firms, let him make out a table of the weight and size and horse-power of engines and boilers of various sizes. Sometimes he can help himself by drawing curves. Also he ought to know something of the **prices paid for energy.** The price paid for them by a labourer is excessive, compared with the price paid

for the same amount of work done by an engine. When intelligence enters largely we can understand why the price should be high. At page 252 I have gathered together a few facts on the price of energy, such as every practical man ought to keep in his head.

Work done by a steam engine where coal is cheap is almost cheaper than by any other agent. We can hardly compare this with the cost of energy from a turbine unless we assume the waterfall as given for nothing, so that the cost of energy will only depend upon interest and depreciation on the cost of the machinery and wages for attendance. A good modern engine of about 1,000 horse-power working under a constant load night and day, gives one horse-power for about a farthing per hour, or about £9 per year, in a country district where land, coal, and wages are cheap. This price is greatly increased as the engine is smaller and as the load is less constant, so that small steam engines in towns are more expensive than small gas engines, whose power including all charges may be put at 1d. per hour per horse-power, being only half this when the engines are of about 100 horse-power. For small powers, gas engines or oil engines are particularly to be recommended, principally because they may be so readily started and stopped and require so little attention.

A horse-power is equivalent to 746 watts.

A not unusual charge of an electric company is 5*d.* per Board of Trade unit. A Board of Trade unit is 1,000 watts for one hour, or 1⅓ horse-power for one hour; that is, the cost is 3¾*d.* per electrical horse-power hour. This great charge is mainly due to the fact that the output of an electric station fluctuates very greatly. The plant is there all the time, sufficient in size for the maximum demand, and yet for twenty hours out of the twenty-four there is a demand for very little power. It is for the same reason that the cost of a horse-power hour from an hydraulic company is 2*d.* to 4*d.* It is the great comparative cheapness of power from well-designed steam engines which is most prominent in all calculations that we make; power from coal is 500 times as cheap as power from the best manual labour, and it is in consequence of this fact that there has been such an enormous development of manufactures in the last 150 years.

**4.** When did man begin to utilize the energies of Nature, other than his food, in the production of mechanical power? The earliest dwellers in mountains must surely have used the potential energy of lifted rocks when their foes were conveniently placed underneath them. Did even the early Egyptians use either wind or water power? They probably let the wind propel small boats. The military engines and

ships of the Greeks and Romans, wonderful contrivances, were actuated by men and animals. It is true that Hero of Alexandria, 120 B.C., used steam to turn a re-action wheel, and the Egyptian priests used the pressure of vapours in performing their mysteries, and there was some knowledge of the pressure and heat properties of fluids, but it was not till the fifteenth century that we began to use Nature's stores of energy. The yew bow of England stored sufficient energy to cause an arrow to penetrate light armour. The cross bow stored much more energy, and knights could no longer safely attack the rank and file of an army. But the first heat engine, a gas engine using gunpowder, a gun, may be said to begin the history of our subject. Here the useful energy produced from heat is the kinetic energy of a projectile. We have no more efficient heat engine for obtaining ordinary mechanical power than were even the first **forms of guns.** If we could only convert kinetic energy easily into the other mechanical forms of energy, we should probably return to the gun form.

FIG. 1.—HERO'S ENGINE.
Boiler below. The right hand support is a pipe with stuffing box conveying steam to the hollow sphere.

**5.** But for a student of our subject who is a beginner, its **mere history** is probably one of the very worst of studies. The student of history fails to notice that traffic has always steadily increased on common roads, and that although railway traffic may steadily increase it may become less important again than the road traffic, and he does not notice how the value of a thing depends on many other things. Hero (120 B.C.) described a steam turbine, Fig. 1; Branca (1629 A.D.), led steam by a pipe from a boiler to impinge on the vanes of a wheel to drive it, Fig. 2. These inventions are looked upon with good-natured contempt by the man who speaks of the gradual improvements of the steam engine through Solomon de Caus, and that unfortunate victim of a worthless king the Marquis of Worcester, as well as through the pumping engines of Savery, Newcomen and Watt. Great improvement there certainly has been, but as to its exact nature I should prefer the judgment of the man who studies carefully the latest form of the steam engine, and gets to know

s defects before he indulges in the luxury of a study of history. s he reads the history he will note that the nature of what was lled *improvement* depended upon the environments of engineers, and hat these used to be very different from what they are now. He ill note, for example, that the most complete drawings of the best odern steam engine would have been worthless one hundred and fty years ago. Why, some of the oldest steam boilers had shells of one with metal plates between the fire and water; then through opper and cast-iron they gradually became of riveted wrought iron steel, the improvement not being in our conception of a boiler, but tools and methods of manufacture. Let us remember that even

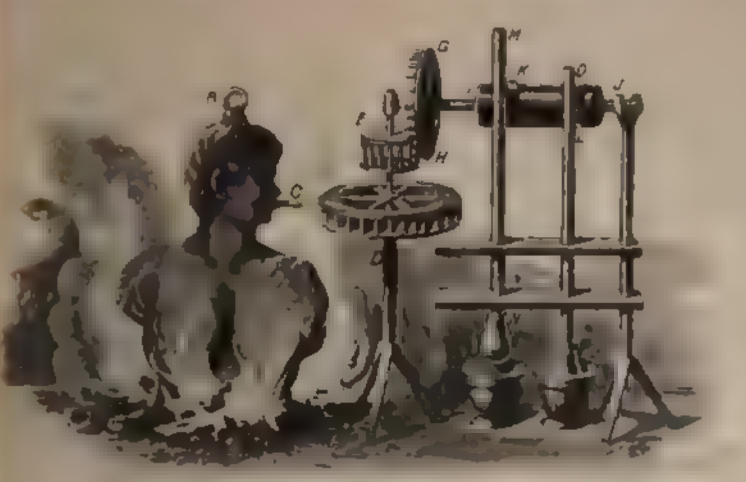


Fig. 2.—Branca's Engine.

Watt was jubilant if his cylinder was not more than $\frac{3}{8}$ of an inch untrue in its bore.[1] It is for the understanding engineer one of his most instructive lessons to go through the historical collection of models in the **South Kensington Museum**; for the young student it may not by any means be a good lesson. The instructed man will notice that the modern type of engine may be the result of gradual improvement on the old Watt pumping engine, but it is just possible that it has retained certain characteristics of the old pumping engine which are unnecessary and hurtful, and which would certainly not be

[1] In boring a cylinder the limits of error now allowed by Messrs. Willans and Robinson are ± 0·05 of a millimetre or 2 mils, and there is less error allowed in other parts of an engine. The metric system of measurement is in use in these excellent shops; its introduction has given no trouble whatsoever.

visible if it had developed from another primitive form. In the seventeenth century there was one work to be done of enormous importance, requiring much power. There was a great evil, a new evil. Had it been an old evil it would have been let alone. Mines were being sunk deeper than ever they had been before; thousands of horses had constantly to be employed to keep them free from water. Here was the new evil: everybody saw the need for great power, nobody wanted power for anything else. Hence the **pumping engine** was developed, and it was only when it showed its power to do other things as well as pump, that men ventured to prophesy

> "Soon shall thy arm, unconquered steam, afar
> Drag the slow barge, or drive the rapid car."

It is useless to consider what would have happened if it had been absolutely necessary to drive great factories in the time of Branca. Why! the very engine of Branca, almost without improvement, has lately been brought into use, and already competes in economy with the very best steam engines of equal power. There is a great deal of virtue in a revolving wheel. It may go at great speed, and yet not shake the framework which supports it, even when this framework is light. The very earliest engine, that of Hero, was really a revolving wheel, a reaction turbine, and as I write this [April, 1897] I have received a letter from a friend in Newcastle to say he had just been out on the new Parsons' turbine steam boat and that it proves to be the very fastest boat that has ever gone through the water, although only 100 feet long. And furthermore, at much smaller speeds, the very best other boats vibrate so much that a man in the stern can hardly keep himself upright, even when holding on hard, whereas at its highest speed the **Turbinia** has no vibration. See Fig. 56.

**6.** The English railway carriage was a developed stage-coach, and consequently even at the present day many of these carriages have shapes, ornamentation and uncomfortable arrangements of their space, which look ridiculous to a person ignorant of the history of their **gradual development.** Use and wont have made us fond of them, and in argument we defend their every defect as if it were really a virtue. The original steam boiler was shaped like a domestic copper or kettle, and remained so even when flues were used; when fitted to steamers it took the shape of the steamer, but it was still merely a superior sort of kettle, and although the value of high pressure was known, high pressures were not used, because they would require boilers radically different in shape. Even now the locomotive boiler

is as nearly of the shape used in Stephenson's *Rocket*, as it can be kept: it is quite absurd to think that this shape would be chosen by an unprejudiced engineer (if such a person could be found) if he were asked to design the most suitable form of boiler for its purpose. I have, perhaps, no right in such a book as this to ask how long it will be before the locomotive boiler is made so that it will not contain more steam and water than are sufficient for a few minutes' work of the engine, but it seems to me that at present one half of all the valuable properties of an engine are sacrificed to a dislike for radical change.

Throughout applied physics we find this **conservative tendency.** In so far as it makes us cautious and afraid to adopt new-fangled and untried notions, it is useful and good; there is safety and certainty in a well-known thing, whose defects are well-known, and have already been guarded against. It is only excessive and persistent shrinking from all alteration that I condemn. When I was an apprentice I was taught that there was something almost sacred in the necessity for **beams and parallel motions** in the best steam engines; they were merely the lineal descendants of the beams of Newcomen's engines, and had no more to do with the real efficiency or good working of the engines than the two hind buttons are to the fit or fastening or beauty of a frock coat. These buttons are the lineal descendants of the buttons that used to fasten back the coat flaps of our ancestors.

**7.** When Aladdin first discovered the power at his command it is remarkable how conservative he was in his notions. He made the genius bring him silver dishes, because he started in the silver dish line, and there is one of the most interesting of lessons in the fact that although each of his silver dishes was worth sixty pieces of gold, he sold each of them for one piece of gold over and over again. Aladdin's imagination had to be stirred by a violent emotion before he could make the genius work in other ways for him. Even at his best I believe that Aladdin never took full advantage of the power of the wonderful lamp. His finest palace was probably just an ordinary house, made very large and stuck over with precious stones, as vulgar as Milan Cathedral. The engineer, far more than Aladdin, needs to have his imagination developed, because Aladdin's power was unlimited, whereas, great as the stores of Nature are, they are not all for the engineer to develop. It is possible that future scientific men may discover some way of developing them, but so far as we can see there is no great store of energy available for man which is in any way comparable with coal.

For the last twenty years I have lifted up my voice occasionally

in the hearing of a not unbelieving but a half-hearted generation, to warn men of the time to come, when their great stores of energy **will be exhausted.** The chancery law of England is destroying invention in all but small details; but if I am right in my beliefs, it would be worth while for our government to hand over a few millions of money to its best scientific men, telling them to squander it in all sorts of experiments, in an intense search for some method by which instead of only from one-twelfth to one-hundredth of the energy of coal being utilised, nine-tenths of it might be utilised. If I am right, almost all the social and political questions which excite us now will be of small importance on the future of the human race, for the wild competition of nations and people for luxuries must gradually during the next four hundred years become a struggle for mere existence.

**8.** Eighteen hundred years ago Rome had numerous well-to-do citizens, and was surrounded with comfortable villas; but throughout the Roman Empire the well-to-do citizens were very few in comparison with their poor dependants or slaves. To-day, every town in England is becoming surrounded with comfortable villas; millions of people live in comfort, hundreds of thousands lead luxurious lives. But this is not only the case in England: throughout France, Germany, Italy, America, indeed all over the world, we find signs of enormous increase in numbers of a class of people who are well beyond the necessity of working for their living—people who are, we hope, developing art and literature, and the moral instincts of the nations, because they are beyond sordid cares. The phenomenon is peculiar to our own time. It was never known before in the history of the world. We also see the general population of the world increasing at an astonishing rate, and the proportion of people who may be called poor is not only less than it ever was before, but is exceedingly less. All the waste places of the earth are beginning to blossom. Irrigation has changed the yellow sand of North Texas and New Mexico and Arizona, of New South Wales and South Australia and Queensland, to green verdure, and they are filling up with people. Much of this is, we may hope, permanent; but in so far as it depends upon outside demand for agricultural produce, it will die. It would not be fair to say that the whole phenomenon is due to the steam engine. I take it that when a nation or group of nations is let alone from outside influence, the growth of its wealth increases by what we call the compound interest law, or rolling snowball law—increased wealth produces love for settled government, and settled government leads to increased wealth. But this sudden

development is surely greatly due to the steam engine. The poorest woman can easily buy clothing material and other goods that used to come on camels' backs in small quantities from the looms of India for the ornamentation and delectation of emperors and their nobles only. Quite common men live now in houses furnished with luxuries of which no potentate of the Middle Ages could dream.

I think it to be evident that very much the greater part of all that goes to make up **our civilisation** is directly or indirectly to be traced to our utilisation of coal, and it is just as evident that when our stores of coal get exhausted the greater part of all this wealth and evidence of civilisation must disappear. The world will not be left in its old state. The old state was like that of an earnest poor young man with great hopes, the new state will be that of the spendthrift, whose fortune has gone but whose expensive habits remain. Then will come the time of great struggle for Niagara by all the civilised nations of the earth; the water power of the West of Ireland will form a new centre of civilisation, as will the hills of Switzerland and all places of high tide round the coasts of the world. Then will be the time when men will try to utilise the stores of energy which now seem to be insignificant or hopelessly out of our reach: the direct radiation from the sun or the internal heat of the earth.

I am sure that the mind of no engineer ought ever to be quite free from this incubus—that **we are wasting our coal** with enormous rapidity: that a heat engine is essentially uneconomical. But this book is altogether about heat engines, and when in future I shall speak of the economy of a steam engine, I shall compare it—not with that of the perfect engine about which we know so much, but of which not one cheap specimen has yet been made, and not even with the most perfect heat engine imaginable—but with the perfect steam engine.

I am about to speak of the steam engine as it is—not even as I hope and imagine that it may become before it finally disappears. I shall speak of our best engines which act by reciprocating motion with cylinders and pistons, and in much the same sort of way whether we see them of many thousands of horse-power, driving our largest and fleetest ships, or whether they are of the smallest size, driving a few printing presses.

**9.** There is one part of my subject which must be left out: I shall speak in Chap. XXIX. of the **balancing of engines**, but I shall not be able to say much about the effects of want of balance. The study of the steam engine is really a branch of applied mechanics

and of heat. The study of vibration is also a branch of applied mechanics, but it is such a different branch that it goes usually under another name—sound or acoustics; its special study in regard to steam engine effects is so little advanced that I shall do my best to avoid mentioning it in the body of my book. In fact, I must content myself with the following general observations on the subject.

In Great Britain an annoying defect may remain unreformed for a century, but let it be called a **nuisance** by a chancery court and reform is very rapid. Large steam engines are now working in towns—not merely in the slums, but in the districts inhabited by rich people. We are first told that really we must produce no smoke, and instantly we use mechanical stokers or better grates and flues, and we refrain from forcing the fires, and get rid of smoke although for a hundred years every engineer has declared the thing impossible. There is a vast difference between being asked to try to get rid of a nuisance and being told by the policeman that we must stop working if we create a nuisance. We find it necessary to use non-condensing engines in towns because condensation water is expensive; and of course our blast pipe becomes an organ-pipe nuisance; we find that all window frames within half a mile are really microphones—we have remedied this defect of our engines because the only alternative was to stop working.

There is a defect that is put up with in locomotives and in ships which is ever so much worse in a large town, and it has been declared to be a nuisance. Consequently every young station engineer has already acquired an astonishing amount of cunning in diagnosing it and mitigating its effects. It is the vibration produced by reciprocating engines. Of course the only real remedy is the use of a steam or gas turbine, sure to be applied in the long run; but capital has given momentum in the direction of reciprocating engine manufacture, and a complete change towards turbine manufacture must be slow.

Now, in the old days of slow moving engines, the vibrations due to masses moving with accelerations were not important. The vibratory forces are quadrupled when the speeds are doubled, and compact engines must run at high speeds: hence our troubles.

We notice that rotating masses may be perfectly balanced quite easily. But it is a very different thing with reciprocating masses: to balance them needs careful study, and in many cases it seems almost impossible. I have stood on the frames, or rested my teeth against a pencil touching the frames of the best balanced

engines now in the market, and could not detect any vibration; and yet when two, three, or more such engines are working in one station, their slight effects coalesce and there may be very considerable vibration of the ground. Indeed, it may be considerable in one part of the station and hardly noticeable in another part. Again, I have examined sets of flats in a large mansion near a central station, using my "tromometer," which is very sensitive; I have gone from room to room, getting small indications of motion, and I have found that one room was in considerable vibration when its surrounding neighbours were quite quiet.

The student of acoustics does not need to be told that this room was really accidentally tuned to the vibration, and just as one string of a pianoforte will respond to a suitable faint note, just as a ship will roll dangerously if the waves are in tune with it, so this room responds to the faint impulses produced by distant engines.

A householder lays his complaint: the flowers on his dining-table are quivering always; the glass and metal ornaments are always rattling. The cunning young station engineer comes to inspect the quivering room; he says nothing at first; he goes about observing, touching, listening, and he finds some opportunity of slyly moving the heavy piano. No, he declares, he feels no vibration. Curiously enough, the complainer also feels none, nor perhaps is he likely to do so until he moves that piano exactly into the same spot again.[1]

When vibratory impulses act upon a thing, we speak of its forced vibration and also of the natural vibration which it has of its own. Its forced vibration will be small or great, depending upon whether the frequency of the forced vibration is far different from or is nearly equal to the natural frequency.

Young engineers, spurred by necessity—the mother of all reform —know a great deal: they would know ever so much more if they studied acoustics a little, and more particularly if they studied the simpler parts of the mathematics of vibration. The engineer who is a good mathematician will study Lord Rayleigh's *Theory of Sound.* I believe that a study of my own books *The Calculus for Engineers* and *Applied Mechanics* will give to the observant young engineer the sort of mathematical knowledge that he wants, and he will be fairly well fitted to fight the new nuisance if he adds a knowledge of some such book as Tyndall's *Sound.*

[1] It is interesting to study the vibrations induced in a rough model of a ship suspended by springs by model engines placed on it in various positions. We ought to be able to balance the engines more or less, and to change their sequence. The effect of synchronism of the engine periods and the natural vibration of the ship, the positions of the nodal points, &c., can only be studied in this way.

When the pitching, rocking, and tugging vibrations of a locomotive pass along a train, nobody complains; everybody feels that discomfort is part of what he has paid for. The railway shareholders pay a larger coal bill, and find it impossible to exceed certain speeds, that is all. When for every instant during the twenty-four hours, one's state room on a passenger steamer is shaking, not merely on account of the racing of the screw, but on account of the badly balanced engines, as no chancery court has declared the thing a nuisance, we expect to find this vibration on every ship that floats. Before the time of Charles the Second people did not know how miserable they ought to feel with unlighted streets; and folk who live with pigs in mud cabins are proverbially oblivious of their misery.

**10.** Having now vented all my anger upon the defects of the steam engine, it becomes my business to incite students to the study of it.

It is my intention to make this elementary account of our subject one which will be really useful to the practical engineer. But I warn my reader that he must do some work; he must try to get exact ideas. It is all very well for men and women who trifle with a subject and call it study, to frankly skip the dry part (or worse, to pretend to understand it), but the practical engineer knows that his ideas must become exact, **he must be able to make calculations.** His life is a war with Nature; he wants to coerce Nature in all sorts of ways. The careful training in calculation, what is it but a sharpening of one's weapons? I suppose that in the old days it was rather a nuisance to have to mend one's armour, to sharpen one's sword, to mend the spring of one's cross-bow. Preparatory work of this kind must always have been a bore; but the man who neglected it got knocked on the head. I must therefore ask the student to work steadily through the examples in arithmetical and graphical computation on mechanical and heat energy and to begin these at once. The more difficult exercises are in smaller type.

[Added 1901. On one square foot of Egypt the heat energy received in one year from the sun is about $10^9$ foot-pounds or 500 [illegible]. We now get 1 actual H.P. for 20 lb. total weight of [illegible] and boiler, and for such a special use as that of a [illegible] we might get it for 8 lb.]

## CHAPTER II.

### THE COMMONEST FORM OF STEAM ENGINE.

**11.** IT is difficult at first to take in the idea that fluids act on the solid bodies which they touch, with great force. The atmosphere through which we move so easily, presses with a force of 15 lb.[1] on every square inch of our bodies; but there is a balancing pressure from the inside of our bodies, and so we do not feel the pressure as a load. A boy who experiments with a sucker, and who uses more scientific methods of exhausting the air from a space, so that the pressure due to the outside atmosphere becomes more evident in various ways, will gradually get to know something about **the pressure of fluids**. Lectures and reading teach almost nothing unless we also see and make experiments.

I have sometimes closed a very small strong vessel with water in it, put it over a gas flame, and stood at a distance to watch, or rather, to hear it explode, when the pressure of the steam became great enough. It is said that the great force which steam may exert became known to Watt through the behaviour of his mother's kettle. I doubt this. Steam escapes too easily from a kettle. Even neglected boilers fail to explode in ninety-five cases out of one hundred, because even carefully riveted joints give way and leak rapidly.

**When water is boiled** in a kettle, its temperature is always about 100° C. (or 212° F.), because it is under atmospheric pressure. Giving more heat to the water does not raise its temperature, it only causes some more water to boil away. Up a mountain its temperature is less, because the atmospheric pressure is less; and

[1] Really 14·7 lb. per square inch, or 2,116 lb. per square foot, is what we take to be the standard pressure of our atmosphere. The real pressure of the atmosphere varies from day to day.

the lowered temperature of boiling water is often noted by travellers as indicating the height of a mountain.

When we use a strong kettle or boiler which is closed up, we may get very much higher temperatures and pressures. When we know the temperature we know the pressure. Students will do well to try this for themselves in the way described in Art. 179. Boilers (see Chap. XI.) are so constructed that (1) they may be able to withstand the very great pressures usually employed;[1] (2) large quantities of coal may be rapidly and completely burnt in them, its heat being, to as great an extent as possible, given to the water. We particularly want from a boiler **steam which is dry**; it must contain as little water as possible (a cloud consists of drops of water, so does the visible stuff which has come from the spout of a kettle; we want our steam to be transparent, to have no condensed steam present). There are drops of water in the steam of the boiler, because of the spray due to the violent ebullition which is always going on;[2] this we call "priming," and by careful ways of taking the steam into the steam-pipe, we greatly get rid of it. Again, unless the steam-pipe from the boiler to the engine is well covered with a non-conducting covering, some steam will condense. The electric companies by better clothing their steam-pipes have greatly diminished their coal consumption. We often give the condensed steam a chance of settling by passing it through a **separator** (Figs. 3 and 4); but do what we will, we find that the steam reaching the engine contains some water. The steam supply to the engine is controlled by the stop or regulation valve, the hand wheel of which may be turned by the engine driver. There is also in many engines a throttle valve, which is kept closing or opening more or less by the governor of the engine. The governor admits more steam if the engine is going too slowly, and closes off the steam a little if the engine is going too quickly.

Many of the small engines on board ship are supplied with steam from the main boiler through reducing valves. Steam from the Belleville boiler is always supplied to engines after a reduction of about 60 lbs. per square inch in pressure by a reducing valve to dry it.

[1] Pressures of 250 lbs. to the square inch are not yet common, but pressures of 200 in compound and triple expansion engines are quite common. Even pressures so great as 165 have long been common in locomotives, and yet in these there is usually no compounding. Single expansion engines seldom use a higher pressure than 110 lbs. per square inch.

[2] A pound of low-pressure steam is of very great volume compared with a pound of high-pressure steam; hence violent ebullition and priming are more usually found in low-pressure boilers. But it is for this very reason that artificial help to the circulation is more needed in high-pressure boilers

In passing through valves, steam loses pressure, because of friction, but it has a tendency to become drier. This tendency to superheat is not very great, however, under ordinary circumstances. Instead of relying upon throttling, it is far better to let a part of the steam pipe be kept heated either by the hot furnace gases or by a special furnace. In non-condensing


FIG. 3.—SEPARATOR.

W[illegible] steam [illegible] dry steam [illegible] water [illegible]


FIG. 4.—[illegible]

Wet steam [illegible] round [illegible] dry steam [illegible] through the [illegible] The water [illegible]

[illegible] which [illegible] water is too [illegible] Ordinary **steam traps** [illegible] the same way.

engines where we do not mind if air and other gases get mixed with the steam it is better to have a gas jet burning inside the steam-pipe being supplied with air and gas under pressure. When by any means we not only remove all water from the steam, but raise the steam to a higher temperature still, we say that it is **superheated**.

Many men have the notion that if one part in ten of the stuff entering a steam engine is water, it only means a lost effect of

10 per cent. This is very wrong. He might as well say that if one man among ten sailors entering a ship has cholera, it only means a loss of the labour of one man in ten. The fact is, the condensation and consequent waste going on in a steam engine cylinder would hardly have a chance of beginning if the entering steam were dry. The formation of a skin of water on the metal inside the cylinder is of enormous importance in causing more steam to condense there, and so destroying the efficiency of the steam engine (see Chap. XXXV.). Anyhow, it is very important that the supply of steam to an engine should be dry, and even that it should be more than just dry, that is, superheated. It will be noticed also in all the figures of the best cylinders in this book that not only are they well covered with non-conducting felt and wood, but there is also a well-drained steam jacket. This jacket communicates freely with the boiler, and it gives so much heat to the outside of the cylinder that no skin of water is likely to form itself on the inside surface. In three-cylinder engines all the cylinders and receivers are jacketed; the student will see that if all the jacket steam comes from the boiler, the low pressure cylinders have a better chance of keeping dry than the high pressure.

I feel sure that it is very important to show a beginner by direct experiment how great may be the force exerted by steam. Various experiments may be suggested. If there is an experimental boiler in the laboratory with a large safety valve, loaded with a dead weight, (as in Fig. 181), or even by a weight acting through a lever (as in Fig. 182), the student may get to know of these great forces by noting the force required to keep a valve closed. I have sometimes used a piece of apparatus like a small Bull engine (Fig. 21), lifting a weight.

**12. Ordinary Steam Engine.**—Steam engines have been of many forms, but the simplest, the direct-acting form, has survived the others. Forty years ago this sort of engine was thought unsuitable where economy of energy was important. It was used in locomotives because it was simple in construction, and not liable to get out of order. It was getting to be used in ships, partly for the same reason, but mainly because it occupied less space than the then preferable beam engines, with their parallel motions and other complicated contrivances for lessening frictional and other losses. But when a large factory engine was required, nobody dreamt of using a direct-acting engine. Later when, at length, it was recognised that there were far more serious losses in engines than those saved by parallel motions, direct-acting engines were used even in factories,

but valve motions worked by tappets or corliss or other complicated gears were used with them, and the locomotive type of engine was still scornfully thought to be suited only for very small powers. Now-a-days it is recognised that the simple construction of the locomotive engine and the simple locomotive slide valve motion may be employed in the very largest engines, where we aim at the very highest economy, and hence it is that the **old despised type of engine** is not only the easiest to describe, but the most important for students to understand.

**13.** Fig. 5 shows a small stationary engine, whose **cast-iron cylinder** A B is closed at the ends by castings E and F bolted on. It has no steam jacket, and the lagging of felt and wood which is used for clothing it and keeping it warm is not shown. This cylinder is very carefully bored out to be exactly circular in section. We are so particular about this that if a large cylinder is to lie horizontally, we bore it in the horizontal position, and if it is to be vertical, we bore it in the vertical position. The boring of a large cylinder in the shops ought to be observed by a student, who must note not only the mechanical arrangement of cylinder and boring bar, but also the speed of cut and the rates of feed both in roughing and finishing.

The shapes of some cylinders are shown in other figures.

There are two flat openings or ports at the ends, hardly visible in the figure, through which steam may be admitted or exhausted. In our engine the steam exhausts or rushes off when released, to the atmosphere, because Figs. 5 or 15 is evidently what is called a non-condensing engine. In a condensing engine the exhaust is to a condenser, a vessel kept in a cold and nearly vacuous condition. In many cylinders there is only one port for each end, see C and C in Fig. 5, whereas in others, such as Fig. 23, the steam is admitted and exhausted from each end by separate ports, called the admission and the exhaust ports. This is much better, because the exhaust steam is much lower in temperature than the entering steam, and the entering steam tends to condense on the surface metal of these passages, a much more serious matter than it may seem to be. What we call the valve motion is simply the contrivance which automatically admits steam into the cylinder on one or the other side of the piston at proper times, allowing it to escape at proper times. In Fig. 5 no valve motion is shown The student must assume that there is a boiler which generates high pressure steam as fast as it is needed, and that this steam is brought through a supply pipe to the steam chest or valve chest,

being admitted to either end of the cylinder through some kind of valve.

**14. The piston** shown in Fig. 5 is much thicker and more clumsy than is usual in larger specimens of engine. Other forms are

Fig. [illegible].—Horizontal Steam Engine.

shown in our figures, and there also we see how the piston is made steam-tight. What is wanted is that if there is high pressure steam, say in A, Fig. 5, and mere exhaust pressure in B, the steam from A shall not pass the piston into B. The cast-iron block, which is the main portion of the piston, is a very slack fit for the cylinder; three

cast-iron packing rings are shown upon it. These split rings are sprung into grooves in the block, and are always trying to get larger and press gently outwards all round against the cylinder's surface so as to create as little friction as possible, and yet to remain steam-tight. In Fig. 28 there are three rings, and it will be seen that, what with their nice fitting in the grooves, and the places of split being far apart, steam has difficulty in getting past the piston. In large pistons like Fig. 6, the springiness of the rings is not alone relied upon. Notice the various ways that are taken by different makers to produce the necessary steam-tightness without too much friction.[1] In spite of all efforts even in the most elaborate construction of pistons, such as are shown in Figs. 6, 28, 29, &c., there can be no doubt that considerable leakage takes place. If the engine is held fast in a particular position, and if high-pressure steam is admitted to one end and the other is open to the exhaust, there is such good packing of the piston in good engines that the leakage is so small as to be very difficult to measure; but unfortunately there can be no doubt of a pretty considerable leakage past the piston when it is in motion. It seems that the leak is a leak of water; the steam condenses, comes past the piston as water, and evaporates on the other side, and this state of things is mainly due to the piston in its motion passing over parts of the cylinder metal which are sometimes hot, sometimes cold.

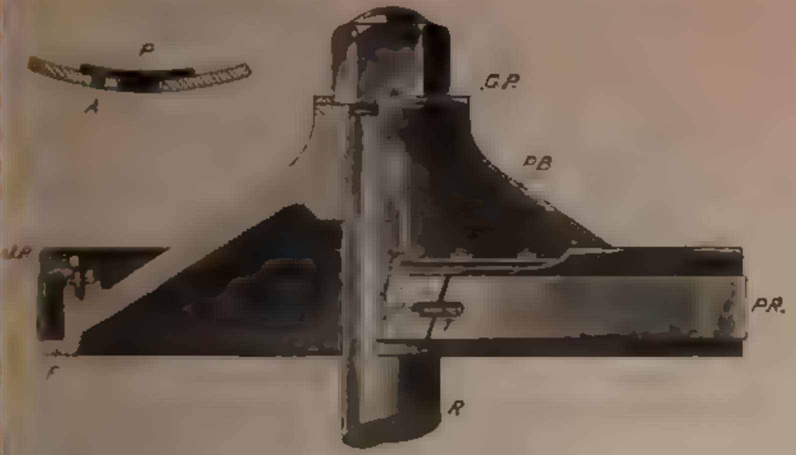
The student will notice that there are many shapes of piston. The conical shape of body of Fig. 6 is adopted for large pistons as being thought better for strength and lightness. Later on it will be seen that lightness is a very necessary quality in the moving parts of engines. As to the strength, let the student think of the great forces due to steam acting on a piston. Even a locomotive piston such as Fig. 59 is often 18 inches in diameter. Consider one only 12 inches in diameter. Let the pressure on the side to which steam is admitted be only 100 lbs. per square inch in excess of what is on the exhaust side. The total resultant force in the direction of motion of the piston is 100 lbs. × the area of a circle 12 inches diameter, or 100 × 112, or 11,200 lbs. A total force of 5 tons! A student who has experimented with a model of the Bull engine shown in Fig. 21, may perhaps understand how great such a force is and the significance of its greatness, and yet our piston is small

[1] I have proved in my book on Applied Mechanics, that the usual method of construction of piston rings is quite wrong. The ring ought to be cut, clamped smaller, and in this condition turned to the size of the cylinder, and if so made it will press uniformly all round, not otherwise.

and the steam pressure moderate. I have known men who could lift 400 lbs. with two hands. I can readily lift a man (with both hands) whose weight is 150 lbs. Think of a force which is 75 times as great as this. And notice that the steam will exert it even when the piston moves very rapidly, if the boiler will only generate steam fast enough and if the pipes and opening into the cylinder are large enough. The piston rod *R* is very firmly fastened to the piston. The nature of this fastening will be gathered from Figs. 6 and 36. Every one knows how apt some part of one's bicycle used to get loose in spite of the great experience of manufacturers. Have you ever been troubled with a shoe-tie getting loose? I have been tormented with the tying of the load of a packhorse getting loose. All kinds of lock-nuts, and locking arrangements have been invented because a fastening is so apt to get loose, even when the load on it is not great, if the load keeps altering. Now the fastening of the piston and its rod has to stand pushes and pulls each of 5 tons, altering twice or many more times every second, sometimes as in marine engines for months, and it must not get loose. Therefore you must treat with great respect the style of fastening which has been found to stand such trials. Figs. 6 and 36 show some kinds of fastening which are found to last well.[1]

In most cases before the piston has travelled over the whole stroke the admission of steam is stopped; the steam already admitted must expand and its pressure gets less than it was originally; but there is nothing very wrong just now in supposing that the steam is admitted freely at 100 lbs. pressure to the end of the stroke. At or a little before the end of the stroke it is allowed to escape to the exhaust, and high pressure steam is admitted on the B side of the piston, and consequently there is a force of 5 tons (leaving the small area of the piston rod out of the calculation) forcing the piston back again.

[1] British engineers deserve their great success. Their work is tested not merely by an appearance of goodness such as a fraudulent plumber is quite able to give to the worst of jobs. Good work is the result of honest earnest effort, such as has never before been exercised in any profession in the whole history of the world. Users of the Willans engines tell me that they will run for many months continuously with no other care than proper lubrication. Mr. Crompton told me this morning (July, 1898), that an engine had just been opened at Kensington for the first time after a 21 months' run (during lighting hours), and it was found not only to need no renewal of any part, but no sign of wear could be detected anywhere, and the engine was started without anything being done to it. Surely this reputation of English engineering is worth maintaining. It may be in the power of foreigners to obtain more orders for ships and engines, but it is our boast that when work is ordered it is well done.



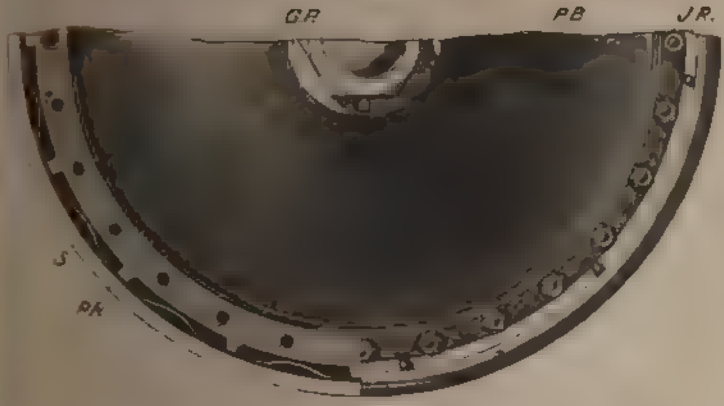





FIG. 6.—PISTON.

Marine piston of [illegible] cast steel body P.B. with [illegible] packing ring P.R. which is pressed outwards by coach springs S between packing ring P.R. and flange F. The surfaces between ring and packing ring and flange are steam tight. Spiral springs are often used instead of coach springs.

The ring [illegible] and nuts and split pins locks on the nuts.

The gun metal tongue piece P.T. has set screws [illegible] at one side [illegible] the [illegible] and is itself [illegible] with square [illegible] and split pins. All the set [illegible] B are [illegible].

In [illegible] engines [illegible] pistons are [illegible] for [illegible] at the bottom of the piston. Marine pistons are [illegible] treated similarly on account of the vessel rolling.

**15.** Consider then this force of 5 tons alternately pushing and pulling the piston rod, changing 100 or possibly 400 times per minute, the whole mass of piston and rod starting, getting up speed, stopping, and coming back again in the same fashion with great rapidity, and you will see why it is that we have a very powerful agent to deal with. The piston must be strong, its fastening to the piston rod must be strong, and the rod itself must be strong. The rod passes steam tight through the cylinder end *F*, because of the steam tight packing of the stuffing box and gland *G*. In small engines **the stuffing box** as Fig. 7 is filled with rope yarn, or asbestos rope, which the studs and nuts of the gland *G* keep squeezing so that it presses gently out against the rod. Sometimes in such a case a very thin sheet of brass or copper is between the packing and the rod, and this keeps the rod polished.

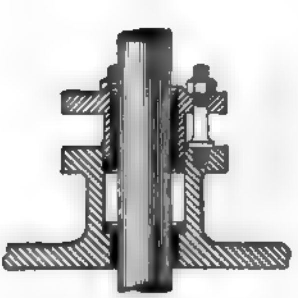


FIG. 7.—A COMMON FORM OF GLAND AND STUFFING BOX.

In the figures we see in how many different ways different manufacturers pack their stuffing boxes. Thus, for example, in Fig. 9 we have one form of metallic packing used in very large marine engines. *HH* are half rings of white metal squeezed between bronze rings *J*, a number of springs in the frame *K* at the end maintaining the pressure. The white metal is squeezed against the rod *A* keeping it steam tight. The gland *F* is forced by four studs and nuts *CC* to compress ordinary packing of asbestos in the stuffing box *FG*, and that these may never be tightened up unequally, each nut has a spur pinion as part of it, gearing on a central spur ring; turning one nut means turning all four. *F*, *G*, and the bush at the inside are bronze. Ordinary stuffing boxes have merely a brass neck bush at one end and the gland is either of brass or cast-iron, faced with brass (see Fig. 7). Packing for pump rods, &c., is of gasket (interwoven strands of hemp and cotton) or an elastic core of india-rubber surrounded by canvas. For steam rods asbestos rope is generally used.

**16.** We see then that the piston rod is pushed and pulled alternately with great forces, and that by means of the connecting rod *L* and the crank *MN* the crank shaft is kept rotating. The fly-wheel *R* keyed upon the crank shaft keeps the motion steady. If any student has difficulty in seeing how the reciprocating motion of the piston rod and cross head *H* is converted into rotatory motion by a connecting

rod and crank, let him examine any sewing machine, or foot lathe, or an ordinary cycle. He will also learn from these things the steadying effect of the fly-wheel.

The piston and its rod move with a motion of mere translation. That is, every point has a path of the same length as and parallel to that of any other point.

This is what we mean by our rough and ready statement, "the piston moves in a straight line." It is very important that the end of the rod should be guided so as to move in a straight line and so it terminates in *H* **the cross head.** The nature of the guiding is evident in Figs. 5, 15, 43, 47, &c., which show many forms of slides and slippers fastened to ends of piston rods, and their guides. The arrangement differs in different forms of engine and must be studied in connection with the shape of the frame. Notice in this example, Fig. 5, how the cylinder is fastened to the frame *P*, and the shape of the guides *KI*. The cross head is

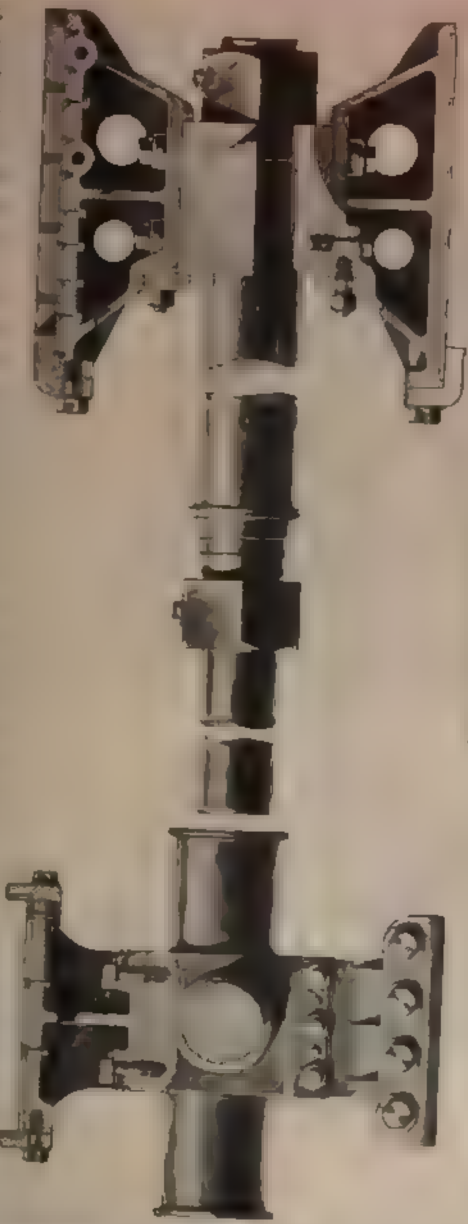


FIG. 8. SLIDE AND PISTON ROD.

This rod, with its cross pin,—to work on small forked end of connecting rod—is suitable for open guides. It shows detachable slippers carrying white metal for the [illegible] faces. A closed slide has [illegible] bearing surface only important. The two slippers are shown by mistake.

a strong pin connecting $H$ and the end of the connecting rod $HM$, at the other end being the crank pin $M$, the crank $MN$ being fixed on the revolving crank shaft on which the fly-wheel $R$ is keyed.

**17.** The nature of the reciprocating motion of $H$ and the piston when $M$ revolves uniformly is well known. It is evidently necessary for $HM$ to keep as nearly constant in length as possible, and the student must ask himself these questions: 1. The ends of **the connecting rod** must fit the pins at $H$ and $M$ always nicely, but there must be wear; how are the end fittings adjusted so that the distance between the pins keeps constant? 2. The forces at these pins alter quickly in high speed engines: in fact, blows may be said to take place; how are the keys, cotters, and other fittings of the ends prevented from shaking loose?

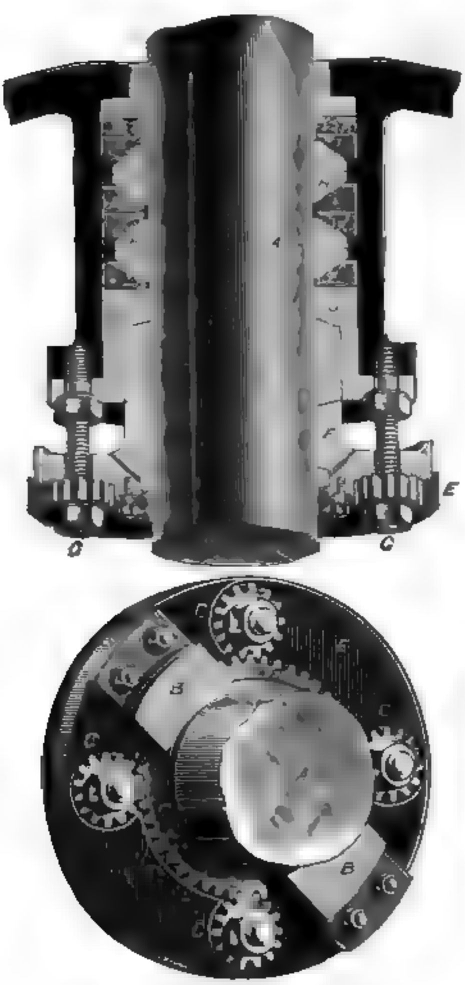


FIG. 9.—MARINE ENGINE. GLAND AND STUFFING BOX.

The figures tell this story themselves. Thus Fig. 10 shows half in section and half in elevation the end of a rod, fitting the steel crank pin $A$. The gun-metal "brasses" or steps $BC$, are kept tight on the pin by the key $H$ and cotter $G$, which fasten the strap $SF$ to the butt $E$. This kind of rod used to be common; it is not suited to withstand the loosening action which occurs in modern high speed engines.

Now look at Fig. 42 or the rod of Fig. 11, whose "big end" fits the crank pin and whose small forked or "gudgeon" end, with two

brasses of gun-metal, fits the cross head with its slipper blocks shown in Fig. 8 upon the piston rod end. Notice how the crank pin brasses, cylindric out and in, are lined with white metal because of the excessive friction, and how they may be adjusted by filing the distance pieces. Notice how the *cap* and *jaw* are fastened together. Bolts are thinned down to have a less section than at the screw thread, except where the bearing surfaces are; they stretch therefore instead of fracturing at the thread. Spare brasses are usually carried on ships, so that if heating has occurred and the white metal has "run" it may be replaced. It is as common to shrink the end of the rod upon the pin or gudgeon, and the head of the piston rod is forged, part of the piston rod becoming a slipper slide whose base

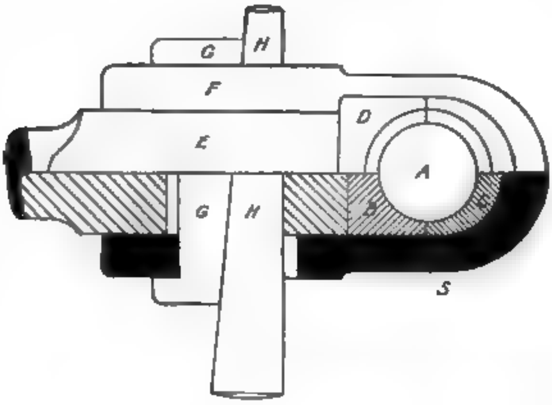



FIG. 10.—CONNECTING ROD END. For slow speeds, with steel loose strap FS, held by gib G and cotter H.

carries a gun-metal slipper faced with white metal. A slide often has a guide only on one side of it. The hollow space in the guide has cold water circulating in it for coolness in many large marine engines.

**18.** In small engines we have all sorts of **frames and guides.** The frame, all one casting, of which four views are shown in Fig. 43, has bored guides *BG*. There are two bearings, *BB*, on the frame, for the crank shaft, and the fly-wheel would be overhung, as shown in Fig. 15. This form may be used vertically as a wall engine. Fig. 44 shows the "girder-frame" of a larger engine (up to pistons of 12" diameter) also with bored guides. Fig. 47 shows the cast iron frame of a large vertical engine with two flat guides.

The careful student will notice if he examines old types of engines that an important change has been going on in the arrangement of

metal in the frames of engines, so that by its mere inertia it shall tend better to prevent vibration of the ground, and also that the whole frame shall act rather as a tie rod or a strut than as a bracket.

**19.** The crank shaft $N$ and crank with the crank pin $M$, are shown in Fig. 5. The pedestals (or pillow blocks) are very much like pedestals of ordinary shafting, except in this—the loads on ordinary shafting are usually merely vertical loads. On a crank shaft there are horizontal forces, due to the pushing and pulling forces of the connecting rod, and consequently the cap is not always placed vertically above the journal.

In the figure I show an over-hung crank, one bearing of the shaft is on the frame, the other detached from the frame would be supported beyond the fly-wheel. Fig. 15 shows a crank between the two bearings, the fly-wheel being over-hung. The reason why the part away from the crank pin is often made massive is because a lopsided rotating thing is **out of balance.** Let a student illustrate this for himself with the following piece of apparatus. Arrange a disc of wood which may be revolved at a high speed, and let there be a piece of lead fastened to it somewhere, so that the centre of gravity of the rotating part is not in the axis of rotation. It will be found that the frame and indeed the table on which it rests, gets into a state of vibration, and it is evident that this is due to the unbalanced centrifugal force of the lead. Now place an equal piece of lead exactly opposite to the first, and just as far away from the axis, and we find on rotating the disc that there is balance. Such experiments as this are very instructive. We can make a small body balance a much larger one by placing it further away from the axis. There is much more than this to be said about the subject of balancing. A rotating mass is not in balance unless its centre of gravity is in the axis of rotation, but this is not always the sufficient condition for balance, and students must refer to Chap. XXIX. They will there find that rotating masses may be perfectly balanced; that is, there need be no vibratory forces acting in the framework of the machine. Again, it is found that an engine like those shown in Figs. 5 or 15, sets the engine-bed and foundations and the ground in vibration because of the reciprocating motion of some of its parts. It is found that we get a fair approximation to the actual state of things if we suppose the piston, piston rod, cross head, and half connecting rod to move with a reciprocating motion in the centre line of the engine; these I shall call the reciprocating part; the forces on the framework due to this can only be balanced by another reciprocating part moving exactly in the opposite way. It is

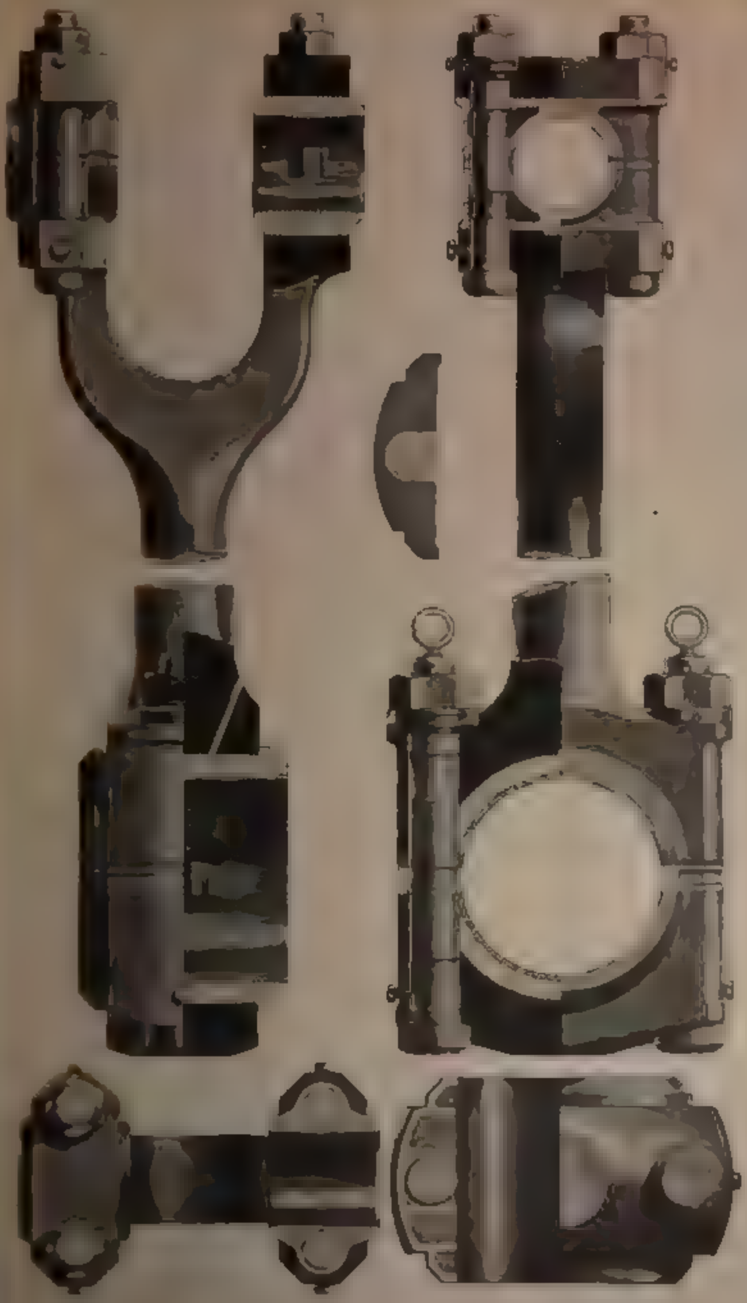


FIG. 11.—STRONGEST FORM OF MARINE ENGINE CONNECTING ROD.

very seldom indeed that we find the reciprocating parts of an engine balanced, and this is why in certain parts of London the electric light companies have been compelled to replace reciprocating engines by steam turbines. A rotating part may be made to balance a reciprocating part, but this introduces reciprocating forces in a direction at right angles to the first. This is how the endlong forces are balanced in a locomotive. There are up and down or pitching forces unbalanced in the best locomotives, but the endlong forces are balanced, and these are more important than the others, because when they are not balanced the locomotive tugs at the train instead of drawing it steadily. A very badly-balanced locomotive burns so much more coal per train mile that even the ordinary poor sort of balancing is of considerable importance. The bad balancing of the engines on a torpedo catcher or any other modern swift vessel greatly aggravates the annoyance due to vibrations produced in other ways, as for example, from the propeller (because it has not many blades) or from the action of the sea upon the hull of the vessel.

·**20. Knocking or Backlash.**—It will be noticed that however good may be the fit of a brass to a pin, when the forces between them are suddenly reversed, there is a blow; this is of course greatly increased by bad fitting, as when brasses get worn. Hence it is worth while sacrificing other advantages if by so doing we can be certain that the forces, however they may vary, never change in direction; that is, if it is invariably one side of a brass which is always acting on its pin or journal. It will be seen in Art. 65, that when steam is only allowed to act on one side of a piston, and if there is plenty of cushioning, the piston rod may never be required to exert a pull; it may always be kept exerting a pushing force at every part of the revolution of the engine, and it is mainly for this reason that single-acting engines are in use. When a single-acting engine is vertical as the Willans engine (Art. 236) for example, the mere weight of the moving part is important in preventing backlash. In this engine, however, the reciprocating forces are so great that ordinary cushioning has to be supplemented by an air-cushion.

**21.** It is to be noticed that we cannot be absolutely certain of the length of the connecting rod; also, other parts of the engine alter slightly in length, because of unequal expansion by heat, and hence it is necessary to allow of a little **clearance** at both ends of the cylinder. The actual volume of the clearance, that is, the volume which must be filled by fresh steam at the very end of the stroke, may sometimes be approximated to if we have the working drawings of the engine; but I prefer to measure it by placing the engine in

the dead point position, to fill up the clearance space with water, and then to run off this water and measure it.

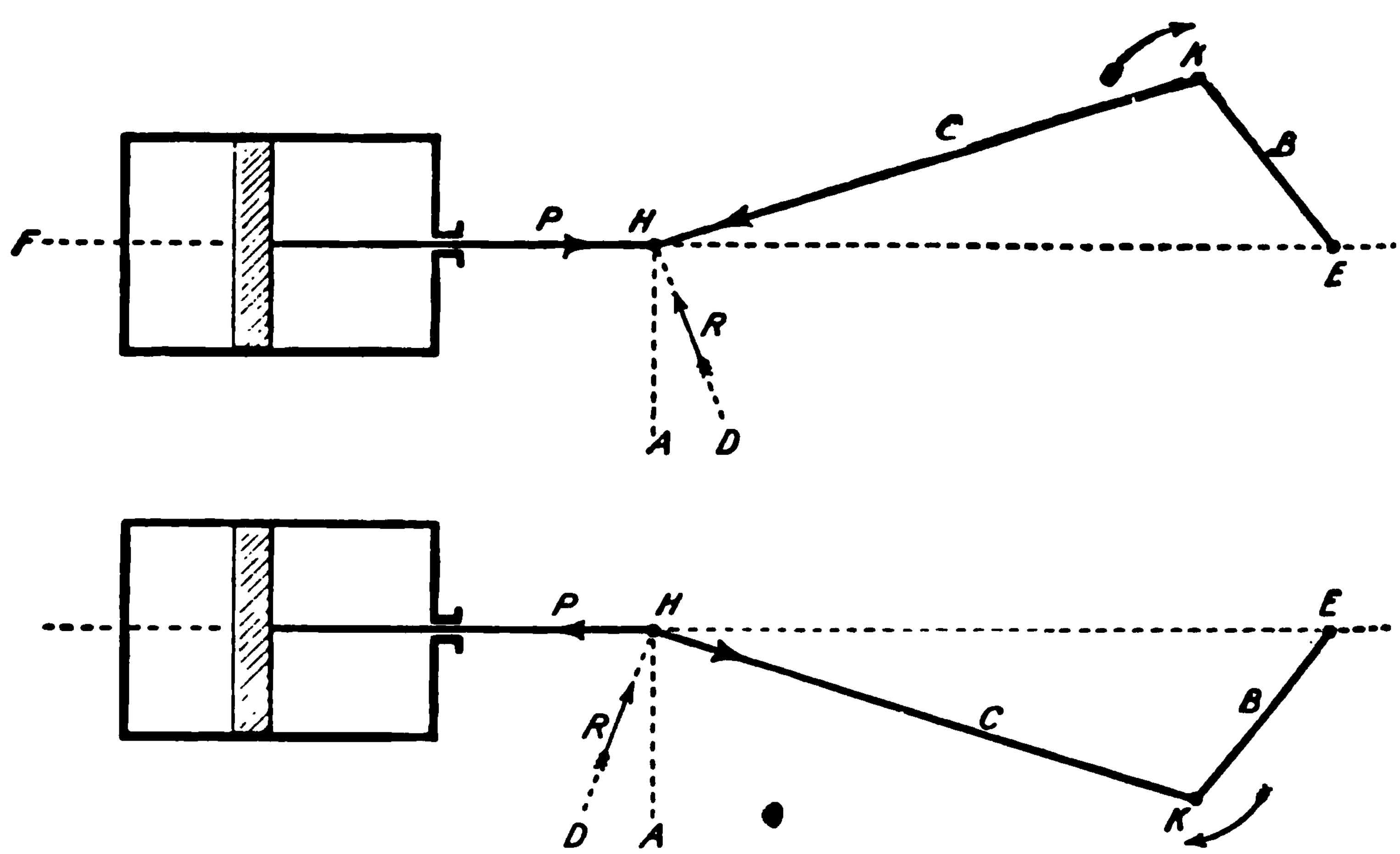
**22.** It is to be noticed that the steam acts not only on the piston, but also on the end of the cylinder. The cylinder is bolted to the engine-bed, and this is held down to concrete or brick-work or masonry foundations. Great stiffness is needed in these parts to withstand the effects of such rapidly reversed great forces. In marine engines the power is transmitted by the crank shaft to the propeller. In locomotives it is transmitted by the crank shaft, and through the driving wheels to the places where these touch the rails. The friction must exceed the pulling force, else there will be slipping. In factory engines the fly-wheel is often a great spur wheel, driving a smaller mortise spur wheel. In this case the fly-wheel is always built up of many parts, keyed and bolted together, because a single casting so large would not be true enough. In the smaller factory engines the fly-wheel is used as a drum, from whose rim the power is taken off by a belt or by ropes, as shown in Figs. 15 or 144.

Many special machines, such as dynamo electric machines, are **driven direct**; the engine and dynamo are on the same bed-plate, and the four sets of brasses for the four bearings (two for the engine and two for the dynamo) are bored out at one operation, great care being taken to get them exactly in line.

**23.** Fig. 12 shows a skeleton drawing of Figs. 5 or 15. If a student thinks for himself he will see that if $P$ is pushed in the direction of the arrows, the cylinder is pushed back. This is why the cylinder and the crank shaft must be firmly held on one framework or engine-bed. Of course if the bed were to yield in its length quite readily, there would be no turning of the shaft. The skeleton drawing brings home to us also the fact that the end of the piston rod or cross head $H$ ought to be guided; for the pushing force of five tons in $P$ is resisted by the push in $C$, and it is obvious that guides for $H$ are needed to exercise an upward guiding force, such as is shown by the arrow head. The slide is **pushed downward** on the guide. Now let the student make another skeleton drawing like Fig. 13, which is merely what Fig. 12 becomes when the crank has made half a revolution further. The piston rod is now pulling the slide, and the connecting rod pulls the slide also in its resistance to motion, so that again the force of the guides on the sliding block is upward. Hence if we are sure that the direction of motion shall always be the same, a closed slide with one slipper rubbing on one stout guide may take the place of the two or four guide bars which we see in Figs. 5, 47 or 62. Just as $C$ pushes $H$, so it

pushes the crank pin $K$; the push in $C$ multiplied by the perpendicular distance from $E$ to $HK$ is what we call the *turning moment* on the crank shaft.

**24.** It is of very great importance for a student to study (not so much with mathematical exactitude as to have working notions) this **turning moment** for every position of the piston. It may be done, perhaps, by making many skeleton drawings; but it is far better to have a working sectional model such as is shown in Fig. 101. If there is a workshop available, a student will very readily make a sufficiently good model for himself with a few laths of wood and wood screws. I myself have used with students a large model in which the distance from $A$ to $K$ is 6 feet. It has a connecting rod which




FIGS. 12 AND 13.

may be lengthened, the distance from $K$ to $A$ also being altered; the distance of the piston $P$ from the end of its stroke may be measured with great accuracy, and also the angle turned through by the crank from $O$ $O$, its *dead point* position. First, we study the mechanism, noting how travel of piston and angle of crank are related to one another (see Art. 67). Second, we study the forces acting in the several parts, and particularly the turning moment on the crank shaft. Third, we notice that the weight of the conducting rod must modify our calculations a little, but not much. Fourth, we notice that the forces must be rather different at one speed of rotation of the shaft from what they are at another, because it requires force to set a body in motion, and to stop it an opposite kind of force. Notice the great difference between this and the previous effect due

to mere weight of connecting rod. It may be said that all this is a mere matter of calculation. Now it is true that we can learn a great deal by mere mathematics, but what we often learn is merely how to pass examinations; it is a student's business to learn to think, and he may be quite sure that he will never really think about or understand the steam engine till he has experimented, observed, and handled either real parts of engines or such a model as I have described.

**25.** However great the pushing or pulling force on the piston or connecting rod may be, there are two positions, the two ends of the stroke, in which there is no turning moment on the crank shaft. These are the *dead points*, well known to all ladies who work sewing-machines, and to men who work foot lathes or bicycles. And the turning moment varies greatly during a revolution. Hence, to equalise this and also to make sure that we can start an engine from any position whatsoever, it is usual to duplicate everything, there being two engines working the same shaft, their cranks being at right angles, so that when one is at its dead point the other cannot be so. When three cylinders work the same crank shaft their cranks usually make angles of 120° with one another.

Fig. 62 is an example of the **coupled engines** of a locomotive, the cranks being at right angles. Donkey engines used for crane work on board ship have two cranks at right angles and no fly-wheel, so that they may be easily stopped and started from any position. Any person who watches such an engine working must see how important is the steadying function of the fly-wheel of an ordinary engine. Engines in hydraulic power stations are often stopped and started automatically by the rising and falling of the accumulator weight acting on a throttle valve, and this needs coupled engines. Some of our figures show three cranks on the same shaft. Not only do we in these ways get a more uniform turning moment on our shaft, but we find it easier to balance the forces which act on our framing and foundations. This is one reason why triple cylinder engines are now so largely used, but it is not the most important reason.

**26.** We see that if steam is in $A$, Fig. 5, at great pressure coming from the boiler, and if the steam has escaped from $B$ to the atmosphere or to a condenser so that the pressure in $B$ is small, the piston is being pushed from left to right and the crank turns in the direction of the hands of a watch. The fly-wheel has great inertia, and so the crank moves beyond the "dead point" position. If now steam is admitted to the $B$ side of the piston and exhausts from the $A$ side, the piston is moved from right to left. We see then that a great

force acts on the piston in the direction of its motion if steam is properly admitted and exhausted to and from the *A* and *B* sides alternately, the crank moving uniformly if the fly-wheel is large enough.

I have said that the pressure is calculated on the cross section of the cylinder, and does not depend upon the mere shape of the surface exposed to the steam. The student ought to be quite sure that this is so. Neglecting friction, due to motion of the fluid (quite negligible here), a fluid presses at right angles everywhere to any surface as shown in Fig. 14. But it will be found that all the lateral pressures balance one another, and the resultant force on the piston is just the same as if it were quite flat. Perhaps this will be the more evident if we imagine the piston, say that of Fig. 14, to be weightless and frictionless, and that steam of the same pressure is admitted on both sides of it. Although one of these is flat and the other is not, we cannot imagine that the piston will tend to move. The proof is given in books on applied mechanics. See also Art. 113.

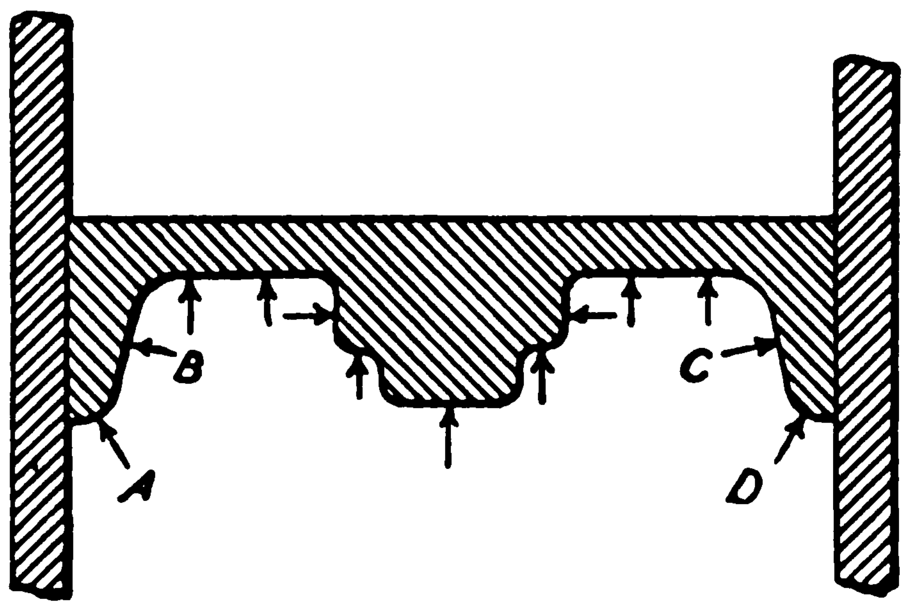


FIG. 14.—PRESSURE ON A PISTON.

**27.** We have not spoken yet of the effect of the **piston rod.** Let the student work these exercises.

EXERCISE 1. The absolute pressure (pressure above that of a perfect vacuum is said to be absolute) in the space *A*, Fig. 5, is 167 lbs. per square inch, and the absolute pressure in *B* is 17 lbs. per square inch; the cylinder 12 inches in diameter (112 square inches in cross section), and the piston rod is 2¼ inches in diameter (2¼ × 2¼ × ·7854, or 4 square inches in cross section). What is the resultant force on the piston?

*Answer.* The force on the *A* side is 112 × 167, or 18704 lbs. The force from the *B* side is (112 − 4) × 17, or 1836 lbs. on the piston itself, and if we take the atmospheric pressure outside to be 14·7 lbs. per square inch, as this acts on the piston rod, there is also a force resisting the motion of 4 × 14·7, or 59 lbs., so that the resultant force is 18704 − 1836 − 59, or 16809. Our rough and ready calculation when we neglected the area of the piston rod, gave us 16800 lbs., and so was in error to only a very small extent.

EXERCISE 2. Steam in *B* is at 167 lbs. per square inch, and there is exhaust in *A* at 17 lbs. per square inch, take the same sizes as before. Here the resisting force on the *A* side is 17 × 112, or 1904 lbs. Steam in *B* acts on the annular area 112 − 4, or 108 square inches, the force being 108 × 167, or 18036 lbs., together with the atmospheric pressure on the piston rod of 14·7 × 4, or 59 lbs. Thus the

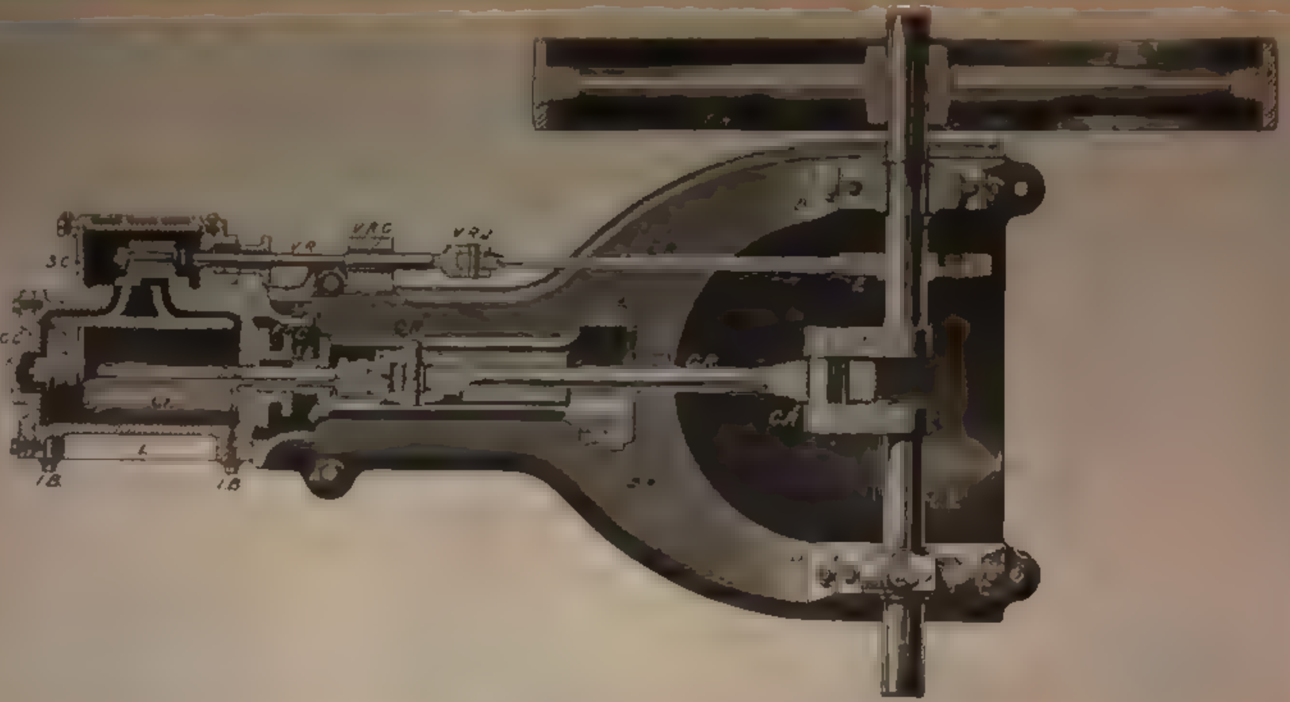



Fig 15 - Plan of Small Horizontal Engine

Showing slide valve S.V., steam or valve chest S C , valve rod V R and guide V R G , eccentric E , eccentric rod E R , crank shaft C S , flywheel F. W , crank shaft pedestals B., cross head C H , back and front cylinder covers B C C. and F C C.


FIG. 16.
Side view of eccentric and valve of Fig. 15, showing how the valve uncovers the ports. The edges A and B are shown also in Fig. 20.

resultant force from right to left is 16191 lbs. Notice that it is the area of the piston rod which has caused the above rough and ready answer to be too great by nearly 4 per cent. It is usual to neglect the area of the piston rod in such calculations.

**28.** It is the function of a **valve gear** to admit and exhaust steam to and from the spaces $A$ and $B$ at the proper instants. We might imagine four valves—one admitting steam from the boiler to $A$, another exhausting it, and a similar pair to and from $B$. Thus in Fig. 23 there are the two steam valves $A$ and $B$ which admit steam from the space $F$ to which it comes from the boiler and another two, $C$ and $D$, which release steam to the exhaust space $E$, which communicates with the atmosphere or a condenser. The valves are cylindric, filling cylindric seats, and it is the very effective but complicated *Corliss gear* which gives them their proper motions.

**29.** In a very great many engines a **slide valve** is used like $SV$, Figs. 15 and 16, the face of the valve and its seat being plane. The eccentric disc $E$ is keyed on the crank shaft so that the straps and rod $ER$ cause the valve to get a reciprocating motion, a thing easy enough to understand when seen, and not to be easily understood without being seen. Fig. 20 shows in 13 views the motions of the piston and valve. Steam is admitted to the steam chest $SC$ all round the back of the valve, which slides steam tight on the seat. In Fig. 15 steam is rushing from $SC$ through the left-hand port to the space to the left of the piston, whereas any steam which may exist in $Cy$ is free to escape by the right-hand port to the exhaust passage, which is cast as part of the cylinder. Another view, a cross section of the cylinder and valve through this exhaust

passage is shown in Fig. 18. Let the student examine and sketch and draw a real valve. I have attempted to give an idea of its shape in Fig. 19. On the valve seat there are three openings or

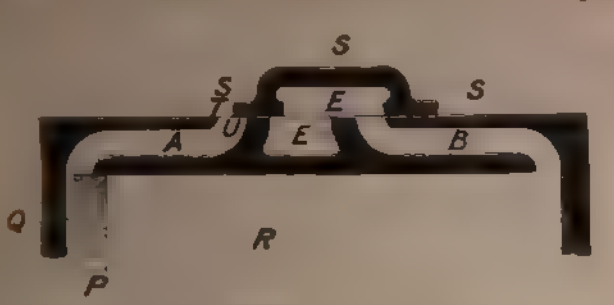



Fig. 17.—Slide Valve and Seat

In the position shown, steam is entering from the steam space S through A to the space Q; any steam in R is exhausting through B to E.

the ends of passages. The narrow $P_1$ leads to one end of the cylinder, the narrow $P_2$ to the other end, and the broader middle one $E$ to the exhaust. Looking down on the back of the valve, Fig. 16, when it lies on its seat, we see how as it moves it uncovers and covers up again the ports $P_1$ and $P_2$, so that steam may get into them or get


Fig. 18.

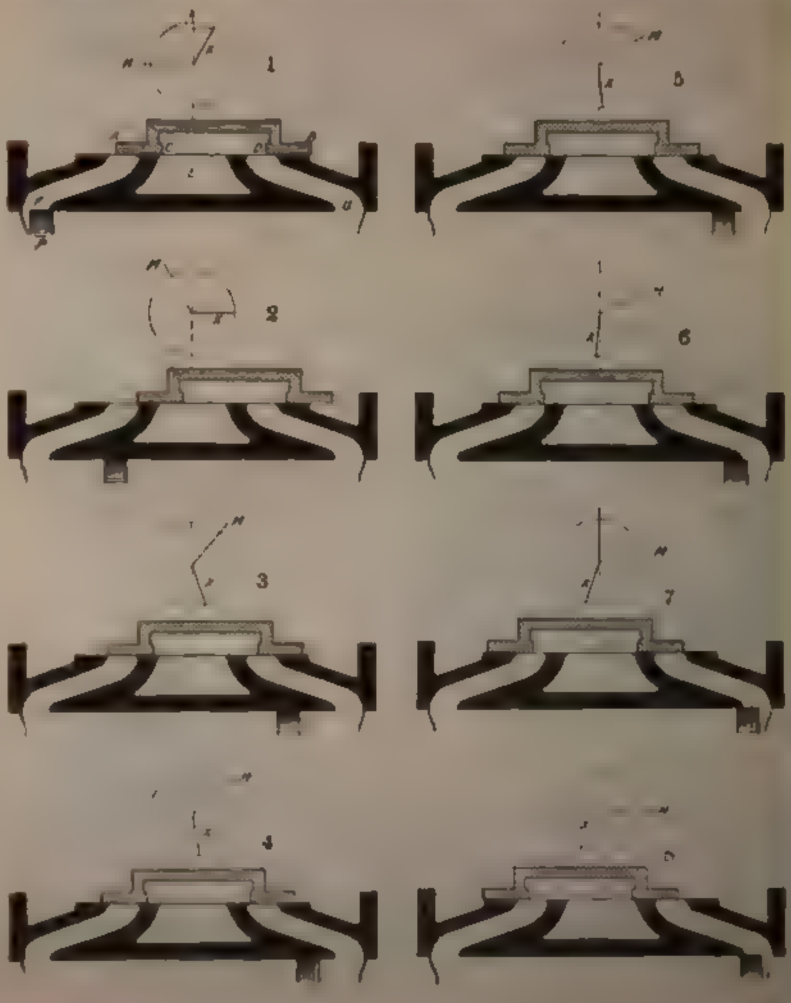
Perspective of section of cylinder through exhaust passage E. Valve not shown.


Fig. 19.

Showing slide valve lifted above the ports P1 and P2 and exhaust space E which it usually covers.

cut off and underneath the valve we see by the section, Fig. 18, how steam reaches $E$ from $P_1$ or $P_2$ when it is necessary to exhaust. It will be found by Fig 20, 1 and 2, if we keep our eye on what occurs in the space to the left of the piston $P$ that steam is admitted

freely as the piston travels from left to right until in 3 we see that it is cut off. As the piston travels on and no more steam is admitted, as the volume of the steam gets larger, its pressure gets less, and it continues to get less till we have the position shown in 6 or 7. Here the steam is *released* and begins to rush away to the exhaust; in 8 we may imagine that even if the time is short, so much steam has got away that the pressure is practically the same as in the exhaust. Now the piston begins to turn back, to move from right to left, and as it moves, the left-hand space is freely open to the exhaust, and the pressure in it is low and remains so till we get to 11. The exhaust now closes, and what is called *cushioning* begins. As the piston



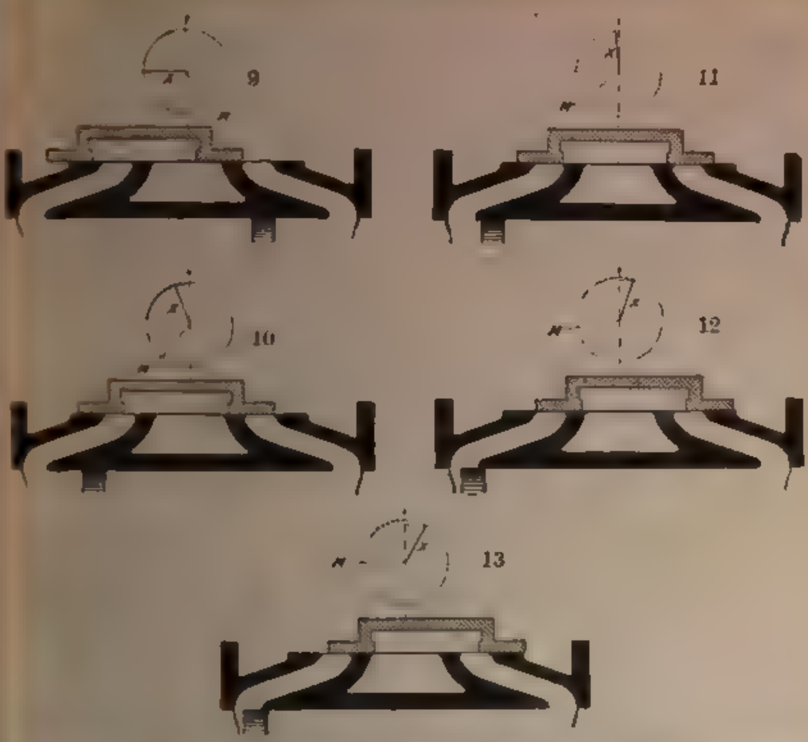



FIG. 20.—RELATIVE POSITIONS OF ECCENTRIC SLIDE VALVE AND PISTON.

As the crank M turns clockwise through one revolution, the valve and piston take these positions. The position of the crank M is shown for each, and X shows the position of the eccentric which, as in Fig. [illegible], works the valve directly. X is ahead of M by an angle, which is 90° + the *angle of advance*. In this case the angle of advance is [illegible].

makes the space smaller, any steam in this space gets to have a higher and higher pressure until, in the position of 12, fresh steam is admitted just before the beginning of the new stroke. This cushioning and admission before the end of the stroke are just as important in high-speed engines in bringing the massive reciprocating piston, &c., to rest, as a thick feather bed would be in preventing one getting hurt in jumping from a window.

**30. To ensure the study** of the diagrams of Fig. 20 let the student draw upon paper a curve showing his notion of how the pressure alters in the left-hand space. If he will measure the distance of the piston (any point of it) from the end of its stroke and call it $x$ at any instant, and at the corresponding time try to get a notion of the steam pressure in the space, he will see that the following numbers are about right. I take the entering steam to be at the absolute pressure of 100 lbs. per square inch, and the exhaust steam at 17 lbs. per square inch (as if it were a non-condensing

engine, the exhaust being a little greater in pressure than the atmosphere). If the crank of an actual engine made one turn in about two minutes, and if we had a pressure gauge to show the pressure in *A*, we could observe these pressures. But in truth they were measured in a very different way on an engine making 10[illegible] revolutions per minute.

Students will note for themselves how reasonable it is to assume that the pressures are fairly correct. I take the length of the crank to be 0·5 feet.

FORWARD STROKE.

| $x$ | 0 | 0·1 | 0·2 | 0·3 | 0·4 | 0·5 | 0·6 | 0·7 | 0·8 | 0·9 | 1·0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $p$ | 100 | 100 | 100 | 100 | 100 | 100 | 97 | 85 | 63 | 50 | 23 |

BACK STROKE.

| $x$ | 1·0 | 0·9 | 0·8 | 0·7 | 0·6 | 0·5 | 0·4 | 0·3 | 0·2 | 0·1 | 0 05 | 0·0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $p$ | 23 | 19 | 18 | 17 | 17 | 17 | 17 | 17 | 17 | 19 | 28 | 100 |

The student will now plot $x$ and $p$ as the co-ordinates of points on squared paper to any scale he pleases, and see what sort of figure he obtains. He will note that the points of admission, cut off, release, and compression may not seem to be very distinctly marked: this is because the pressures were measured on a quick moving engine whose valves closed comparatively slowly. The best kinds of valve gear close the valves very quickly. We have an instrument called an *indicator*, which draws such a curve as this for us, showing the pressure on either side of the piston for all positions of the piston, even when the engine revolves at 350 revolutions per minute; it is easy to understand that it is of great use to the engineer whose slide valve and piston are out of sight. For one thing, it enables him to see if his valve is admitting, cutting off, releasing, and allowing compression to begin just at the right periods.

Notice in the above that the distance $x$ does not exactly represent the volume of the steam to scale, because, even when $x$ is $o$ and the piston is at the end of its stroke, the space has some volume which we call **the clearance**. We cannot let the piston come quite up to the cylinder end, and besides the passages have some volume. We try to get the volume of the clearance space as small as possible (and of as little surface as possible because of condensation when fresh steam is admitted), but in the following approximate calculations (chap. III.) I shall assume no clearance.

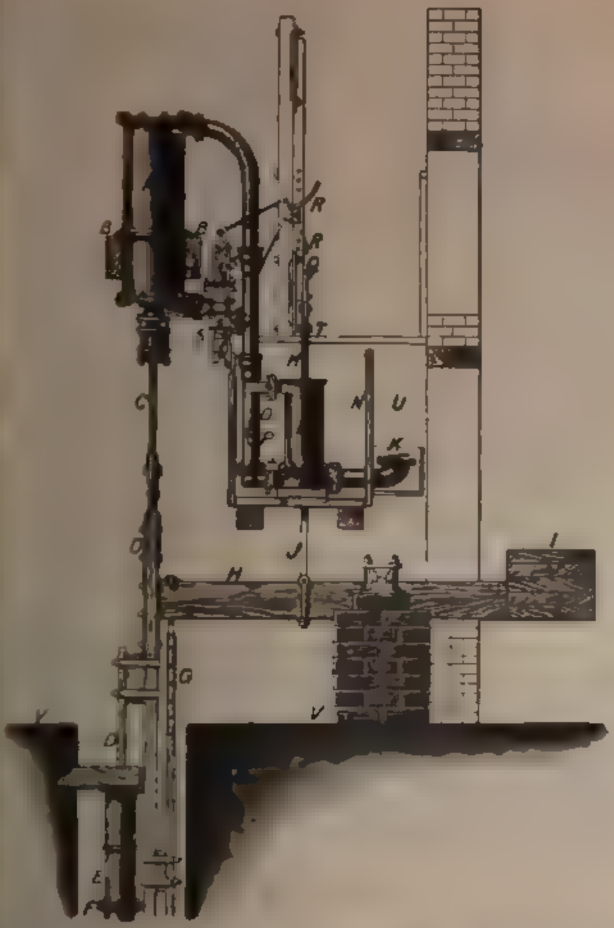


FIG. 21. BULL'S PUMPING ENGINE.

Pump rods D attached by piston rod C to piston in cylinder A, lifted up by steam pressure admitted also to piston, and produced below it an descent by the pipe condenser P in the tank N and air pump I. The lever H enables weights to be adjusted, and also drives air pump, which also is a plug rod regulating the valves.

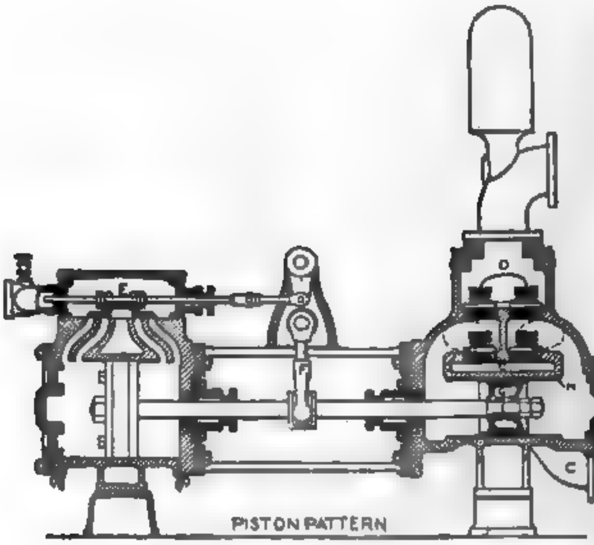



Fig. 22.—Worthin[...]
Steam Pump.

In this, as in the m[...] copies of it, the st[...] and pump pistons [...] on one rod. As u[...] now, it is double, t[...] is there are two r[...] two steam cylind[...] and two pumps. [...] rod of one moves [...] arm F, and this wo[...] the slide valve E of [...] other, so that there [...] pause at the end of e[...] stroke, allowing [...] pump valves to [...] and close gradu[...] The pump has a li[...] H, which may be th[...] or thin for high or [...] lifts. Water is pum[...] from C to D.

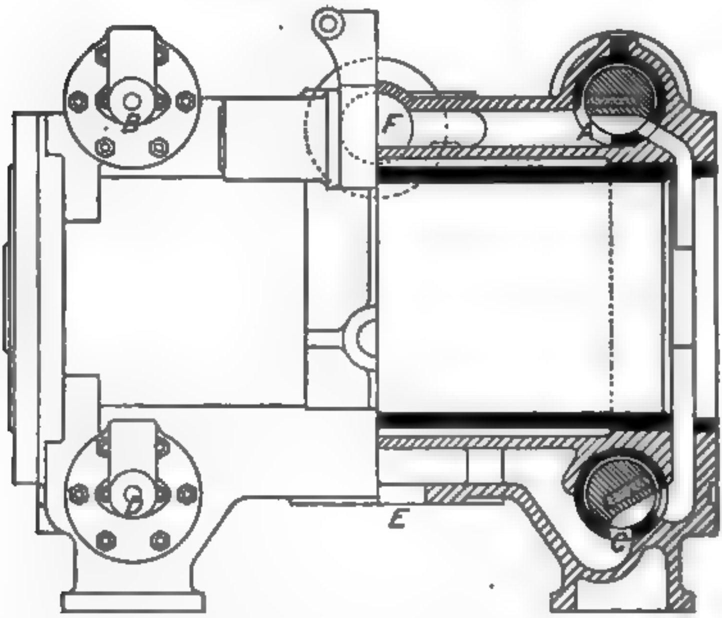


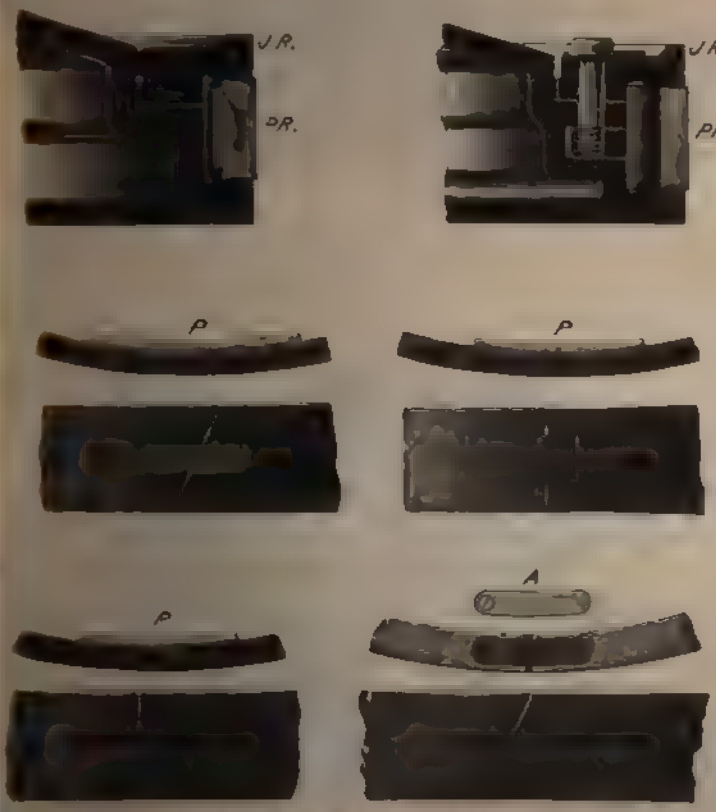

Fig. 23.—Cylinder of Engine with Corliss Gear.

Showing the liner steam jacket, steam ports A and B and exhaust ports C and D Steam ent[...] at F, and is exhausted at E. The valves are cylindric slides rotated by rods from a wristpl[...] The governor disengages the admission valves, so that they shut off quickly, earlier or later in [...] stroke depending on the work being done by the engine.

*Note.*—When A is open C should be shut not as shown.


Fig. 24.

Piston with hollow cast-iron body, with single packing ring B, pressed out with many springs. Packing ring B is fastened down by the pins C. D is the tongue.




Figs. 25 and 26.—Piston Rings and Tongues.

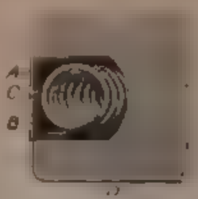



FIG. 27.—PISTON PACKING.

Two rings A and B are pressed outwards and apart by a continuous spiral spring C. This is to prevent the usual leakage at the top and bottom flat surfaces between the ring and body.

FIG. 28.—PISTON PACKING.

The junk ring is screwed down so that the piston rings just fit the grooves, and the nuts it in position are secured by split pins.

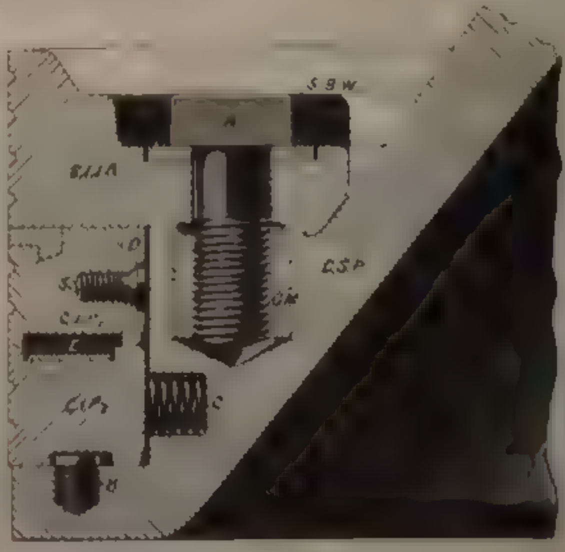



FIG. 29.

An elaborate piston packing for the high pressure cylinder of a marine engine.

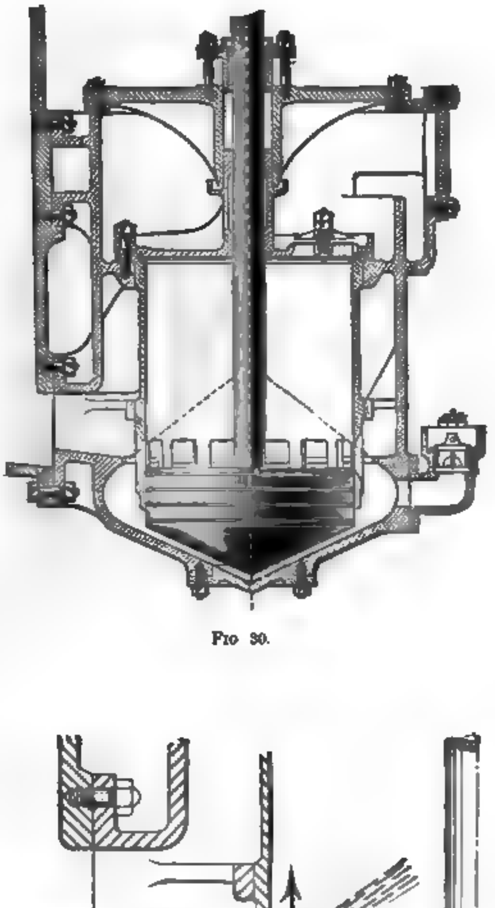


Fig. 30.

Figs. 30 and 31 show a sectional elevation of the Edwards Air Pump. "In this pump foot and bucket valves are entirely dispensed with. The condensed steam flows continuously by gravity from the condenser into the base of the pump. Upon the descent of the bucket the water is projected silently and without shock at a high velocity through the ports into the working barrel. The air has a perfectly free entrance into the pump, and the action of the water tends to compress that already in the barrel and to entrain or carry in more air with it, thus increasing the efficiency of the pump (see Fig. 31). The rising water is followed by the rising bucket, which closes the ports, and, sweeping the air and water before it, discharges them through the valves at the top of the barrel."

"The advantages claimed for the Edwards pump are :—

"(1) Simplicity of design and reduction in the number of valves required.

"(2) Increased efficiency. The water flows to the pump *by gravity* and is there dealt with mechanically, and is in no way dependent upon pressure in the condenser to force it into the pump ; thus it becomes possible to obtain a higher vacuum than would otherwise be the case.

"(3) Free air inlets ; there being no foot and bucket valves to obstruct the entrance of the air to the pump.

"(4) A regular quantity of water to be dealt with at each revolution of the pump. This is a very important point, more particularly with high-speed pumps."

(The blocks for Figs. 30 and 31 were lent by Messrs. the Edwards Air Pump Syndicate.)

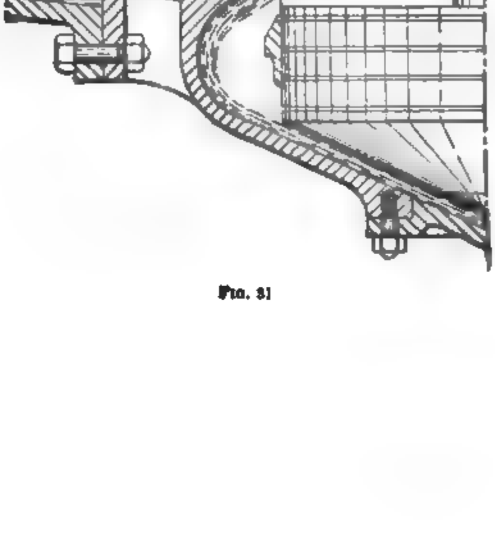


Fig. 31

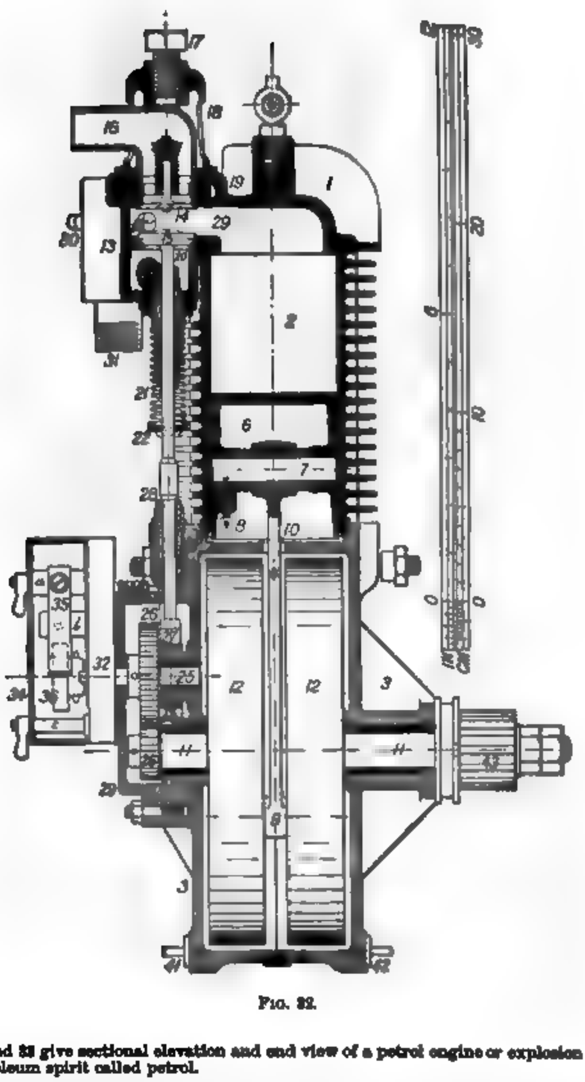



FIG. 32.

FIGS. 32 and 33 give sectional elevation and end view of a petrol engine or explosion engine the light petroleum spirit called petrol.

14 is an induction valve which opens on the down-stroke of piston 6 when pressure in cylin is less than that in pipe 16, admitting air and petrol; 29 is the clearance space into which mixture is compressed to about 65 lbs. per square inch. The charge is then fired by an el spark from ignition plug 30. The piston is driven forward; on the return stroke the valve opened by cam 33 and the contents of cylinder exhausted. The cam 33 is on shaft 25, which, thr the spur wheels 26, revolves at half the rate of the shaft 11 of engine, so that exhaust takes

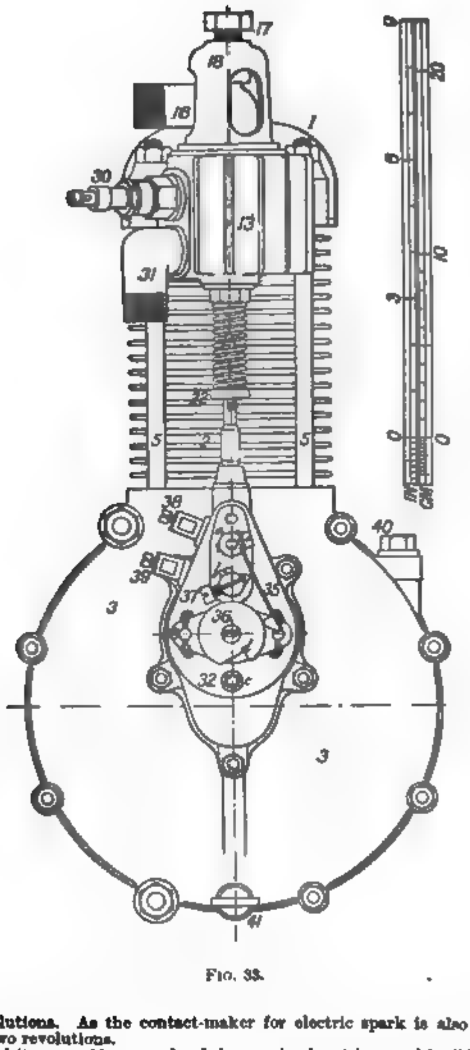



Fig. 33.

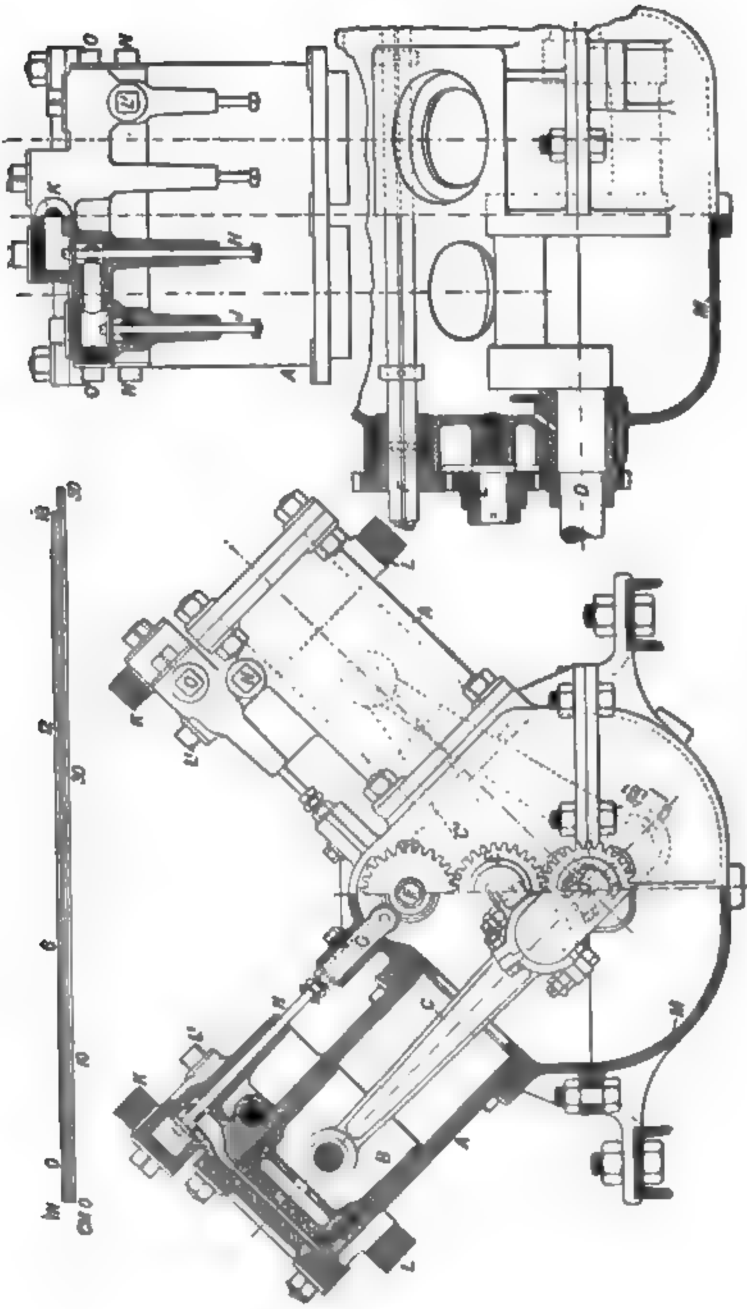
every two revolutions. As the contact-maker for electric spark is also on shaft 25 ignition place once in two revolutions.

The cylinder 2 and its cover 10 are made of close-grained cast-iron and is ribbed to allow sufficient tion to keep cylinder cool. The body 3, containing shaft, &c., is of aluminium. Valves 14, 15 f steel, and connecting-rod 9 is a steel forging with gun metal bushes at its end.

The crank is formed by means of a fixed pin joining the two fly-wheel discs 12 and 12.

The main shaft 11 has gun metal bushes for bearings.

(These figures are taken from *Motor Vehicles and Motors*, by special permission of Mr. ment.)



THE SERPOLLET DIAGONAL FOUR-CYLINDER STEAM MOTOR. RATED AS 10 H.P.

FIG. 34.—Transverse sectional elevation.
FIG. 35.—Longitudinal sectional elevation.

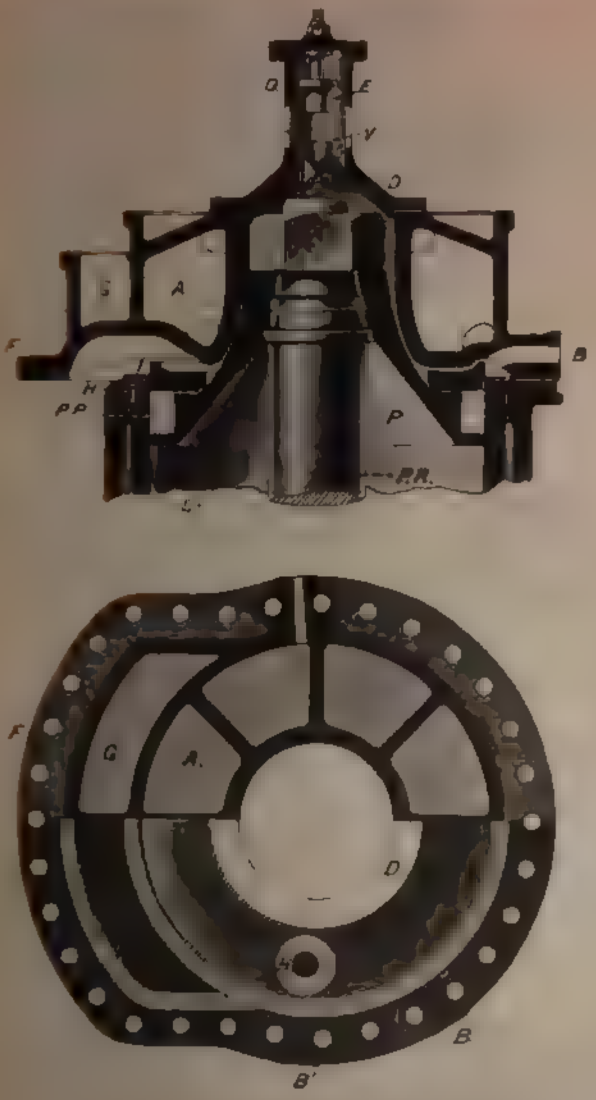



FIG. [illegible]. COVER FOR HIGH PRESSURE CYLINDER.

[illegible] of the steam port, and [illegible] central [illegible] P P is the space for the piston rings or packing.

E

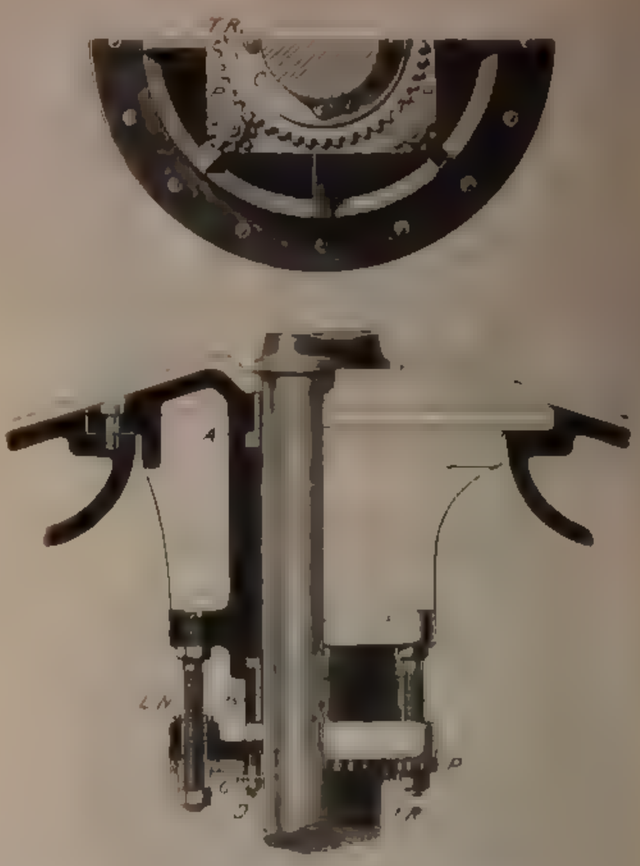



FIG. 3.—HIGH PRESSURE CYLINDER STUFFING BOX.

[illegible]


Fig. 38.—Cylinder Shell

This is for [illegible] intermediate pressure cylinder. The door carrying the stuffing box is not shown, but the position it would occupy is easily seen by reference to Fig. [illegible] stuffing box. The [illegible] is at D, the liner not being shown. The valve seat is of hard cast iron or steel, and is fastened down with countersunk gun-metal screws, being well [illegible] which [illegible]. Cast iron is found better than gun-metal for the valve seat. The liner C L is also made of hard cast iron and the remainder of the cylinder of soft cast iron.


Fig. 39.—Fastening of Liner to Cylinder Shell

[illegible] the liner C L is fastened by being screwed firmly to the cylinder base [illegible] whether steam is blowing through.

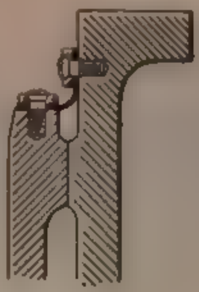


FIG. 46.—FASTENING OF LINER TO CYLINDER SHELL.

Expansion may be allowed for by using a copper ring having one row of screws in the cylinder shell and the other row in the cylinder liner.

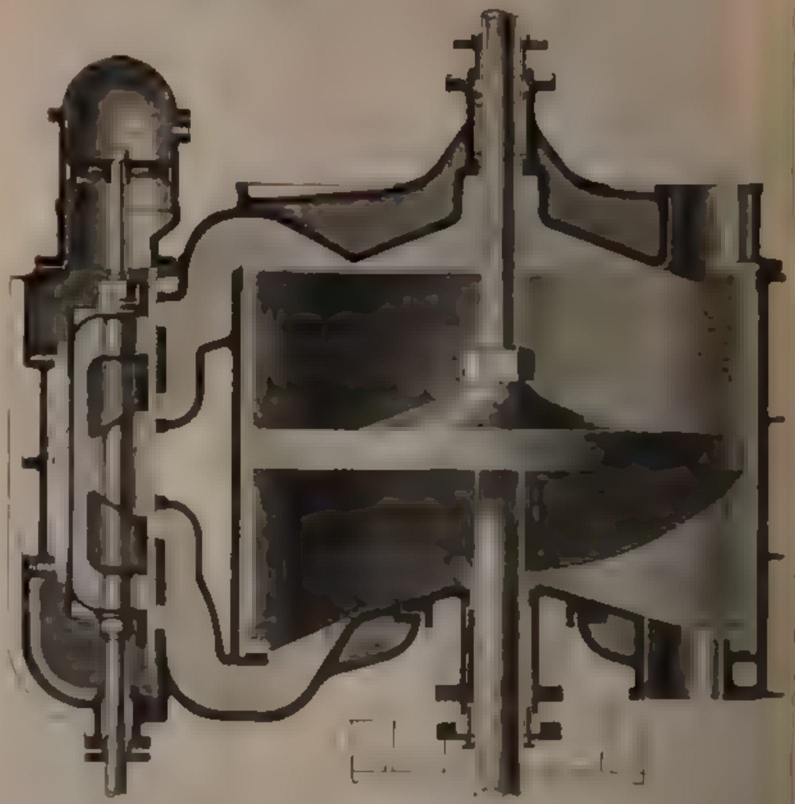


FIG. 47.—MARINE ENGINE CYLINDER.

Showing the steam jacket, double ported valve with [illegible] piston and relief frame. The [illegible] a complicated casting, the liner and the cover are the three important parts of a cylinder. The cover [illegible]

The [illegible] cylinder cover are getting to be [illegible]

[illegible] from the bottom of [illegible] cylinders and valve chests [illegible] worked by levers from the starting platform and discharge into the condenser and into the feed tank. Two manholes are [illegible] the manhole covers are omitted in the figure. Safety valves and pressure gauges are fitted to all receivers.

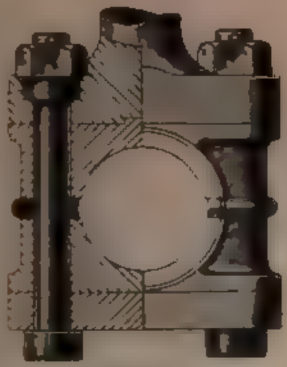


Fig. 41.—Connecting Rod End.

The end of this connecting rod is made T shaped and the brass is recessed into it. Between the [illegible] is a thick liner,—often accompanied by thin sheets of brass or tin,—and adjustment for wear [illegible] the thickness of the liner. There is a plate or cap at the outer end, and long [illegible] together as shown.

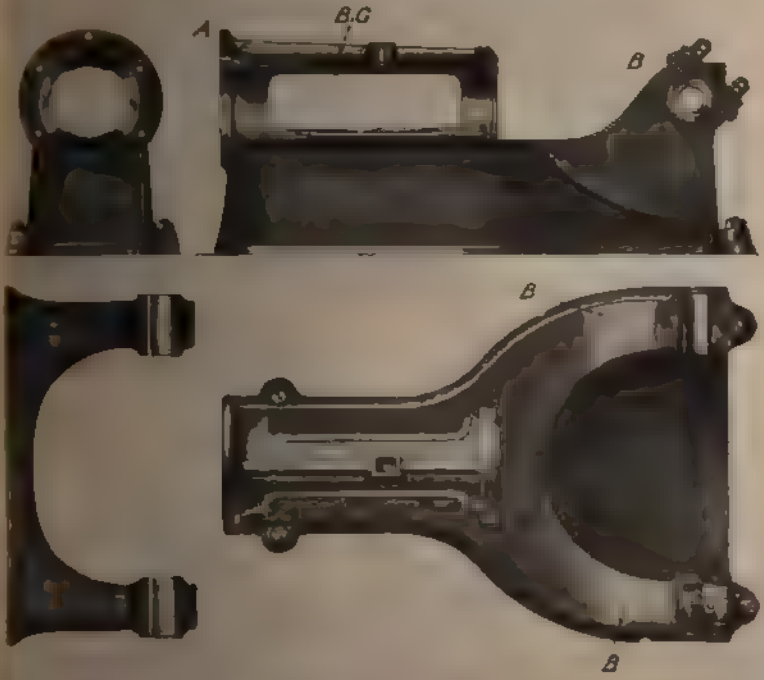



Fig. 42. Frame for Small Engine.

With bored guide. Cylinder (not shown) overhung. Fly wheel overhung.

FIG. 11. FRAME FOR SMALL ENGINE.

Girder type, with bored guide. Cylinder (not shown) overhung. Separate crank shaft bearing.

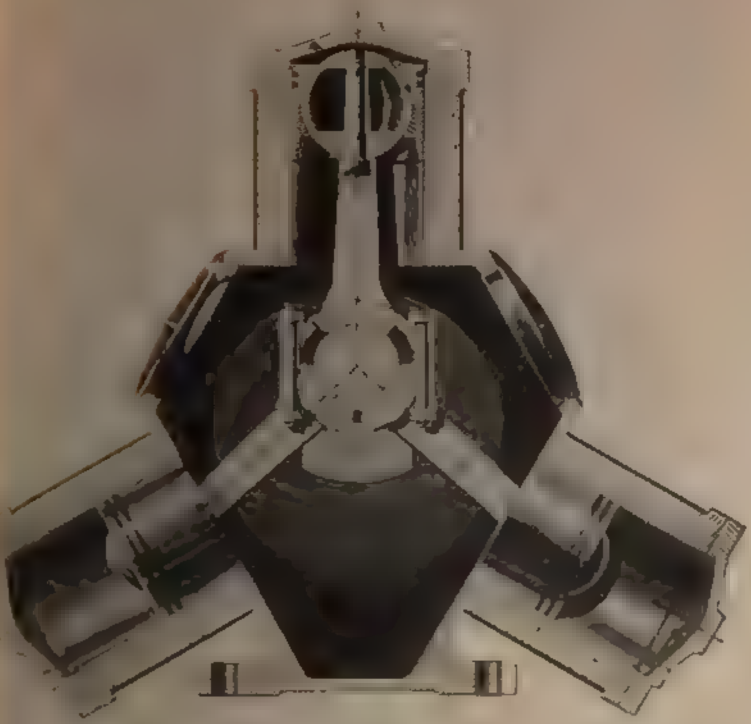


Fig. 45.—Small Brotherhood Steam Engine.

There are three single-acting cylinders with trunk pistons, driving the same crank. The valve [illegible] is not shown.

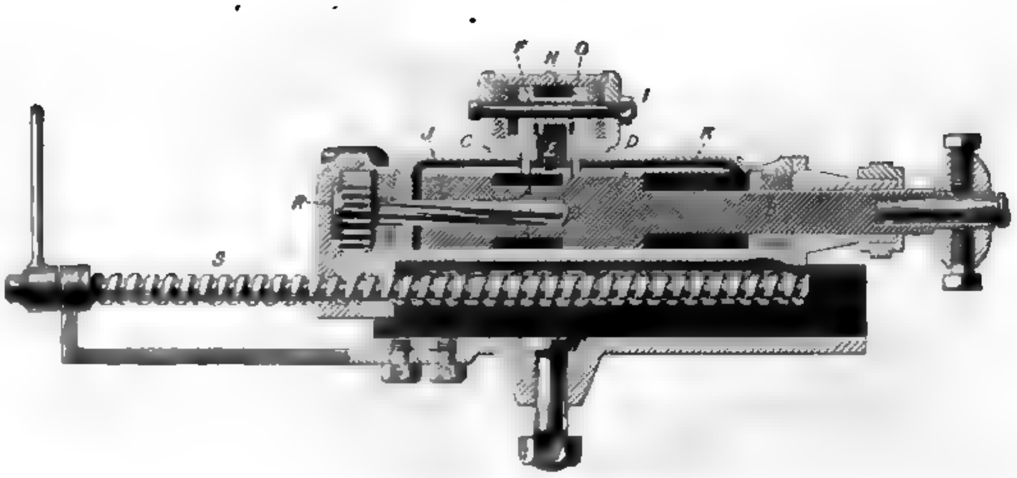



Fig. 46.—Steam Rock Drill.

The drill is attached to the piston rod B, and the screw S feeds it. During every stroke the piston drives the ratchet R through the bar with twist grooves R B, and pawls prevent back rotation in the return stroke, so that the piston and drill rotate. The piston valve F G admits steam alternately to the end of the piston, the valve being held fast on its seat while the stroke is being made, because there is steam pressure on one side and exhaust pressure in one of the passages C or D on the other side. C and D exhaust from the ends of the piston. The waste space round A is always open to the exhaust, and thus either C or D has exhaust pressure in it, the other having steam, the larger parts of the piston opening one of them and closing the other at the same time; and this also determines whether the space at the end of the valve shall be open to steam or exhaust, and therefore how it shall move under the steam pressure. The whole contrivance is most ingenious.

*Note.*—The bars separating exhaust space E from the steam ports adjacent to it have been drawn too narrow, so that the covering bars on valve F G will not close these ports.

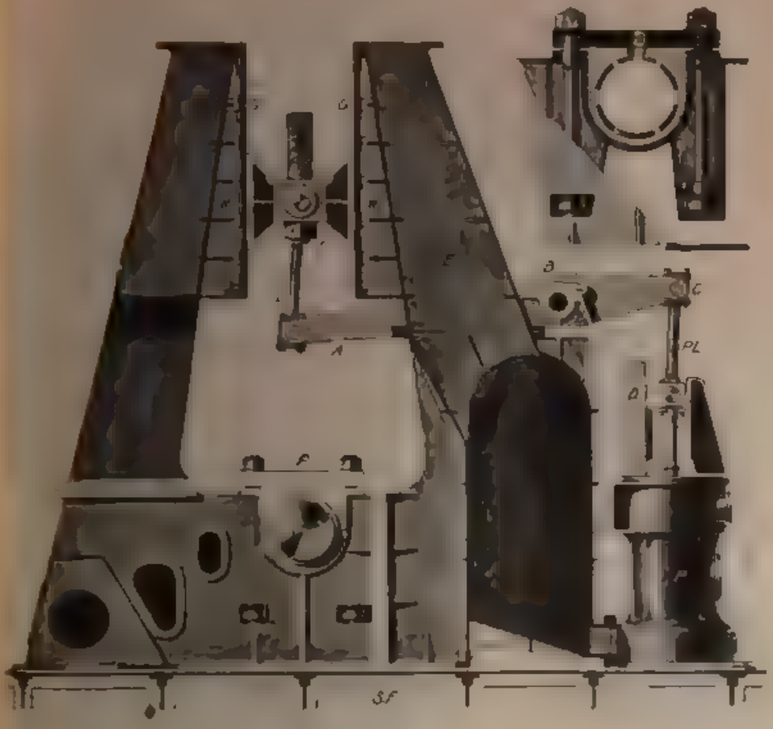



FIG. 47. MARINE ENGINE FRAME.

This shows the common arrangement of the frame in [illegible] engines. It rests upon the ship [illegible]. It has open slide bearings suitable for the [illegible] shown in figure. P R being [illegible] rod the other end of which is [illegible] to the [illegible] the cylinder which rests [illegible] frame [illegible] Water [illegible] the [illegible] bearings if necessary. The [illegible] brasses of [illegible] easily removed by taking [illegible] when they [illegible] displacing the shaft.

[illegible] pumps [illegible] A B [illegible] from the crosshead [illegible] A B C. Many engineers prefer an independent engine to drive the [illegible] pumps, which are [illegible] with [illegible] also independent [illegible] pumps. The surface [illegible]

[illegible] common form of frame is like half the above [illegible] only [illegible] guide) with a steel stay bar in [illegible]

Fig. 45.—Marine Engine Crank.

In a large marine engine crank shaft the pins and shaft are hollow. A, the annular lubricat[illegible] [illegible] centrifugal force through the tube B to the bearing surface through the hole [illegible] This system of lubrication [illegible]

[illegible] counterweights [illegible] fitted to the cranks of [illegible] balance the rotating parts [illegible]

The crank shafts [illegible] parts [illegible] cylinder [illegible]

In smaller engines the crank shaft [illegible] Many engines [illegible] part by shrinking the webs on to the shafts and the pins into the webs, driving or screwing small pins into the joints.

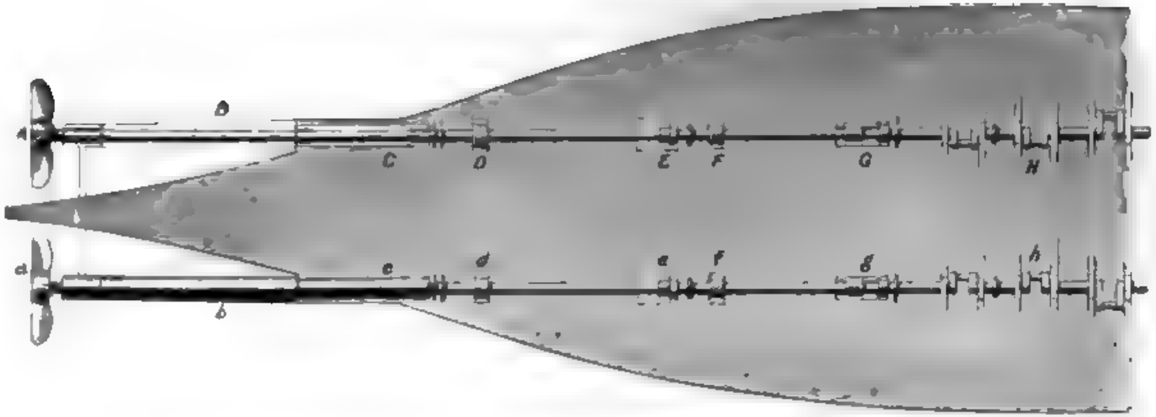



FIG. 49.—SECTIONAL PLAN OF TWIN SCREW STEAMER.

Showing the position of the propeller shafts and crank shafts along the steamer. Two three-cylinder engines at H and *h*. G and *g* are thrust blocks. F *f*, D *d* are ordinary bearings. C *c* are the stern tubes with casings or sleeves B *b*, and there are outer bearings close to the propellers A *a*, carried by one bracket.

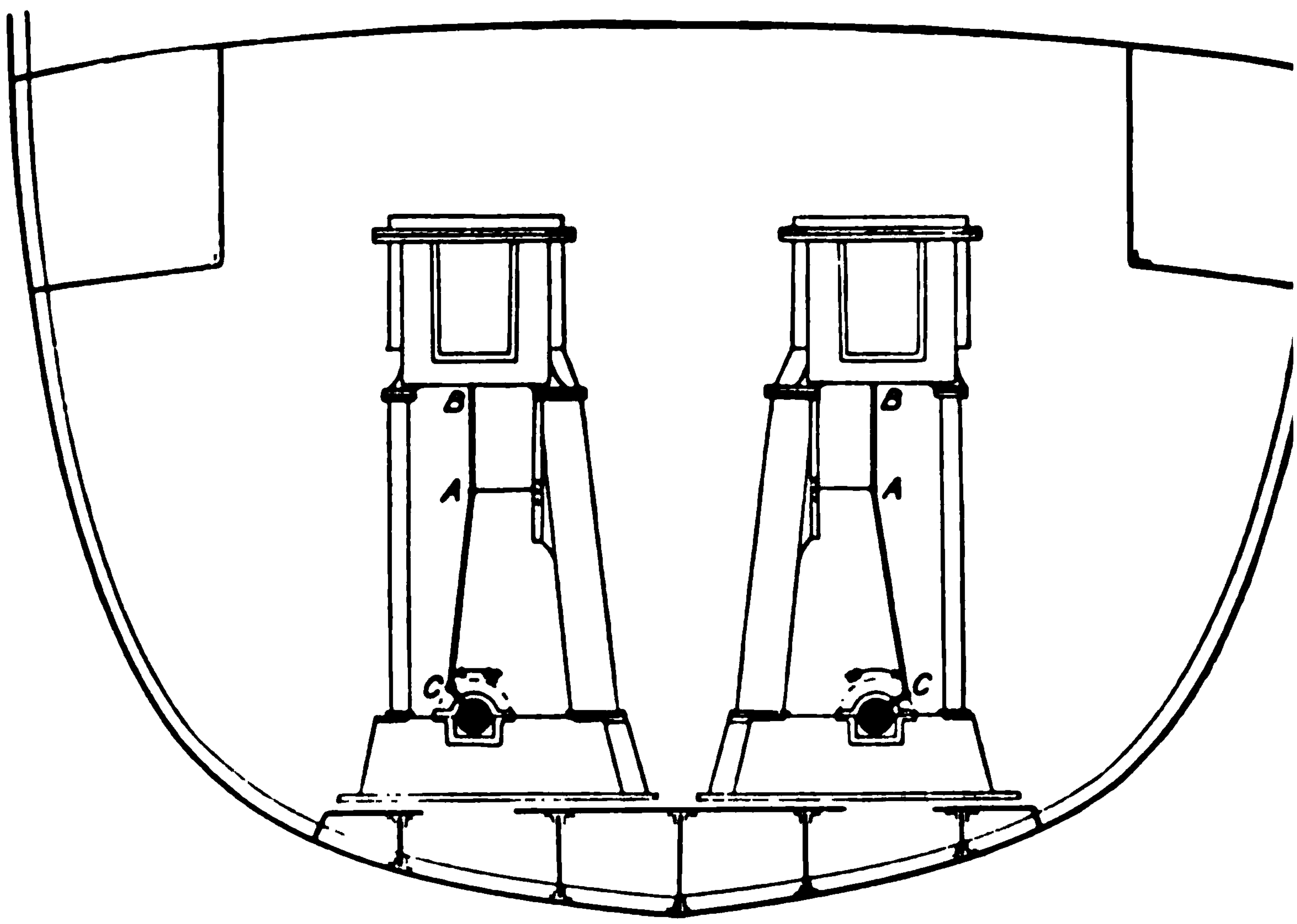



FIG. 50.—CROSS SECTION OF TWIN SCREW STEAMER.

Showing position of shafts and engines in the hull. In ships of war, coal bunker prot


Fig. 61.—Thrust Block.

The thrust block is attached to the ship frames near engines [illegible]

There are seven collars on the shaft S which rub on horseshoe surfaces lined on one side with white metal; these are easily withdrawn for repair. [illegible] for wear by the nuts N N on the side bolts passing from B to B. Wedges at the ends allow for fore and aft adjustment [illegible] water to circulate as shown, filling the whole trough. Ordinary plummer blocks are [illegible] The [illegible] the weight of the shaft S. Lubrication is by syringes from the overhead [illegible] places [illegible] the thrust block [illegible] when the engines are put astern it is sometimes thought advisable to set a few of the shoes to [illegible] shoes have been provided, one set taking the ahead thrust and the other set taking the astern thrust [illegible]

The propeller shaft is [illegible] forged steel, flanged couplings being forged on, except the sternmost length, which has a loose coupling, as it has to be fitted in from the outside through the stern tube. Each plummer block is provided with a water and an oil box.

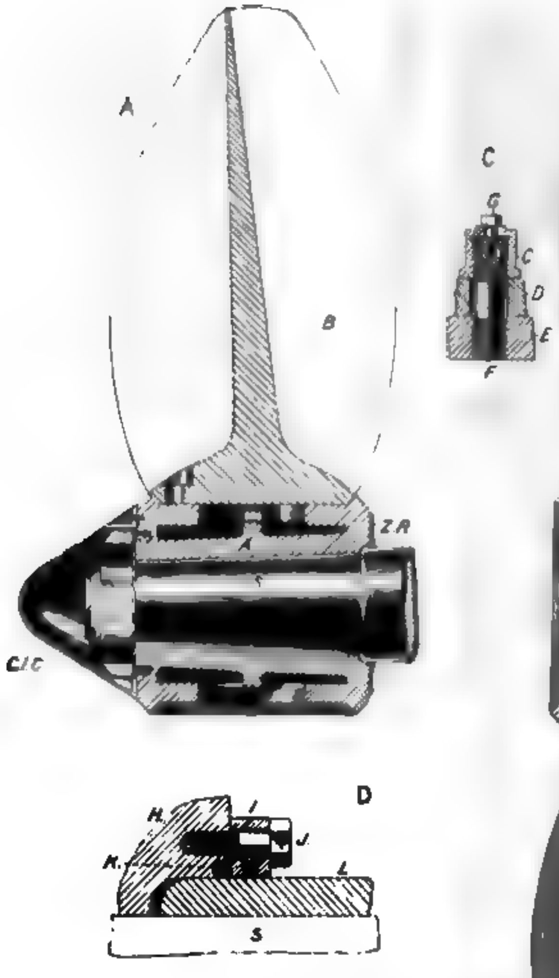
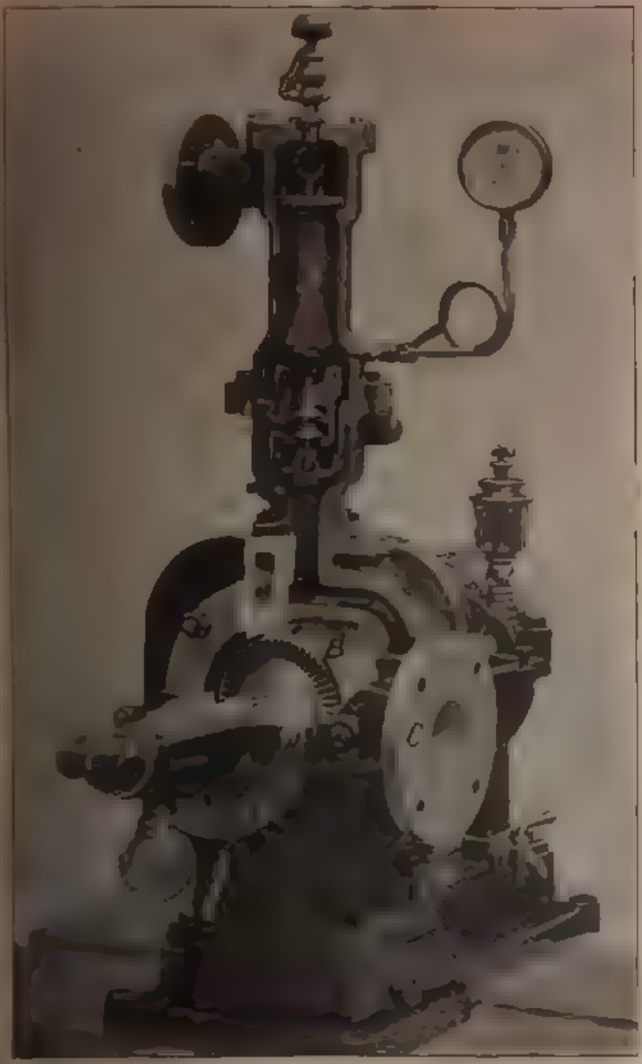



FIG. 52.—SCREW

The figure shows a four-bladed screw, the blades B being made of manganese bronze, and secured by bolts to a cast steel hub A, which in its turn is secured to the conical end of the propeller shaft by two keys, a cup nut, end, and keep plate, and to protect it from sea water there is a conical tailpiece over all. The propeller shaft passes through a gun-metal stern tube, which is fitted into the ship builder's stern tube fixed to ship frame.

There is a stuffing box where the shaft leaves the inner end, the rubbing surfaces being of lignum vitæ. Each blade is recessed into the boss, and all the bolts are made flush, with keep-plates to lock them. There are often means of slightly altering the pitch by changing the angle of the blades by making the bolt holes a little large. Everything is of gun-metal or manganese bronze.




[illegible]

[illegible] to show construction [illegible] through pipe A. At B the steam [illegible] through [illegible] and pass through the [illegible] of the wheel [illegible] the exhaust [illegible]


Fig. 54.

Section showing mouthpiece and vanes of wheel.

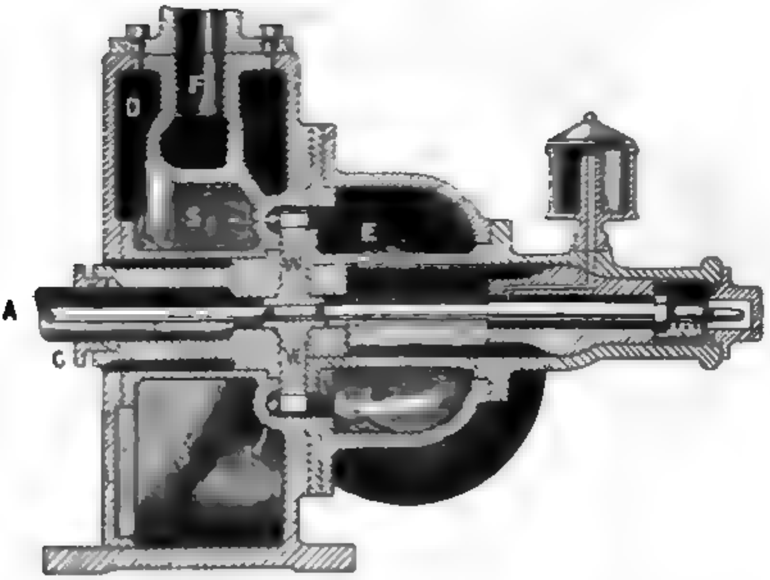
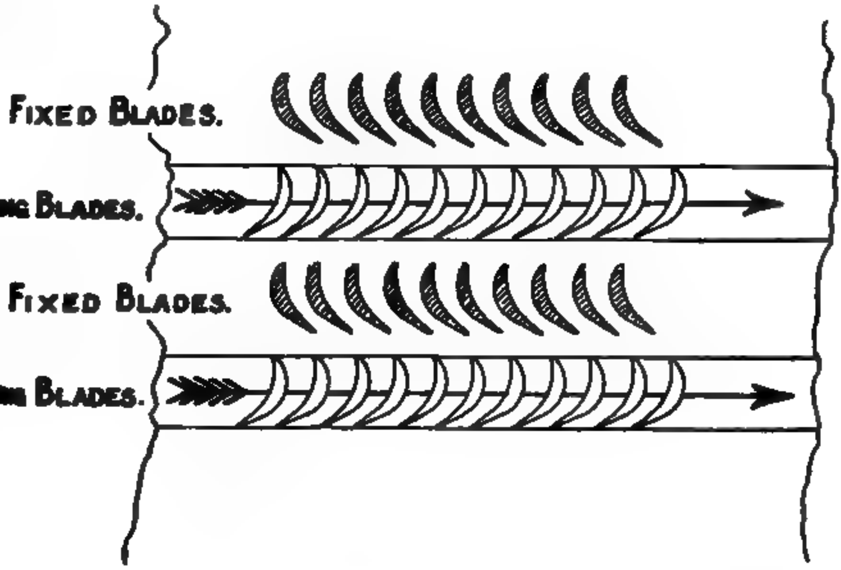



Fig. 55.—Vertical Section of De Laval's Turbine

coming down F finds its way through the chamber S to the mouthpiece J J, where it
the wheel W, as shown in Fig. 54, and is then exhausted through the chamber E.
se power Laval turbine at 15,000 revs. per minute is said to have used less than 20 lb.
brake horse power hour.



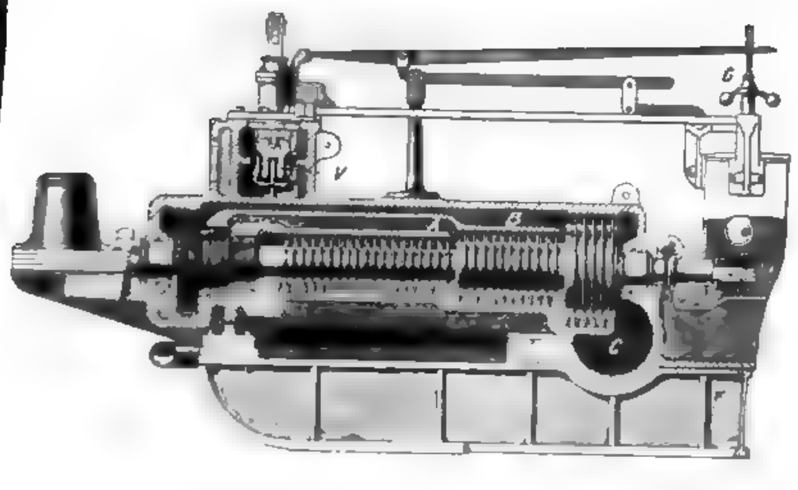




FIG. 56.—PARSON'S AXIAL FLOW STEAM TURBINE.

The axial flow turbines of "The Turbinia" are of about 2,100 horse-power, with a probable consumption 14½ lbs. of steam per horse-power hour.

Steam enters through the valve V, and is led to the turbines flowing axially along the series and is exhausted through C. It continually enters moving blades from fixed blades, and fixed blades from moving blades, each pair being shaped on the well-known principles of construction of an axial flow turbine. The diameter gets greater as the pressure gets less. There are interesting arrangements for diminishing friction and taking up end thrust.

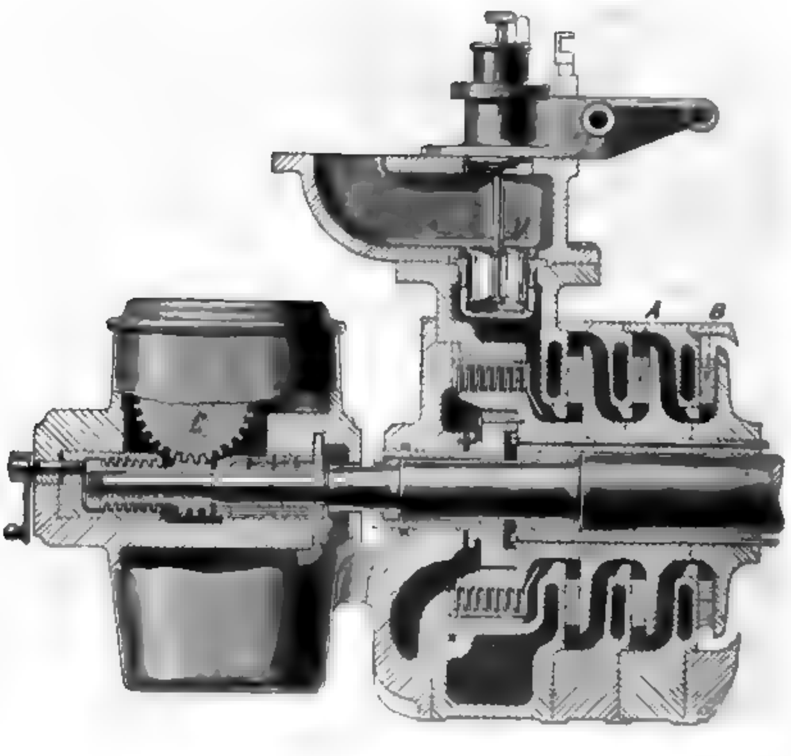



FIG. 57.—PARSON'S TURBINE (RADIAL FLOW).

Steam enters through the valve V, and is led by passages to the centre of the first turbine having passed through outwards, it is led on to the centre of the second one A, and again to a thir one B, until, having passed through a series of those turbines which are fastened to the shaft, it i at or below atmospheric pressure and is exhausted. In its radial passage the steam is guided b fixed vanes into moving ones, and left behind by these to flow again through fixed ones, so that ea radial passage in itself means a passage through many turbines. The shapes of the fixed an moving vanes are well known to the student of radial flow turbines. Revolving discs on the lef take up the end thrust. Speed 5,000 revolutions per minute.

Concentric sleeves on the left have oil circulating between them pumped in. The governin is by longer or shorter gusts of steam being supplied. From Prof. Ewing's tests I find W = 6 + 17·7 E if W is total feed in lbs. per hour and E is the electrical horse-power given out by th driven dynamo; the highest E being 137, and the highest gauge steam pressure being 108 lbs. pe square inch slightly superheated. The air pump was driven by a separate engine. Some measure ments made (1899) on a 1200 Kilowatt Turbine Electro-generator gave a consumption of 19·28 lbs. o steam per Kilowatt hour.

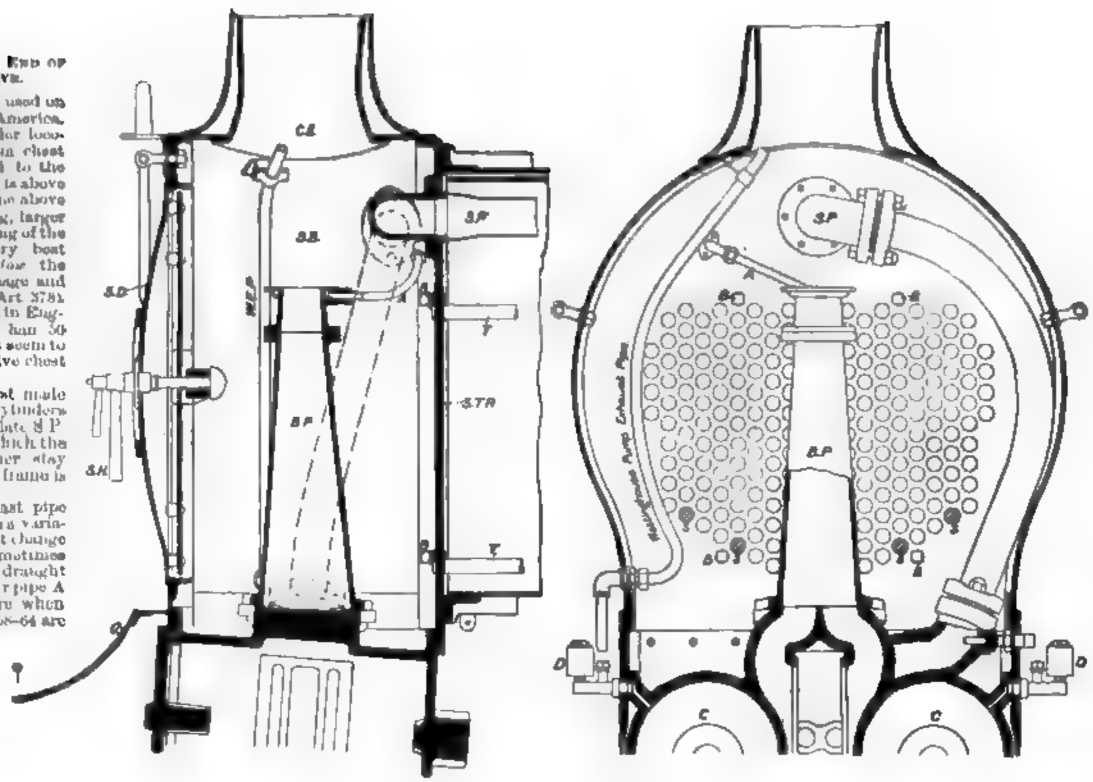



**Fig. 58.—Cylinder and Smoke-Box End of an Inside Cylinder Locomotive.**

Outside cylinders are much more used on the continent, and are universal in America. This shows how, in an inside cylinder locomotive, the cylinders, with the steam chest between, in one casting, are bolted to the frame plates. When the steam chest is above or below each cylinder, or there is one above or below with the valve faces sloping, larger cylinders may be used, but the working of the valves is not so simple. The very best position for the slide valves is *below* the cylinders, as this conduces to drainage and great lessening of condensation (see Art. 378).

The gauge of rails being 56½ inches in England the frames cannot be more than 50 inches apart, hence **19-inch cylinders** seem to be the largest size **possible if the valve** chest **is between.**

**The surfaces of the frames are first** made true, then the bored and finished cylinders are fitted, and, after the spectacle plate S P (Fig. 59) (with openings through which the connecting rods work) and another stay plate and foot plate are fitted, the frame is complete.

The shape and position of the blast pipe B.P. is of enormous importance, even a variation of ⅛ inch in size producing a great change in the coal bill. The chimney is sometimes prolonged downwards to get less draught through the upper tubes. The blower pipe A produces a blast to quicken the fire when the engine is not working. Figs. 58–64 are copied from complete drawings of an inside cylinder locomotive lent me by Mr. Pickersgill, of Aberdeen (Great North of Scotland Railway). **The descriptions overlap.**

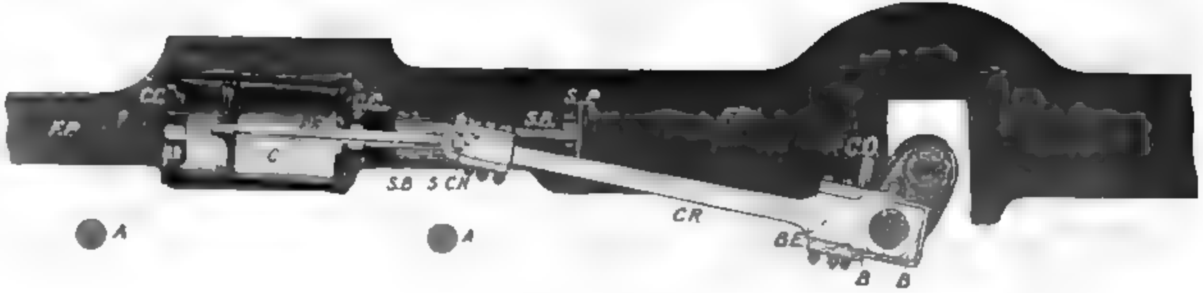



FIG. 59.—PART OF LOCOMOTIVE FRAME WITH CYLINDER, CONNECTING ROD AND CRANK.

There are four steel slide bars S.B for each crosshead C.H. which are fixed by studs and nuts to the cylinder cover C.C., and bolted to the spectacle or stay plate S.P., which is a steel casting usually. By using four slide bars the end of the connecting rod C.R. need not be forked, and less vertical space between the top and bottom bars is required than if (as is more common) only two bars are used, as then they must be far enough apart to clear the connecting rod. It is getting to be very common now-a-days to use only one slide bar, the slide being a box formed on the crosshead. The big end of the connecting rod B.E. is fastened, by means of the two bolts, to a steel strap in which are two brasses B.B., which are adjusted to the crank pin J by the cotter C.O. But connecting rod ends are of other forms as in marine engines.

Horn plates and axle boxes hold the coupled wheels rigidly at right angles to the frame plates. In passenger engines the other wheels are free to take the curvature of the rails by use of radial axle boxes or of bogies. In this form of engine only one pair of axle boxes are guided exactly by the frame or girder. Horn blocks, grooved for oil, are fitted, and the whole work is very carefully tested for truth before the cylinders are finally fixed; holes being now bored in the frame in line with those in the cylinder casting. The axle boxes with their brass steps filled with white metal, bearing only on the top, are very carefully fitted to the hornplates, and afterwards fitted to the journals.

The side plates, &c., being on, the boiler (Fig. 202) is now brought complete and tried on. The smoke box tube plate S.T.P. (Fig. 58) is bolted to the cylinders,—this is the only rigid fastening; brackets on the fire-box end merely rest upon the frames and allow of ⅜ inch expansion. The tubes are now fitted at the same time as the slide bars and engine gear underneath; in fact the upper and under work go on at the same time till the engine is lowered by a crane upon its axle boxes, which are now bedded on to the axles, and run underneath the engine on their wheels, the springs being inserted.

Drain cocks, worked by levers from the foot-plate, are not shown. The cylinders C are 1¼ inch thick to allow for re-boring when grooved through bad lubrication, or ashes entering from exhaust. Brass glands, hemp packing and studs are not shown, neither are the studs which fasten the cylinder covers and valve chest covers to the main castings.

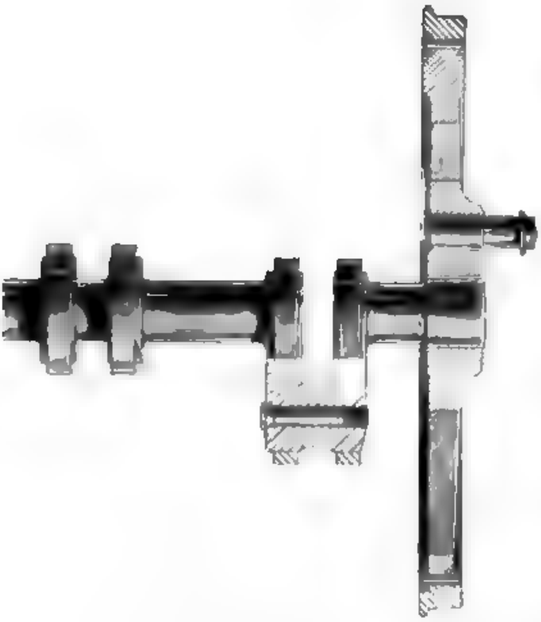
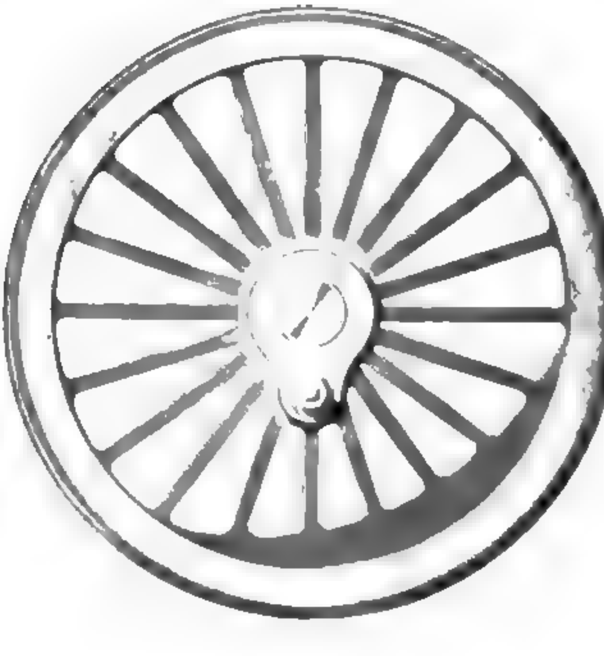


FIG. 60.—CRANK AXLE AND DRIVING WHEELS.

The crank axle is made of steel, all in one piece, the cranks being strengthened by shrinking iron hoops upon them. In some engines strength is given to the web of the crank by making it circular instead of oval. The driving wheels are of cast steel and have separate steel tyres; now usually 3 inches thick and even more. This suits the single frame form of engine. The old double frame is only used now where special strength is needed, and has 6 or 8 journals on the driving axle besides four eccentrics, and is costly. The single frame has one axle box for each wheel.

cylinder engine the driving axle is straight, the cranks being on the driving wheels; the valve e, and the frames are of quite a different shape from those of an inside cylinder engine.



I.
HEEL

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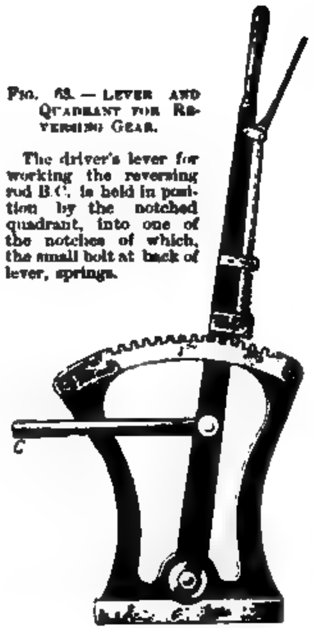


Fig. 63.—Lever and Quadrant for Reversing Gear.

The driver's lever for working the reversing rod B C. is held in position by the notched quadrant, into one of the notches of which, the small bolt at back of lever, springs.

A screw and hand wheel are often used instead of the lever, and relay arrangements are coming into use as in marine engines.

Piston valves and flat slide valves with relief frames are now advantageously used in many locomotives. A relief valve to allow air to enter

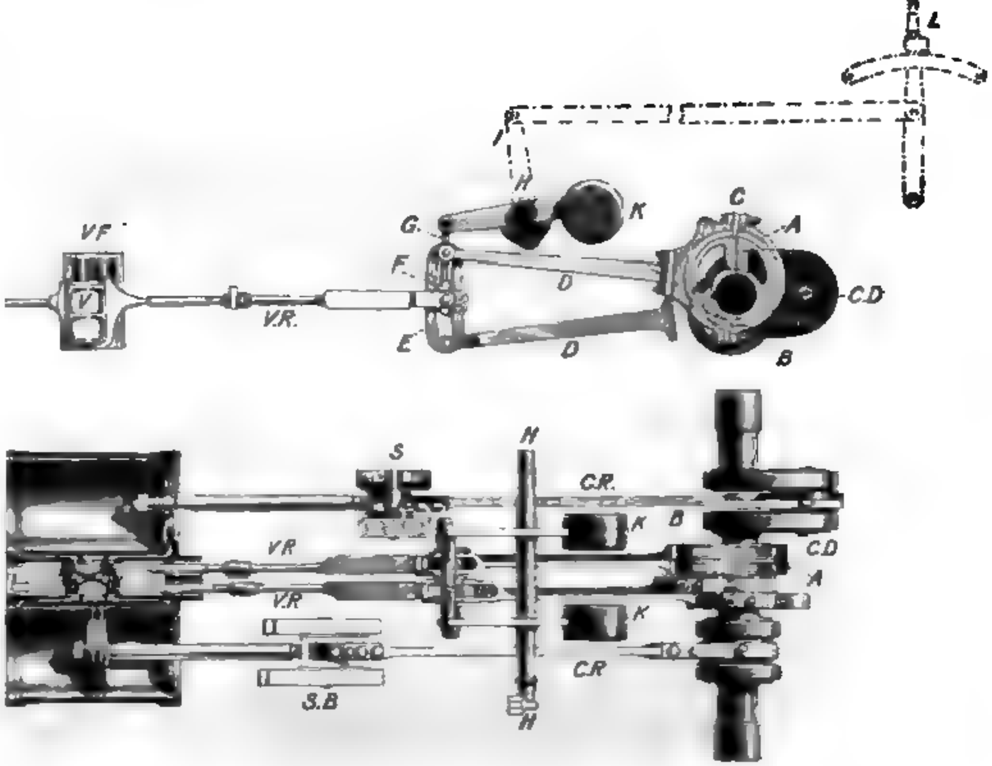



Fig. 62.—Inside Cylinder Locomotive Valve Motion.

... eccentric sheave with a brass liner and B is the back eccentric. The lever L turns the weigh bar

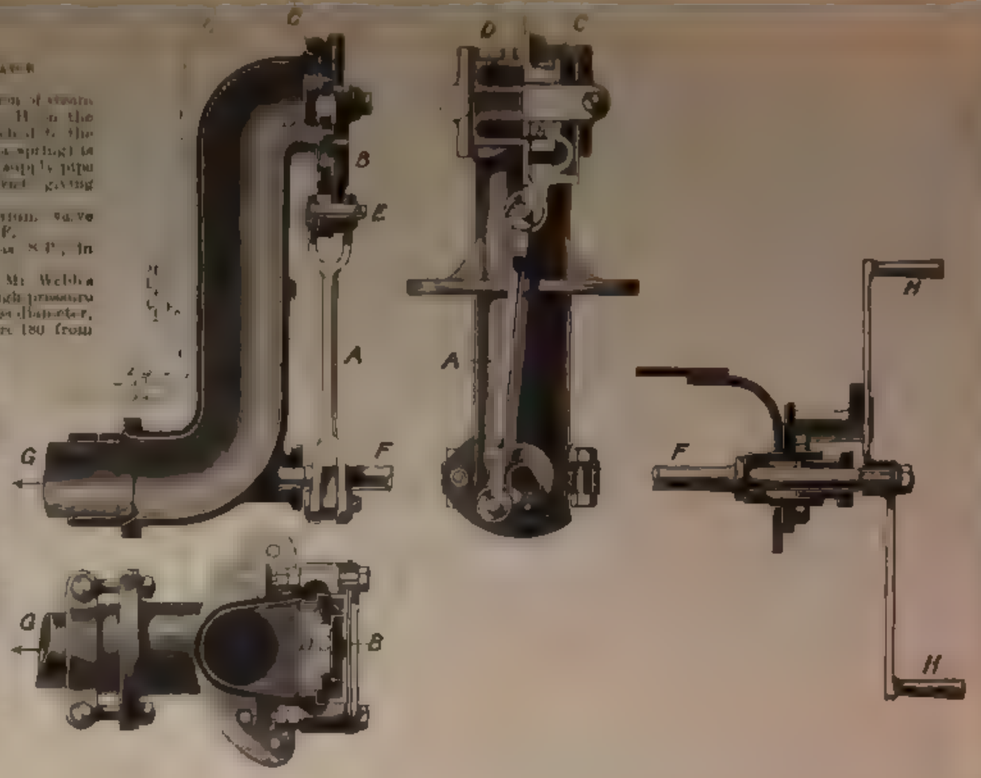
Fig. 64. [illegible] Regulator

The regulator valve is to give a gradual admission of steam to the cylinders and is worked by the handle H on the spindle F which is a crank with a pin attached to the rod A. The outer valve [illegible] is opened first [illegible] admit [illegible] steam to the engine supply pipe [illegible] E [illegible] against the main valve [illegible] giving a larger [illegible].

The [illegible] has an equilibrium valve worked by a [illegible] similar to F.

The [illegible] of the steam pipe is shown as [illegible] in Fig. [illegible] the valve [illegible].

[illegible] drawings of one of Mr. Webb's [illegible] compound [illegible]. The two high-pressure cylinders [illegible] low-pressure [illegible] in. diameter, all of [illegible] stroke. The cranks of the [illegible] are 180 from those of [illegible] high. The valve [illegible] is worked directly by Joy gear, and it works the valve [illegible] of the corresponding [illegible] through a lever at the front end. The smoke box is divided into two by a horizontal partition and each part has its own [illegible] so that there [illegible] bearings there is a third [illegible] crank axle. It is supposed that [illegible] compounding and tandem, it is only when we exceed two [illegible] that it is possible to [illegible] a locomotive [illegible] XXIX [illegible] fairly well, and [illegible] to run at a high speed without [illegible] only two [illegible] little starting power but this difficulty was removed by allowing steam at starting to be supplied to the low pressure cylinder through a reducing valve at the same [illegible] high, so that the cylinders work [illegible] in the ordinary way. This is exactly [illegible] with what is done in traction electromotors running them either in series or parallel.



## CHAPTER III.

### THE VALUE OF EXPANSION.

**31.** BEFORE studying carefully the various forms of valve gear which are in use, the student must get to know **what it is that we want** the gear to effect. Let him imagine four cocks, $A_1$ and $E_1$ to admit steam, and exhaust it on the side $A$, Fig. 5, $A_2$ and $E_2$ to admit and exhaust on the side $B$. Imagine changes to occur slowly, so that we may consider what is occurring at our leisure.

1. $E_1$ closed, $A_1$ open, $A_2$ closed, $E_2$ open, and let us for simplicity call the pressure in $B$ zero, as if the exhaust were to a perfect vacuum. Let there be steam pressure of 100 lbs. per square inch in $A$; cylinder 1 foot in diameter, or area of piston 112 square inches, so that the total force on $D$ is five tons. If $D$ moves through 2 feet under this force, the length of the crank being 1 foot, the work done upon $D$ is 11,200 lbs. $\times$ 2 feet or 22,400 foot pounds. If we neglect friction and loss of energy by concussion, &c., this **energy is given to the crank shaft.**

2. $A_1$ closed, $E_1$ open so that all the valuable 100 lb. steam rushes off, and the pressure in $A$ is 0: $E_2$ closed $A_2$ open, so that the pressure in $B$ is 100. As the piston moves over a distance of 2 feet, the work 11,200 $\times$ 2, or 22,400 foot pounds, is again done on the piston, and communicated to the crank shaft. Hence in one revolution we have 44,800 foot pounds given to the crank shaft.

Now, some men who know very little of applied mechanics[1] seem to think that somehow the angularity of the crank causes this work

[1] Muscular exertion and fatigue occur when a man merely supports a load without doing work in lifting it higher. Any person who confounds such fatigue with what we call *work* in our calculations is sure to get misleading notions. An iron column may support a load and nobody thinks that work is being done.

be greatly wasted. In so far as it causes friction and shocks, there some loss, and the loss due to friction and shocks is serious enough, but this is very different from the imaginary loss of which some men speak. Except for friction, the work done upon the piston is all communicated to the crank shaft, and is given out by the crank shaft. The work done upon the piston per minute, and therefore the horse-power, may be calculated if we know the pressures of the steam on the two sides of the piston at every instant during a revolution of the crank. This power is called the indicated horse-power, from Watt having invented an instrument called an indicator, to register the pressures. The power given out by the crank shaft may be measured by a brake or dynamometer. The brake horse-power is generally about 0·85 of the indicated power in a good engine working at its best load, so we see that the loss due to friction and shock seems large. The loss of energy by friction is often great at slide valves. Observe that we imagine our engine to go slowly, the four cocks being turned at the proper instants by a boy. The indicator would tell whether the boy performed his work properly. If he failed to close two and open another two exactly at the end of a stroke, the indicator would act as a tell-tale.

**32.** Let us suppose now that the boy cuts off steam before the piston gets to the end of its stroke. There will be less work done on the piston. But let us see exactly what will happen. Suppose he **cuts off steam at half stroke,** only allowing half the quantity of steam to be used. Notice that this steam at 100 lbs. per square inch is not all thrown away when cut off takes place, it continues to act on the piston, although with less force. Its pressure per square inch will vary in some such way as this:—

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Travel of piston in feet | 0 | ·5 | 1 | 1·25 | 1·5 | 1·75 | 2·0 |
| Pressure | 100 | 100 | 100 | 80 | 67 | 57 | 50 |

The steam thrown away then is only 50 lbs. steam, and we have evidently had far more work out of our steam per cubic foot.

Suppose the boy cuts off at **one-third of the stroke,** we shall find that the pressure falls in some such way as this:—

| | | | | | | |
|---|---|---|---|---|---|---|
| Travel of piston in feet | 0 | 0·33 | 0·67 | 1 | 1·5 | 2 |
| Pressure | 100 | 100 | 100 | 67 | 44 | 33 |

Here we have only admitted one-third of the quantity of steam, and yet a fairly good force has been acting on the piston during the whole stroke, for the steam thrown away at the end still has a pressure of 33 lbs. a square inch. Surely a student must see already what it was that Watt discovered in his use of expansion. The thing to study is evidently "how much work is done per cubic foot of steam?" We know that it is greater as we cut off earlier; but how much greater is it?

**33.** If we could only tell in all such cases as the above what is the *average* pressure during the stroke, we should quickly know what we want. But the student, who has worked exercises like those of Chap. XV., already knows how to find the average pressure in the above cases. Let him take them as exercises, drawing curves to show $p$ the pressure for each point of the travel. Now, the average represents the work done in a stroke, because it has only to be multiplied by 112 square inches, and by 2 feet for the answer to be in foot pounds. I have done the exercise myself, and I find the following results:—The student must do it himself. The volume of the cylinder is $2 \times 112 \div 144$ or 1·56 cubic feet.

1. No expansion. 1·56 cubic feet of steam used in one stroke. Average pressure 100 lbs. per square inch. Work done in one stroke $100 \times 112 \times 2 = 22{,}400$ foot pounds, or 14,400 foot pounds per cubic foot of steam.

2. Cut off at half stroke. 0·78 cubic foot of steam used. Average pressure 85 lbs. per square inch. Work done $85 \times 112 \times 2 = 19{,}040$ foot pounds, or 24,400 foot pounds per cubic foot of steam.

3. Cut off at one-third stroke. 0·52 cubic feet of steam used Average pressure 70 lbs. per square inch. Work done $70 \times 112 \times 2 = 15{,}680$ foot pounds, or 30,200 foot pounds per cubic foot.

The three answers you have obtained show then that by cutting off steam at half stroke we get 70 per cent. more effect; by cutting off steam at one third stroke we get 110 per cent. additional effect to what we get with no expansion.

**34.** Now, the figures I have given only *illustrate* the good effects of expansion. There are several reasons why they are to be looked upon with suspicion. In the first place the fall of pressure after cut off is assumed to be according to this law;—*when steam has double the volume it has half the pressure, or pressure × volume, keeps constant.*

What right have I to assume any such law of fall of pressure? My right will be discussed later. It is sufficient to say that when a steam engine cylinder has a steam jacket, the pressure does not

ain, I took no **back pressure.** This means that I took an whose exhaust was a perfect vacuum. Now, if the engine was a **condensing engine**, the back pressure would probably be 3 lbs. quare inch; subtract this therefore, and instead of the average res, 100, 85, 70, we ought to take 97, 82, and 67. It is evident his will make no great difference in our notions of the value ansion; but a student ought to work out the actual figures.

ain, if the engine is **non-condensing,** it exhausts into the phere, whose pressure is 14·7. Inasmuch as the passages are not enough to allow infinitely rapid escape of the exhaust steam, st take a back pressure greater than 14·7. In practice we find 6·5 in slow moving engines and 18 in very high speed engines nmon; let us therefore take 17 lbs. per square inch as the usual ressure in non-condensing engines. The average pressures in ove three cases now become 100 − 17, or 83, 85 − 17, or 68, 0 − 17, or 53 lbs. per square inch. Let therefore a student out the figures in the following table.

he will work out exactly in the same way what occurs when t off at one-fifth and one-tenth of the stroke, he can complete ble as I give it. Also I have a reason for giving the fourth n of numbers; it is this;—

5. Engineers are much too apt to speak only of **indicated r and work.** We shall see presently that it is very easy to re with more or less accuracy the true pressure of steam on iston of an engine by means of the indicator, and from this lculate the indicated power. But the power actually given

approximation we may take the friction to be represented by a back pressure of 10 lb. per square inch on the piston, in addition to the real back pressure as shown on an indicator diagram. This is what I have done in column 4, subtracting 27 lbs. per square inch from 100, 85, 70, 52, and 33, which are the average pressures as computed on the assumption of no back pressure.

| | $p_m$ | Work done per Cubic Foot of Steam. | | | |
|---|---|---|---|---|---|
| | | No back pressure. | Back pressure 3 lbs. per square inch. | Back pressure 17 lbs. per square inch. | Back pressure 27 lbs. per square inch. |
| No cut off . . . . . . . . . . . | 100 | 14400 | 13900 | 11900 | 10500 |
| Cut off at half stroke . . . . . . | 85 | 24400 | 23600 | 19500 | 16700 |
| Cut off at one-third of the stroke | 70 | 30200 | 28900 | 22800 | 18500 |
| Cut off at one-fifth of the stroke . | 52 | 37300 | 35200 | 25100 | 18000 |
| Cut off at one-tenth of the stroke | 33 | 47400 | 43100 | 23000 | 8600 |

Column $p_m$ gives the mean pressure during the stroke, assuming no back pressure. From this each back pressure must be subtracted, to get the true average pressure which must be multiplied by the area of the piston (112 square inches) and the length of the stroke (2 feet). This is the work done by the steam admitted. When we cut off at one-fifth of the stroke, the volume of steam admitted is one-fifth of the whole volume of the cylinder. The volume of the cylinder is $2 \times 112 \div 144$ cubic feet.

**36.** It is often assumed that an elementary student can understand quite easily all sorts of abstruse principles of thermodynamics and other parts of physics, whereas the simplest calculations of the above kind are looked upon as belonging to the higher study of the steam engine.

But this book is written to guide a teacher who wishes to make his students really think about the fundamental facts, and I wish it to be understood that the average student has no difficulty whatsoever in making the above simple calculations if he knows about force and work; that is, if he has studied a little applied mechanics. When there are a number of students, let them be divided into sets of three or four. One set of men takes the initial pressure of the steam as 50, the next as 100, the next as 150, the next as 200 lbs. per square inch, and instead of cutting off merely at one-half, one-third, &c., there ought to be cutting off at all sorts

of other periods of the stroke, so that all the students may help in producing a table of numbers giving valuable information. I have found this exercise one of enormous value. The drill-sergeant kind of teacher will get possession of some such very complete table, and show it to students who have not calculated it. If my system takes root I can imagine text books written, by the mere reading of which a man will be supposed to study the subject. He will look at some such elaborate table; he will even think that he understands it perfectly, and unfortunately it will be difficult to prevent his passing written examinations. It is truly wonderful what difficult looking questions men may answer, and get full marks for in examinations, when, all the time, they have no real knowledge of the most elementary facts about the subject.

**37.** The student will now examine his results. He will see that in:

I. Condensing engines. The **indicated energy** per cubic foot of steam is greater and greater with more expansion, as far as the above table goes. He will notice also that in every case the condensing engine has an advantage over a non-condensing engine.

II. Non-condensing engines. The **indicated energy** per cubic foot of steam is greater when we cut off at $\frac{1}{5}$ than when we cut off at $\frac{1}{10}$ of the stroke, and indeed there is no great difference between cutting off at $\frac{1}{3}$, $\frac{1}{5}$, or $\frac{1}{10}$ of the stroke.

III. Non-condensing engines. The **brake energy** per cubic foot of steam is not very different for cutting off at $\frac{1}{2}$, or $\frac{1}{3}$, or $\frac{1}{5}$ of the stroke, but is decidedly less when we cut off at $\frac{1}{10}$ of the stroke; in fact, less than if we had no expansion.

IV. Notice that what I say about indicated energy in non-condensing must be pretty much the same as for brake energy in condensing engines. Indeed, taking 14 lbs. as the extra or frictional back pressure in a small condensing engine is probably taking too little, because the driving of the air and feed and circulating pumps in such an engine is a large addition to the resistance.

When therefore the student hears some foolish unpractical man talking of the virtues of unlimited expansion, let him cite some such figures as we have given above. Don't let any one talk of the discrepance of theory and practice when what he calls his theory is based on no natural facts. The old Cornish pumping engine, which is still found to work satisfactorily, seems not to have ever cut off earlier than $\frac{1}{4}$ of the stroke, and Watt himself usually cut off at from $\frac{1}{2}$ to $\frac{2}{3}$ of the stroke.

**38.** But it will be found in Chapter XVII. that there are three

other drawbacks upon the numbers (like those given in the second column of our table), often cited as exhibiting the virtues of great expansion, and these are :—

First. By greater expansion it may be that we do get greater work per cubic foot of steam; but we are using a large cylinder (and therefore a large engine) for comparatively little total power. Surely mere economy of steam is not the whole of the economy which ought to be studied. Interest and depreciation on **cost of an engine** are important.

Second. The actual quantity of fresh steam entering the cylinder is greater than what we stated above, because of the **clearance space**.

Third. When we cut off at $\frac{1}{2}$ or $\frac{1}{4}$ of the stroke, the quantities of steam used are really not represented by $\frac{1}{2}$ and $\frac{1}{4}$ of the cylinder volume. When we have greater expansion our cylinder is colder before steam is admitted, and a good deal of the newly admitted steam is condensed in heating up the cold cylinder. When therefore we indicate less steam we are actually wasting more, and thus there are two reasons for the percentage loss being greater. As Watt knew very well, this **condensation of steam** entering the cylinder is the most serious trouble before the maker of steam engines. It depends upon the range of temperature or the difference in temperature between admission and exhaust steam and upon the time that elapses before cut off, and its effect is less at higher speeds; engines going at 400 revolutions per minute have only about half the relative condensation of engines going at 100 revolutions per minute. To diminish the range of temperature it is thought well to let the steam expand in **two or three cylinders**. Thus in Fig. 65 we have a triple cylinder engine. The steam admitted to $H$, the high pressure cylinder is at 200 lbs. per square inch, and the exhaust is about 75. This exhaust steam enters a receiver, $A$, a mere space kept warm, as indeed the cylinders also are, by steam jackets. In the most recent engines the volumes of the connecting pipes are thought to be sufficient *receiver* volumes as shown in the figure. In each receiver the pressure varies somewhat, depending upon the size of the space. Steam leaves $A$ and is admitted to a second or intermediate cylinder $I$ at 70 and exhausts from $I$ at about 27 lbs. per square inch into another receiver, $B$. Steam leaves $B$ and is admitted to a third or low pressure cylinder $L$, at 25 lbs. and exhausts to the condenser. One cubic foot of steam admitted to $H$ becomes 16 cubic feet before it is released from $L$. Expansions of 1 to 20 are common and the volumes of the three cylinders are usually as

1 : 2·7 : 7. If this great expansion occurred in only one cylinder it would mean a very great range of temperature of the cylinder, and therefore much condensation of fresh steam every time of admission.

Any student who wishes at this early stage to get **a rough approximation to the effect of condensation** in a well-arranged cylinder, that is, a steam jacketed cylinder working under very good conditions, at 100 revolutions per minute, will find that if he assumes that condensation produces pretty much the same effect as if we had a back pressure of 10 lbs. per square inch, in addition to the above-mentioned back pressures, he will arrive at numerical results which do not badly represent the results of experiments. I need hardly say that this is given as only a very vague direction to students, because the conditions of even well-arranged jacketed cylinders vary very

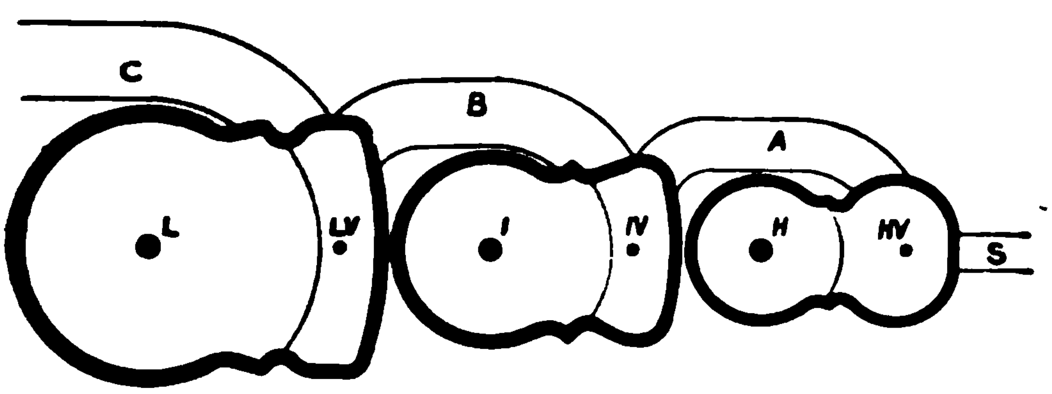



FIG. 65.

Plan of modern three cylinder vertical engine, working three cranks, 120° apart. The pistons, &c., are of the same mass.

Steam comes from the boiler by S, and is admitted by a piston valve H V to the "high" cylinder H, exhausting by the pipe A. This steam from A is admitted by a double ported slide valve IV to the "intermediate" cylinder I, exhausting by the pipe B. This steam from B is admitted by the double ported slide valve L V to the "low" cylinder L, exhausting by the pipe C to the condenser.

greatly. It will be worth while for students to complete the above table by adding a new column of numbers labelled "Back pressure 37 lbs. per square inch," as giving a fairly good general notion of the brake energy per cubic foot of steam, when condensation in well-arranged cylinders is taken into account in non-condensing engines.

**39.** Now let a student imagine himself to be the boy who is in charge of the four cocks. Unless the engine moves slowly he will be quite unable to open and close the cocks exactly at the right times. But let us consider what are these right times.

He is told, let us suppose, to cut off steam exactly at one-third of the stroke. Notice that he ought to cut off with **great quickness** when the proper time arrives. Why? Because it may be shown by calculation that he ought to be admitting steam either at its full steam chest or boiler pressure, or not at all, and if he closes the cock slowly the steam will be *wire-drawn* as it is called. It is for the same reason that the steam pipe and passages must be wide.

Again, a boy of judgment would **admit steam** just a little before the end of the stroke, because his passages are not infinitely large; he would **release steam** also before the hypothetical time, because at the very end of the stroke the back pressure ought to be as small as possible, and the exhaust passages are not infinitely large. It is exactly for this reason that if a theatrical performance is to occur exactly at 7 o'clock the doors must be opened well before 7, and if everybody is to be out of the theatre at 11 o'clock they must begin to go well before 11 o'clock. And the quicker the speed of the engine the earlier must the admission and release take place and the more sharply must the boy cut off his steam. There is much judgment required also in regard to the **closing of the exhaust.** At a certain period in the back stroke the steam is no longer allowed to escape, the exhaust valve is closed; what steam remains in the cylinder is squeezed smaller and smaller, and it therefore increases in pressure and acts as a sort of cushion, which helps very materially in bringing the massive piston and other moving parts to rest, for it is to be noticed that the piston is at rest at the ends of its stroke and is moving very quickly in other positions, and in Chapter XXIX. it will be found that the bringing of these parts to rest so quickly is a serious tax upon the strength of the fastenings, &c. Now a cushion of steam at the end of the back stroke is a wonderful help. Besides, if the cushion of steam could only be squeezed up to the pressure of the entering steam, it is to be noticed that the clearance space would not cause the loss that it usually causes in needing to be filled with fresh high pressure steam.

If he thinks of one side of the piston only it is quite enough for one boy. He must think of **doing four things** exactly at the proper instants, and these four things may be called: *Admission* just before the beginning of the stroke. *Cut off* to be very quick and at the right instant. *Release* well before the end of the stroke. *Compression or cushioning* to begin well before the end of the back stroke.

About 160 years ago, when the oldest Newcomen pumping engine moved very slowly, boys did perform the proper operations, and there is a story told (it is probably untrue, but this is of no consequence to my present purpose) about a boy named Humphrey Potter, who, when in charge of the engine-room, much desired to play marbles upon the engine-room floor, which was well suited to that interesting game. A friend used to come and jeer at him, playing marbles in his sight. Thereupon he invented **the first valve motion.** His master one day entered the engine-room and saw the guileful Humphrey playing marbles. His first duty, that of punishing

[H]umphrey, was strenuously performed, and only then did he observe [th]at the engine was faithfully performing its duty and that the [in]genious Humphrey had so arranged certain sticks and strings that [t]he valves were opened and closed at the proper periods by the [a]utomatic action of the engine. Ask not how the inventor was re-[w]arded. Had he not already had all the reward that a true inventor [e]ver gets, the swelling emotion of seeing his invention a success?

**40. Four cocks or valves** were employed in the old engines, [a]nd they are employed still in the best stationary engines for this [r]eason; the steam passage and valve ought not to be the same as [t]he exhaust passage and valve, because the surfaces are pretty [l]arge, and they are greatly heated by the incoming steam, and [g]reatly cooled by the exhaust steam. There ought therefore to be a [s]team passage and steam valve for each end of the cylinder, and also [a]n exhaust passage and an exhaust valve for each end of the cylinder, [i]f we aim at greatly reducing cylinder condensation, and if we [d]o not mind extra expense, and when we use expensive Tappet [m]otions, and Corliss and other trip gears we can perform the four [o]perations, admission, cut off, release, cushioning, with great ac-[c]uracy in the ways most desired.

One of the most important things to notice about a four (mush-room) valve arrangement is this, that the **leakage of steam** past the valves must be exceedingly small compared with what it is past a moving slide valve. It is almost certain that much of what is called the missing water in a cylinder using a slide valve is really direct leakage past the valve as well as past the piston, and not condensed water as is usually supposed.

**41.** I have said that it is sufficient for many purposes to say that the **friction** of the steam engine and also the **effect of condensation** and leakage may be represented by a back pressure. My justification for this is given in Chap. XVII. If the student is satisfied later, with the correctness of these assumptions, let him note the great simplicity which they introduce in considering what is the most economical ratio of cut off. They are sufficiently correct for us to say in general, that, considering them as part of the total back pressure, the best value of $r$, the total ratio of expansion is

$$\frac{\text{Initial pressure of steam}}{\text{Total back pressure}}$$

and this is true for single or double or triple expansion engines, if $r$ is the total ratio of expansion.

The more usual ways of dealing with friction and the missing quantity will be described in Chap. XVII.

**42. Important exercise.** The student knows how, by actually drawing a hypothetical diagram of pressure, to find the average or effective pressure $p_e$ during the stroke. Thus when cut off is at one-third of the stroke he found that $p_m$ is 70 per cent. of the initial pressure,[1] and he subtracts the back pressure from this to get the average pressure. Let him work the following exercise very carefully.

An engine whose piston is 12 inches diameter or 112 square inches in area, has a crank 1 foot long. The steam is always cut off at one-third of the stroke. The back pressure is 17 lbs. per square inch. Sometimes the boiler pressure is low, sometimes it is high; take the following as the initial pressures of steam in the cylinder, 140, 120, 100, 80, 60, 40 lbs. per square inch. The engine goes at 100 revolutions per minute. Find in every case the hypothetical horse-power $I$ and the weight $W$ of indicated steam per hour.

The student is still neglecting cushioning and clearance, but he is about to obtain results which are of great practical use when we compare them with one another, although they differ in obvious ways from the results of actual trials. The volume of the cylinder at cut off is $\frac{112}{144} \times 2 \div 3$ or 0·52 cubic feet. I have taken from the table, Art. 180, the volume $u$ in cubic feet of 1 lb. of each of the kinds of steam we here deal with, so that we calculate easily the *weight* of steam used per stroke, as there are $100 \times 2 \times 60$ strokes per hour, and so we calculate $W$ the weight of steam per hour. The average pressure multiplied by $112 \times 2$ is the work done in one stroke. Multiply by 200 and divide by 33,000, and we find the horse-power done on the piston.

Now plot $W$ and $I$ on squared paper, and see if you obtain such a law as

$$W = 14{\cdot}2\ I + 400.$$

Our hypothetical conditions are different in many ways from actual conditions. The most important is that there is great leakage past a slide valve or a piston when it is in motion; also there is much condensation going on before cut off in a cylinder, also there is loss due to the clearance. It is then quite a wonderful thing that when we regulate in the above way, letting the initial steam pressure alter, but not altering the cut off, the weight of steam per hour and the

[1] He took an initial pressure of 100; he must prove that this is so for any initial pressure.

indicated horse-power when plotted on squared paper give points lying in a straight line. This is the **Willans' law,** which is found to hold in single cylinder, and in compound and in triple cylinder engines, condensing and non-condensing, single or double acting, with and without steam jackets. It is a law of great practical value to us in our calculations.

This calculation is one which ought to be made by the very beginner, and he ought to repeat it for a back pressure of 3 lbs. per square inch, so as to be able to compare condensing and non-condensing engines. See Arts. 158 and 161.

| $p_1$ the initial pressure. | $p_m$ or $0{\cdot}7\,p_1$. | $p_e$ or $p_m - 17$, the average effective pressure during the stroke. | $I$, the indicated horse power. | $u$, the volume in cubic feet of one pound of steam. | Weight of steam used in one-stroke. lb. | $W$, Weight of steam used per hour lb. |
|---|---|---|---|---|---|---|
| 140 | 98 | 81 | 110 | 3·2 | 0·162 | 1,960 |
| 120 | 84 | 67 | 91 | 3·7 | 0·140 | 1,690 |
| 100 | 70 | 53 | 72 | 4·4 | 0·118 | 1,400 |
| 80 | 56 | 39 | 53 | 5·5 | 0·095 | 1,150 |
| 60 | 42 | 25 | 34 | 7·0 | 0·075 | 880 |
| 40 | 28 | 11 | 15 | 10·3 | 0·051 | 610 |

If the student will add to this table another column showing $W \div I$, he will see why it is that such an engine is less efficient when its load is light.

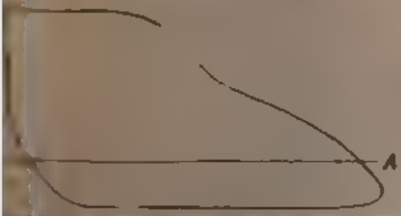
## CHAPTER IV.

### THE INDICATOR.

**43.** In Art. 30 we showed how we imagined that the p
steam might alter during the motion of a piston. We desi
how the pressure of the steam does alter in an actual e
so we use the indicator, which is just as important in
information about what goes on inside a cylinder as the p
stethoscope about the inside of a patient's body. Before
vented it (keeping it secret for a long time) he had alrea
pressure gauge on the cylinder, and his engines moved slow
for him to observe with his eyes how the pressure alter
piston moved; but modern engines revolve so fast that a s
ing instrument is absolutely necessary. The indicator ha
barrel or cylinder like *Cy* of Fig. 73, which communicates
main engine cylinder through a short pipe from *B*. The p
the steam causes the piston *P* to rise by an amount
determined by the stiffness of a spiral spring because there
atmospheric pressure above it. The piston rod acts on the le
and hence the rise of the pencil *P P* indicates the pressu
steam to scale. It is interesting to watch the jerky up a
motion of the pencil *P P* when it is indicating the press
ordinary steam cylinder. The barrel *D*, on which a piece
has been wrapped, rotates for about $\frac{3}{4}$ of a revolution and b
as the cord or cat-gut, which is wound round it near the l
pulled and let go again, and so we see that if the end of su
gets a miniature motion of the piston of the steam engine
line is drawn upon the paper like that which we see in Figs. (
up and down position indicating pressure at any instant, l
position indicating position of the piston of the steam engi
instant. A diagram is usually from $2\frac{1}{2}$ to $3\frac{1}{2}$ inches long a
$1\frac{3}{4}$ inches high. Before one little sheet of paper is repla
fresh one, the indicator cylinder at *B* is made to communi

atmosphere so that the pencil may draw a straight line like $AA$ Fig. 66. This line is called **the atmospheric line.** It tells us position of the pencil when the pressure was atmospheric, and we that pressure is to be measured right angles to it.

Figs. 67 and 68 show two ways




—Indicator Diagram, Condensing Engine.

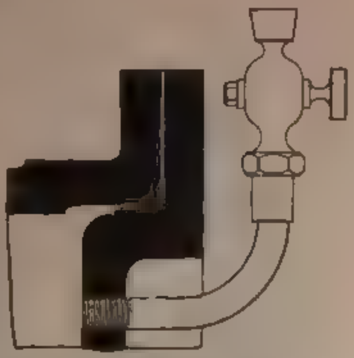


Fig. 67.—Methods of Connecting the Indicator.

which the indicator is usually connected to the cylinder: unless we are sure that the load is very steady, two indicators must be employed. Places too close to steam ports are to be avoided. Plugs are screwed in the holes when the indicator is not being used. In Fig. 69 one indicator, $EPD$, is placed so that by means of the three-way cock $C$ (shown also at $C$ and $D$ Fig. 73) it may communicate by the pipe $CG$ to one end of the cylinder, or by the pipe $CH$ to the other end, or else with the atmosphere. These pipes must not be less than ½ inch internal diameter.

Fig. 68.

Now inasmuch as the pipes $CG$ and $CH$ are of some length and as condensed water sometimes gets entangled in them, we do not altogether like this arrangement because of the greater chance of error. It is, however, very convenient, because we get diagrams from the two ends of the cylinder on one sheet of paper, as shown in Fig. 70 for example.[1]

[1] Let a student think this matter out for himself. Suppose there is a long tube, which is filled with water. Say the length is steam whose pressure is

*A* (Fig. 73) shows the outside appearance of a **Crosby Indicator**, and *B* shows it in section. *D* also seen in Fig. *A* is a hollow brass cylinder on which a sheet of paper may be quickly placed or taken away, and students ought to practise doing this. It will be noticed

Fig. 69.—The Taking of Indicator Diagrams.

that by pulling the cord *B A* Fig. 69, and letting it go again, the paper revolves under the pencil. Now *BA* is pulled by some part of the engine which gives to the paper an exact representation of the motion of the piston of the engine. Thus in Fig. 69 *B* is a point in a lever

rapidly altering; the length *b* is water; the length *c* is, say, air. Note that the rapidly altering pressure of *A* is not at any instant the same as the pressure of *c* at the same instant, and hence if it is *c* that communicates with the indicator the result must be wrong. There is less likelihood of this happening if the pipes have sufficient slope to let all water drain back easily from them into the cylinder. It is easy for a teacher to arrange an apparatus to illustrate this source of error.

the lower end of which $F$ gets the horizontal motion of the cross head $K$, while it moves up and down a little in a slot, the end $J$ being fixed. Again by the method of Fig. 71, $A$ is pulled by a point $B$ of the lever $DBF$, $D$ moving about a fixed centre, and $F$ getting motion from the crosshead $E$ by means of the rigid rod $EF$, or as in Fig. 72 $E$ is the trunk end of the piston of a gas engine moving $F$ in the direction $EF$. The point $B$ pulls the cord $BA$. Other ways of giving to the paper barrel a motion which is very nearly a miniature of the motion of the piston of the engine will strike the thoughtful student, and he will find it an excellent exercise to test by skeleton drawing what is the amount of inaccuracy in each method.

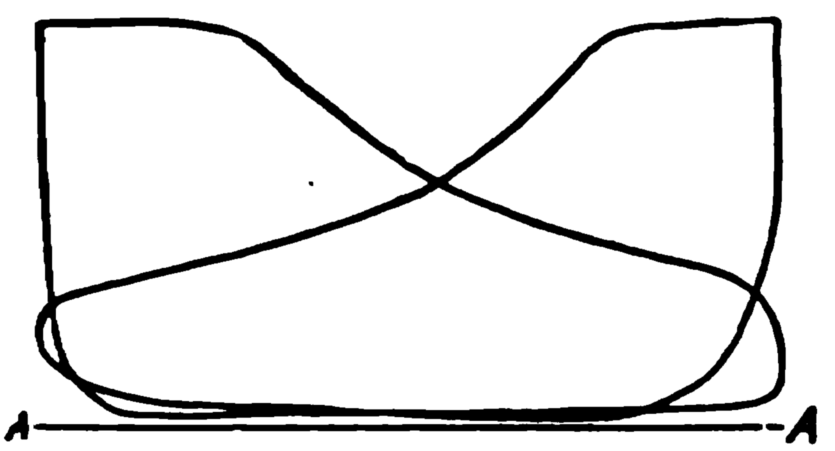


FIG. 70.—SPECIMEN CARD NON-CONDENSING ENGINE.

If the student will reflect a little he will see that the effect of the spring $DS$, which causes the paper cylinder to come back when the string allows it, together with the **inertia of the cylinder,** causes the pull in the string to vary a good deal, and therefore the string alters in length; consequently the paper does not get a true imitation of the motion of the piston. This is one of many defects of the indicator, and students will find it instructive to try a rather yielding kind of string so as to exaggerate the evil. In practice some people now use steel wire or steel strip instead of string or catgut.

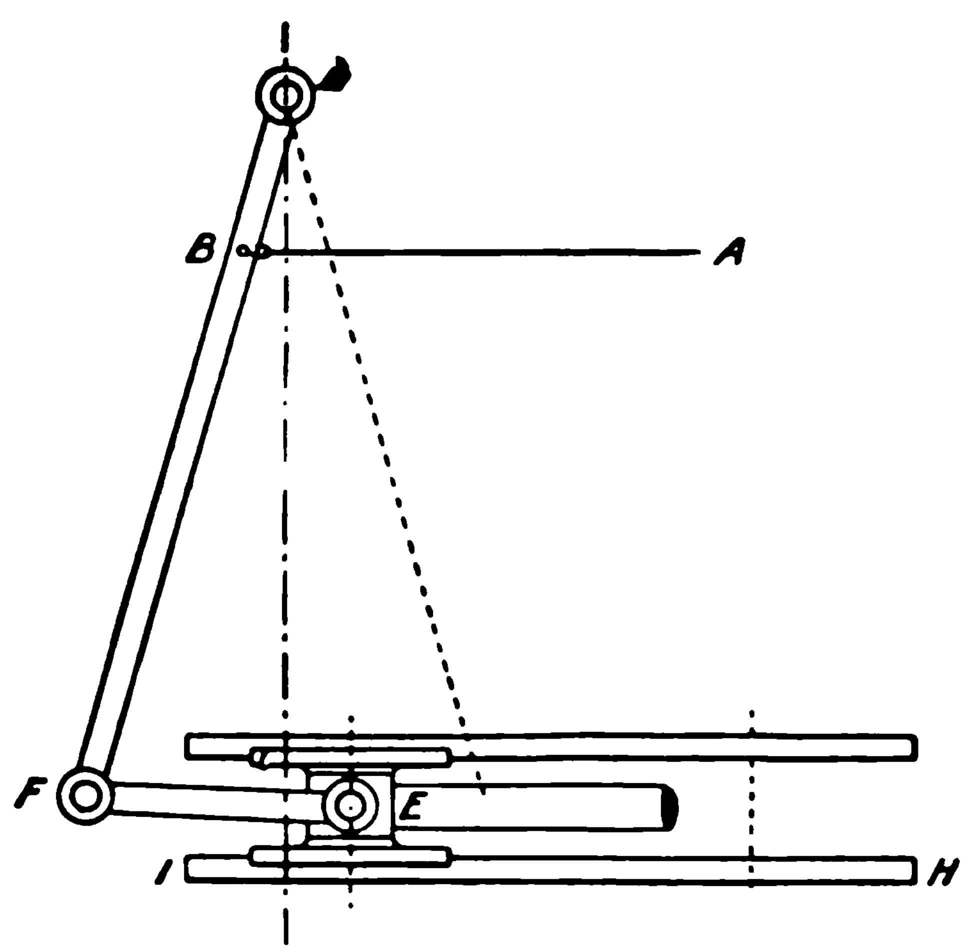


FIG. 71.—HOW THE CORD IS CONNECTED.

**44.** The student ought to make a study of any indicator which he may have opportunity to examine. If he has a choice, let him choose one of the very latest forms suitable for use with engines running at high speeds. If such an instrument is capable of showing pressure

to a good scale, at high speeds, the very greatest care must have been given to its design, and it is worthy of study as a specimen of good instrumental construction. The Crosby Indicator of *A*, *B*, *E*, Fig. 73, is of good design.

*Cy* is the outside cylinder. *Cy. P.* the cylinder proper in which the piston *P* moves steam tight and yet without friction. *Cy. P* is

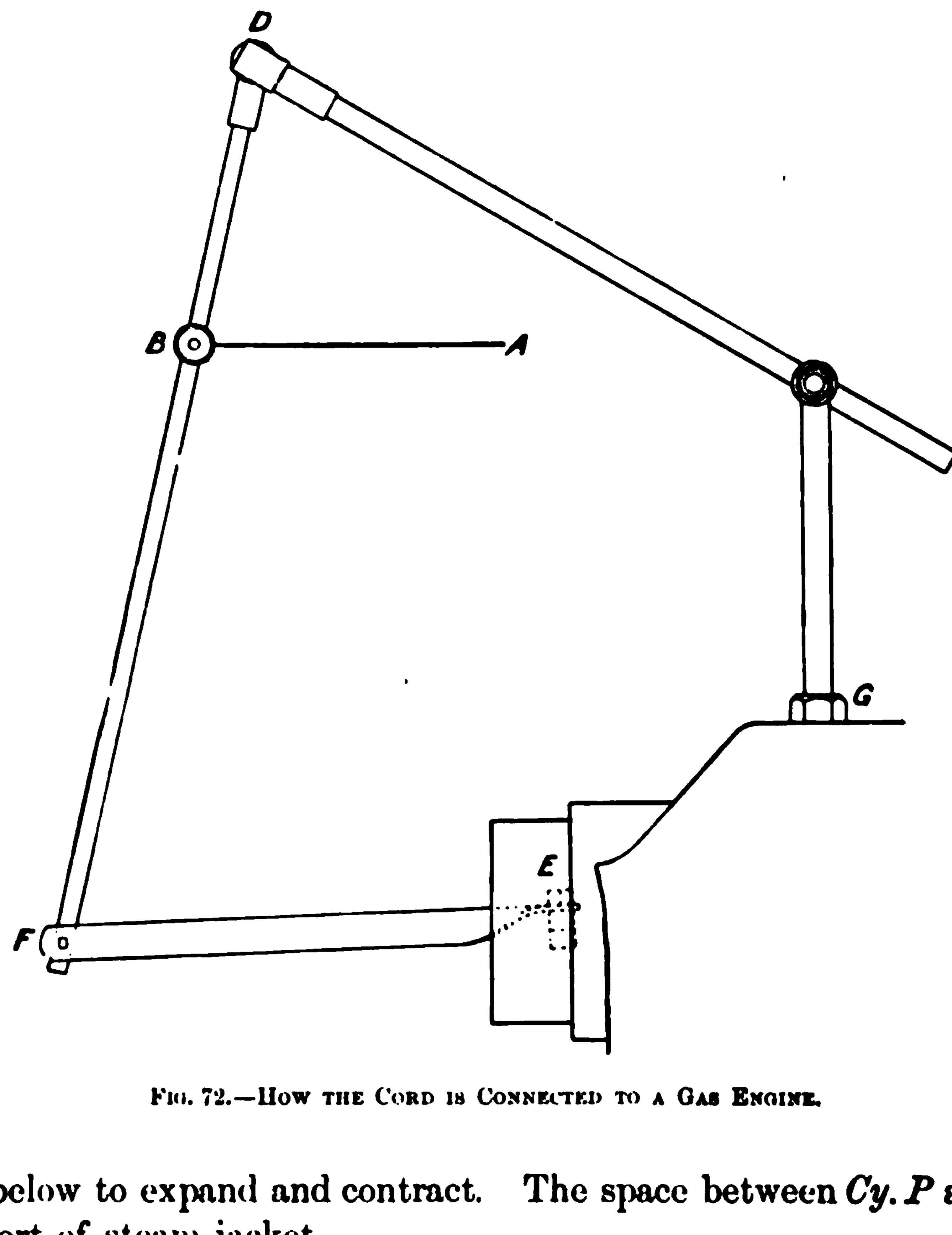



FIG. 72.—HOW THE CORD IS CONNECTED TO A GAS ENGINE.

free below to expand and contract. The space between *Cy. P* and *Cy.* is a sort of steam jacket.

Like all the other moving parts of this indicator the piston *P* is made **as light as possible.** It is of thin solid steel hardened and ground to a slack fit for *Cy.P.*, with shallow channels on its outside for gathering condensed water which forms an excellent packing, with very little friction and practical steam tightness. Its central socket in one piece with the rest extends upwards more than downwards. The lower part receives the piston screw *P.S*; the upper part is slotted

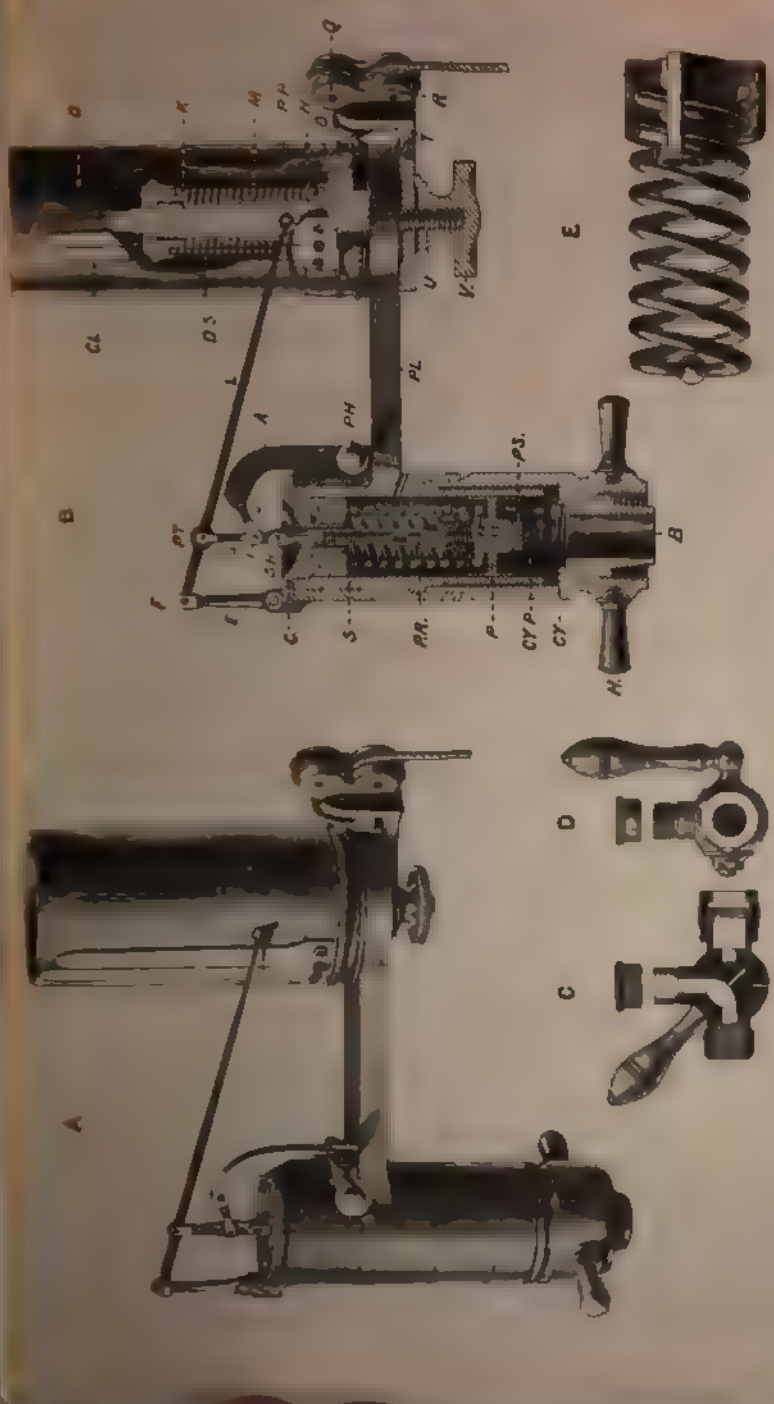


FIG. 73.—THE CROSBY INDICATOR

to receive the bottom of the spring with its central ball; the hollow steel piston rod is screwed in at the top making a firm job. The swivel head *S H* is screwed into the top of the piston rod more or less depending on the required level of the atmospheric line of the diagram.

The cap *C* bushed with steel, screws into the cylinder and into the head of the spring and holds the sleeve *S*, &c., in place. The sleeve turns freely on the cylinder and carries by the arm *A* the fixed end of the link *J I*, which, with the links *E* and *G* and the lever *F. P T. P P* form the parallel motion, which causes the pencil point *P P* on the lever *F. P P* to have a vertical motion, which is six times that of *S H*. In fact the horizontal motion of *F* destroys the horizontal motion of *P P*. There is atmospheric pressure above the piston, and the pressure below it is that which we wish to indicate.

The piston rises through a distance which is proportional to the pressure in excess of the atmospheric pressure, or it falls if the pressure underneath is less than atmospheric. It is very important to test with a good pressure gauge if the motion of the pencil really indicates pressure to the proper scale: the student will readily see how this may be done. If the spring is altogether removed it is easy to move the pencil up and down on the paper, and in this way test if its motion is truly at right angles to the direction of the atmospheric line.

The springs, made each of one piece of steel wire as shown at *E*, are supplied of such stiffnesses that 1-inch motion of the pencil represents either 4, 8, 12, 16, 20, 30, 40, 50, 60, 80, 100, 120, 150, or 180 lbs. per square inch, and a student ought to become expert in altering from one spring to another. Notice that the Crosby spring is right and left-handed, and it therefore has no tendency to press the piston laterally against the cylinder when it is compressed. Boxwood scales of pressure to measure diagrams with are supplied, to correspond with the springs, and the box usually contains also screwdrivers and other tools which are likely to be needed.

The student ought also to examine a drawing of the Richards Indicator, which he can now have no difficulty in understanding. It dates from 1862, and is still in use for engines which make not more than 130 revolutions per minute. Observe in this as in all other good indicators that the cylinder in which the piston moves is separated by a steam space from the outside case, and so is not likely to condense steam inside it.

**45. The errors of indicators** are due to :—

1. The stiffness of the spring alters with temperature, and

e average temperature of the spring is not known, and is fferent in different cases. The error due to this cause may be much as 2 per cent., but a careful man may reduce it to almost othing.

2. Through defects in the parallel motion and in the spring itself, e vertical motion of the pencil may not be exactly proportional to e pressure in all positions. This may be tested at one or two eady pressures, marks on the paper being tested by the scale, and ompared with readings of a good pressure gauge.

3. Bad fitting of the parts through bad workmanship or much use.

4. The inertia of the paper barrel and weakness or strength of its pring, and also friction, combined with the yielding of the cord ometimes causing the travel of the paper to be too great, sometimes oo little; in both cases the motion of the paper being no miniature f that of the crosshead of the engine.

5. Friction, whether at joints of the parts moved by the piston or between the pencil and paper.

**46.** By means of $PH$, which is on the easily fitting sleeve $P H A S$, e cause the pencil to touch the paper or we can withdraw it. In a modern engine going at from 150 to 300 revolutions per minute, it s hardly possible to make the pencil touch the paper and to remove it without tracing out several diagrams. If the contact is continued and if there is a steady load on the engine, the pencil will trace out the same diagram many times, and when the indication (sometimes called "a card") is removed, the paper seems to have only the one line upon it. After allowing the indicator to be warmed up, and seeing that the paper barrel is not clicking against its stops, putting knots in the cord if necessary to get it to the proper length, the usual operations as in Fig. 69 are:—1. Unhook cord $A B$ or use the disengaging device supplied on some indicators; take off old card; put on a new blank paper (you will become expert in this by practice). 2. Turn the cock $C$ so that there is atmospheric pressure underneath the indicator piston; touch paper with pencil and draw it back. 3. Turn cock $C$ so as to communicate with one end of the cylinder, touch paper with pencil and draw it back. 4. Repeat for other end of cylinder. Now disengage cord and remove the paper or card. It will perhaps look like Fig. 78 if the engine is a condensing one, $A A$ being the atmospheric line. It will perhaps look like Fig. 70 if the engine is non-condensing, $A A$ being the atmospheric line. It is usual at once to write on a diagram the time (date, hour, and minute) at which it was taken, and such other information as may be known, such as the number of revolutions of the engine per minute, the

description of the cylinder, &c. Sooner or later these ought to be **written on the diagram:** 1. The boiler pressure at the time. 2. The condenser pressure or vacuum. 3. The scale to which pressure is represented. 4. The diameter and area of the piston and piston-rod. 5. The length of the stroke or twice the length of the crank. 6. Revolutions per minute. 7. All information as to the machines being driven by the engine which may be necessary. It is evident that the information on the card from a locomotive or marine engine, and especially from any particular end of a particular cylinder of an engine, must be very varied to be complete. It is very seldom made sufficiently complete, and hence come doubts and misrepresentation. It is well for the young engineer to learn at once that there is hardly any little scrap of information bearing on the test being made that ought not to be noted at the time.

**47.** If the spring is not stiff, it will represent pressure to a sufficiently large scale, but at a high speed of engine there will be ripples due to the **natural vibration** of the indicator itself. If these ripples get to be too great, as in Fig. 74, a stiffer spring must be substituted. Some men press the pencil firmly on the paper; this kills the ripples, but the **friction destroys the accuracy** of the diagram. Can the student suggest why it is that solid friction like this always makes the **diagram too large?** On admission the pencil rushes up too high, and it stays too high because of the solid friction; it rushes too low and it stays too low during the exhaust for the same reason. Some of the most interesting experiments for students who have a small steam engine to work with are these:—

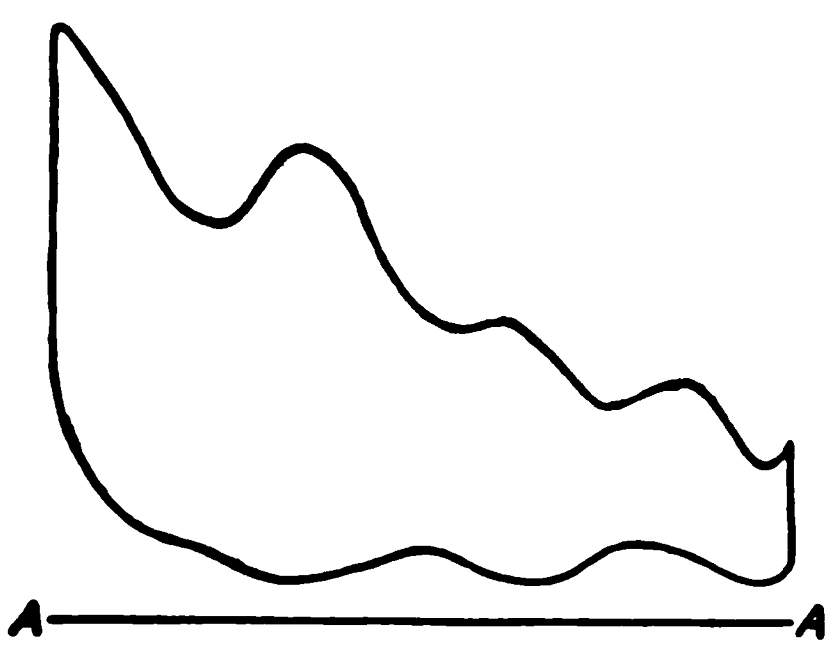



FIG. 74.—SHOWING EFFECT PRODUCED AT HIGH SPEED.

1. Without changing the valve motion, let an engine run first slowly, then faster and faster, and take a diagram at each speed. Note how the wire drawing increases as the speed increases, and how important it is to release and admit well before the end of the stroke at the higher speeds.

2. Note how ripples begin at high speed, and how they become great enough to upset the diagram altogether, so that a stiffer spring must be used.

3. At some slow speed, alter the valve gear in various ways, in each case noting the character of the diagram.

**48.** Vertical distances represent pressure to a scale which depends upon the spring that is used. Let a line *O O* be drawn parallel to the atmospheric line *A A*, and below it at a distance which represents 14·7 lbs. per square inch; then distances measured vertically from *O O* will represent absolute pressures.

The **information given us** by an indicator diagram, if it accurately represents pressure at every part of the stroke on one side of the cylinder, is very varied and valuable.

1. It tells us if our valve motion is doing its duty, admitting steam just before the beginning of the stroke at *B*, cutting off without too much wire drawing at *D*, releasing at *E* well before the end of stroke, and cushioning at *H*.

2. If the pressure of the initial steam at *C D* is very much less than that of the boiler, there is a loss due to the smallness of the supply-pipe or its length.

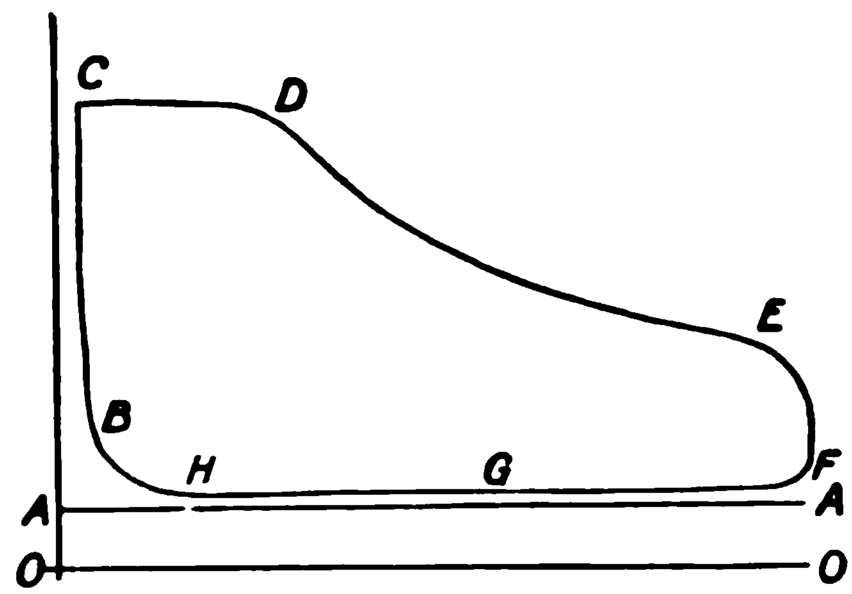



FIG. 75.—SPECIMEN DIAGRAM NON-CONDENSING ENGINE.

3. If the pressure in the back stroke *F H* is not nearly atmospheric in Fig. 75, or nearly the same as that of the condenser in Fig. 78, the exhaust passage is not large enough, or else there was much steam condensed during admission, which is now boiling away during exhaust, and so maintains a high exhaust pressure.

4. The shape of the expansion curve *D E* gives us very valuable information which I do not care here to enter upon.

5. It enables us to calculate the indicated horse-power.

These are only a few of the things about which the indicator diagram gives us information. The indicator may be applied also to the valve chest or the condenser.

*Questions.*—1. If you notice that the admission pressure at *C* is much less than the boiler pressure, what do you infer? 2. If you notice that the pressure at *D* is considerably less than at *C*, is this more likely to occur at high speeds, and why? A gradual fall from *C* to *D* is very different from what is shown in our figure. 3. If the pressure at *F* is much greater than *H*, what may we infer?

In the diagram Fig. 75, the admission begins somewhere

about $B$, the cut off about $D$, the release at $E$, and the cushioning begins at $H$.

Sometimes from $B$ to $D$ is called the steam line or line of admission, $D\,E$ the expansion part of the diagram, $E\,F\,H$ the exhaust line, and $H\,B$ the cushioning or compression.

**49. To calculate the indicated horse-power,** that is the mechanical power exerted by the steam on the piston, we had better neglect here the area of the piston rod. Let $A$ be the cross sectional area of the cylinder in square inches. Consider the space on the left-hand side of the piston (Fig. 5). If Fig. 76 is the diagram, we see that we must find the average value of all such absolute pressures as are represented to scale by $B\,C$ ($C\,C$ is the zero line of pressure drawn to scale 14·7 lbs. per square inch below the atmospheric line $A\,A$) during the forward or ingoing stroke. We must find the average

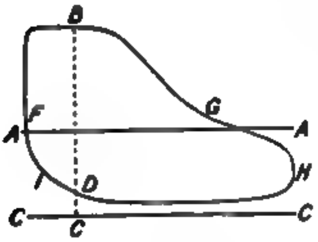



FIG. 76.

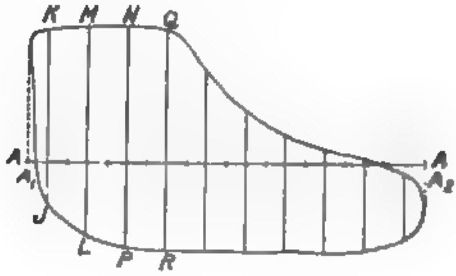



FIG. 77.

value of all such absolute back pressures as $D\,C$. We must subtract the second from the first, and call the answer the effective pressure $p_e$. In fact, the steam does work on the piston in the forward stroke; the piston does work on the steam in the back stroke, and hence we must subtract. Now very little thought will show that instead of taking the averages of the $B\,C$ forward pressures, and subtracting the averages of the $D\,C$ back pressures, we can at once take the average of the $B\,D$ or difference pressures. Hence, all that we have to do is to find the average breadth of the diagram $F\,B\,G\,H\,D\,I$ (breadth being considered to be at right angles to the atmospheric line); and the scale tells us the effective pressure $p_e$.

To get the average we often use **a planimeter** as described in Art. 131. But a **very common plan** is the following:—We draw the two bounding lines of the diagram, lines at right angles to $A\,A$, to cut the atmospheric line in $A_1$, $A_2$, then $A_1A_2$ is the length of the diagram.

I notice that in my figures of diagrams I sometimes show the mospheric line prolonged as from $A$ to $A$, Fig. 81. Now in truth ıe indicator will show it ending at $A_1$ and $A_2$. The ends $A_1$ and $_2$ being faint, perhaps it is always wise to draw the bounding lines f the diagram as I have described.

$A_1A_2$ is divided into ten equal parts, and in the middle of each art a breadth is drawn. The lengths of the ten breadths $KJ$, $ML$, $P$, $QR$, &c., are measured, (usually they are measured at once by the oxwood scale supplied, in pounds per square inch; but they may be neasured in inches, and only the average reduced to pounds per square nch,) and written at the side, added up and divided by ten to get he average value. Notice that if the diagram has a loop the readths of this part are negative. When the average pressure $P$ is known, it must be multiplied by the area $A$ to get the total effective force on the piston; this multiplied by the length of the stroke (twice the length of the crank) in feet, gives the work done in every stroke; multiplied by the number $N$ of strokes per minute (or really revolutions of the crank), and divided by 33,000 we have the horse-power indicated on the left-hand side of the piston. The rule is easily remembered in the form

PLAN ÷ 33,000

If we know the average effective pressure on the other side of the piston, we may calculate the horse-power developed there also, or we may take $P$ to be the average of the two, and take $N$ to be the total number of effective strokes per minute, there being two in every revolution. In many modern, high-speed, single-acting engines the steam acts only on one side of the piston.

The two diagrams are often on the same card as in Fig. 78.

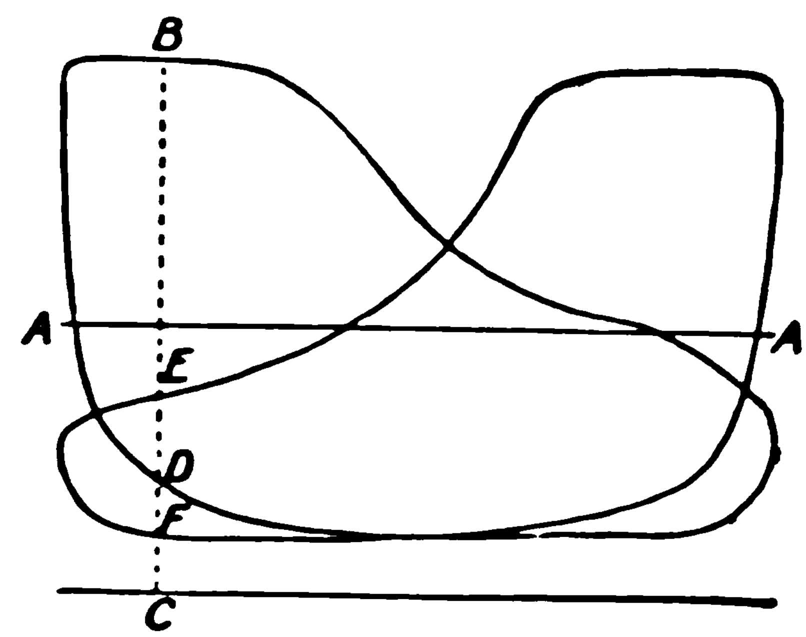


FIG. 78.

**50**. A student ought not to pass too easily over this subject; it is very simple, but let him be sure that he really does understand it, and is not merely taking a thing for granted because everybody says that it is so. Now we may look at the thing from another point of view. Find **the actual forces from left to right**, acting on the piston of Fig. 5, in its forward or ingoing stroke, that is when going from left to

right. At a certain instant the pressure is $BC$ on one side and $CF$ on the other side, so that $BF$ represents the real pressure which, if multiplied by the area of the piston, gives the total force from left to right. Similarly in the back stroke when the piston gets to that place, the force *from right to left* is represented by $EC - DC$ or $ED$ per square inch. An enquiring student ought to make a diagram which shows these values for every position and it ought to be in pounds per square inch, to the same scale as the indicator diagrams. From the diagrams of Fig. 78 I have found the result shown in Fig. 79. This diagram shows to scale what is the force from left to right acting on the piston at every part of its stroke. The length of the stroke being $OO$; at the place $C$ in the forward stroke, $HC$ is the force from left to right, and in the back stroke the force is really from right to left, and is of the amount shown in $CJ$. A student who wants to make a thorough study of the elementary facts concerning steam engines will not fail to make a diagram of this kind. Note that the average total breadth of this diagram at right angles to $OO$ is the sum of what we called the effective pressures on the two sides, and its area is the sum of the areas of the two diagrams of Fig. 78.

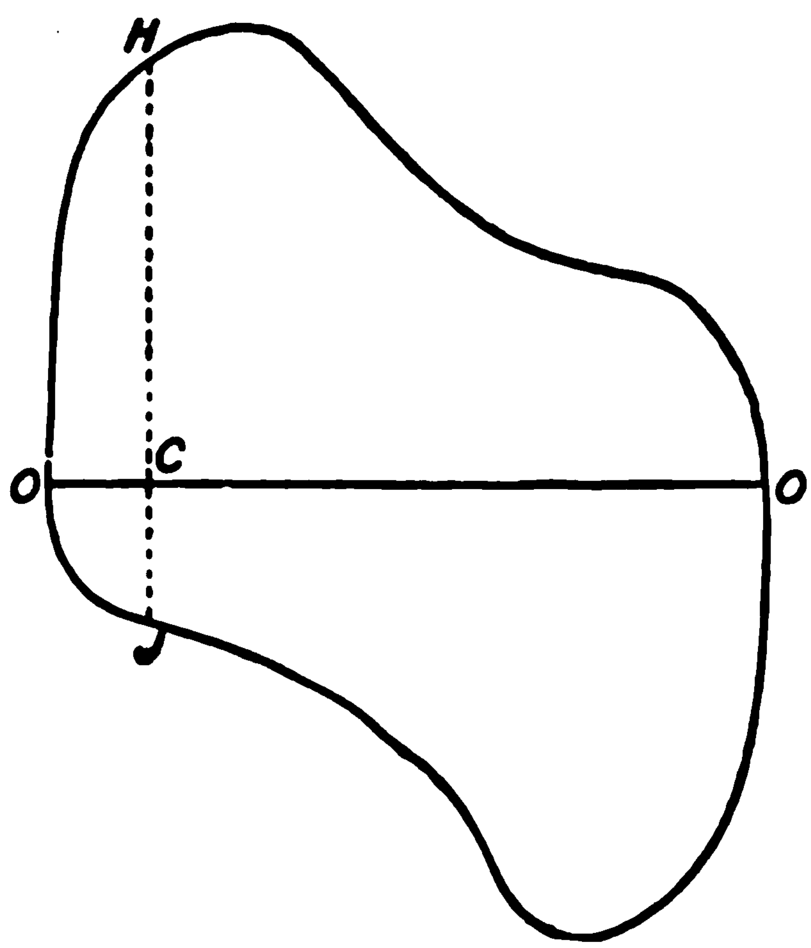



FIG. 79.

## CHAPTER V.

### THE INDICATOR, CONTINUED.

#### A SET OF EXERCISES.

**81.** I do not see how any student can work carefully through a set of exercises like the following without acquiring a fairly good knowledge of the theory of the steam engine. He will ever afterwards be glad to have done such work.

Fig. 80 shows the diagrams from the two ends of a cylinder of 18 inches diameter, crank 15 inches long, 120 revolutions per minute; a steady load was maintained for four hours. Boiler pressure 38 lbs. per square inch by gauge, 52·7 lbs. per square inch absolute. The area of piston is $18^2 \times \cdot 7854$, or 254 square inches. The working volume of the cylinder is $254 \times 30 = 7620$ cubic inches, or 4·41 cubic feet.

**The clearance space** for left-hand diagram (for the side of the piston remote from the crank) was just filled by 13·2 pints of water, or 457 cubic inches; this is 6 per cent. of the working stroke. The clearance space for right-hand diagram was found to be 533 cubic inches, or 7 per cent. of the working stroke.

I show a scale of pressure because I do not know to what scale the engraver will bring the diagram. The scale for volume is of no consequence.

1. What is the **average pressure** from each diagram? Work by taking ten equidistant ordinates and test your answers by planimeter.

*Answer.* 31·2 and 30·3 lbs. per square inch.

2. What is the **indicated horse-power** of the engine?

Neglecting the cross sectional area of the piston rod. The cross sectional area of the cylinder is $9^2 \times \pi$ or $18^2 \times \cdot 7854$, or 254 square inches. The average of the two average pressures $\frac{1}{2}(31 \cdot 2 +$

30·3), or 30·75 lbs. per square inch, and hence the average total pressure on the piston in the direction of its motion is $254 \times 30{\cdot}75 = 7800$ lbs. As the stroke is $2 \times 15 \div 12$, or 2·5 feet long, the work in one stroke is $7800 \times 2\frac{1}{2}$, or 19,500 foot-pounds. As there are $2 \times 120$

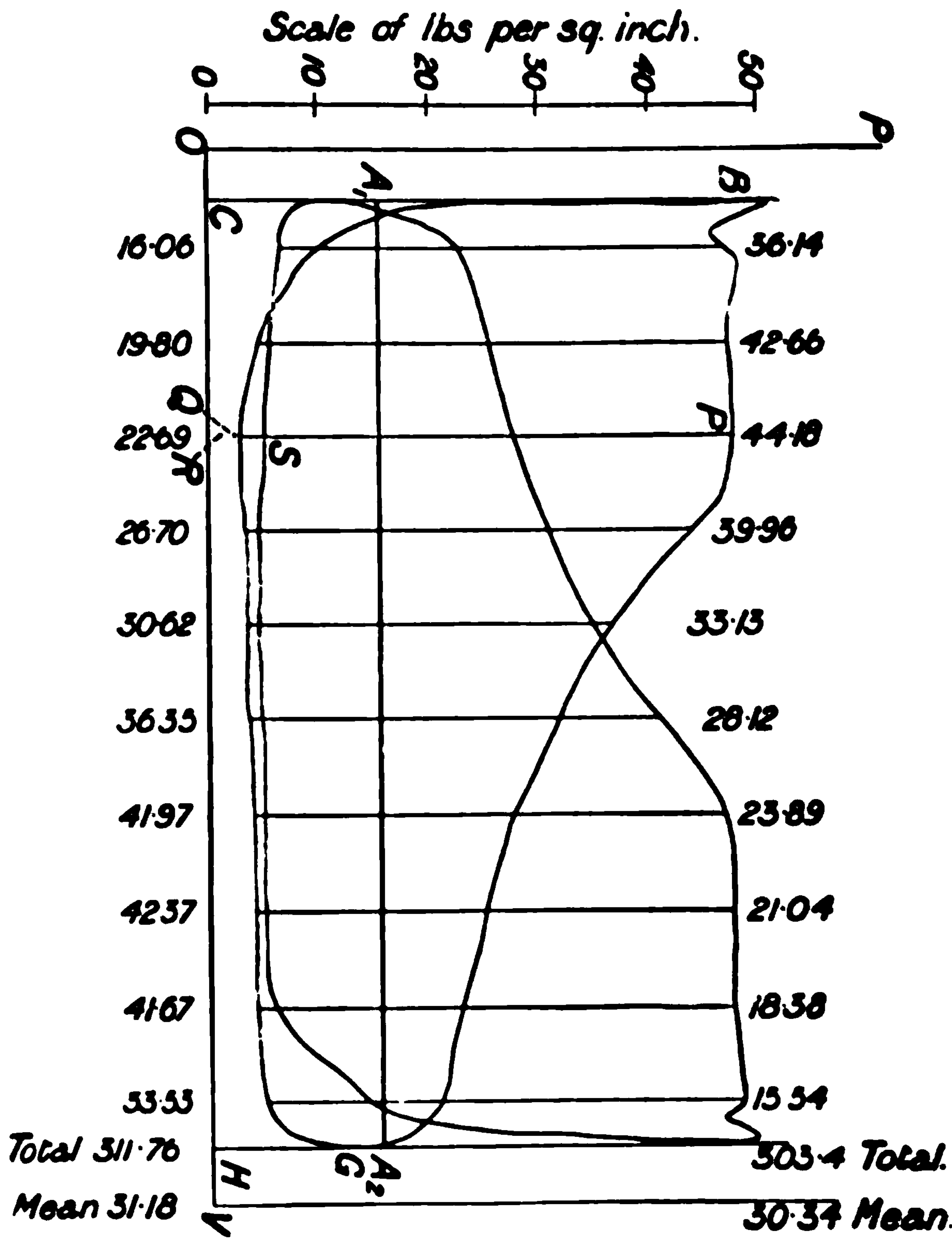



FIG. 80.

strokes per minute, the answer is $19{,}500 \times 240 \div 33{,}000$, or 142 horse-power.

3. The load on the engine having been kept nearly constant for four hours, the following measurements were also made, beginning and ending with approximately the same kind of fire and the same amount of water and same pressure, &c., in the boiler, it was found that 2176 lbs. of coal had been used during the four hours, or 544 lbs. of coal per hour. Hence the **consumption** is 3·8 lbs. **of coal** per hour per indicated horse-power.

A water meter was employed to measure the quantity of feed water supplied to the boiler, it was found to be 242 cubic feet in the

ur hours. Although the feed water was tested and found to be at 18° F. we may take it that its weight is nearly the same as if cold · 62·3 lbs. per cubic foot, hence $242 \times 62{\cdot}3 \div 4$, or 3770 lbs. of steam as supplied to the engine per hour (except for leakage), and hence e get one indicated horse-power for $3770 \div 142$, or **26·6 lbs. of team** per hour. It is to be noticed that of steam of 52·7 lbs. pressure ie consumption by a perfect condensing engine using the Rankine 'ycle (see Art. 214) is 10·2 lbs. per horse-power hour, so that our **fficiency Ratio** is $10{\cdot}2 \div 26{\cdot}6$ or 0·38. Also from the next xercise we see that in our engine there is an expenditure of 288 inits (F.) of heat per minute per horse-power.

4. How much **water** is evaporated **per pound of coal**, assuming that the steam contains no water as it leaves the boiler?

*Answer.* 6·93 lbs.

Note that 1 lb. of feed water at 118° F. converted into steam at 52·7 lbs. per square inch (or 284° F. as may be seen by the table Art. 180) needs $1114 - 118 + {\cdot}305 \times 284$, or 1083 units of heat.[1] Our usual standard of evaporation is the conversion of 1 lb. of water at 212° F. into steam at 212° F., or 966 heat units, and hence as for every pound of coal we have 6·93 lbs. of steam, we have $6{\cdot}93 \times 1083 \div 966$, or 7·77 standard evaporation pounds of steam.

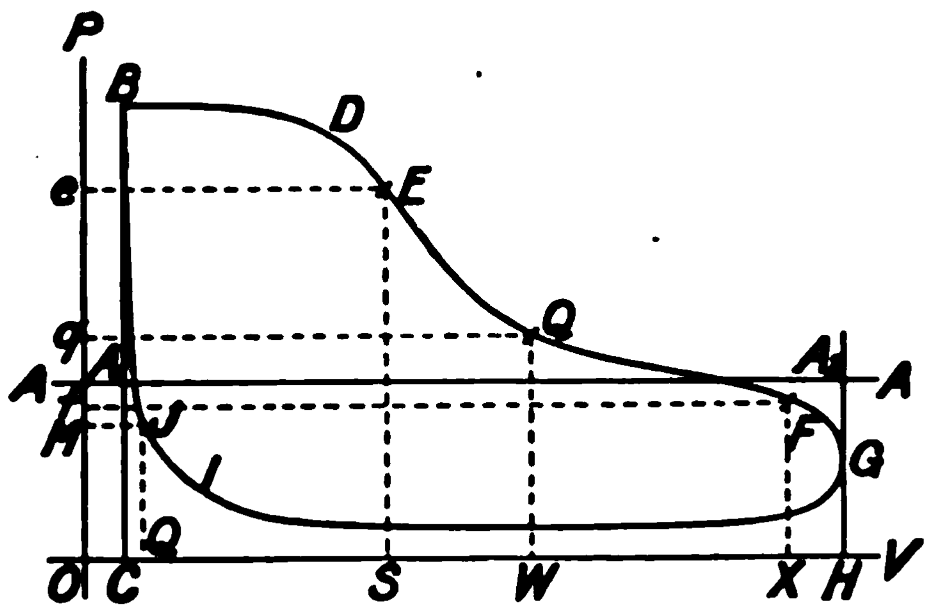



Fig. 81.

5. Draw the zero line of pressure $OH$ Fig. 81. Draw the perpendiculars $BA_1C$ and $A_2GH$ touching the ends of the diagrams. Make $OC$ the same fraction of $CH$ that the clearance space, 457 cubic inches, is of the working volume, 7620 cubic inches. Now draw $OP$

[1] 1 lb. of water at 32° F. raised in temperature to $\theta$° F. and then converted into steam, receives $\theta - 32$ units of heat as water, and the latent heat $1114 - 0{\cdot}695\,\theta$, or altogether

$$H = 1082 + 0{\cdot}305\,\theta°.$$

1 lb. of feed water at $\theta_1$° F. converted into steam at $\theta_2$° F. receives the heat $1114 - \theta_1 + {\cdot}305\,\theta_2$.

These are Fahrenheit heat units suiting Regnault's results. Multiply by 774 to convert into foot-pounds. (See Art. 177.)



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so that we can measure our pressure and volume to scale vertic from $O H$ and horizontally from $O P$. Note that $O H$ represents volume (7620 + 457) ÷ 1728, or 4·673 cubic feet.

6. I have marked the points $E, Q, F$, and $J$, Fig. 82; what are **true volumes and pressures** at these points? My answers the numbers in the first two columns of this table.

| | Volume in cubic feet. | Pressure in lb. per square inch. | Weight of steam present in lb. | Percentage of water stuff which is really steam. |
|---|---|---|---|---|
| At E . . . | 1·715 | 44·77 | 0·185 | 77·8 |
| At Q . . . | 2·865 | 29·62 | 0·209 | 87·8 |
| At F . . . | 4·323 | 21·69 | 0·235 | 98·8 |
| At J . . . | 0·5905 | 13·05 | 0·020 | |

7. Look up the volume of 1 lb. of steam at each of the al pressures and state the **actual weights of steam** present. ] at $E$, steam of 44·77 lbs. per square inch measures 9·28 cubic to the pound; we have 1·715 cubic feet, therefore we have 0·185 lb steam present at $E$. Make out the rest of the above table in same way.

8. At $J$ we see that 0·02 lb. of steam is in the cylinder be admission of fresh steam; at $E$ we have 0·185 lb. present, how n is *indicated* as having entered?

*Answer.* 0·165 lb.

9. Find at $E''$ and $J''$ of the right-hand diagram, Fig. 82, v weight of steam is indicated as having entered on that side of piston.

*Answer.* The volume at $E''$ is 1·97 cubic feet at 45·38 lbs. square inch and its weight is 0·2153 lbs., at $J''$ ·019 lb. of steam i cylinder before admission.

| | Volume | Pressure | Weights |
|---|---|---|---|
| At E″ . . . | 1·97 | 45·38 | ·2153 |
| At J″ . . . | ·5423 | 13·45 | ·0190 |

10. We see then that 0·165 + ·196, or 0·361 lbs. of steam are dicated per revolution of the engine; is not this 0·361 × 120 × or 2599 lbs. per hour of **indicated steam?**

But we saw that 3770 lbs. of steam per hour really left the bo and hence 1171 lbs. per hour, or 31·1 per cent. of all the steam lea the boiler, is **missing** or not indicated just after cut-off.

11. 500 lbs. of water per hour is measured as coming from the steam jacket, and it is estimated (no matter how it is estimated just now) that 130 lbs. of steam leaks away per hour from joints in pipes, &c.; this leaves 3140 lbs. of water as entering the cylinder every hour, and so we have $(3140-2599) \div 3140$, or 17·2 per cent. of the steam **is condensed** either in the cylinder or on its way to the cylinder.

Why should the cylinder itself condense so much steam as we find that it condenses? This is now the **most important practical question** for the engineer.

12. We have assumed that 3140 lbs. of water stuff enter the cylinder per hour, or $3140 \div (60 \times 120)$, or ·436 lb. in one revolu-

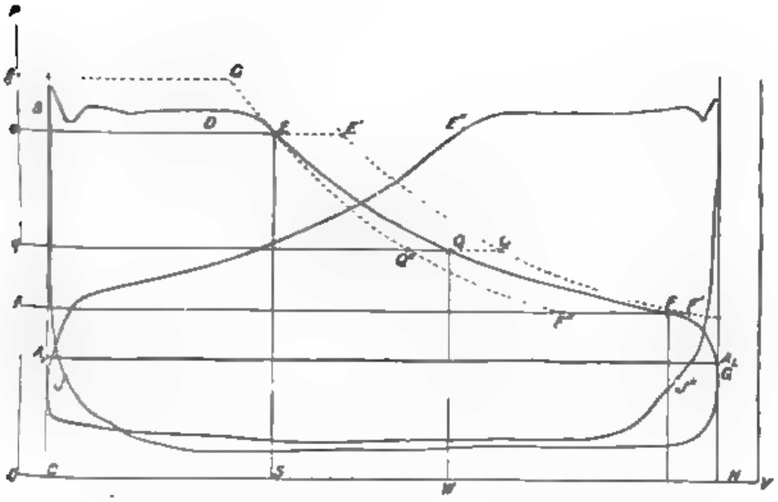



Fig. 82.

tion. Assume that this is equally divided between the two sides of the piston as the average pressures are nearly equal, so that ·218 lb. of water stuff corresponds to the left-hand diagram shown again in Fig. 82. The steam in the clearance space before fresh admission was 0·02 lb. Assume that there was no water present in the clearance space. Then at $E$, or $Q$, or $F$ the total amount of water stuff present is 0·238 lb. *Question.* If it were all steam what would be its volumes at the three pressures 44·77, 29·62, and 21·69?

*Answer.* 2,268, 3·268, and 4·386 cubic feet.

Let the points $E'$, $Q'$, and $F'$ represent these to the volume scale

of the figure. We can now complete the table of Exercise 6. We see that $E'R$ would be the volume of the water stuff corresponding to the point $E$ if it were all steam, but only the volume $ER$ is steam, and hence $ER$ is to $E'R$ as the amount of actual steam is to the whole water stuff present. Similarly $QT$ is to $Q'T$ as the actual steam is to the whole water stuff at $Q$. Similarly $FW$ is to $F'W$ as the actual steam to the whole water stuff present at $F$.

I have an easy rule for drawing such a curve as $E'Q'F'$ (see Art. 185) when any point, say $E'$ in it is given. I will not give it here, but surely a thoughtful student can have no great difficulty in inventing such a rule when he sees that one is needed. Hint; at any point $Q'$, the distance $Q'q$ represents the volume, and $QW$ represents the pressure of the same weight of steam as is shown in the same way at $E'$. Hence (see (9) Art. 180)

$$ES \times E'e^{\,1\cdot0646} = QW \times Q'q^{\,1\cdot0646}.$$

is the law showing the relations of these quantities to one another.

Does condensation or evaporisation occur from $E$ to $F$? *Answer*, Evaporation.

13. Students may be interested to know that during the above four hours' test the average power leaving the crank shaft was measured as a torque of 5033 pound feet at an angular velocity of 120 revolutions per minute, or 754 radians per minute; that is the useful power given out by the crank shaft was 5033 × 754, or 3,795,000 foot pounds per minute, or 3,795,000 ÷ 33,000, or 115 horse-power.

The power given by the steam to the piston was 142. The useful power is 115, and hence the **efficiency of the mechanism** of the engine is 0·81, or 81 per cent.

14. During the above four hours the average power leaving the dynamo machine which was driven by the steam engine was measured as a current of 730 ampères at an electrical pressure or voltage of 100 volts. This is 730 × 100, or 73,000 *watts* (called by the electrical people 73 *units* sometimes), and as we know that 746 *watts* are equivalent to 1 horse-power, the power electrically given out was 73,000 ÷ 746, or 98 horse-power. The **efficiency of the shafting and dynamo** is 98 ÷ 115, or ·852, or 85¼ per cent.

15. During the test the electric power was sent through wires to incandescent lamps; 4½ per cent. of the power leaving the dynamo was converted into heat in the wires, that is, the drop in voltage

was from 100 to 95·5, so that 93·6 horse-power was given out as heat and light. When electric motors instead of lamps received the electric power in some similar tests, they gave out 85 mechanical horse-power to drive machinery.

16. A pound of the coal when carefully burnt was found to give out 15,300 (Fahr.) units of heat; each heat unit is equivalent to 774 foot-pounds, and hence each pound of coal means a supply of energy of 15,300 × 774, or say $12 \times 10^6$ foot pounds. For each pound of coal there was a supply of 6·93 pounds of water, and each pound of water had received 1083 heat units, so that the steam per pound of coal has 7505 heat units, or 5,809,000 foot-pounds. One indicated horse-power for one hour is 33,000 × 60, or 1,980,000 foot pounds. This work is done by 3·8 pounds of coal, and hence the indicated work for 1 pound of coal is 1,980,000 ÷ 3·8, or 521,000 foot pounds.

The useful work transmitted from the crank shaft per pound of coal is 81 per cent. of this, or 422,000 foot-pounds. The electrical energy leaving the dynamo machine per pound of coal is $85\frac{1}{8}$ per cent. of this, or 422,000 × ·852, or 359,500 foot-pounds. The heat and light energy given out by the lamps is $95\frac{1}{2}$ per cent. of this, or 343,000 foot pounds.

We may therefore make some such statement as the following:—
The total energy obtainable from a pound of coal is disposed of in the following way:—

5,809,000 foot-pounds to steam, 6,191,000 foot-pounds wasted in chimney and by radiation.

521,000 foot-pounds to piston, 5,288,000 foot-pounds to condenser and by conduction and radiation.

422,000 foot-pounds from crank shaft, 99,000 foot-pounds wasted in friction of engine.

359,500 foot-pounds to electric light leads, 62,500 foot-pounds wasted in shafting and dynamo.

343,000 foot-pounds given out as light and heat by lamps, 16,500 foot-pounds wasted in leads.

52. In the above table we note the great waste in converting the steam energy into indicated work. Part of this loss occurs in the steam jacket; **most of the waste** will be accounted for if we measure the heat given to the condensing water. Measuring the number of pounds of condensing water used per hour, and its rise of temperature, it is easy to calculate the heat received by it from the exhaust steam. See Art. 138.

Students may be interested in some of the results of four other

four-hour tests made on the same engine, without altering its cut or speed, but with different steady loads.

| Indicated horse-power. *I.* | Power transmitted from crank shaft. *B.* | Electrical horse-power. *E.* | Water in lb. per hour. *W.* | Coal in lb. per hour. *C.* |
|---|---|---|---|---|
| 190 | 163 | 143 | 4805 | 730 |
| 142 | 115 | 96 | 3770 | 544 |
| 108 | 86 | 69 | 3080 | 367 |
| 65 | 43 | 29 | 2155 | 218 |
| 19 | 0 | — | 1220 | — |

Although this was a single cylinder engine, and therefore not v economical, the results are well worthy of study, because there relationships among the numbers which are the same as those find in any engine which is governed, as this one was, **by throttli the steam**, or in some other way lowering the initial pressure of steam.

Thus for example let the student plot the values of $W$ and $P$, $W$ and $E$, or $B$ and $I$ on squared paper. Let him also find the coal water per hour per indicated or transmitted or electrical horse-pow and let him meditate on his answers. He is gathering mate for a very thorough practical comprehension of the steam engine.

And now I should like to think that the average student ha chance of making all the measurements which I have described. E if only a small steam engine is available, an earnest teacher will fi that he can let students make tests of great value to his students.

**53.** At Finsbury it was **a regular part of the Session's wo** for two students to attend to the machinery every Wednesday, fr the lighting of the fire at 7 A.M. to 9.30 P.M. Whatever part of t stoker's or engineer's work they could be entrusted with, they d They regularly took all the measurements necessary for calculati indicated horse-power, actual horse-power given out by engine, fe water, coals, &c. They made elaborate reports of all that was do during the day. Few people seem to know how much roughly co rect information may be obtained easily from the study of an ordina working engine, for I want it to be understood that this was specially arranged laboratory steam engine.

An **exercise of considerable interest** may here be mentione A batch of twenty students (who had already had the above kind experience) would have a day's measurements. They knew exact what each of their duties was beforehand. Their watches agree

When any observation was made, the time was noted, and each student stayed twenty minutes at each kind of observation, and then went on to another. When he went to another job he found two or more men there to instruct him if he needed instruction. He reduced all his own observations. At any instant there would be—Two men checking the speed indicator by counting, and also taking temperature of hot well; two men measuring feed water; three men taking indicator diagrams; two men observing pressure gauges, one on boiler, one on exhaust in engine room, one on vaporising condenser on roof of building; two men weighing coals, &c.; two men observing actual power given out by engine, and transmitted through dynamometer coupling; two men measuring electrical horse-power given out by dynamo machine, which was the only thing driven by the engine through a long shaft. The engine was run for four hours at a time under a steady load.

All the observations were entered in a great table as soon as they had been reduced. Students who took diagrams had to make separate reports on the nature of the expansion curve, the missing water, the state of the valve motion, and many other things. Such a field day as this was, I found, worth many lectures in bringing home to students what actually occurs in machinery. It is to be remembered that these students had previously obtained the calorific power of the fuel; some years they took samples of the furnace gases, and analysed them in the chemical laboratory; every year they tested the instruments used for measuring feed water, the transmission dynamometer &c., before the field day.

Imagine a student to go through this easy work and arrive at the above results: take into account the impossibility of his doing the work without understanding it. Surely any one can see how very different must be the notions of a student after this kind of experimenting from those of a man who merely reads a book or listens to lectures. I affirm that **simple experimental work** of this kind is absolutely necessary for the elementary student if he is to get sound notions not merely concerning steam engines, but about energy questions in general.

### 54. *More Exercises.*

17. Try if there is a **law of expansion** of the simple form $pv^k =$ constant. At a point like $Q$ (Fig. 82), $Q\,W$ represents the pressure, and $Q\,q$ the actual volume of the expanding steam to some scales. If there is such a law as the above, it is easy for the student to prove that the actual scales of measurement are of no importance. I there-

fore measure the distance $Q\,W$ in inches, and call it $p$, and I measure $Qq$ in inches and call it $v$. Measurements like the following ought to be made at many points from $E$ to $F$. My measurements are made, not upon the diagram as engraved, but upon my own copy of this diagram. When the table has been made out let the student take the common logarithms of all the measurements.

| $p$ | $v$ | log. $p$. | log. $v$. |
|---|---|---|---|
| 4·46 | 3·34 | ·6493 | ·5237 |
| 4·11 | 3·73 | ·6138 | ·5717 |
| 3·78 | 4·12 | ·5775 | ·6149 |
| 3·44 | 4·6 | ·5366 | ·6628 |
| 3·19 | 5·08 | ·5038 | ·7059 |
| 2·96 | 5·58 | ·4713 | ·7466 |
| 2·67 | 6·3 | ·4265 | ·7993 |

He will now plot ·6493 and ·5237 as the co-ordinates of a point on squared paper, and get a point for each pair of numbers. It is evident that if there is such a law as

$$pv^k = const.,\ \text{or log. } p + k \text{ log. } v = C$$

then the plotted points must lie in a straight line, and so the test is quite easy. In the present case I find that a straight line seems to lie evenly among the points. We may reasonably say therefore that the law is true. Assuming it to be true I see from my own squared paper that if *log. p* were 0·65, *log. v* would be ·525,

$$\text{so that } \cdot65 + \cdot525\,k = C \quad . \quad . \quad . \quad . \quad (1)$$

again if log. $p$ were 0·4, log. $v$ would be 0·833,

$$\text{or } \cdot40 + \cdot833\,k = C \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Subtracting (1) from (2) we find $-\ 0{\cdot}25 + \cdot308\ k = 0$, or $k = 0{\cdot}81$, and so the law of expansion is very satisfactorily shown to be

$$p\ v^{\,0.81} = \text{constant}$$

**55.** In the next Exercise we are going to study what goes on in the water and steam in the cylinder during the expansion from $E$ to $F$ (Fig. 82). It is assumed that at every point such as $Q$, we know that there is the volume $Q\,q$ of steam, and $Q\,Q'$ represents the extra volume that there would be of steam if the water were all steam. We shall consider what would take place if the whole amount were 1 lb. (we know that we have only ·209 lb. ÷ ·878, or 0·238 lb. present, or 0·209 lb. of steam and 0·029 lb. of water). We assume that all

e steam and water is at the same temperature, and a student ust decide for himself what value he may place upon results based this assumption, which is certainly wrong, but which seems to be e only one on which we can base calculations.

Assuming (as is usual, but in my opinion, wrong) that there is **o water present at the beginning of the admission,** we see at during admission there is 0·222 lbs. of steam condensed; we ay take it that the latent heat of this condensed steam is given p to the cylinder during the admission, but at what rate this is one at every instant of the admission we do not know, although e may speculate about it. Again, during the release the stuff is artly in the cylinder and partly in the condenser; in the condenser, eat is being rapidly given out by the condensing steam; in the ylinder whatever water remains is probably boiling away, receiving eat from the metal of the cylinder. It seems when we consider the vaporation going on from $E$ to $F$ (Fig. 82), that there is no great likelihood of much water being present during the exhaust.

*Use of MacFarlane Gray's Diagram.*

EXERCISE 18. Let Fig. 83 be a $t\phi$ diagram (see Art. 203).

Points on the curve $AB$ are plotted to the figures headed $\phi_w$ in the table, Art. 180. Points on the curve $CD$ are plotted to the figures headed $\phi_s$ in the same table.

The curve $EQF$ on the $t\phi$ diagram, Fig. 83, corresponds with the curve $EQF$ on the indicator diagram, Fig. 82. It is drawn in the following way. To find the point $Q$. Find the temperature corresponding to the pressure at $Q$ and draw $qQ'$ to correspond; divide $qQ'$ in $Q$, Fig. 83, in the same proportion as that in which $Q$ divides the distance $qQ'$, Fig. 82. Find the other points in the curve $EQF$ in the same way.

The $t\phi$ diagram tells us

1. If the expansion from $E$ had been adiabatic $qQ'' : Q''Q'$ in the $t\phi$ diagram would have been the ratio of the amounts of steam and water present at $Q$. Hence, in the indicator diagram make $qQ'' : qQ'$, as $qQ'' : qQ'$ in the $t\phi$ diagram, and so get the curve $EQ''F''$, Fig. 82. This is what the real adiabatic expansion indicator diagram curve from $E$ would be when we deal with the proportion of steam and water which we know to be present at $E$.

2. The line $O\phi$, Fig. 83, is really supposed to be drawn at $-461°$ F., or $-274°$ C., so the student must imagine the dotted ines in the diagram to be very much longer than they are shown. Indeed, on the temperature scale the point $O$ marked $-461°$ F.

ought really to be looked upon as a zero of temperature, and instead of 200° F. we ought to read the absolute temperature 661. On the complete diagram, areas measured right down to the line $O\,\phi$ represent heat received.

Thus each pound of water stuff from $E$ to $Q$ **receives heat from the metal** of the cylinder of an amount represented by the area $E\,L\,M\,Q\,E$, and the total amount of heat received during expansion from $E$ to $F$ is the area $E\,L\,N\,F\,Q\,E$. The scale to which heat is represented is always easy to find because the rectangular area $e\,m\,n\,E'e$

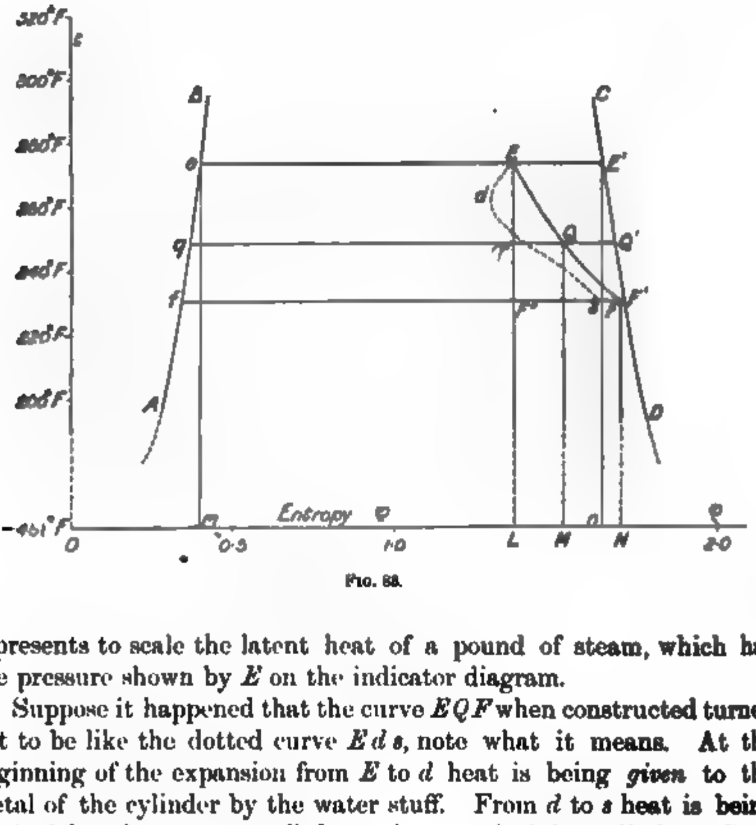



Fig. 68.

represents to scale the latent heat of a pound of steam, which has the pressure shown by $E$ on the indicator diagram.

Suppose it happened that the curve $E\,Q\,F$ when constructed turned out to be like the dotted curve $E\,d\,s$, note what it means. At the beginning of the expansion from $E$ to $d$ heat is being *given* to the metal of the cylinder by the water stuff. From $d$ to $s$ heat is being *received* by the water stuff from the metal of the cylinder. Such curves carefully studied show us how heat is exchanged between metal of cylinder and the water stuff. Students must work many exercises in this way in spite of the fact that we cannot prove that there is no water present before admission.

**56.** My students sometimes draw the complete $t\phi$ diagram cor-

ponding to our indicator diagram. The assumption throughout that a pound of *water-steam* is present in a vessel of changing ume, and the amount of it in the state of water is exactly known each point. The volume of the water is neglected. If the in- ator diagram were really correct, and if we could be sure that the mperature is always the same in any part of the water and steam, ch a diagram might be very interesting. I am sorry to think that

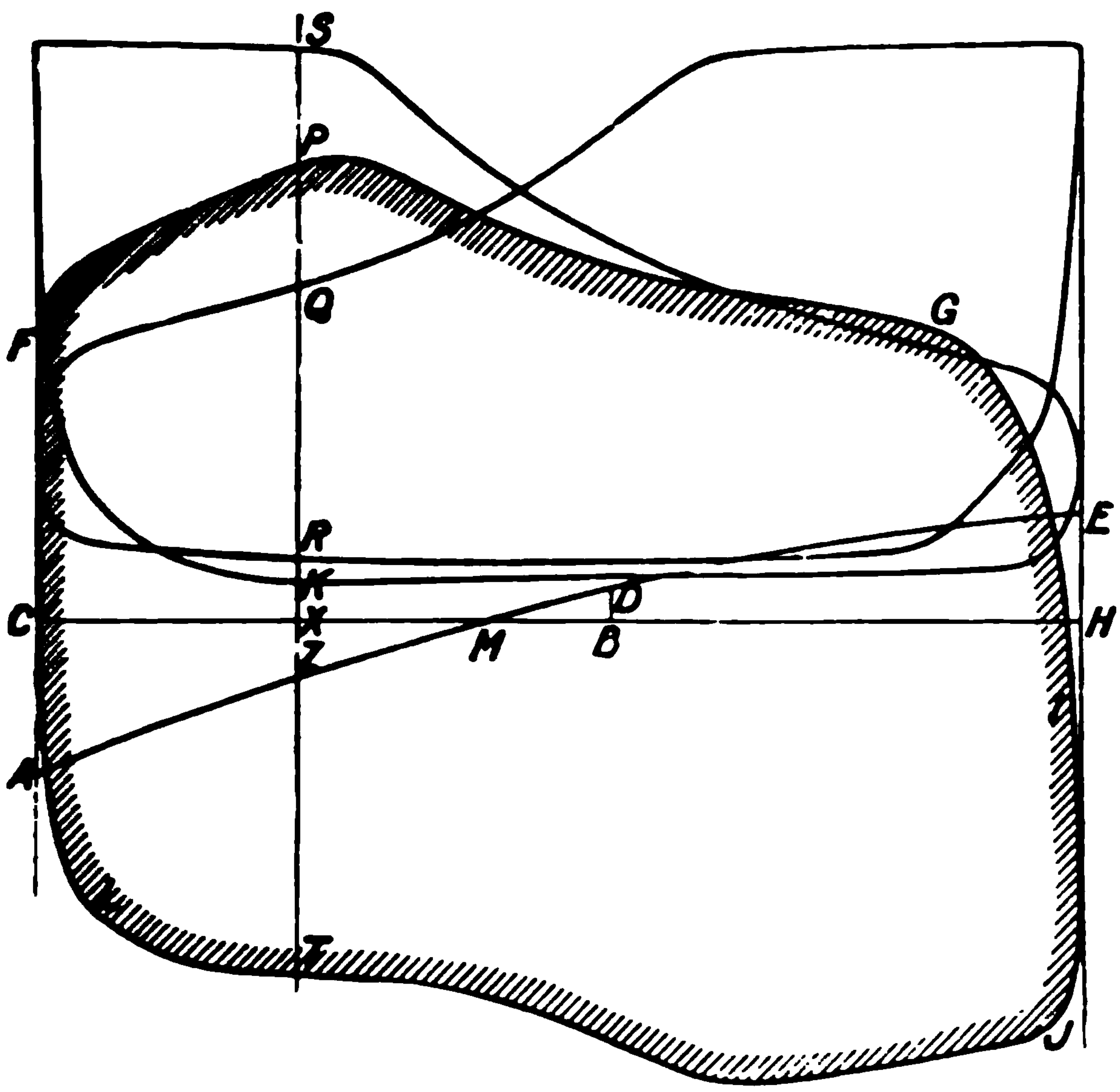



FIG. 84.—DIAGRAM OF FORCE AT CROSS HEAD.

some philosophers are apt to forget that this consideration renders most of their speculations useless. But the much more important assumption that we do actually know how much water is present at any point is really untenable.

**87.** EXERCISE 19. In many of the above exercises upon our indicator diagrams we have gone on the assumption that all the stuff inside the cylinder is at the same temperature. This is, of course, untrue, like many other assumptions which we make in our desire to calculate something; but reasoning on even wrong assumptions may give rise to useful suggestions. It is a more absurd assumption still that the material of the cylinder is non-conducting, and yet if we imagine some water in a non-conducting cylinder to represent the metal of a real cylinder which is heated and cooled, although the assumption is wrong, it leads to suggestions that may be of use.

Using the numbers given us by the above diagram, and assuming that we have no knowledge of the actual amount of water present, I have worked out (Art. 216) what must have been the amount of water present before admission,

and also the amount condensed during admission. In fact, the shape of a small portion of the expansion curve tells us the value of these things if the cylinder is non-conducting.

**58.** EXERCISE 29. **Following the method of Art. 50, make a diagram showing for every point of the stroke the force (divided by the piston area for convenience) acting on the cross head end of the connecting rod, if the weight of piston, piston rod and crosshead is 450 lbs., and if the connecting rod is 6¼ feet long.**

First let us find the forces which the cross head must exert to maintain the motion of these parts, if there was no steam pressure on either side of the piston and if the crank shaft were driven by an outside agent.

The crank pin velocity is $v = 2\pi \times 1\frac{1}{4} \times 2 = 15{\cdot}71$ ft. per second. The accelerations at the ends of the stroke are, numerically—

$$v^2\left(\frac{1}{r} \pm \frac{1}{l}\right) \text{ or } 15{\cdot}71^2\left(\frac{1}{1{\cdot}25} \pm \frac{1}{6{\cdot}25}\right) = 236{\cdot}8 \text{ and } 157{\cdot}9.$$

The moving mass is $\frac{450}{32{\cdot}2}$ in engineers' units, or 14. Hence the forces at the ends of the stroke are 3315 and 2211 lbs. Reducing these to the scale of pounds per square inch on piston we have 13 and 8·7.

We see by Art. 339 that when the crank is at 90° with the line of centres, the piston is 0·125 feet to the *right* of its mid stroke, and its acceleration is 39·5, giving to the scale of pressure a force of $\frac{39{\cdot}5 \times 14}{254}$ or 2·18 lbs. per square inch.

The engine is horizontal, so that the mere weights of the parts (neglecting the connecting rod) do not enter into the calculation. I have made $CA$ represent 13 lbs. per square inch, $HE$ 8·7, and I found the point $B$ 0·125 feet to the right of the mid stroke, and made $BD$ represent 2·18 lbs. per square inch. I drew the curve $ADE$ through the three points, and take its vertical distance anywhere from $CH$ to represent the force which at the crosshead would give to the moving mass the acceleration which it possesses. To the same scale it is now evident that the total force *from left to right* (that is towards the crank shaft) on the crosshead is shown by distances of points on the diagram [illegible] $CH$. To find such a point as $P$ I take the distance $SR$ (from $S$ on one diagram to the back pressure point $R$ on the other diagram), subtract from it [illegible], and let $XP$ represent the answer. This is very easy to do with the edge of a strip of paper: it is easier to do than to describe. Again, make [illegible], and we find [illegible].

In Art. [illegible] I have shown the nature of this diagram for a single acting engine.

**59.** [illegible] connecting rod of our engine is [illegible] from the crosshead and 35 inches from [illegible] mass is distributed in the following way—[illegible]th of [illegible] 147·2 lbs., at the crank pin. It [illegible] XXIX [illegible] if we replace the real connecting [illegible] exceedingly tedious and difficult [illegible] quickly, and the error is small. [illegible] crank pin is always radial, [illegible] of the crank it may be neglected. [illegible] the turning moment on the crank [illegible]

[illegible] forces due to acceleration, [illegible] proportion of $\frac{450 + 128{\cdot}8}{450}$. This

combined with the old steam force diagram gives a diagram not very unlike *CFPGIJTLC*. At any instant let the force (the diagram ordinate multiplied by the area of the piston) be called $F$ pounds, in the position shown in Fig. 86, $AO$ being the line of centres, and $OH$ a line at right angles to $AO$. It is easy to show that the turning moment on the crank shaft due to $F$ is $F \times OH$ if we neglect the weight of the rod and friction. It is therefore necessary for the student to draw such a figure for many points in the piston stroke, and to multiply each value of $F$ in pounds by each distance, such as $OH$ in actual feet. It is well to make a diagram in which the abscissæ represent angles passed through by the crank.

If the crank shaft gives out power uniformly during the revolution, the height of this diagram above its average height represents the acceleration of its velocity to scale. It is useless to pursue the matter further, when the connection of the shaft with driven machinery is by belting or elastic mechanism.

**60.** Exercise 22. The engine is on the same shaft as the armature of a dynamo machine; the whole mass moved is like a fly-wheel, weighing 3 tons, with an average radius of 5 feet. What is its **fluctuation of speed?**

*Answer.* The mass of the wheel is $3 \times 2240 \div 32{\cdot}2 = 209$. Its moment of inertia is this multiplied by $5^2$, or it is $I = 5225$ in engineers' units. Each of the excess moments in the diagram, divided by $I$, gives the acceleration. I know that the speed is very nearly uniform, and it will save trouble and produce almost no error to assume that equal angles passed through by the crank represent equal times. Hence the area of the acceleration diagram from $\theta = 0$ represents the gain of velocity. The graphical method of proceeding is easily understood; the tabular method described in my *Applied Mechanics* ought also sometimes to be employed.

| $\theta$ | Turning moment, $M$. | $E$, or excess of $M$ above average $M$ | $\int E{\cdot}\delta\theta$. | Gain in turns per minute, since $\theta = 0$. |
|---|---|---|---|---|
| 0 | 0 | −6212 | 0 | 0 |
| 5 | 630 | −5585 | −29500 | −·0344 |
| 10 | 1400 | −4815 | −55500 | −·140 |
| 15 | 2380 | −3934 | −83700 | −·215 |
| 20 | 3300 | −2911 | −101000 | −·258 |
| 25 | 4740 | −1475 | −112000 | −·292 |
| 30 | 5350 | —862 | −118000 | −·302 |
| 35 | 6360 | +145 | −119000 | −·310 |
| 40 | 7350 | 1141 | −116000 | −·302 |
| 45 | 8210 | 1998 | −108000 | −·275 |
| 50 | 9060 | 2846 | −96200 | −·249 |
| 55 | 9900 | 3688 | −79900 | −·206 |
| 60 | 10580 | 4368 | −59700 | −·155 |

It will be seen that I have not divided all the values of $E$ by $I$ to get angular acceleration; again, instead of totting up $\int$ acceleration $\times\ \delta t$, the gain in radians per second, I have totted up $\int E{\cdot}d\,\theta$, taking $\delta\,\theta$ in degrees, as it saves unnecessary labour. This represents the gain of angular velocity to some scale; I want it in revolutions per minute. Now the gain in revolutions per minute is evidently $\frac{60}{2\pi}\int_0^t \frac{E}{I}\,dt$ if $t$ is in seconds $= \frac{60}{2\pi}\int_0^\theta \frac{E\,dt}{I\,d\theta}\,d\theta$. As $\theta$ is in

degrees, $\frac{dt}{d\theta} = \frac{\text{time of a revolution in seconds}}{360}$, sufficiently nearly constant for our purposes; this is $\frac{0\cdot5}{360}$ or $\frac{1}{720}$. We have therefore to multiply the numbers in column (4) by $\frac{60}{2\pi I}\frac{dt}{d\theta}$, which is $2\cdot58 \times 10^{-6}$, to get the numbers in the fifth column.

EXERCISE 23. State the maximum and minimum and average turning moments on the crank shaft.

Of course as we have neglected friction, the actual turning moments are less than these. In fact, Exercise 13 tells us that they are 81 per cent. of these.

**61.** EXERCISE 24. For eight positions of the crank show to scale (1) the actual force on the crankpin, (2) that component of it which acts at right angles to the crank, (3) that component of it which acts radially.

(2) Is already drawn to scale, for it is the turning moment on the crank shaft in pound feet divided by 1·25 feet, the length of the crank. Let $BS$ drawn at right angles to the crank $OB$, represent it, make the connecting rod (produced as shown) direction $BR$ be the diagonal of the rectangle $BSRQ$, then $BQ$ is the component towards $O$ of the push of the connecting rod on the pin. But we have also the centrifugal force (only to be calculated once) of 147·2 lbs., this is $\frac{147\cdot2}{32\cdot2} r (4\pi)^2$, or 903 lbs., and $QW$ represents it in amount and direction, hence $BW$ represents (3), and completing the rectangle $BWTS$, $BT$ represents (1).

**62.** I have now described some of the exercises which I usually ask a student to work for me. After such a course of study he may feel that he really has thought a little about the steam engine. I know a great deal about the average student; he has read books and looked at the figures in the books, and he has heard descriptions of how calculations are made; he has that sort of knowledge of his subject which is possessed by a newspaper writer.

How often must we say these things before teachers and students get to believe us. When I was very young I used to think that the views I imbibed from magazine articles were my own, although they changed with the moon. After a popular lecture I thought I knew a subject nearly as well as the lecturer. No man learns to think by mere reading or listening to lectures; he only learns priggishness, and his method of study is exactly like Mark Twain's telescopic method of climbing Mont Blanc. It is very weak in me to publish in this book such figures as Fig. 83. A student ought not to see any such figure unless he has drawn it himself, and then his knowledge would have the exquisite flavour given by discovery.

**63.** There are fifty other useful exercises which might be described. For advanced students I may suggest the following.

EXERCISE 25. The indicator diagram (Fig. 82) gives the pressure of the steam for every position of the piston. By means of the table (Art. 180) write out the

temperatures of the steam. Find the angle made by the crank with its dead point for each of these positions, and draw a curve showing temperature of the steam on the left-hand side of our piston for every position of the crank.

This curve is interesting in itself. But now let the student take the values of twenty-four or thirty-six or more equidistant ordinates, and by any of the well-known methods express the temperature as a function of the time in Fourier series (see Art. 316). They will give greater interest to the considerations of Art. 229.

EXERCISE 26. In a position $AB$ (Fig. 85), if the connecting rod is at right angles to the crank, the push in it is great, and the centrifugal force upon

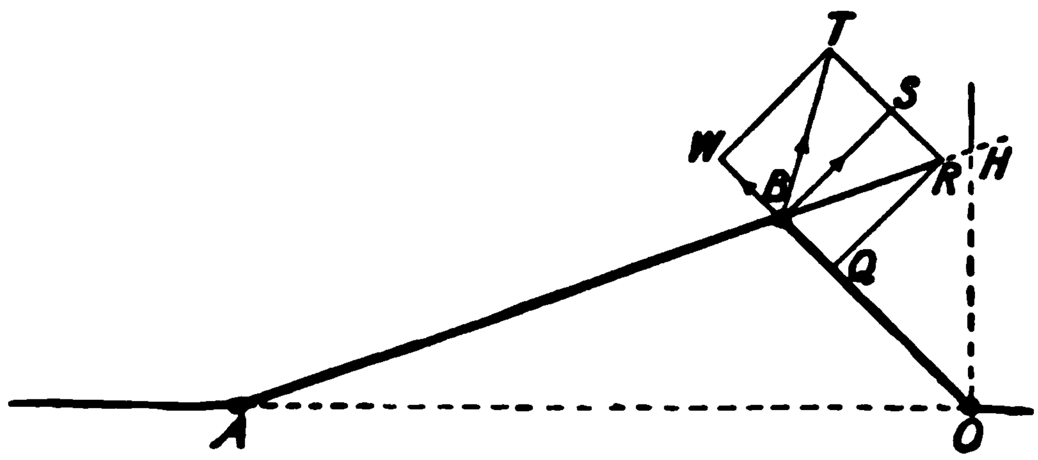



FIG. 85.

it is great, and presumably this is its position of greatest weakness; consider now its strength.

It will be noticed that in a horizontal engine the figure shows the best direction of motion, because the weight of the rod opposes centrifugal force when the rod is a strut. In an engine which must work at full power in either direction of rotation, the weight of the rod ought to be considered as well as the centrifugal force. In vertical engines the weight of the rod may be neglected. This exercise is one that ought to be worked out on the principle described in books on applied mechanics.

EXERCISE 27. At any instant what are the stresses in every part of the frame of the engine? If the engine runs at very high speed we must take elastic

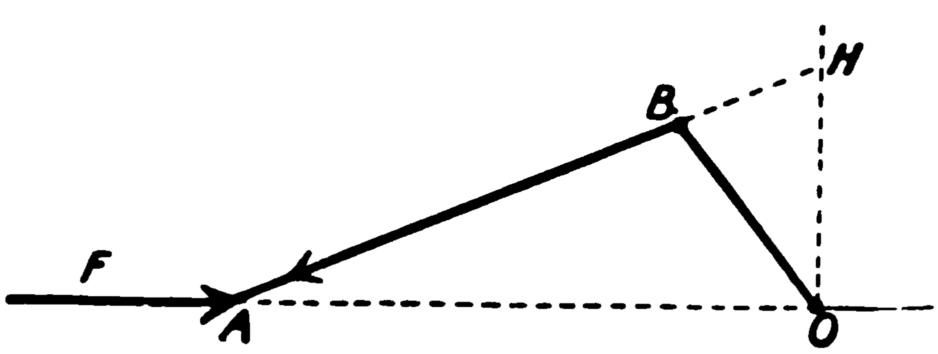



FIG. 86.

vibratory effects into account; but at speeds up to 400 revolutions per minute in such engines as are in the market, we may neglect such effects.

The indicator diagram enables the forces on the cylinder to be calculated, the above diagrams enable all the other loads on the frame to be calculated. It is usual to consider these at only one or two positions, when their effects are likely to have the greatest stress-producing effects. The calculation belongs to that part of applied mechanics which is called machine design, and no general rules may be given concerning it. The frames of modern engines differ from older engines greatly in the regard paid to considerations of this nature, but also greatly to ease of manufacture and fitting.

**64.** Exercise 28. At any instant find the forces with which the fran the engine acts upon its supports. Neglecting all the steam pressures the ge principle is this—

If $m$ is any small portion of the engine which is moving, and it ha acceleration in a direction which I shall call $x$ of the amount $\ddot{x}$, then $m\ddot{x}$ be regarded as a force in the direction $x$. If the resultant of all such forc found, this is the resultant of all the forces acting on the ground or other port. The general theory is given in Chap. XXIX. A triple expansion en has often two low-pressure cylinders, and in this and in quadruple expa engines there is always a good opportunity of effecting a partial balanc properly spacing the cranks and adding to the masses of the smaller pis small models being relied upon rather than calculation.

**65.** *Single-Acting Engine.* In Art. 58 we found the diagram showing force acting towards the crank at the crosshead of our double-acting en The most important reason for using single-acting engines is that this force

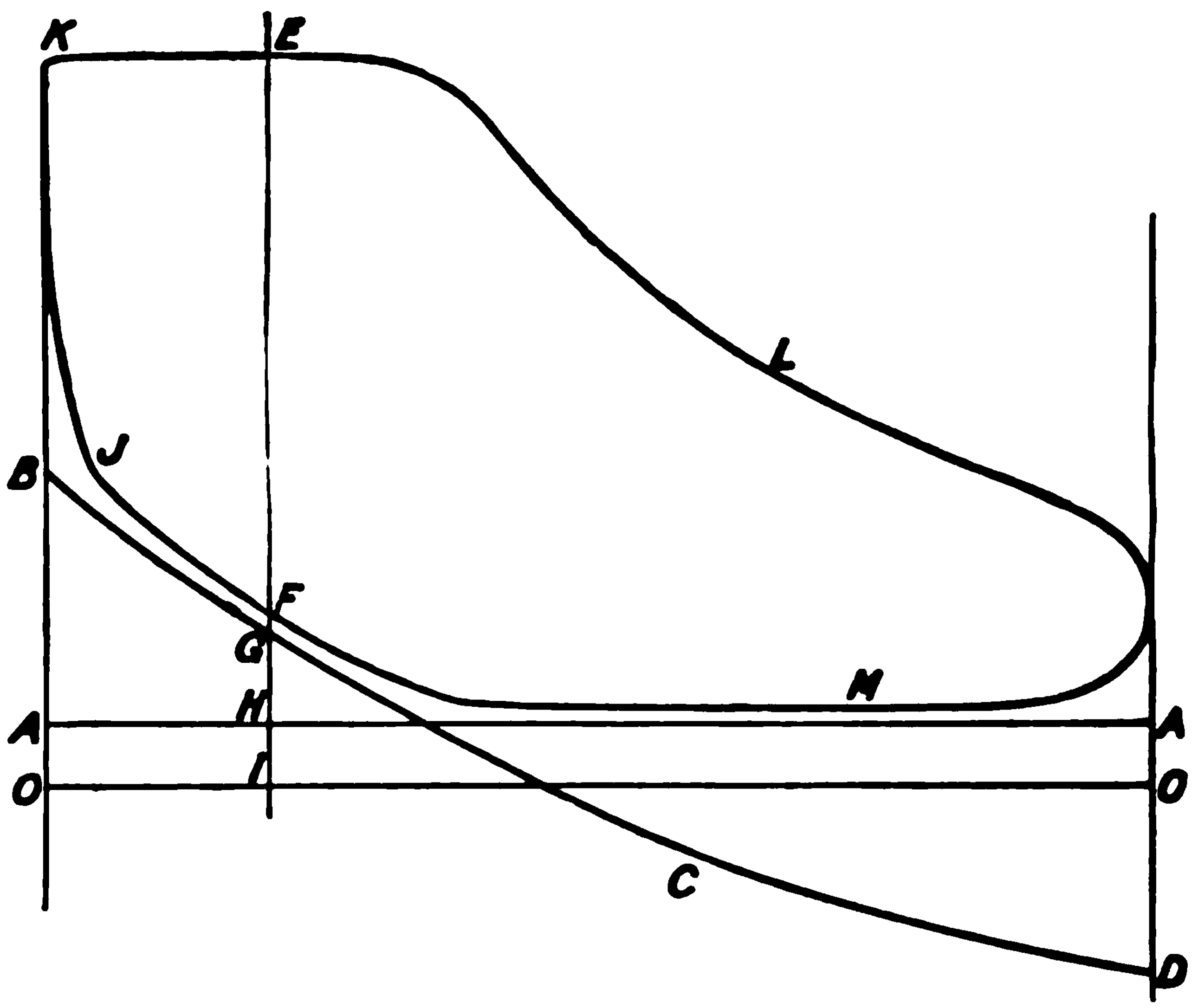



Fig. 87.

be always in one direction, so that "knocking" is not possible. This is i trated in Fig. 87, where we have $J\,K\,E\,L\,M\,F\,J$, the diagram. In this case is the atmospheric line, and the other side of the piston is exposed to atmosp pressure always; hence the force due to the steam itself is $E\,H$ in the for and $F\,H$ in the back stroke, always in the same direction. Let us suppose cylinder vertically above the crank and the steam as acting on the upper si the piston. Let all forces be reckoned per square inch of piston. Let represent the weights of piston, piston rod and crosshead. In most ordi engines $A\,O$ represents from 2 to 6 lbs. per square inch of piston. Let $O\,B\,C$ be the acceleration force diagram, which must be *subtracted* from the down forces. We thus see that $E\,G$ and $F\,G$ are the downward forces at the c head, and they do not change sign.

But if the speeds were greater, so that $B\,C\,D$ were to cut $J\,F\,M$, we sh

be compelled to give more cushioning. When the speeds are much greater we prolong the piston rod and let the cushioning effect of air behind an auxiliary piston supplement the ordinary cushioning effect of the steam. It is well worth while for a student to work out an example in which $OB$ is three times as great as what is shown, noting exactly how much air cushioning effect is necessary to prevent reversal of force.

If we desire also to prevent knocking at the crank pin, we take care that a proper proportion of the mass of the connecting rod is supposed to exist at the crosshead, thus increasing the ordinates of the $BCD$ diagram, calling the result a crank pin diagram. **All knocking may then be prevented** in a single-acting engine, but this is impossible in double-acting engines; which are never, therefore, run at a high speed; but in double-acting engines we can often utilise the inertia forces to alter the point in the crank pin path at which the knock occurs, so that it shall not produce such serious effects. It will be noticed that in all

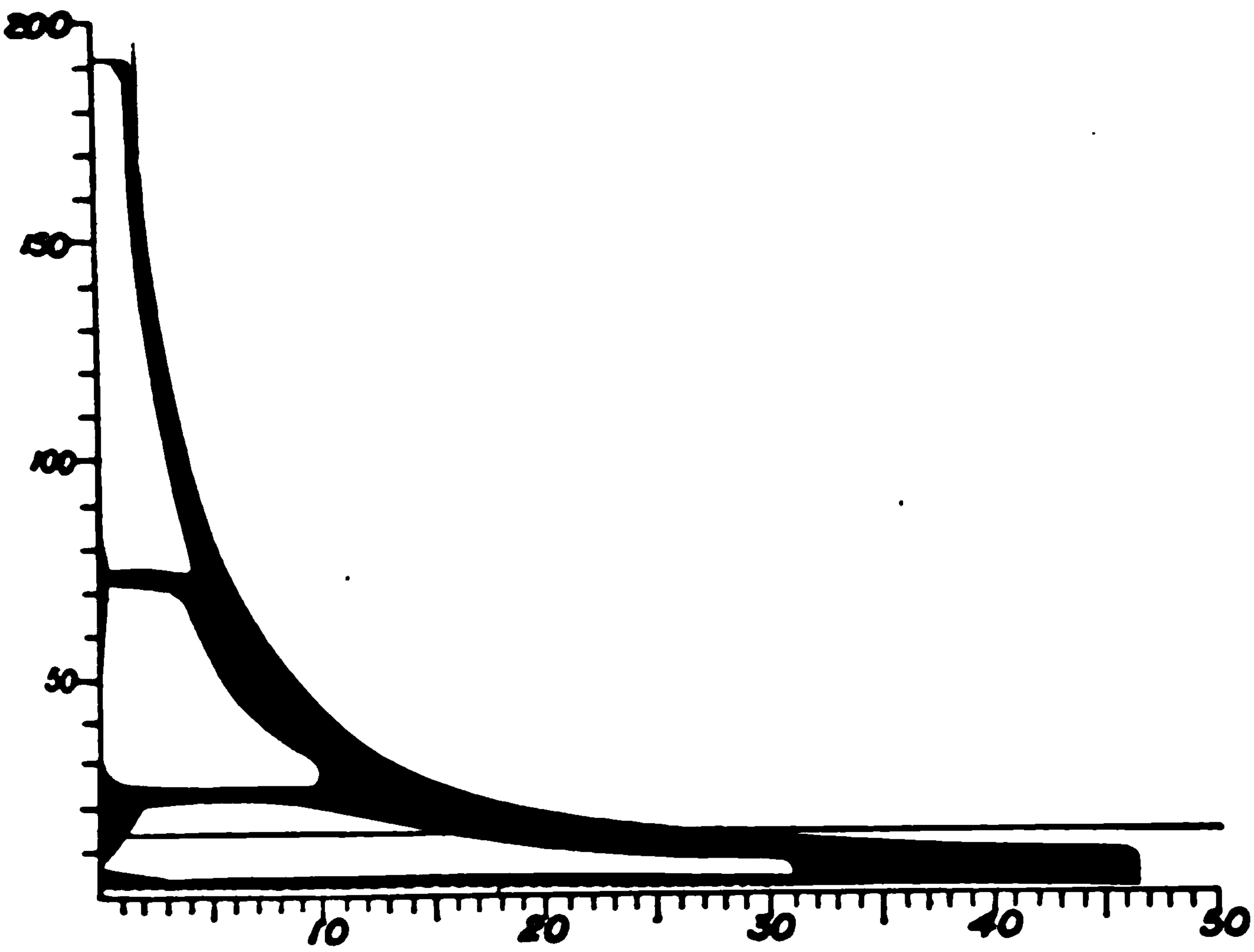



FIG. 88.

cases the inertia effects tend to equalise the turning moments on the crank shaft, but I am not disposed to think this a very important matter.

**66.** *Double and Triple Expansion Engines.*—It is a good exercise for a beginner to take a particular case, say that the cut off is at two-fifths of the stroke in each cylinder; to assume volumes for the spaces which receive the steam exhausting from one and admitted to the next; for each position of the crank, to note what the steam in each cylinder is doing, and to draw the hypothetical indicator diagrams on the assumption that $pv$ remains constant. I need not give examples of the answers as they are very easily arrived at.

Such exercises as these are easily worked out. In a compound engine, cutting off at half stroke in both cylinders; prove that by cutting off earlier in the stroke in the low pressure cylinder, more work is done in this cylinder and less in the high pressure cylinder; also it tends to remove the "drop" of pressure in the high pressure cylinder at release (absence of "drop" is not desirable).

The actual diagrams from each cylinder may be treated separatel[y]
Arts. 51–56, and this is the best way, because in the expansion part w
what is most to be studied, we really deal with different quantities of s[t]
the three cylinders, the steam in each clearance space being different
For some purposes, however, it is thought well to show them all on o[ne dia-]
gram, to the same scale of pressure and volume. Now the total volume o[f]
steam being known for each, it is easy to make them equal to scale.

Figs. 88 and 89 are examples of diagrams which have been so reduce[d]

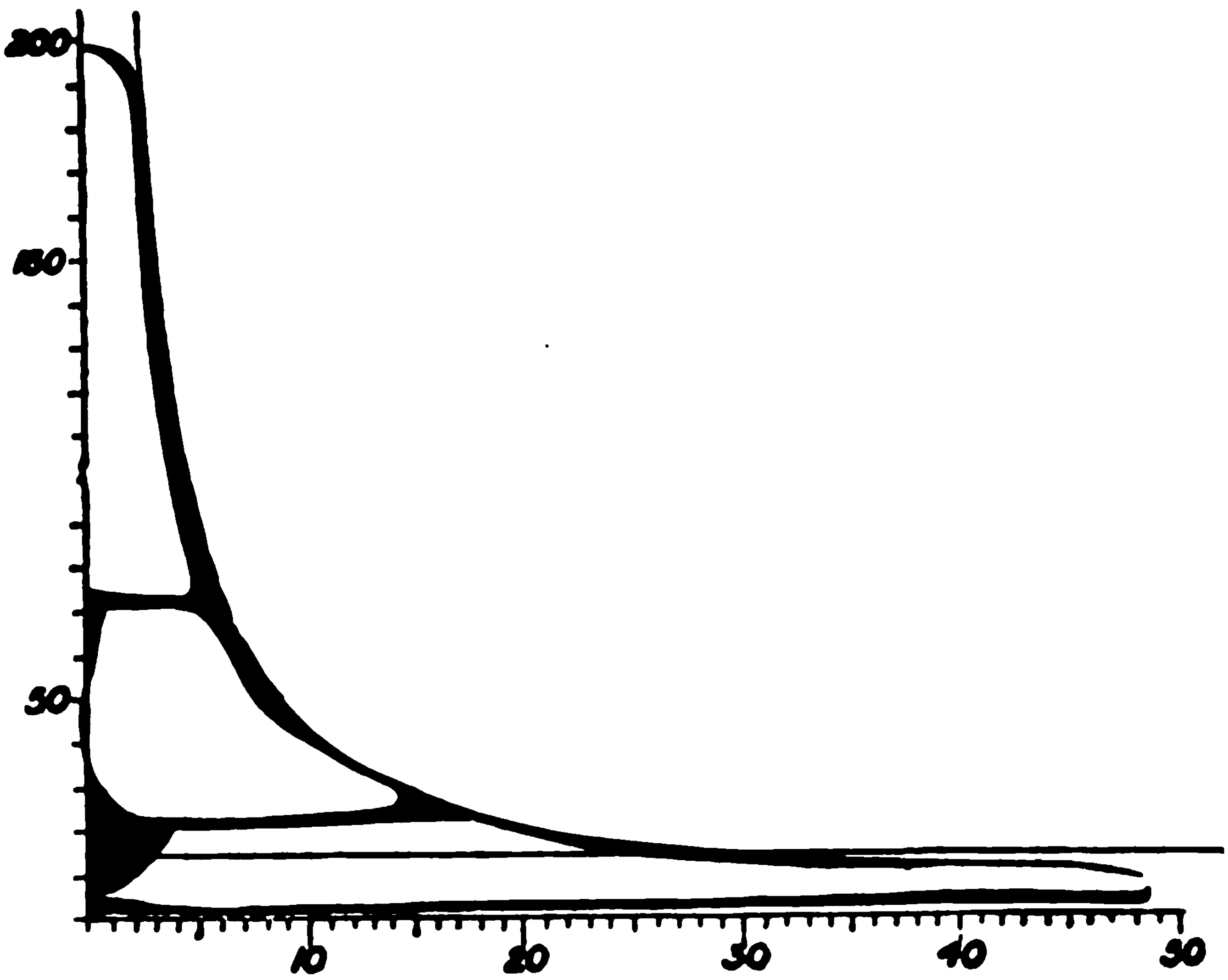



FIG. 89.

are copied from a paper by Prof. Osborne Reynolds. Fig. 88 when the
no steam in the jackets ;[1] Fig. 89 when all the jackets had steam of 190 lb
the boiler. The shaded parts represent condensed steam.

As in Art. 51 the saturation curve is drawn on the assumption tha[t]
is no water present in the cylinder at the end of the exhaust. If we onc[e]
that there may be water present, all our calculations are comparatively
I believe that there is almost always some water present, an unknown a

[1] Great care seems to be taken in existing triple cylinder engines to k[eep]
jacket pressure of the I. and L.P. cylinders low ; the steam entering by r[educing]
valves and there being relief safety valves. It is interesting to see so much
taken to produce evil effects.

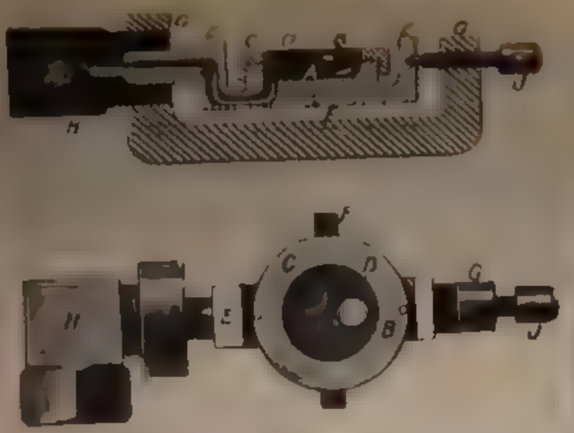



Fig. 90.

**Perry's Indicator** for very high speeds. When the cylinder [illegible] box A, the pressure [illegible] with the very shallow [illegible] steel diaphragm [illegible] a white [illegible] light [illegible] spot [illegible] proportional to the [illegible] plate or a sheet of [illegible] wall [illegible] tracing paper it [illegible] light [illegible] the rapid change of shape [illegible] placed [illegible] one [illegible] I have shown [illegible] speed [illegible] a large [illegible] of students. Mr. [illegible] he had discovered a curious [illegible] water in the cylinder which had greatly troubled him. [illegible] thrown continuously on the wall of an engine room might be useful.

## CHAPTER VI.

### THE RECIPROCATING MOTION.

**67.** ALTHOUGH the student is already supposed to know the motion of a slider driving or driven by a crank by means of a connecting rod, we shall here study the mechanism a little.

In the study of all kinds of link work mechanism, I think that much is to be gained by making **simple models of laths** fastened by pins. I now suppose the student to have made such a model. Figs. 91 and 101 show the kind of model, a somewhat more elaborate model than perhaps the student may make for himself. The end $A$ of the connecting rod $A B$ is guided

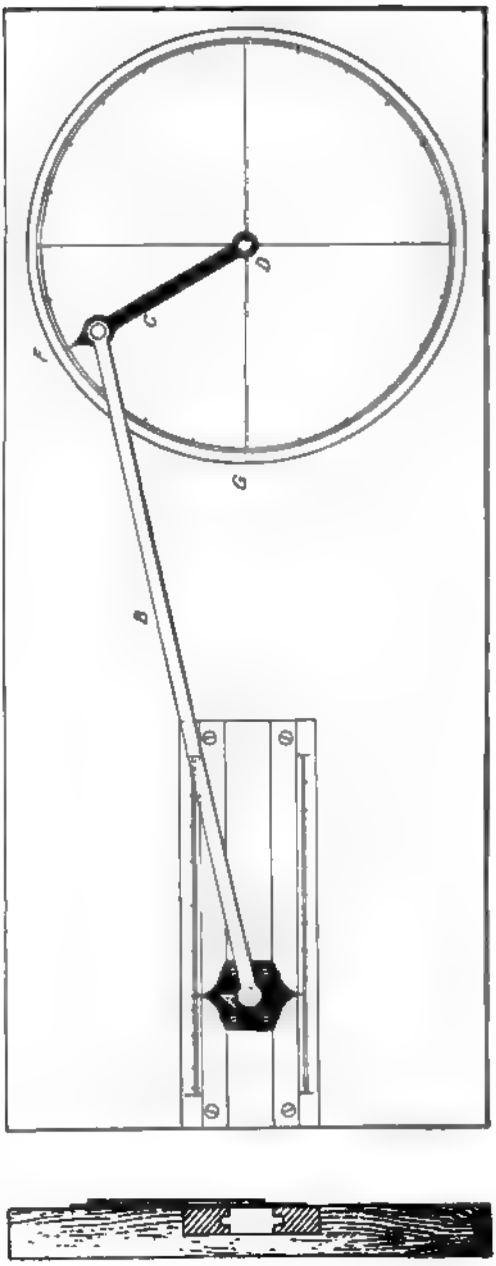



Fig. 91.

to move in the straight line direction $D\ A$; the motion of $A$ represents the motion of any point in the crosshead or piston or piston rod. The other end $B$ of the connecting rod moves in a circular path, whose radius is the length of the crank. The student will find that the straight scales for $A$, and the circular scale $G\ F$, enable the relative positions of $A$ and $D\ C$ to be studied. The following problems ought to be worked in other ways, and the answers tested by means of the model.

The next best method of study is by skeleton drawing.

PROBLEM.—When any position of the crank is given us, to find the position of the piston, or *vice versâ.* Notice that any point in the piston or piston rod or crosshead has exactly the same motion, the whole mass having a motion of mere translation. Let $A$, Fig. 92,

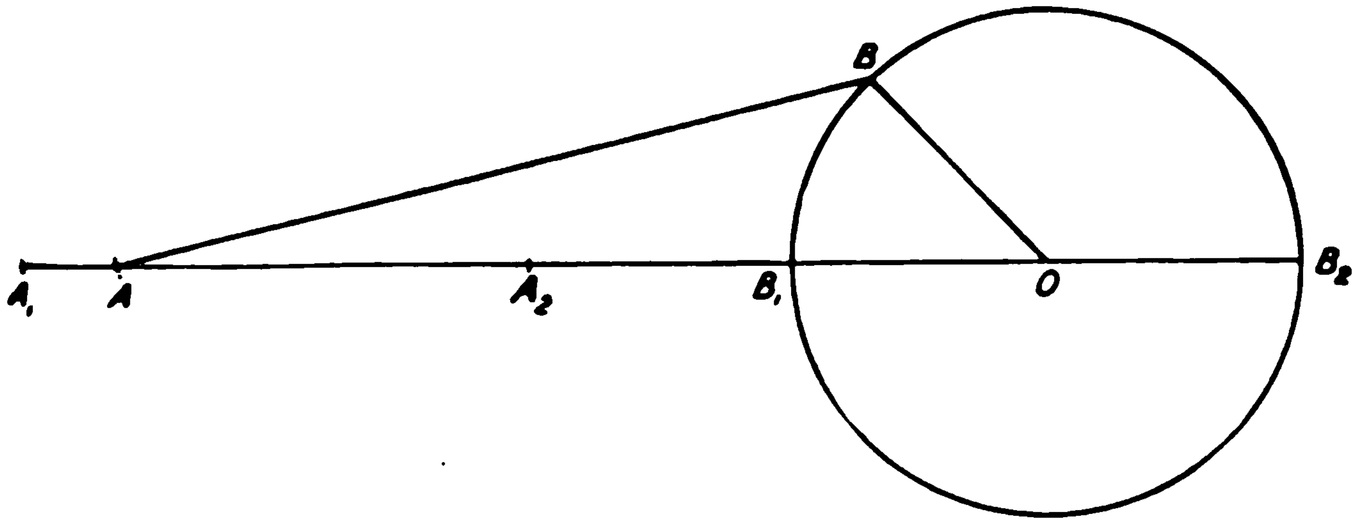



FIG. 92.

be the centre of the crosshead, $B$ the centre of the crank pin. $O\ B$ is the crank, $O$ being the centre of the crank shaft. Let $O\ B$ be drawn in any position, that is, making any angle such as $A\ O\ B$ with the centre line of the engine. Set off the distance $B\ A$ equal to the connecting rod and we have the proper position of $A$.

If $B_1$ and $B_2$ are the dead points of the crank pin, let $B_1 A_1$ and $B_2 A_2$ be each equal to the length of the connecting rod, and these are evidently the ends of the stroke of $A$. The distance of $A$ from the end of its stroke is evidently the same as the distance of any point on the piston or piston rod from the end of its stroke.

If we want to find $A$'s position pretty often we need not always make the above straggling drawing. Once for all, cut a template out of zinc plate or thin sycamore of the shape shown in Fig. 93. The edge $C\ D$ is straight. The edge $E\ D$ is an arc of a circle drawn to a radius equal to the length of the connecting rod, coming down at $D$ at right angles to $C\ D$. The edge $C\ E$ is of any shape we please.

*Answers.* $A_1 A = 0{\cdot}158$ or $A A_2$ $1{\cdot}892$, $A^1 A_2 = 0{\cdot}108$ or $A^1 A_1 = 1{\cdot}842$.

Note that $A^1 A_2$ is less than $A_1 A$.

EXERCISE 2. For angles $A_1 O B = 0$, 15°, 30°, 45°, &c., find the distances $A_1 A$ and $A A_2$.

Find what these would be if the connecting rod were so long that we might regard the arc $D E$ of the template as a straight line; in fact, as if the template were a set square.

Your answers must be carefully checked by the following table:—

| $AO^1B$ . . . | 0° | 15° | 30° | 45° | 60° | 75° | 90° | 105° | 120° | 135° | 150° | 165° | 180° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_1A$ . . | 0 | ·041 | ·158 | ·343 | ·575 | ·834 | 1·100 | 1·352 | 1·575 | 1·757 | 1·892 | 1·973 | 2·000 |
| $AA_2$ . . . | 2 | 1·959 | 1·842 | 1·657 | 1·425 | 1·166 | 0·900 | ·648 | ·425 | ·243 | ·108 | ·027 | 0·000 |

The following table gives the answer in case the connecting rod were infinitely long:—

| Angle $A^1OB$ . | 15° | 30° | 45° | 60° | 75° | 90° | 105° | 120° | 135° | 150° | 165° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_1A$ . . . . | ·034 | ·133 | ·293 | ·500 | ·741 | 1·00 | 1·259 | 1·500 | 1·707 | 1·867 | 1·966 |
| $AA_2$ . . . . | 1·966 | 1·867 | 1·707 | 1·500 | 1·259 | 1·00 | ·741 | ·500 | ·293 | ·133 | ·034 |

Examine and compare the numbers in these Tables.

EXERCISE. 3. Steam is cut off in both in-going and out-going stroke when the crank has travelled 80° from the beginning of the stroke. Through what fraction of the whole stroke has the piston travelled in each case?

What would these fractions be if the connecting rod were infinitely long?

| | 5′ Conn. rod. | Infinite Conn. rod. |
|---|---|---|
| In-going . . . . . . . . . | 0·37 | 0·415 |
| Out-going . . . . . . . . | 0·46 | |

In a very great number of rough calculations it is sufficiently correct for our purposes to drop a perpendicular $B A$ from $B$, the position of the crank pin, upon the line of centres, and to regard $A$ as the position of the piston in its stroke, the ends of the stroke being $A_1$ and $A_2$. It is evident that in this construction the assumption is that the connecting rod is infinitely long.

If $O B$ is an eccentric crank (see Art. 71), it will be found that this construction gives the position of the valve with very great

accuracy indeed, because the eccentric rod is very long compared with the eccentricity of its disc.

The student will notice that if in Fig. 96 $B$ is a crank pin going in a circle round $A$, and if the block $B$ moves in the straight slot $C B$ in the slide, the motion of any point in the slide is like that of a crosshead with an infinitely long connecting rod. This mechanism is sometimes used in small engines.

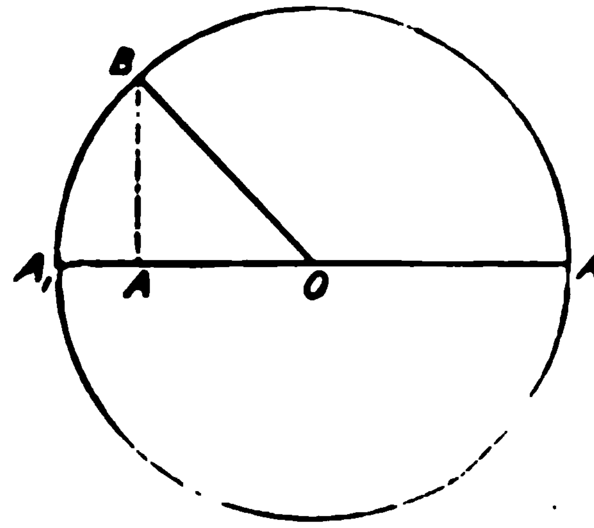



FIG. 95.

Again, if the slot is curved to the arc of a circle, the motion of the slide is exactly the same as that of a crosshead worked by a connecting rod whose length is the radius of the slot.

**69.** It is easy to show that when the piston is at $A$, Fig. 95, the distance $A B$ represents its **velocity** to scale if the connecting rod is infinitely long. The velocity at the middle of the path $O$ is equal to the velocity of the crank pin in its path, and this gives us the scale, because at the centre the velocity is represented to scale by the radius of the circle. It is easy to show also that the **acceleration**, when the piston is at $A$, is represented to scale by the distance $O A$. The acceleration at $A_1$ or at $A_2$ is $0{\cdot}011\ n^2 r$ or $v^2/r$ if the crank is $r$ feet long, making

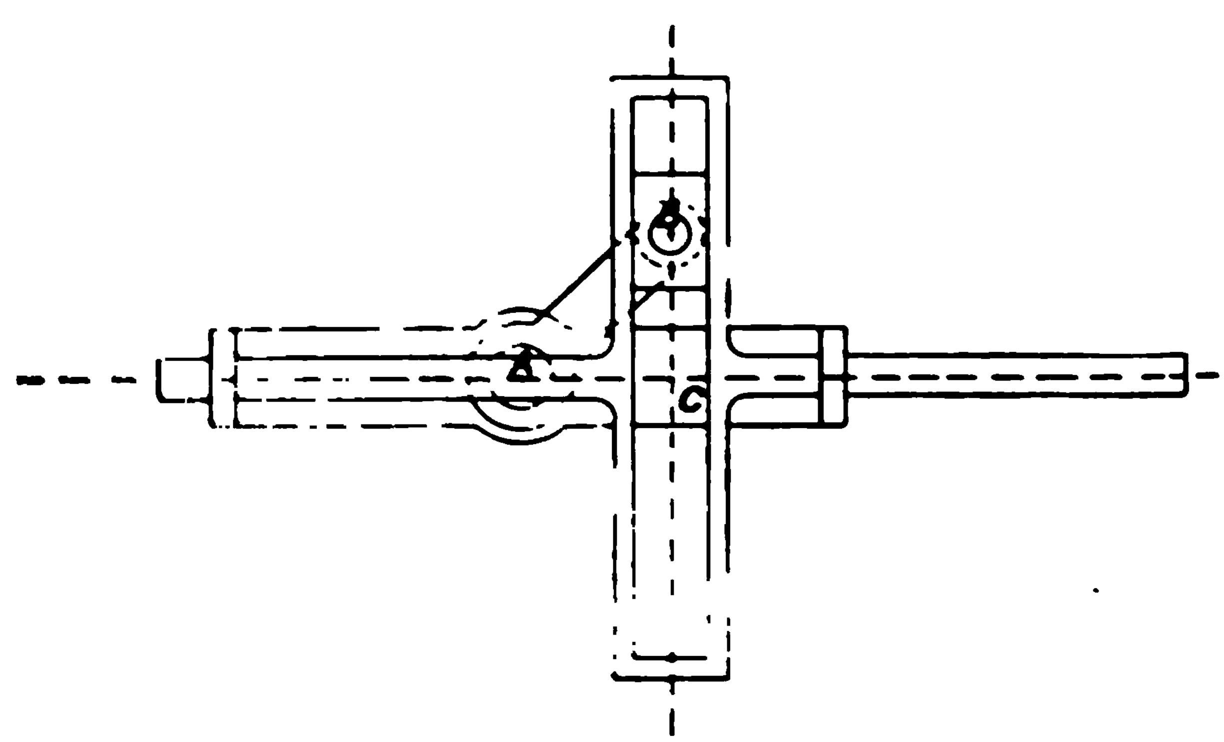



FIG. 96.

$n$ revolutions per minute or if $v$ is the velocity of the crank pin. The student must remember that if the weight of a body is $W$ lb. at London $W \div 32{\cdot}2$ is its mass in engineers' units, and mass multiplied by acceleration is force. It is obvious that the accele-

ting force at the ends of the stroke is of the same value as the ntrifugal force on the same mass if it existed on the crank pin.

**70.** There are many ways of proving the above statements. lere is the easiest, if I may assume that the calculus will in future e taught to elementary students. The fundamental idea of the lculus is that of a rate such as a velocity or an acceleration, nd even the beginner must have this idea.

If then $OA$, Fig. 95, is called $x$, the distance of the piston to the left of its mid stroke, $r$ the length of the crank, and if the angle $A_1OB$ is called $\theta$, then

$$x = r \cos \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the crank moves with the angular velocity $q$ radians per second ($2\pi n/60 = q$, if $n$ is in revolutions per minute, or $2\pi f = q$, if $f$ is what scientific people call the *frequency*, or the number of complete oscillations per second, or $2\pi/\tau = q$ if $\tau$ is the periodic time in seconds), then $\theta = qt$ if we count time $t$ in seconds from the position where $\theta = 0$.

$$\text{Hence } x = r \cos qt = r \cos \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{Velocity } v = \frac{dx}{dt} = -rq \sin qt = -rq \sin \theta \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{Acceleration } a = \frac{d^2x}{dt^2} = -rq^2 \cos qt = -rq^2 \cos \theta = -q^2x \, . \quad (4)$$

Evidently the velocity is greatest at mid stroke, and is $v$, the same as the linear velocity of the crank pin. The acceleration is greatest at the ends of the stroke, and is then equal to the centripetal acceleration of the crank pin, $rq^2$ or $4\pi^2f^2r$ or $4\pi^2n^2r/3600$ or $v^2/r$.

Notice that the acceleration is numerically equal to $q^2$ times the displacement $x$. This is the characteristic of simple harmonic motion (called S. H. M.), that the acceleration is proportional to the displacement. The subject, like that of periodic functions in general, is very fascinating, and its study is one of the most important for all engineers.

# CHAPTER VII.

## HOW THE VALVE ACTS.

**71.** FIG. 97 shows an **eccentric**. I want a student to understand at once that an eccentric disc and rod are simply a crank pin and connecting rod. *O* is a shaft to which the eccentric disc or sheave is keyed so that it rotates with the shaft. The eccentric strap *S E* in Fig. 97 consists of the two parts *S S* and *E E* bolted together so that they embrace the disc *D D* with no fear of their

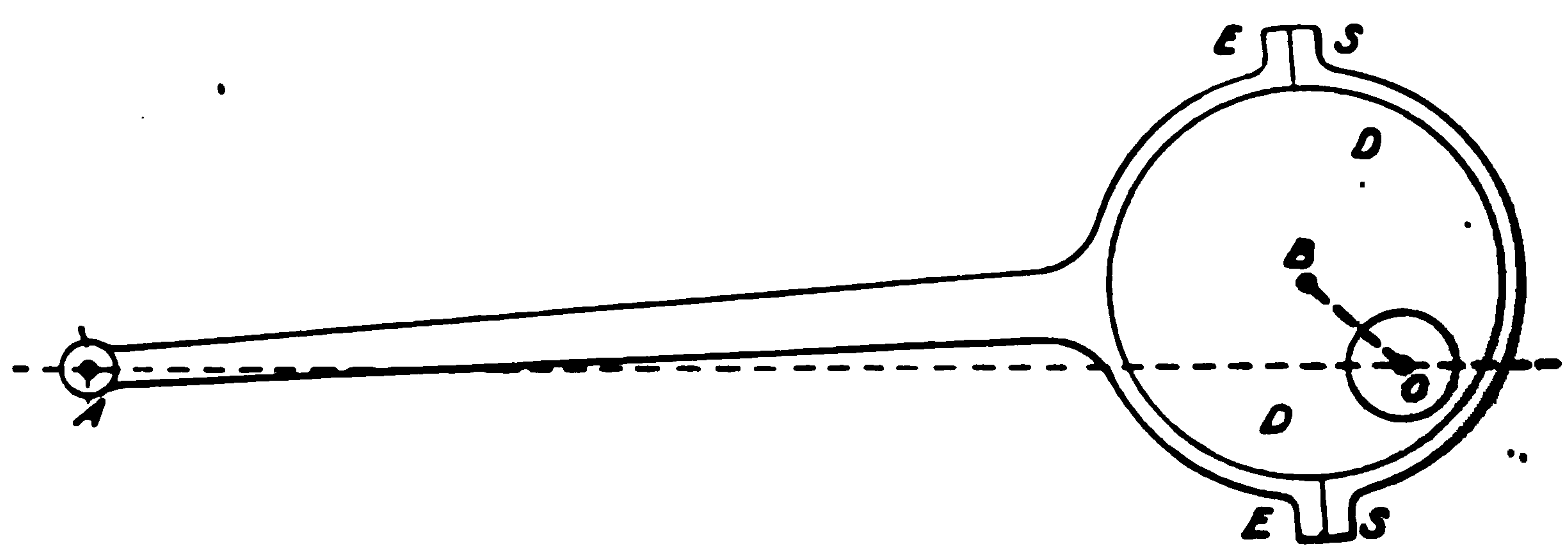



FIG. 97.

slipping off sideways. In fact *S S* and *E E* and the eccentric rod *E A* are all like one rigid piece working the pin *A*.

Now, it is evident that *B*, the centre of the eccentric disc, must move in a circular path round *O*, the centre of the shaft which is fixed, consequently *B* is exactly like the centre of a pin, a very large pin *D D*, and the eccentric straps and rod are simply a connecting rod. It is the great size of the pin *D D* which disguises this fact from a beginner.

Thus, if the points *A B O* of Fig. 98 are in the same positions as *A B O* of Fig. 97, or of *A B O* of Fig. 99, it is evident that their motions are the same. Or another way of putting it:—We are asked to work a pump or slider of any kind from the shaft *F F* Fig. 100, by means of a crank; how shall we do it?

1. We can cut the shaft as in *I*, inserting a crank-pin *D D*. But notice that as we have cut the shaft, we cannot transmit much power through it for other purposes.

2. Do as in *I*, but make the pin larger as in *II*, larger as in *III*,


FIG. 98.


FIG. 99.

larger still as in *IV*. But if this is made the pin at the end of a connecting rod we call the arrangement an eccentric disc and eccentric rod.

Hence we take Fig. 97 to be represented by Fig. 99. We call

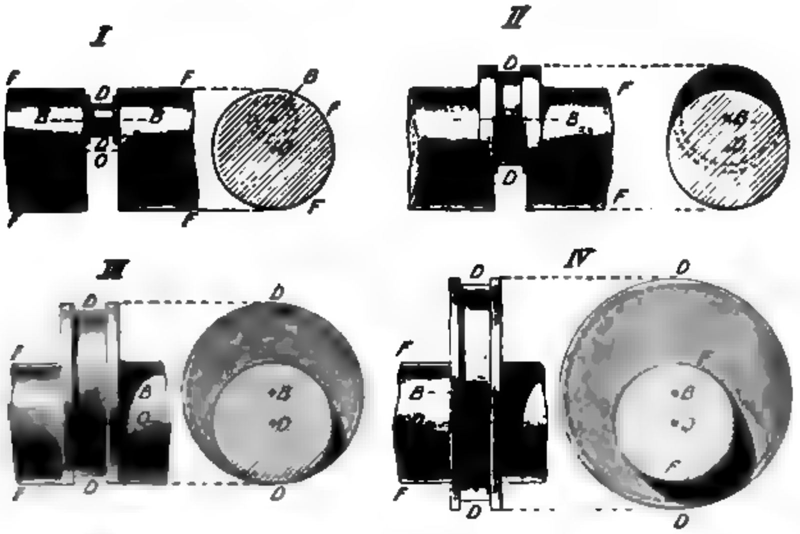



FIG. 100.

*O B* the eccentric crank. *A B* being the eccentric rod which is really a connecting rod. If we drop the perpendicular *B A′* from *B* upon *OA*, the direction of motion of the pin *A*, we may say that *A* is at the distance *O A′* to the left of its mid stroke. In fact, the position

of $A'$ between $A_1$ and $A_2$ represents the position of $A$ between the ends of its stroke.

**72.** EXERCISE 1. An eccentric crank is 2 inches long, when the angle $A\ O\ B$ is 130°, where is $A$? That is, say how far $A$ is to the right of its mid position.

*Answer.* 1·286″ to the right of its mid stroke.

2. In the last case, when $A_1\ O\ B$ is 0°, 45°, 90°, 150°, 220°, 295°, where is $A$? The answers are given in this table.

*Answers*—

| Angle . . . | 0° | 45° | 90° | 150° | 220° | 295° |
|---|---|---|---|---|---|---|
| $A_1A^1$ . . . | 0 | ·586″ | 2·00″ | 3·732″ | 3·532″ | 1·155″ |

**73.** If a teacher wishes to give a thorough understanding of the simplest valve motion to his students, he will have **a model made** something like what is shown in Fig. 101. Let no one think that he can easily arrange a better model. This is the outcome of many years' experience in the teaching of students. It is meant to enable students to understand clearly how *lap* and *advance* affect the distribution of steam. I have found that if a man gets a wrong notion about lap and advance at the very beginning of his studies, it is exceedingly difficult for him to get rid of it, and, although it seems absurd that a man should pick up a wrong notion about this simple matter, it will be found not only possible but probable.

Let then the student have a large model to work with, like what is shown in Fig. 101. He ought to be able to walk all round it, and to make the following measurements:—

1. There is a graduated circle which enables us to measure accurately the angle $R\ K\ H$ which the crank $K\ H$ makes with the line of centres of the engine. I always call this angle $\theta$, **the angle passed through from the near dead point.**

2. There is a graduated scale which enables us to measure the distance of the piston $C$ from the outer end of its stroke.

3. There is a graduated scale which enables us to measure the distance of the valve $W\ W$ from the *middle* of its stroke. I always call the distance of the valve to the right of the middle of its stroke, $x$.

4. There is a graduated circle $L$ which enables us to measure accurately the angle which the eccentric crank is ahead of the main


FIG. 101

Model for measuring the effects of advance and outside and inside lap and travel on the distribution of Steam. *Laps* altered by the screws I and O. Advance by the nut N. Travel by G

crank. For I am enabled by my model to vary this angle, ε could unkey my eccentric disc and key it again in a new pc I do not use keys, however, but fasten it in any position I by means of a bolt $N$ and slot $S$ in (3). When I do chan position of the eccentric disc, I like to know how much ahead main crank it is.

*Ahead*, what do I mean by *ahead*? I mean ahead

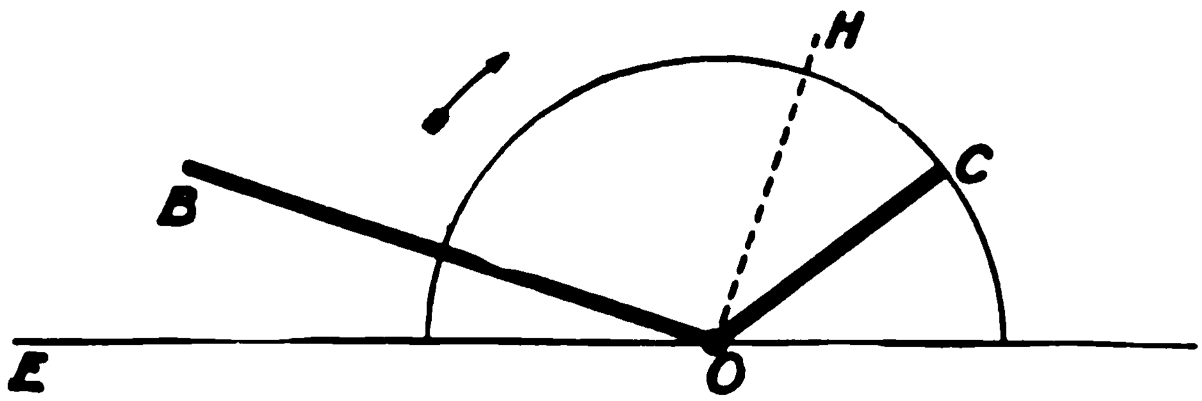



Fig. 102.

arrow, Fig. 102, shows the direction of motion. In the p of things in Fig. 102, $O\,B$ is the main crank, $O\,C$ is the ec crank, and the angle $B\,O\,C$ is what I like to measure. The $B\,O\,C$ is always greater than 90°, and the amount by wh exceeds 90° is called **The Advance.** This is $H\,O\,C$.

In my model it would be difficult to take off one eccentri and put on another; I should like to do something like this bec is important to change the eccentricity of the eccentric, that

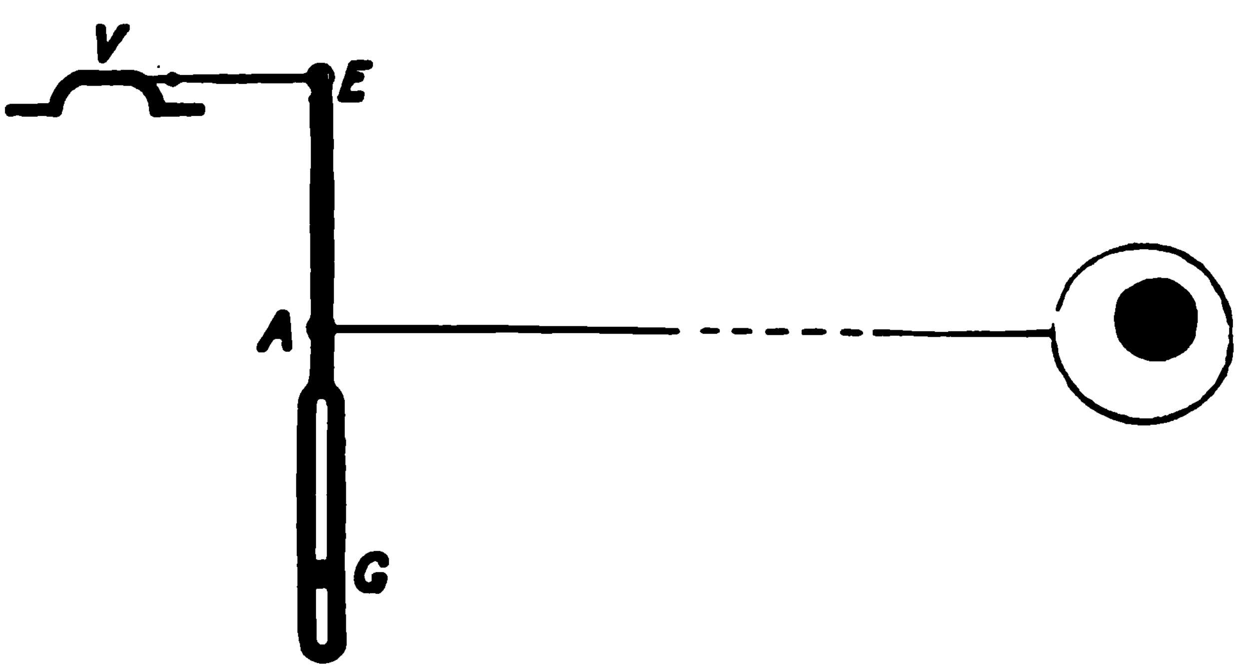



Fig. 103.

length of the eccentric crank. Now, on my model I do what very nearly to the same thing. I do not let $A$, of Fig. 103, wc valve directly. $A$ is a pin on the lever $GE$, the pin $G$ being fi the fulcrum, and I am able to change the position of this fulcr raise it or lower it, without altering the positions of $A$ or $E$.

by changing $G$ we cause the motion of $E$ to be greater or less in amount, but it is always a magnification of $A$'s motion. In fact, by changing $G$, we alter the half travel of the valve just in the same way as if we altered the eccentricity of the eccentric. Therefore on my model I have the power of altering the half travel and the advance.

It might be said that since the eccentric is really a crank, we ought to use a crank for it on the model, and then it would be easy to alter its length and so get a different travel of valve without using a lever. But in the first place, a student would prefer to see on the model an actual eccentric; secondly, the lever is a very good way of altering travel; thirdly, using the lever enables us to put the valve *above* the cylinder and the motions of the parts are all visible to a class of students.

Now when the model is being used let the student imagine that

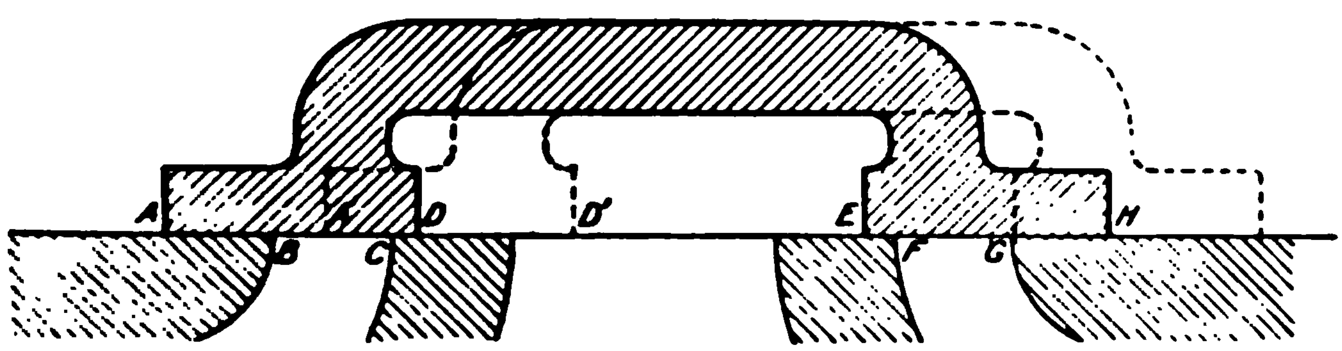



FIG. 104.

the half travel of the valve is really equal to the eccentricity of an eccentric working the valve directly without a lever as the valve is worked in Fig. 15.

I am in the habit of showing the valve motion *above* the piston motion, as in Fig. 106, and the student must get to imagine the main crank $C$ and the eccentric crank $E^1$ to be revolving at the same rate. He ought to make use of such a diagram as Fig. 107, where the eccentric crank $O\,C$ is shown in its proper angular position ahead of $O\,B$ the main crank.

Let the valve be drawn in its middle position as in Fig. 104. The distance $A\,B$ or $G\,H$ is called the outside lap; the distance $C\,D$ or $E\,F$ is called the inside lap. The outside lap is often called **the lap.** In my model I can at once alter the amount of the outside by means of the screw marked $O$, or of the inside lap by the screw $I$, or reduce them to nothing.

The most important thing for a beginner to understand is that **when we alter the advance and the lap and the half travel, we alter the distribution of steam in an engine cylinder.**

I have heard of very few real engines in which an attempt has been made to vary the lap when the engine is running. Here we only indicate such a variation so that students may observe the effect of more or less lap on a mere model.

*Exercises with the Model.*—(1) Let a student adjust so as to have no outside or inside lap. Let the eccentric crank be at right angles to the main crank. This is what we call the *normal valve* with no advance. On working the model it will be found that steam is admitted and cut off at the ends of the stroke so that there is no expansion.

(2) It is now worth while to see what is the effect of trying some lap and no advance, or no lap but some advance, and I leave this to the student himself. He ought to draw the possible indicator diagrams, and this is an excellent exercise even for the most advanced students who know the effects of speed.

(3) Give lap to the valve. Advance the eccentric beyond the normal position; this additional angle, which is the excess beyond 90° by

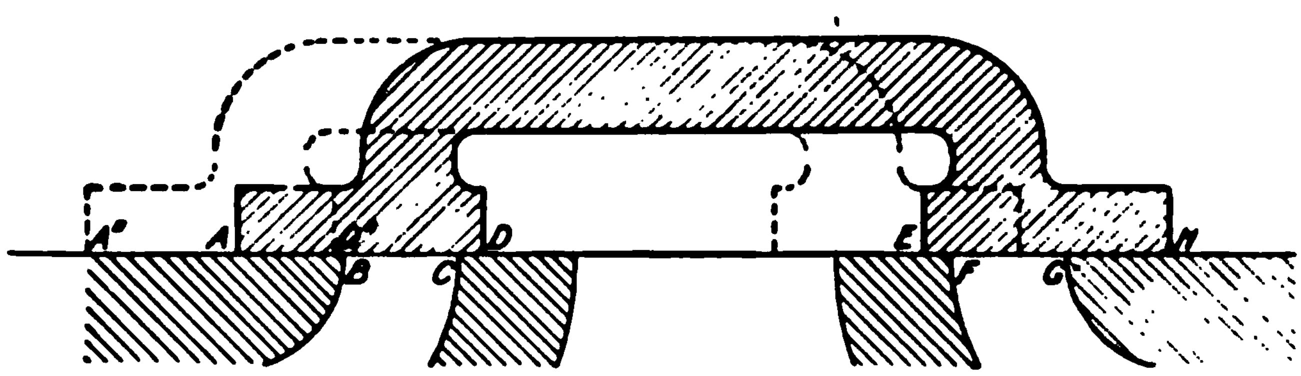


FIG. 105.

which the eccentric is ahead of the main crank, is called *the advance.* It will now be found that we effect our purpose of cutting off before the end of the stroke. After a student has watched the effect he has no difficulty in discovering the reason.

**74.** In Fig. 104 the full lines show the valve in its mid position. I shall speak only of what occurs to the left-hand port $B\ C$. The dotted lines show the valve displaced to the *right* of its mid position by a distance which I call $x$. That is, $A\ A^1$ is $x$. Now the opening of the port to steam is $B\ A^1$, or

$$\text{Opening to steam} = x - \text{outside lap.} \quad . \quad . \quad .$$

If, therefore, for any position of the piston or crank we want to know what is the opening of the port to steam, our only difficulty is in finding $x$.

Again, look at Fig. 105, where the dotted lines show the valve displaced to the *left* of its mid position. I often call this displacement $D\ D'$ by the name $y$, although it is merely a negative $x$. The

ening of the port to exhaust is $CD'$, and $CD$ being the inside ), we see that

Opening to exhaust $= y -$ inside lap . . . . (2).

The problem to be solved is :—For a given position of the piston, hat is the opening of the port to steam or exhaust ?

If we are told the position of the piston, it is easy to find the sition of the main crank (that is, the angle $\theta$, which it makes with e dead point), and hence our problem really is, "When we know here the main crank is, where is the valve ?"

That is, if $\theta$ is given, what is $x$? We have a very easy way of nswering this question, and it must be very clearly understood that his one simple answer is really the key to all the problems which ome before us. If we know the distance of the valve to the ight of its mid stroke, we need only subtract the lap and we at nce know how much opening there is to steam; or if we know the

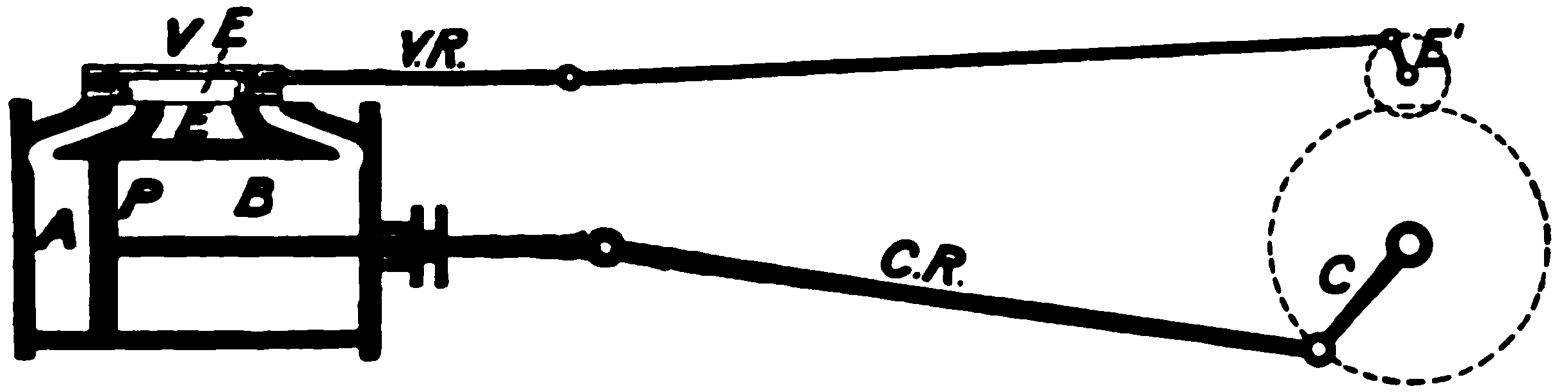



FIG. 106.

listance of the valve to the left of mid stroke, we subtract the inside ap and we see the opening to exhaust.

Think of the valve $V$ being worked directly as in Fig. 106 by the ccentric crank $E^1$ on a shaft on the same level as the valve, this haft revolving exactly at the same rate as the main crank shaft so hat $C$ and $E^1$ are at the same angle with one another always; for ny position of $C$ the position of $E^1$ may be drawn, and the displacenent of the valve is easily found.

Notice that $OB$ (Fig. 107), the main crank position, being given, we draw $OC$ (to represent by its length the half travel of the valve r the eccentricity of the eccentric), and we take care that $OC$ shall e ahead of $OB$ by an angle equal to $90° +$ advance. The student an have no difficulty in seeing that the valve is the distance $OC'$ o the right of its mid stroke, and this is $x$ which we want to know. **Here then is a rule.** We may carry it out as in Fig. 107. $OB$ s given, that is, the angle $AOB$ is given; make $BOD = 90°$, make $DOC$ the advance, let $OC$ be the half travel, drop the perpendicular, nd $OC^1$ is the answer.

**75.** But the rule is thought to be a little too clums
events there is a much simpler rule **invented by Zeuner**
$A\,O\,G$ (Fig. 108) horizontally to represent the dead point
or centre lin
engine. Draw
right angles t
Make the ang
equal to the
and produce $E$
Set off $O\,E$ =
the half trav
$O\,E$ and $O\,H$
meters describ
We now have
gram which

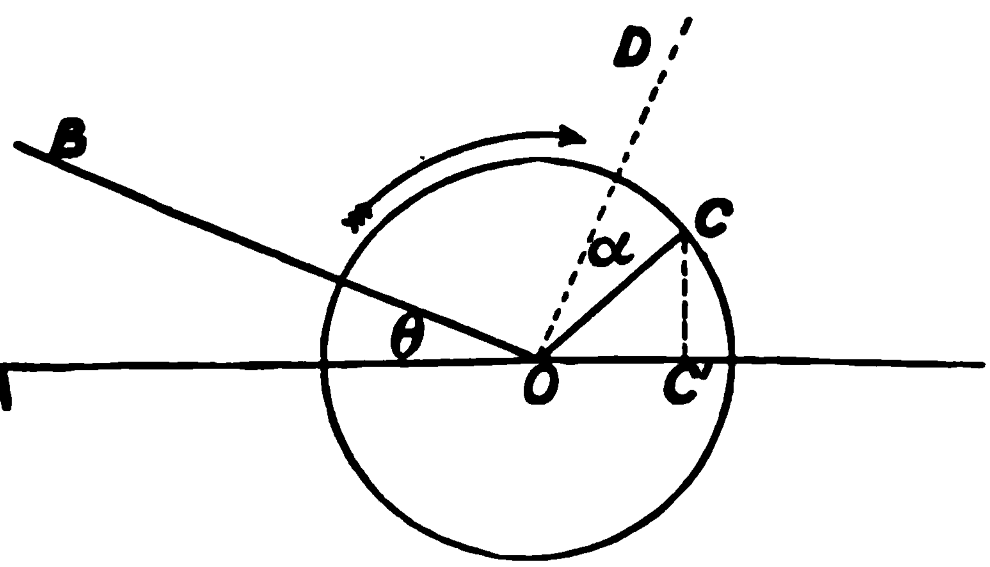



Fig. 107.

what we want with much less trouble than before. It will b
that if we draw the direction $O\,B$ of the main crank from
distance $O\,B_1$, is the very answer wanted by us; the distance
the distance of the valve to the right of the middle of it
when the main crank is in the position $O\,B$. Thus in F
I have shown the main crank in a number of positions.

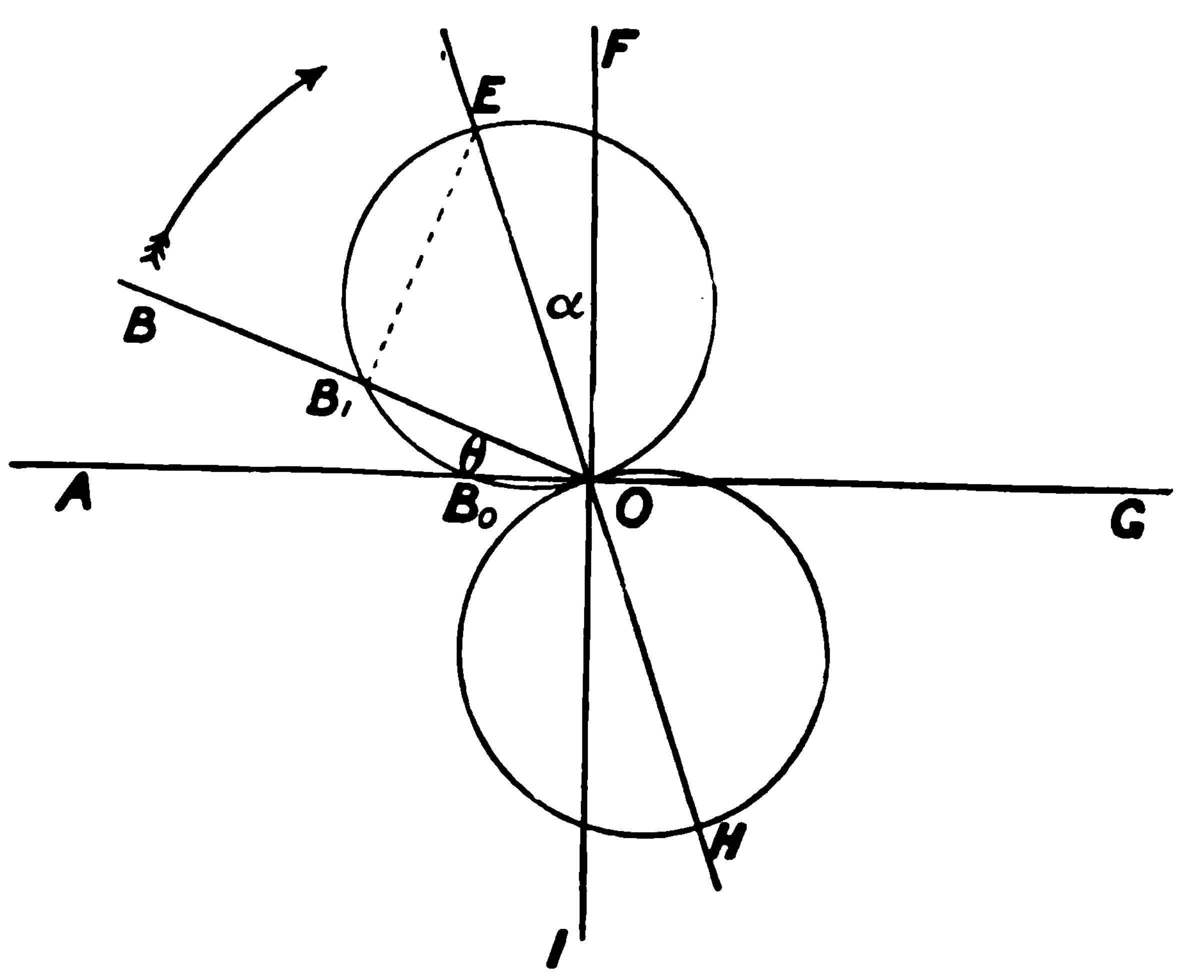



Fig. 108.

distances $O\,B$ represent in every case the distance of the va
the right of its middle position. The distances $O\,B^1$ are dis
to the left of mid position.

if students see how the rule is arrived at they ought to rcises by the method of Fig. 107, and also by this method,

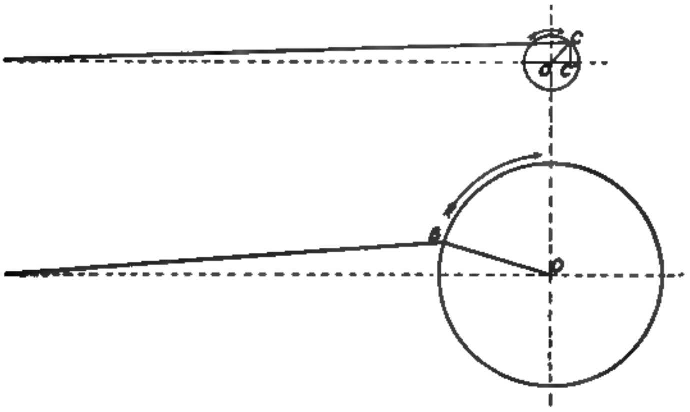


Fig. 109.

ch case they ought to test the accuracy of their answer odel.

if Fig. 111 is a Zeuner diagram, the angle $F O E$ being the the distances $O E$ and $O H$ being each the half travel, $O M$ e dead point With radius al to the out- lescribe the arc With radius l to the inside cribe the arc Now note that these arcs will subtraction (of $x$, of inside lap without giving trouble. Thus is any position ain crank ; $O B$ as $O S$ is the shows at a he opening of

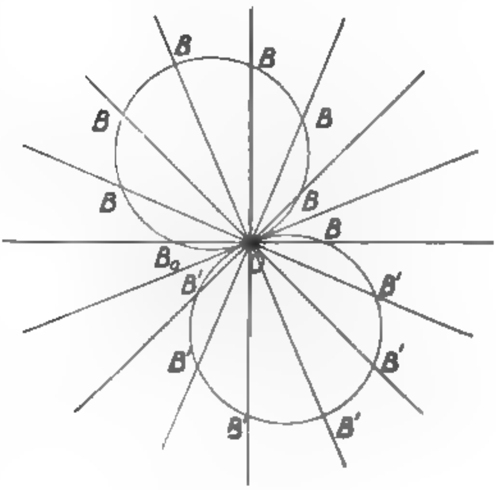



Fig. 110.

to steam. Again, if $O B^1$ is the position of the main $B^1$ is $y$, the distance of the valve to the left of its new

position; and as $O V$ is the inside lap, the distance $V B^1$ shows glance the opening of the port to exhaust.

**76.** Example 1. Find the positions of the main crank wher valve is just opening the port to steam (we call this the admissi when just closing to steam (we call this the cut off); when opening to exhaust (we call this release); when just closin exhaust (or when compression is beginning).

*Answers.* Produce $O A$, $O K$, $O R$, and $O C$, and these are the tions required.

Example 2. Where is the main crank when the port is most to steam? *Answer.* In the position $O E$.

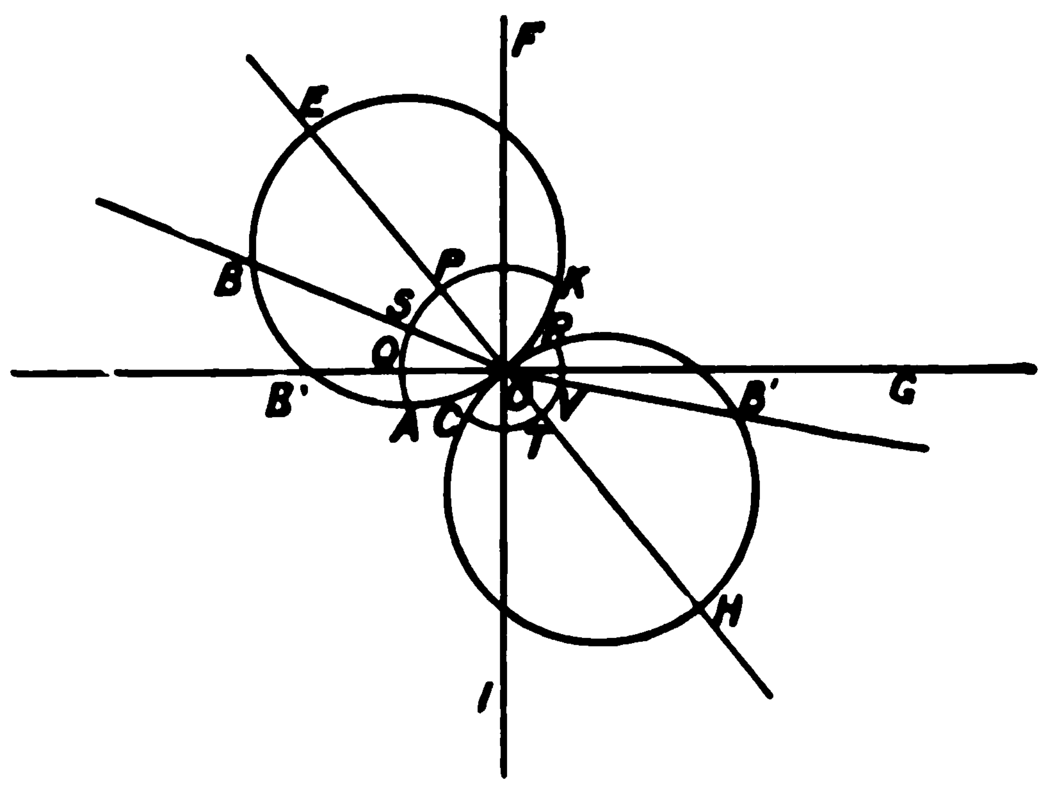



FIG. 111.

Example 3. Where is the main crank when the port is open to exhaust? *Answer.* In the position $O H$.

Example 4. At the beginning of the stroke what is the ope of the steam port? *Answer.* $B_0 Q$. Observe that $B_0 Q$ is called **lead of the valve.**

**77. Proof of our Graphical Rule.** If the student has di Fig. 107 and Fig. 108 for the same half travel, advance and $\theta$, he find that the triangle $C O C^1$ (Fig. 107) is exactly the same as triangle $E O B_1$ of Fig. 108, and of course, if this is so, the rule n no further proof.

$E O$ is the diameter of a circle, and as the angle in a semicir always a right angle, the angle $E B_1 O$ is a right angle. Also we r $O E$ the same as $O C$, or the half travel. Now, in Fig. 107

$$\theta + 90^\circ + a + C O C' = 180^\circ$$

and in Fig. 108, $\theta + a + E O B_1 = 90^\circ$

It is therefore obvious that $E O B_1 = C O C^1$. Hence we have two right-angled triangles, whose hypotenuses are equal, and one other angle in each, therefore the triangles are the same, and $O C^1 = O B_1$. It is easy in the same way to see that the intercepts on the lower Zeuner circle represent displacements of the valve to the left of its mid position.

All that I have stated here might have been given in a few words, and indeed the whole thing is exceedingly simple, but I advise a student to work exercises like the above, and make very sure that he understands the rule and its proof.

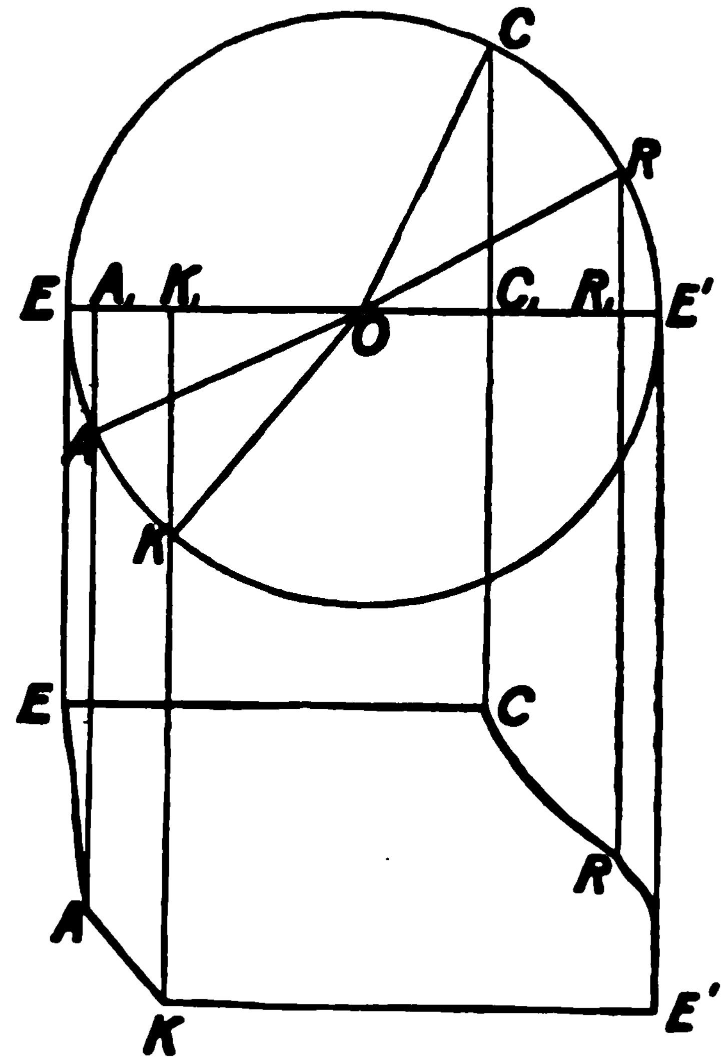


FIG. 112.

**78.** Having the positions of the main crank of the engine when the four events take place, **to draw the hypothetical indicator diagram.** Neglecting the angularity of the connecting rod, it is obvious that this is the answer;—With $O$ as centre describe any convenient circle, $E\,C\,R\,E^1\,K\,A$, Fig. 112, and project from the points, $A, E, C$, &c., in a direction at right angles to $E\,O\,E^1$, the line of centres. Evidently $A_1, C_1, R_1, K_1$ show where the piston is relatively to the ends of the stroke $E$ and $E^1$ when the four events take place. Draw $K\,E^1$ and $E\,C$ parallel to $E'E_1$ to represent the admission and back pressures to any convenient scale of measurement. It is not necessary to indicate the zero line of pressure. Draw the expansion curve $C\,R$, the compression or cushioning curve $K\,A$, the release curve $R\,E'$ and the admission $A\,E$. It is only necessary to draw these reasonably like what such curves usually are, but it is convenient to have sharp corners to remind us that we are studying the times of occurrence of four important events. When a student has inked-in such a diagram as the above, in red, he may, if he thinks it worth while, round the corners, to

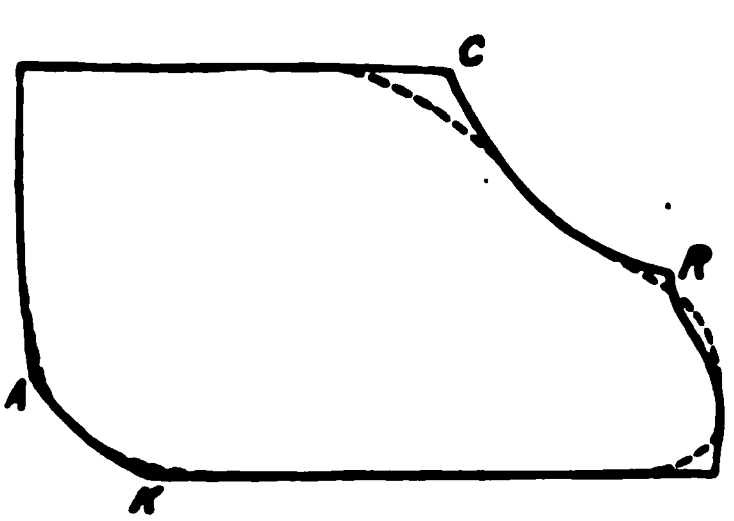


FIG. 113.

show that the events will really occur gradually, with a certain amount of wire drawing, as shown in Fig. 113 in the dotted lines. I advise the student not to draw these dotted lines, however. It is well to know that the inside lap is sometimes a negative quantity in quick-moving engines, and the points $C$ and $R$, Fig. 41, now find themselves on the $O\,B\,E$ circle.

If a student neglects the angularity of the connecting rod, and if there is the same lap for both ends of the cylinder, having drawn the diagram for one end, he has the diagram for both ends; but if he wishes to take account of the angularity of the connecting rod, let him project the points in the figure to the diameter, not by a set square, but by the template of Fig. 93. Thus he will

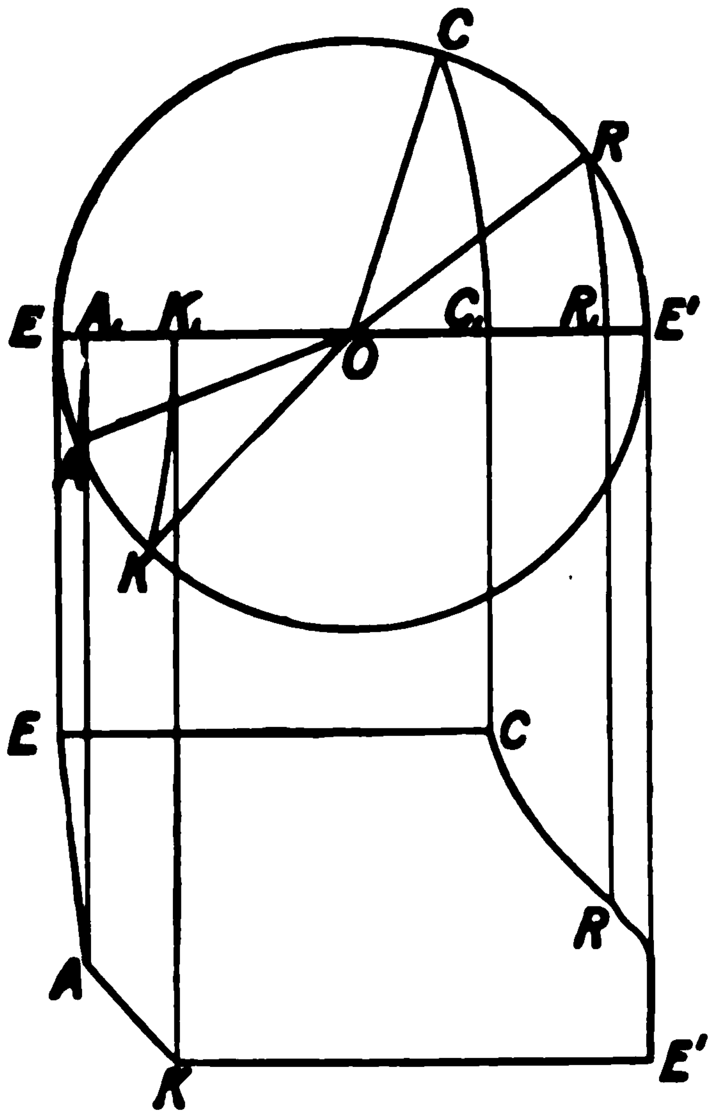



FIG 114.

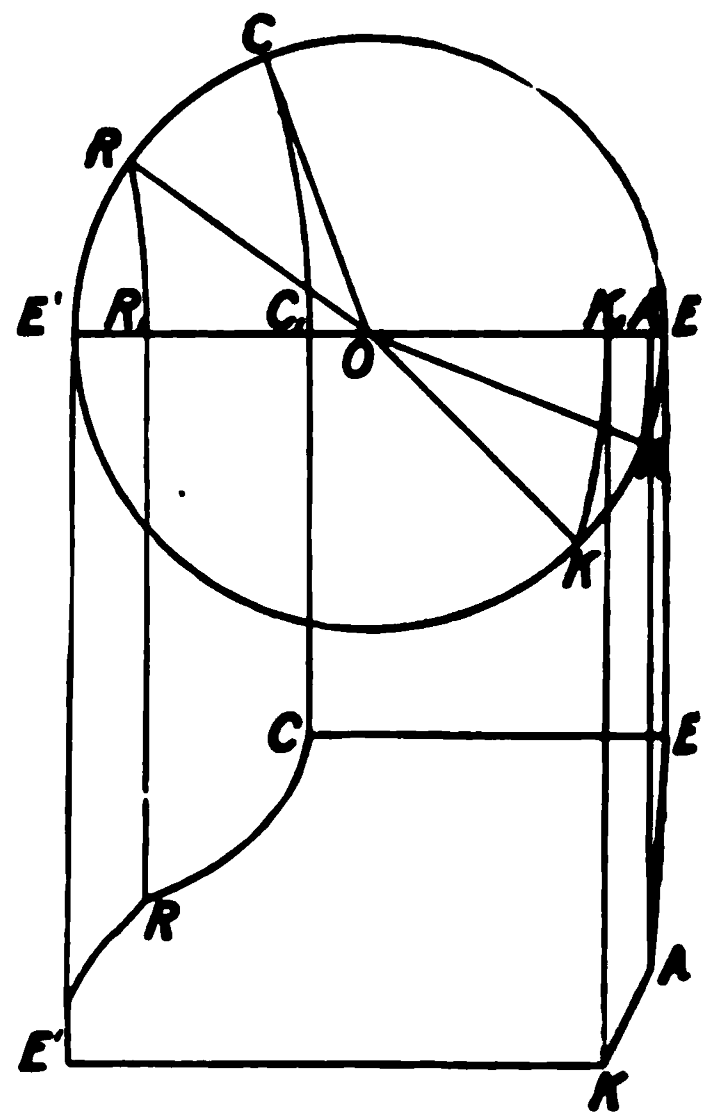



FIG. 115.

obtain the positions of the piston, $A_1$, $C_1$, $R_1$ and $K_1$ (Fig. 114), when the four events occur. If he projects from these points by lines at right angles to $E\,O\,E^1$, and proceeds as before, he will get the more probable indicator diagram. For the other side of the cylinder, instead of taking a short cut to the answer, which ought at once to suggest itself to the student, let him draw the actual positions of the crank when the four events take place; that is, let him set $O\,A_1$, $O\,C_1$, $O\,R$ and $O\,K$ of Fig. 114 forward 180°. Thus he will have Fig. 115; projecting on the line of centres $E\,O\,E^1$ with a template as before, and then projecting with straight lines at right angles to $E\,O\,E^1$, we may draw the diagram $E^1\,C\,R\,E\,K$ of Fig. 115. The student who carries this out must meditate on the fact that in the *in* stroke there is a longer admission than in the *out* stroke. If we want to remedy this, we can do so by diminishing the lap on the out stroke side, and this is often done in marine engines where the weight of the piston and other parts must be lifted in the *out* or *up* stroke.

Instead of drawing two figures like 114 and 115, let the student who has drawn Fig. 114 merely produce the lines $O A_1$, $O C_1$, &c., and draw Fig. 115 on the top of Fig. 114. This will probably bring home to him better the effect of angularity of the connecting rod in altering the diagram. He will have both his diagrams on one sheet of paper just in the positions in which we usually find them when taking diagrams as shown in Fig. 78.

**79. Exercise.** In each of the following cases find the positions of the main crank at admission, cut off, release, and compression. Find also the *lead*. The outside lap is 0·52 inches, inside lap 0·15 inches.

| Half travel in inches. | Advance in degrees. | *Answers in degrees.* Positions of Crank. Admission. | Cut off. | Release. | Compression. | Lead. in inches. |
|---|---|---|---|---|---|---|
| 2·10 | 25 | 10·7° | 39·3° | 20·9° | 29·1° | 0·37 |
| 1·68 | 39 | 21·0° | 57·0° | 33·9° | 44·1° | 0·54 |
| 1·42 | 51 | 29·5° | 72·5° | 44·9° | 57·1° | 0·58 |
| 1·28 | 67 | 43·0° | 91·0° | 60·3° | 73·7° | 0·66 |
| | | Before beginning of stroke. | Before end of stroke. | Before end of stroke. | Before beginning of stroke. | |

For each of the above cases let the hypothetical indicator diagram be drawn by beginners. Advanced students will draw diagrams for the two sides of the piston on the assumption that the connecting rod is five times the length of the crank.

## 80. *Three Important Exercises for Beginners.*

In each of the following cases find the positions of the main crank at admission, cut off, release, and compression. Draw the hypothetical indicator diagrams. Each of them shows the sort of change that occurs when we shift one of the usual gears employed to work slide valves. In each case the lap may be taken to be 0·8″ and the inside lap 0·3″.

I. A Stephenson or Allan link motion, open rods.

II. A Stephenson or Allan link motion with crossed rods.

III. The Gooch or Stewart Finck motion or any of the numerous forms of radial valve gear.

I.

| | | | | |
|---|---|---|---|---|
| Half travel in inches . . . | 2·50 | 2·10 | 1·70 | 1·52 |
| Advance in degrees . . . | 30 | 40 | 50·9 | 69·2 |

II.

| | | | | |
|---|---|---|---|---|
| Half travel in inches . . . | 2·50 | 2·00 | 1·55 | 1·20 |
| Advance in degrees . . . | 30 | 36·3 | 46·0 | 63·2 |

III.

| | | | | |
|---|---|---|---|---|
| Half travel in inches . . . | 2·50 | 2·05 | 1·65 | 1·35 |
| Advance in degrees . . . | 30 | 38 | 49·3 | 66 6 |

The advanced student will draw these indicator diagrams for both sides of the piston, assuming a connecting rod five times the length of the crank. The lesson to be learnt by the elementary student is, however, the more interesting. This is one of the cases in which actual drawing by a student himself is of the greatest importance. If he uses four different colours of ink for each set, and meditates on his results, he will get exact notions of what occurs when we shift from full gear, giving more and more expansion in each of the above cases. He will note that in I. the *lead* increases with more expansion; in II. it diminishes, whereas in all such gears as III. the lead is constant at all grades of expansion. It would be easy for me to give these interesting diagrams, but a student can draw them all in much less than an hour.

I shall now proceed to show how by means of the various gears we can produce the above-mentioned changes in the distribution of steam.

A more advanced treatment of the subject will be found in Chap. XXVIII.

**81.** It is worth while to mention that many small steam engines which we wish easily to reverse (such as steam starting engines, &c.) have no lap and only one eccentric, with no advance. A simple slide valve converts the steam space into an exhaust space, and *vice versâ*.

## CHAPTER VIII.

### VALVE GEARS.

**82.** We now know that if $O\,C$ (Fig. 116) represents the main crank of the engine, and if $O\,A$ is the eccentric crank; knowing the outside and inside lap of the valve, if we also know the angle $D\,O\,A$ (the advance) and $OA$ (the half travel), we know the probable indicator diagram. Now imagine that we have a means of suddenly increasing the advance of the eccentric to $D\,O\,A^1$ and of making the half travel less; let $O\,A^1$ represent it: it is evident that we shall alter the nature of the distribution of steam. Imagine that we have a method of suddenly altering the position of the eccentric $O\,A$ to the position $O\,B$. A student must see that, by doing this we have completely altered the motion of the valve and made it suit a reversed direction of motion of the engine. But there is no great satisfaction in mere book study of this; we must have before us an actual model of an engine, such as my own, in which I am able suddenly to slacken the fastening of the eccentric disc to the shaft and let it change from the $O\,A$ to the $O\,B$ position, and it at once becomes evident that we have reversed the engine. There is no great satisfaction in putting this subject before the student without some kind of model. A bright lad takes in some sort of idea of what occurs, and, mainly by

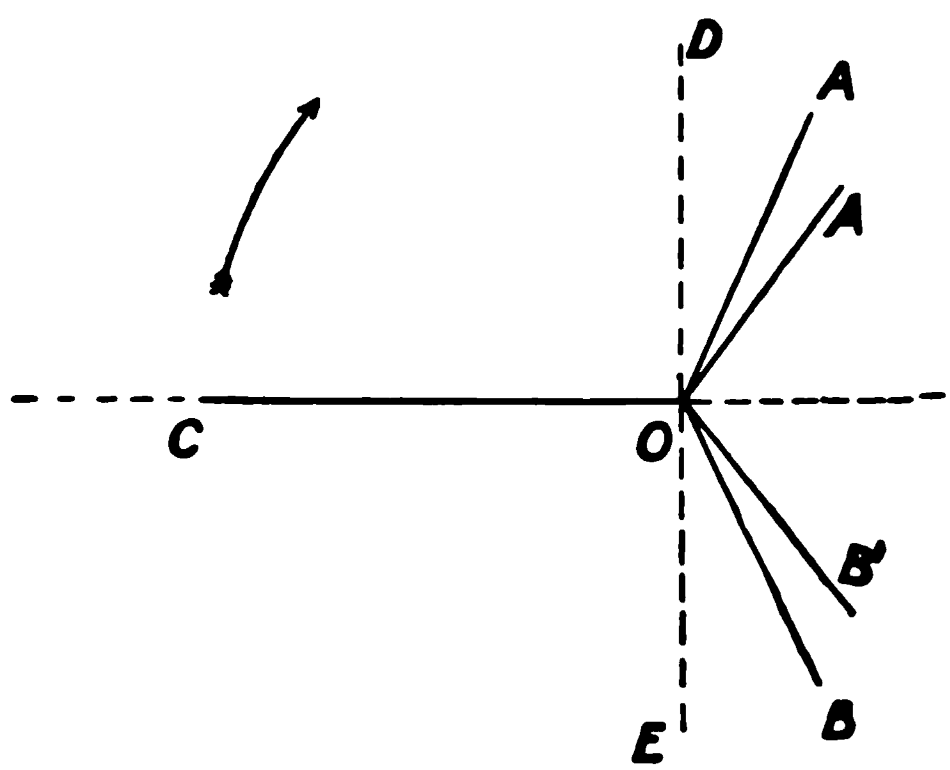


Fig. 116.

faith in his books, believes he sees the whole thing clearly. But what a pity it is that there should not be a model to show one's class exactly how it is that if the valve was going successively through the positions 6, 7, 8, 9, &c., Fig. 20; the shifting of the eccentric alters it so that it is going through the positions 6, 5, 4, 3, &c.

Now this slackening and refastening of the eccentric is a plan that has often been carried out, but we effect exactly the same object in the following way.

**83.** There are six kinds of **link motion** and perhaps ten good kinds of **radial valve gear**. They are employed in working the common slide valve, and they enable us to alter the advance and half travel of the valve, letting the engine go in either direction.

There are two eccentrics *A* and *B*, one in the position *O A* of Fig. 116 (see also Figs. 284, &c.), the other in the position *O B*, being

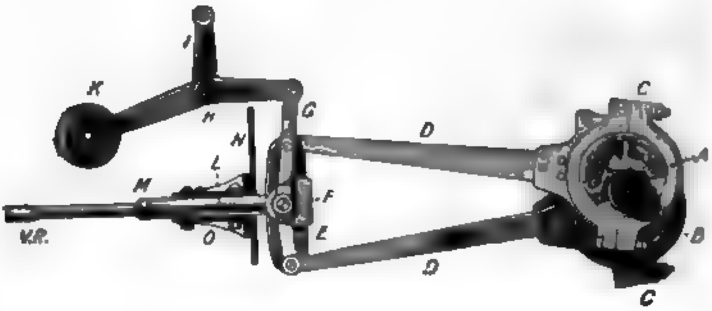



FIG 117.—STEPHENSON ARRANGED FOR LOCOMOTIVE, LINK SHIFTS, BLOCK NOT; LINK CONCAVE TO SHAFT.

symmetrically placed relatively to the main crank. Their rods end in pins on a slotted link, which is hung from the reversing link *G*. When *G* is lowered to the position known as full forward gear the eccentric *A* alone works the valve because a block on the end of the valve rod *V R* keeps in the slot. When *G* is raised high, the eccentric *B* alone works the valve. Fig. 117 shows the link in what is called mid gear, and it is evident that by lifting and lowering the link we have many conditions of working that are intermediate between full forward and full back gear. It is quite easy to show, but it is a little beyond the scope of this very elementary treatment of the subject, that intermediate positions of the gear mean a less travel and a greater advance than the two extreme positions.

It is only when the main crank is in its outermost position, most remote from the cylinder, that we look at the crossing or non crossing of the rods when we want a name for the valve motion, because any one can easily see that what

we call open eccentric rods will appear crossed in certain positions of the engine.

**Gooch's Link**, shown in Fig. 119, is not lifted or lowered, it merely swings nearly horizontally hanging from *B*. A block *E* is lifted or lowered in the slot of the link, and this block is at the end of a radius bar *D* which gives

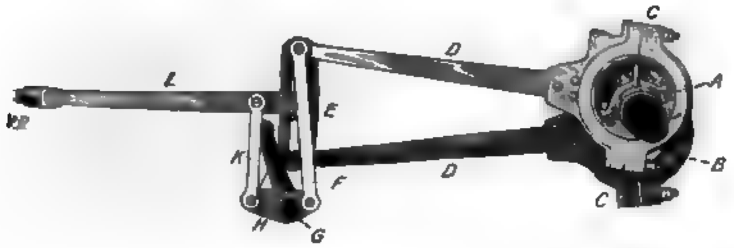



Fig. 118.—Allan arranged for Locomotive: Link Shifts, Block Shifts, Link Straight.
Balanced weigh shaft G causes link to rise and block to fall or *vice versâ*.

the valve its motion. In the **Allan Link** motion, Fig. 118, the straight link *E* is lowered, and the radius bar block is lifted, or the link is lifted and the radius bar block is lowered. We may put it in this way—

To change the gear—

Stephenson, link lowered or raised, *block not*.

Gooch, block raised or lowered, *link not*.

Allan, link lowered when block raised, or link raised when block lowered

In any of these we may have either open or crossed eccentric rods, so that there are really six varieties of link motion. The Stephenson motion with open rods is much more generally in use than any of the others.

Links are either of the "slotted" or "solid bar" or "double bar" forms. In the second and third forms the ends of the eccentric rods may be in the arc of

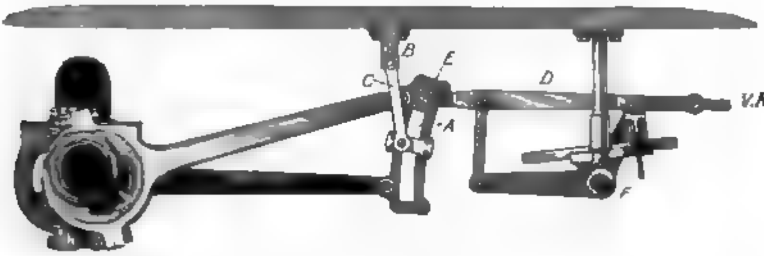



Fig. 119.—Gooch arranged for Locomotive Block Shifts, Link Not, Link Convex to Shaft
Balanced weigh shaft F causes D E to rise or fall.

motion of the centre of the block. Only in the double-bar form, now most general in large marine engines, can the end of either eccentric rod really coincide with the block. But there is no advantage in such coincidence.

**84.** To Shift the Gear.—In locomotives it is very usual to have a lever like that of Fig. 63 on the footplate of the engine. By means of this both engines are shifted in gear at the same time. See Fig. 62.

Note that as there are two cylinders on a locomotive, and therefore two link motions and therefore four eccentrics, the whole gear looks very much more difficult to understand than it really is.

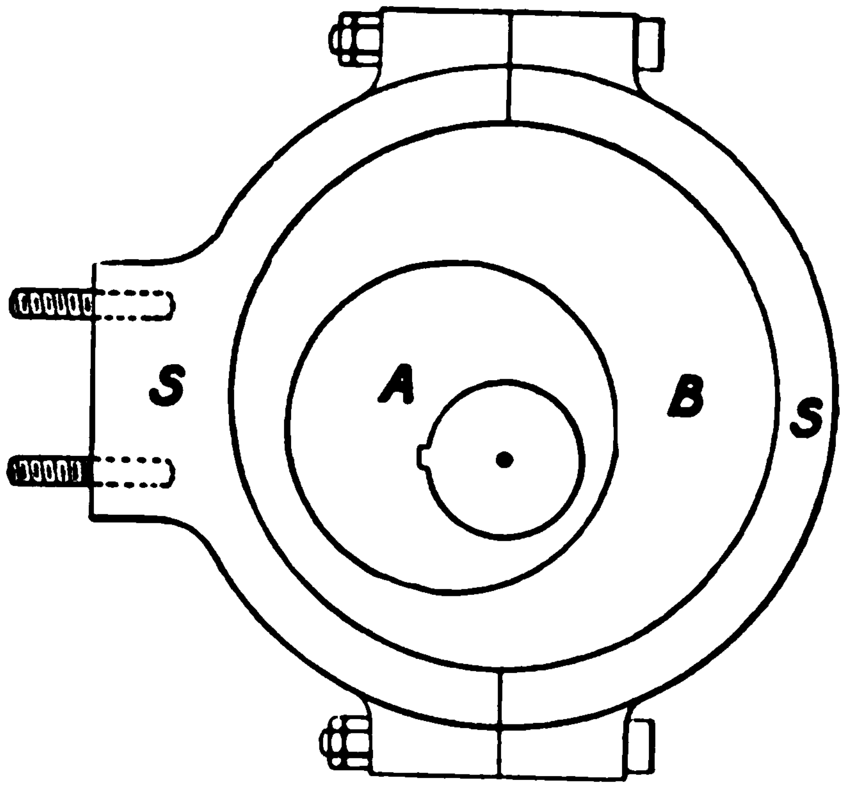



FIG. 120.—ARMINGTON AND SIMS ECCENTRIC.

The Armington and Sims eccentric disc is really in two parts, A and B. The governor causes a relative motion between them, so that sometimes there is a large throw and about 30° advance, and sometimes a smaller throw and a greater advance.

In a stationary engine with link motion, if the engine is small, a reversing lever like that of Fig. 63 is used. But if the engine is larger, so that the gear is more massive, even if its weight is balanced, it is usual to employ a capstan wheel and worm gearing if the gear is to be shifted by hand, and in the largest marine engines a special engine is employed to give power to shift the valve gear quickly. This is one of the things which make a marine engine complicated looking. Imagine 20,000 horse-power to be given out

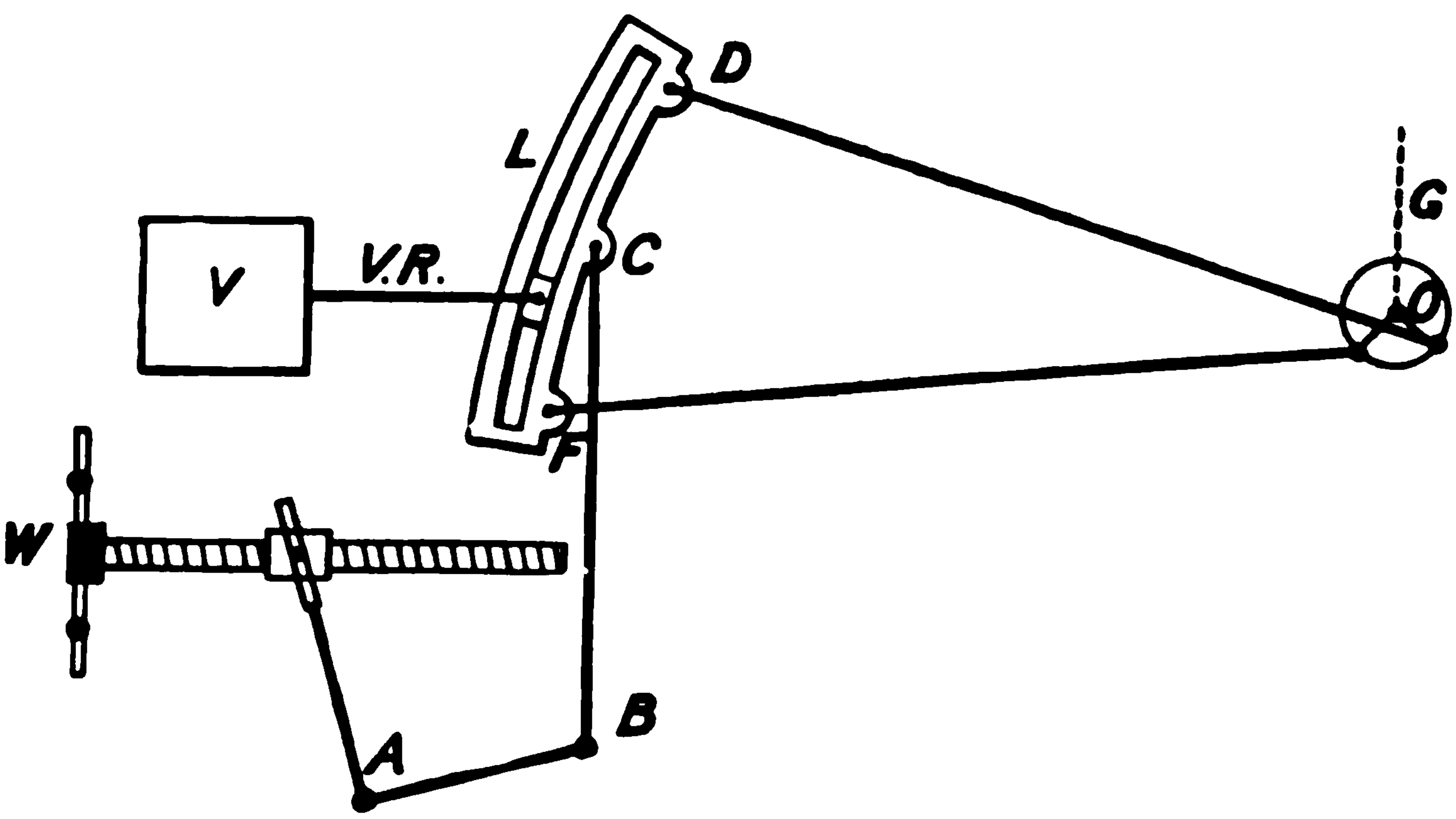



FIG. 121

Illustrating how we shift the links in small engines. Where the weigh shaft A turns it moves all the links.

by two triple cylinder engines which have six link motions with their twelve eccentrics—all to be shifted simultaneously.

Fig. 121 shows the capstan-wheel *W*, turning a screw on which

a nut moves so that the weigh shaft $A$ turns on its axis through an angle, which enables arms $A\ B$ and $B\ C$ to shift each link of two or three cylinder engines at the same time. This is a common reversing motion for small compound engines.

For engines of about 2,000 horse-power and upwards, we sometimes have a hand wheel or lever which moves a slide valve on a small steam cylinder; this admits steam to one side or the other of a piston, whose motion like that of the above nut $W$, causes the weigh shaft to turn.

In Brown's gear there is a "cataract" piston on the auxiliary steam piston rod to prevent too rapid motion. The motion causes the auxiliary engine valve to come back to its shut position. There is always an independent hand gear attached to the steam gear for use in case of accident. Brown uses also a simple governor arrangement which brings all the links to mid gear if the engine exceeds a certain critical speed.

In some modern locomotives the pressure of steam or the pressure of air (in case the Westinghouse brake is used on the train) is used to assist in shifting the gear.

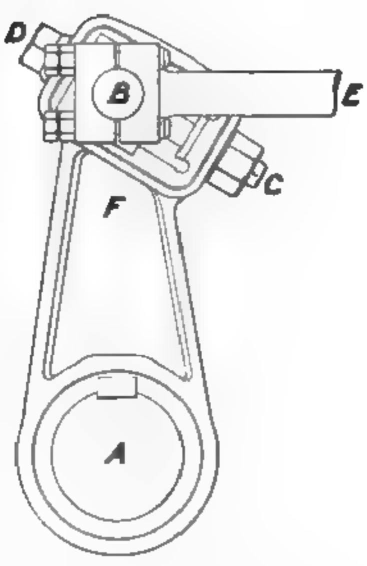



FIG. 122.—INDEPENDENT LINKING UP GEAR.

A is the weigh shaft. B E is the reversing rod leading to any of the links. The screw D C enables the adjustment of any one link to be different from the others. D C is nearly at right angles to B E in the astern position.

**85. Hackworth Gear.** This is the parent of all **the radial gears**. It is shown in Fig. 123, as applied to a vertical engine. The eccentric disc is placed 180° away from the main crank. The block $B$ at the end of the eccentric rod can slide in a slot, which is often straight, but may be curved. The pin $D$ in the eccentric rod works the valve.

To alter this gear we merely alter the angle of inclination of the slot to the horizontal. In forward gear, it inclines upwards as shown. If we wish to reverse the engine, we must turn it round so that it slopes downwards. The hand wheel $C$, and worm $E$, working on the worm sector $F$, are used for reversal or alteration of the gear.

Instead of a slot to guide $B$, we may have a swinging link ($N\ J\ B$) of Fig. 124. It is evident that $B$ will move in the arc of a circle with $N$ as centre. To reverse this form of the gear (called the **Marshall gear**) we have merely to move $N$

round the dotted path. For convenience in doing this, $N$ is usually a pin at the end of an arm ($N L$), and by moving this arm about $L$ as a centre, the gear is shifted. The Angstrom gear also is simply the Hackworth, in which the guidance of $B$ in a nearly straight path is effected by using a parallel motion.

**86. Joy Gear.** Fig. 126 shows the Joy gear. $C$ is the crosshead, $C E$ being the direction of its motion, the centre line of the engine. $K$ is the crank pin. $J$ is a pin on the connecting rod, one end of the link $J L$, the other end $L$ swinging in the arc of a circle, of which the fixed pin $M$ is the centre.

Notice that the point $J$ moves nearly in an ellipse, shown dotted as $J N$. The pin $A$ in the link $J L$ moves in a curve $A P$ which is

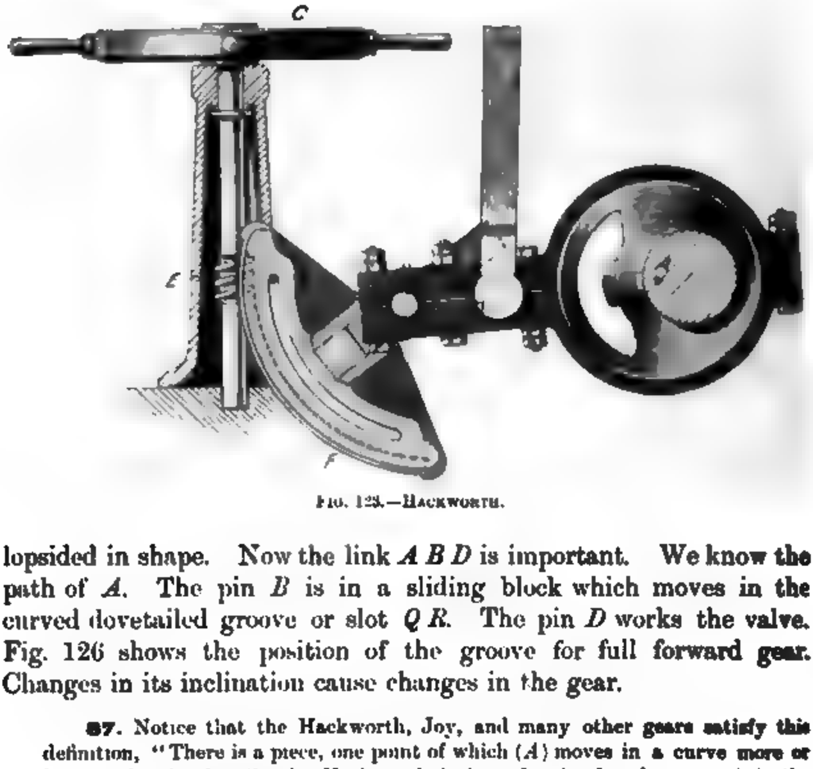



FIG. 125.—HACKWORTH.

lopsided in shape. Now the link $A B D$ is important. We know the path of $A$. The pin $B$ is in a sliding block which moves in the curved dovetailed groove or slot $Q R$. The pin $D$ works the valve. Fig. 126 shows the position of the groove for full forward gear. Changes in its inclination cause changes in the gear.

**87.** Notice that the Hackworth, Joy, and many other gears satisfy this definition, "There is a piece, one point of which ($A$) moves in a curve more or less nearly circular (in the Hackworth it is truly circular, because $A$ is the centre of an eccentric sheave); another point ($B$) has a reciprocating motion nearly in a straight line; another point ($D$) works the valve."

But in studying any of the twenty forms of radial valve gear, the student will find the following definition much more helpful—

There is a piece ($A B$) whose average direction is at right angles to the valve rod; a pin ($D$) in the piece ($A B$) works the valve. Speaking only of motions in the direction of the valve motion or piston motion; $A$ moves either synchron-

nously with the piston or half a period ahead of it, in any case reaching the ends of its stroke as the piston reaches the ends of its stroke; $B$ is a quarter period behind or in front of $A$, being always at the middle of its stroke when $A$ is at the end of its stroke. $A$ is a half period ahead of or behind the crank in the Hackworth gear, and consequently $D$ is between $A$ and $B$. $A$ is synchronous with the crank in the Joy gear and consequently $D$ is in $AB$ produced.

When a student takes up the theory of these gears, he will find that the above definition makes one theory do for all the gears (see Art. 312).


FIG. 124.—MARSHALL.

**88.** If we try, using a single eccentric or any form of link motion or radial valve gear, to cut off earlier than at one-third of the stroke, we get a poor result; there is too much wire drawing at the cut off and the release is earlier than we desire to have it. Some of the gears are better than others in this respect, the intermediate positions giving quicker cut off than a single eccentric would do.

It will be found that in the very largest marine engines we seldom desire to cut off at even so little as one-third of the stroke, and so the common slide valve and the above kinds of valve motion are to be found in these large engines. Cutting off at one third of the stroke in each cylinder of a triple cylinder engine is like cutting off 1/27th of the stroke in a single cylinder.

**89.** When we look at a large triple cylinder engine for the first time, we are confused at the apparent **complexity of its con-**

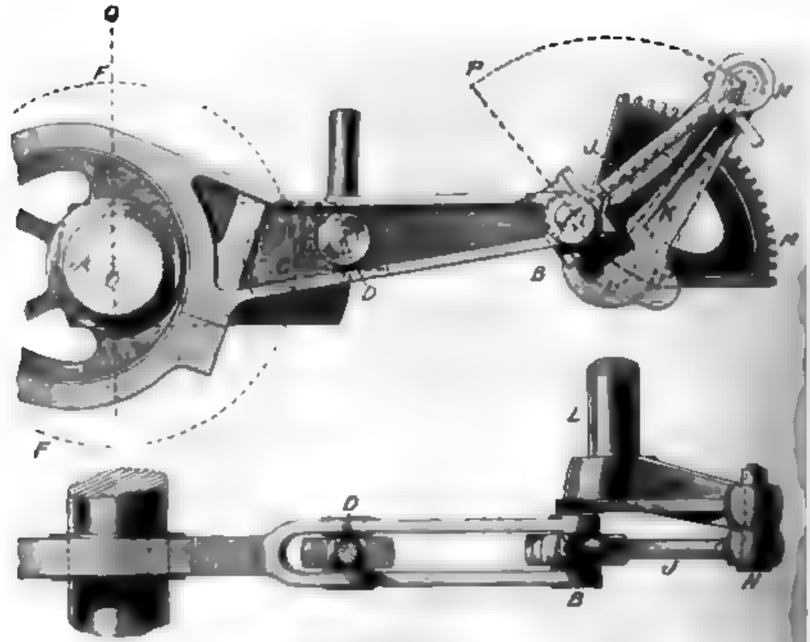



FIG. 125.—MARSHALL.

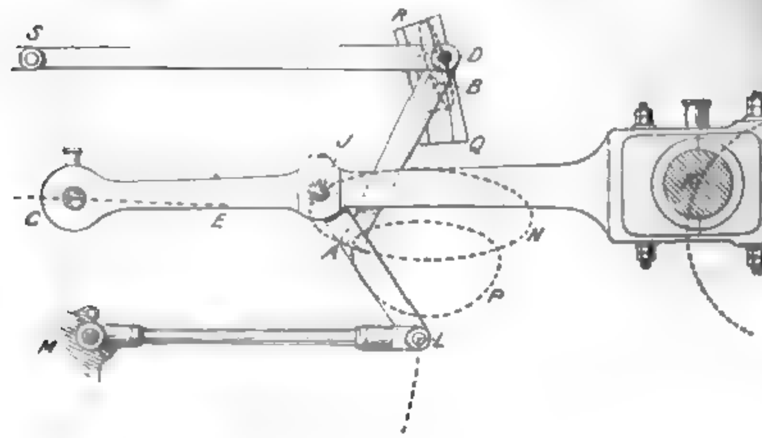
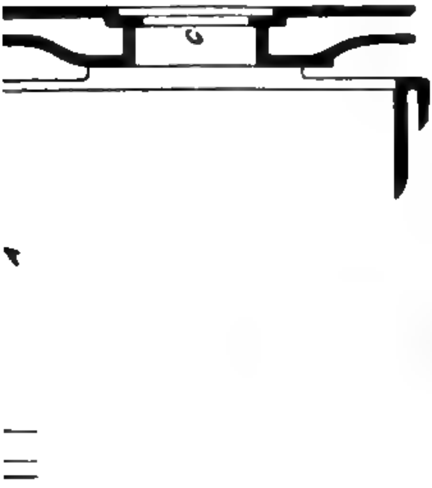



FIG. 126.—JOY.

**struction.** When, however, we consider it carefully, we **see that** it is simple enough. Thus, for example, look at the **great valve** *V*,

), of the low pressure cylinder; it is very heavy, and to take ht off the valve rod, the rod is extended above, so that the e piston $P$ may, because of the steam pressure upon it, the weight not only of the valve and rod, but some of the tion.

in, the valve $V$ is so large that the steam pressure on its ould press its face so tightly against the seat on which it



a. 127.—LOW PRESSURE CYLINDER.

ions of steam ports P, exhaust port E, and steam jackets for double-ported valve.

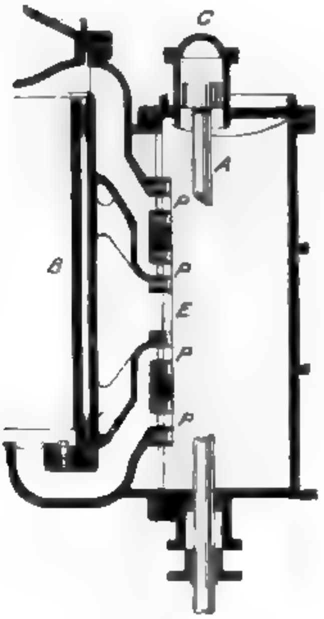



FIG. 128.

Showing valve chest, steel liner for valve seat, balance piston, &c., for a double-ported valve.

that there would be excessive friction, consequently the relief $R$ is applied, which prevents pressure steam from getting on part of the back of the valve. Fig. 130 will show more clearly e ring $R$ of the relief frame slides steam tight on the back of ve. The space at the back communicates with the exhaust, pressure gauge is often provided for testing the packing of g.

t again, the large valve $V$ is apparently complicated in r way. In truth it is only complicated by its parts being d. The space marked $E$ is exhaust, that marked $S$ is steam. . 131 the valve is at its mid position. Let us think of one

end of the cylinder only and the left-hand side of the
Note that the passage *P* has two openings into it inste
merely one, and that whatever is occurring at one of th
occurring in exactly the same way at the other. If a st
will make a model like Fig. 131 of paper, and move it

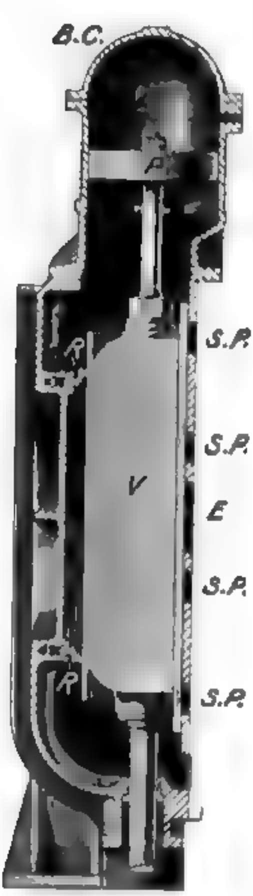



Fig. 129.

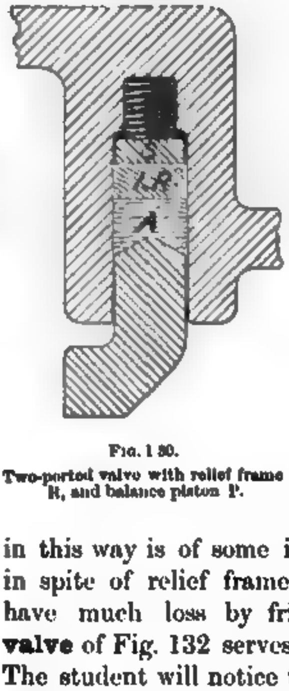



Fig. 130.
Two-ported valve with relief frame R, and balance piston P.

valve seat, h
see at once
this is the
nary locon
slide valve
parts we
doubled. H
easily imag
treble-portec

It is qui
dent that
is less trave
these valves,
large openi
steam and
haust; the
tional work
in this way is of some importance, be
in spite of relief frames we always
have much loss by friction. **The
valve** of Fig. 132 serves the same pu
The student will notice that when the
uncovers the port, there is steam en
the same port through the hollow
from the steam space on the other
This valve must have a raised sea
would be interesting to know what
centage of my readers will make w
models in paper of Figs. 131 and 132.

Fig. 134 shows **a piston valve**
when there are highest pressures. This is merely an ordinar
valve, only that, instead of having a flat face we have a cylin
face, and pressures are very well balanced. The pistons *R*
are packed with rings as ordinary pistons are packed to make
steam tight. The port openings extend all round the pistons
that there are bars across to prevent the packing rings spr
out. Another is shown in the Willans engine (Fig 233).

Sometimes the steam space is at the two ends, the exhaust being between the pistons, surrounding the connecting tube; sometimes the reverse arrangement is adopted, as in Fig. 134. A slight

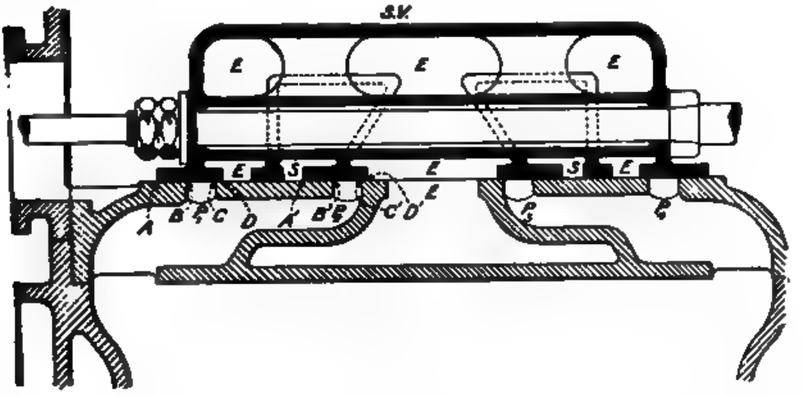



FIG. 131.—DOUBLE-PORTED VALVE IN MID POSITION.

$AB = A'B'$ = outside lap; $CD = C'D'$ = inside lap. The part $A'B'C'D'$ is an exact copy of $ABCD$. The space $S$ is really outside the valve, and is filled with steam. The space $E$ is all exhaust. Whatever occurs at the port $P_1$ occurs at the same time at the port $P_2$.

difference in size of the two pistons allows us to dispense with a balance piston.

**90.** Sometimes **momentum cylinders,** or cushions, are fitted in quick-running engines to supply the forces due to mere inertia of the valve gear. In **Joy's Assistant Cylinder,** instead of the ordinary balance piston we have a piston which is forced by steam in the direction in which the valve is moving,

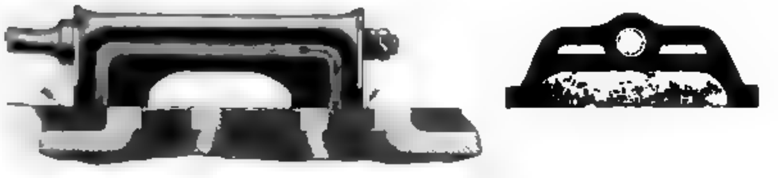


FIG. 132.—TRICK VALVE.

so that the eccentric rods are greatly relieved; there is also the necessary cushioning action at the end of its stroke. In existing marine engines these Joy Assistants may exercise as much as 20 horse-power.

A practical man who understands his engine will not need any hints as to **the setting of valves.** Indeed this merely means that when each crank is at its dead points its valve shall just have the proper amount of lead; not quite the same perhaps for both ends; and this is effected by the nuts on the valve

rods. If the leads at *both* ends have to be increased or diminished, the *adva* of the eccentric must be altered. The position of the valve at mid travel exactly midway between its position at the dead points (see 1 and 8, Fig. 201

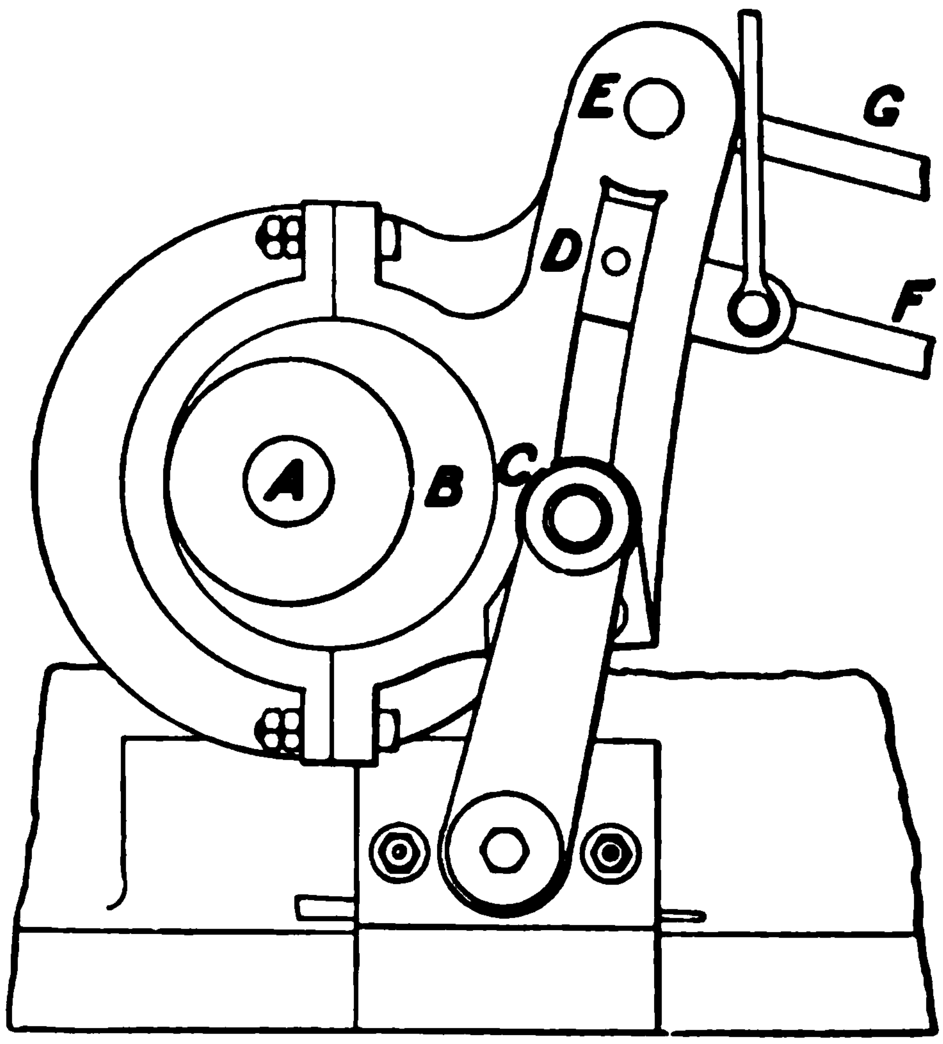



FIG. 188.—THE FINK SLIDE VALVE GEAR.

This is the simplest of valve gears. The valve is worked from the block D by the radius rod F. D may be lifted or lowered in the link slot to reverse or to give different grades of expansion. The link is rigidly part of the eccentric straps, the centre of whose disc B is 180° from the main crank. The point C of the link or eccentric straps is guided to move nearly horizontally in the arc of a circle.
The result is much the same as is obtained by the use of a Gooch link or by Radial valve gear, only that the octaves in the motion (Art. 815) are much more pronounced.
In the figure a pin at E works another slider through the rod G. When this gear is used for reversal, C is a point in the line joining A, and the centre of the slot.

and this ought to be symmetrical over the exhaust port. It is to be remember however that, especially in marine engines, there is more lead and more insi lap at the lower port. There is another reason for greater lead at the low port; it enables the wear of the eccentric straps to be taken up.

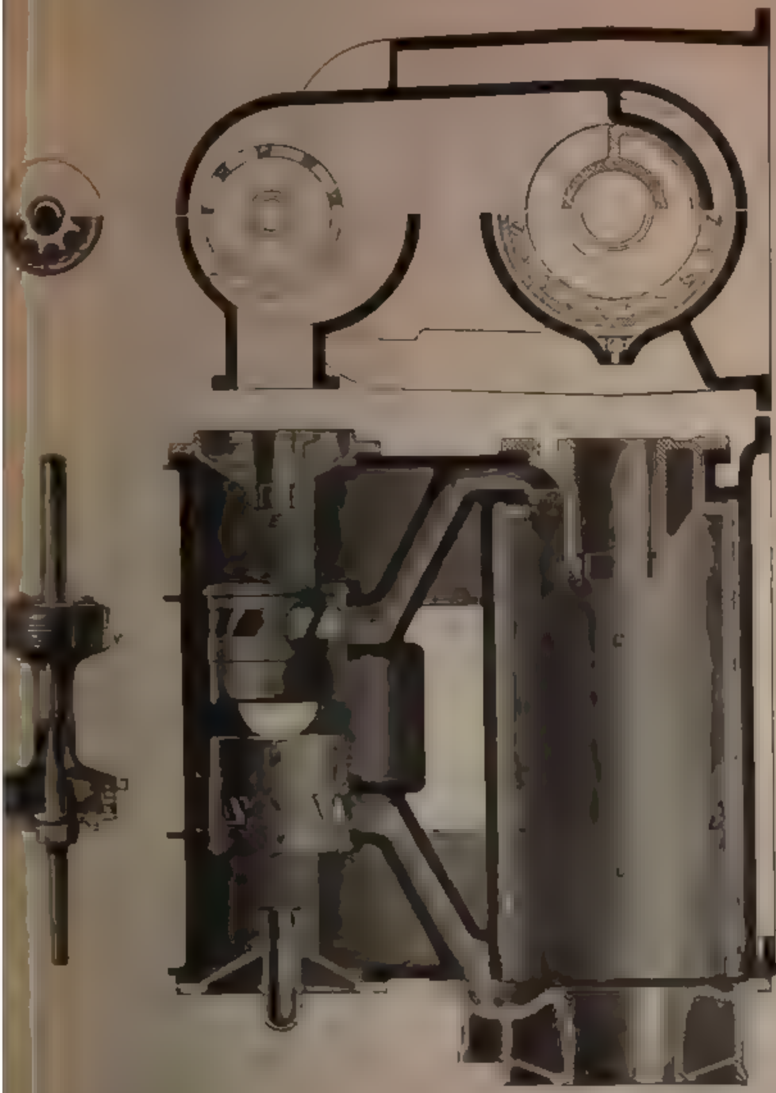


FIG. 158. TRIPLE EXPANSION MARINE ENGINE PISTON VALVE, VALVE SEATS, CHEST AND JACKETS; HIGH PRESSURE CYLINDER.

[illegible] Exhaust space at ends of piston valve; steam space in middle.

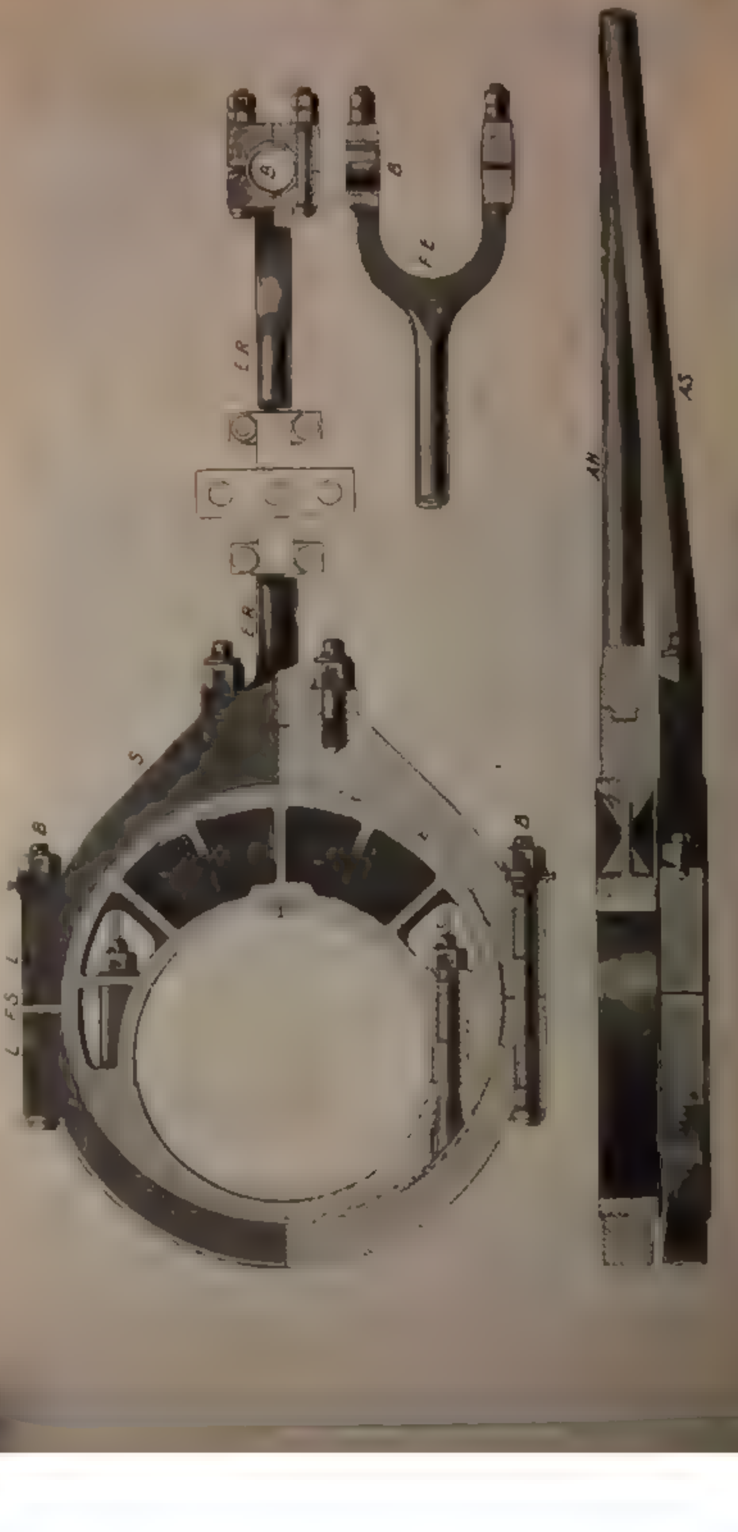



Fig. [illegible].—Ahead and Astern Eccentrics for Link Motion of a Marine Engine.

## CHAPTER IX.

### THE EXHAUST AND FEED.

**91.** WHEN steam escapes from a cylinder, it may escape to the atmosphere, and the engine is said to be **non-condensing.** Sometimes the steam escapes through a pipe in the chimney, so that it may create a draught through the flues of the boiler. This is the case in locomotive engines which are always non-condensing. Sometimes the pipes through which the escaping steam passes are surrounded by the *feed-water*, which thus gets heated before it enters the boiler. Sometimes the feed-water supply enters a box in spray, being pumped off from the lower part of the box. The exhaust steam passes through this box also on its way to the atmosphere. The main objection to this plan is that the feed-water is heated before passing through the pump, and this gives trouble. Also the water takes up objectionable oil from the steam. Weir heats the feed-water by mixing with steam from the lowest pressure receiver of a triple expansion engine. Boiler steam is sometimes used to heat the feed-water, and the increased efficiency discovered is really due to the fact that the method used greatly increases the water circulation. (See Chap. XXXIII.) There can be no doubt of the great benefit derived from heating feed-water in feed-water heaters by the heat in the flues before it enters the boiler, partly because rushes of cold water produce great local strains in the shell and flues, and this is thought to be very important; but as a matter of fact there is usually found to be a saving of something like 10 per cent. in fuel, on the whole. Again, the heated feed-water tends to deposit its sediment in the heater rather than in the boiler itself, and besides, its air gets greatly driven out of it. The air is, however, only objectionable in condensing engines.

The name *Feed-Water Economiser* is more usually given to a number of tubes surrounded by the waste gases which are about to escape up the chimney. They have to be constantly kept cleaned

from soot by means of scrapers. One seldom sees a group of four or five Lancashire boilers without a feed-water heater, and it is commonly said that the feed-water heater is practically equivalent to an extra boiler in steam produced, without extra fuel being needed, (see Fig. 196).

**92.** In a **condensing engine** the exhaust steam passes into a cold chamber called a condenser. This chamber, which may be of any shape, is kept cold sometimes by cold water circulating round it, and is then called a **surface condenser**; sometimes by jets of cold water spraying from a rose-head, inside the condenser on the end of a pipe, the other end of which dips into a neighbouring pond or tank; then it is called a **jet condenser.** It is to be remembered that the weight of cold water needed for condensation is usually taken to be about thirty times the weight of steam to be condensed. Less will do; but in any case the amount of water needed is so great that we never dream of using a condensing engine if the water must be supplied by a water company and has to be paid for.

EXERCISE. 1 lb. of steam at 3·62 lbs. per square inch or 65° C. is condensed by water at 15° C., the mixture being 40° C., what weight of water is used?

In the table of Art. 180 we see that the latent heat of the steam is 561, and the steam not only condenses, but falls 25°, so that altogether the pound of steam loses 561 + 25, or 586 units of heat. The water is raised 25°, and hence its weight is 586 ÷ 25, or 23 lbs.

It is of importance that a student should be able to calculate: 1. How much heat must be given to feed-water in the boiler to produce steam. 2. How much heat goes away from the cylinder in the steam, whether the engine is condensing or non-condensing.

To raise 1 lb. of water to any temperature requires (with enough accuracy for our calculations) 1 unit of heat for every degree. Thus to raise a pound of water from 0° C. to $\theta$° C. needs $\theta$ units of heat. To convert the water into steam at $\theta$° C, without any further increase of temperature, needs $l = 606{\cdot}5 - 0{\cdot}695\theta$ units of heat. This last is called the latent heat of the steam; it was measured by Regnault, and he found this formula to represent his results pretty accurately. A table of values of $l$ is given in Art. 180. Let a student consider steam at 101·9 lbs. per square inch. Its temperature is 165° C. Suppose that a pound of feed-water was at 40° C., it took 125 units of heat in rising to 165° C., and it then took 606·5 − 0·695 × 165 or 492 units of heat to convert it into steam. Altogether it was given 125 + 492, or 617 units of heat in the boiler. Let us suppose that a pound of steam escapes from the cylinder at 17·53 lbs. per square inch, or (according to the table,

page 320) at 105° C., its latent heat is 534 units, and if we imagine it cooled to the temperature of the feed-water, this means 105 − 40, or 65 more units. It therefore would carry off with it 599 units of heat. In a condensing engine if we imagine 1 lb. of steam at say 65° C. converted into water at the temperature of the feed-water (40° C.), we find that it must have 581 units of heat taken from it. Now, I do not say that for every pound of steam produced, we have a pound of steam in the exhaust, because some of the exhaust stuff is water—but the above figures will teach an important lesson, important in all heat engine work, namely, that we take away and waste in the exhaust nearly as much heat as we give to the stuff, so that only a small portion is utilised and converted into useful work.

Having to take away by means of cooling water this great amount of heat from the exhaust steam is a great trouble. It is so great a trouble that we would fain use non-condensing engines on board ship. Why do we not, then? Because, if we let all our steam go off uncondensed to the atmosphere, where shall we get feed-water for our boilers? From the sea; sea water, which deposits salt inside the boiler, even if we are continually trying to avoid it by blowing off. It is, however, the very hard, tight-sticking **deposit** from sulphate of lime which we fear most. This is so insoluble in hot water that it is impossible to use sea water in boilers with pressures higher than about 55 lbs. per square inch (absolute). And this also is the reason why we must use surface condensers. But on land when we can get a sufficient supply of fresh water for the feed, if there is a steady load on the engine, and we use high pressures, there is often found to be no great advantage in having a condensing rather than a non-condensing engine. If, however, the load varies greatly, there is considerable saving in using a condensing engine if we do not have to pay for the condensing water.

Calculations like the above have to be made continually in practical work, and the student ought not only to work numerical exercises, but he ought to make measurements for himself in a heat laboratory. Even one actual measurement of the latent heat in a quantity of steam will give ideas which no practical man ought to be without. It is quite absurd to think that a man who has only this kind of knowledge by hearsay, really comprehends what he talks about. What we continually need to remember is Regnault's total heat $H$, the heat given to a pound of water at 0° C. to convert it into steam at $\theta$° C., and its amount is $H = 606{\cdot}5 + 0{\cdot}305\theta$. $\theta$ units of this is spent in merely heating it as water, and $H - \theta$ or $606{\cdot}5 - 0{\cdot}695\,\theta$ is the latent heat. Notice that there is less latent

heat in high pressure steam than in low pressure, although there is more total heat.

If students do exercises, they ought to take cases such as that of say one-quarter of a pound of water, and three-quarters of a pound of steam—how much heat has produced it? how much heat will it give out in the condenser?

The following method of calculation is very much more suggestive and accurate than the last, and the student ought to work at least one exercise with it carefully.

EXERCISE. An engine uses 17 lbs. of dry saturated steam at 101·9 lbs. per square inch (absolute) per hour per indicated horse-power. How much heat enters the condenser with the exhaust-steam per hour per horse-power? Assume no radiated heat, no leakage of steam, no steam jacket.

The total heat in 17 lbs. of such steam may be calculated, or from the table Art. 180 we see that the heat supplied is 17 × 656·8, or 11,166 Centigrade units. Now 1 horse-power hour is 1,980,000 foot-pounds, or (dividing by 1393, Joule's equivalent) 1422 Centigrade units. Hence 11,166 − 1422 or 9744 units reach the condenser per indicated horse-power hour.

**93.** An injection condenser may be of any shape; the injection water rushes in as spray, and with the condensed steam and air it is drawn out through a foot valve *F V* in Fig. 136, which shows an air pump. Fig. 47 shows how it is worked in marine engines. In the down stroke of the bucket the water passes through the bucket valves *B V*; in its up stroke this water is lifted and passes through the delivery valve *D V* to the hot well. Notice that many light valves are often used in air pumps instead of one large one; this is for quickness, and also that they may lift under very small pressures. Valves are often made of thin sheet brass or phosphor bronze instead of india-rubber. The barrel and bucket are castings, usually in gun metal. The force pump, Fig. 140, feeds the boiler; when the plunger *A* is lifted, water is sucked from the hot well through the valves *G F E* to the barrel of the pump; when the plunger is pushed down, the water in the barrel is forced through other valves to the boiler. The feed pump is usually so large that it would supply more feed-water than the boiler needs.

Intermittent feeding is bad for many reasons; the feed-water ought to be supplied regularly. A good engine-driver will leave the water at a high level in his boiler when he stops his engine for a time. Sometimes, however, when he wants to start, the water may be too low, and it is important to be able to feed the boiler without starting the main engine. This gives us also a reason why a high tank of water is so useful, as we may easily fill the boiler from it. It is usual to have means of independent feeding in all large engines, so that it may go on when the engine is stopped. If injectors (Art. 95) only are used, they ought to be in duplicate. If

FIG. [illegible].—AIR [illegible] LARGE [illegible]

The [illegible] valves [illegible] are [illegible] with [illegible] plate [illegible]

[illegible] at the end [illegible] the [illegible] to [illegible] air [illegible] to [illegible] the efficiency of [illegible]

Note [illegible] the barrel of [illegible] pump [illegible] the valves to be [illegible]amined.

an independent steam pump or other separate boiler feeder be employed, it is usual to order it large enough to supply much more, say twice or three times the actual feed-water; this is done with the object of letting it work slowly so that it may wear well and need little attention. The Worthington **steam pump (Fig. 22)** is usually employed because it gives no trouble and is of easy regulation. In ships there is one main pump in each engine room capable of supplying all the boilers, and there is one **auxiliary pump** in each boiler room delivering only to specified boilers, and with suction from either the feed tanks or the reserve tank or the sea.

If the injection water is dirty we must be careful to strain it, and if we have a purer fresh supply it is usual to use it in preference to the condensed steam as feed-water. We often use a fresh supply when we have a surface condenser, as condensed steam is sure to have oil in it, and the oil does harm to the boiler. Oil sometimes seems to get on the tubes or flues of the boiler in places, preventing the water touching the metal which may get extremely hot at such places. I have sometimes seen places in the crown or just beyond the crown of the furnace which seemed red hot, and I have usually attributed them to patches of oil. **Oil filters** are used in marine engines to free the feed-water of oil, and almost no oil is now being admitted to valve chests or cylinders for lubrication.

A **surface condenser** (see Fig. 138) is usually formed of a great number of $\frac{3}{4}$ or $\frac{5}{8}$-inch drawn brass tubes, $\frac{1}{20}$ inch thick, about 1 inch apart, of zig-zagged arrangement in a brass casing *C C*, and through them cold water is kept circulating as shown by the arrows *C W T* to *C W T* through *C W* (in amount about 70 times that of the feed-water), by means of what is called the circulating pump, which is usually a centrifugal pump. In a marine engine it is the sea water which is kept so circulating, and there is usually an arrangement by which this circulating pump may draw water from the bilge instead of the sea. Driven by the main engines there are also usually bilge pumps; there are then often four pumps forming part of the main engine—air, feed, circulating, and bilge. Fig. 139 shows an independent circulating pump.

The tubes are always kept cold, and the exhaust steam being admitted at *A* into a closed space outside and all round these tubes, is condensed and is drawn away through *B* by the air-pump. The condensed water is the **feed-water,** and needs only an occasional small addition of (fresh, not salt) water, because of leakage and blowing off. Thus the same water is used over and over again, and an engineer need not have more variation of level than an inch in his boiler. Just at the beginning it is thought well


[illegible] screwed stuffing box ends [illegible] condenser tubes to allow expansion, without [illegible] water [illegible] to the condenser [illegible] also stays and fastenings for tube plates

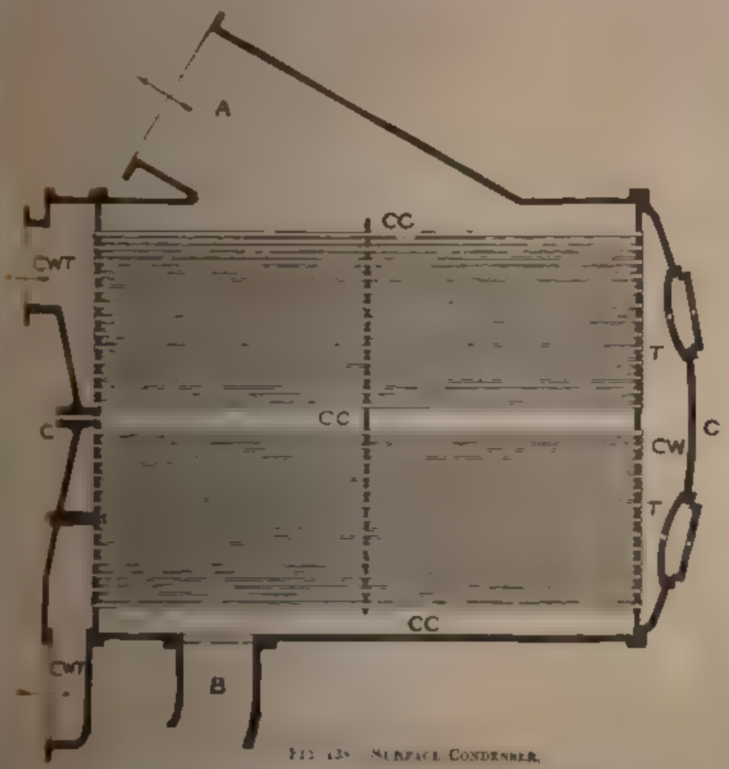



FIG. [illegible] SURFACE CONDENSER.

[illegible] generally cylindrical in shape [illegible] feet long. Tube plates T, 1 inch thick [illegible] water [illegible] the tubes and steam [illegible] There are two advantages in having the [illegible] tubes [illegible] the deposited [illegible] and the [illegible] keeps [illegible] the great [illegible] of water [illegible] joints to leak, [illegible] tendency

[illegible] support as shown.

[illegible] were made very much smaller [illegible] the whole condenser might [illegible] smaller in size.

to use salt water for a short time, as this produces a thin scal
over the inside of the boiler which is thought to protect the pl
against *pitting*.

EXERCISE. In recent practice one square foot of tube conde
about 12 lbs. of steam per hour (sometimes a higher figur
taken). Find the total length of $\frac{3}{4}$-inch tube required for an en
whose maximum indicated power is 1,000 horse, using 16 lbs. of st
per hour per indicated horse-power.

The total area is 16,000 ÷ 12 or 1,333 square feet. One foo
$\frac{3}{4}$-inch tube has a surface $\pi \times \frac{3}{4} \times 12$ square inches or $\pi \times \frac{3}{4} \div 1$
0·196 square feet. Hence 1,333÷0·196 or 6,800 feet length of pipi
required. If each length of tube is 8 feet, we need 850 lengths.

It is usually thought well to employ as large an air pump with a
face condenser as with an injection condenser, although there is n
less water to remove. This is on account of the air which is alw
present in water to some extent, and from which the condenser mus
kept free. Not only does such air spoil the vacuum, but the me
trace of air very materially retards the condensation of the stean

**94.** When water is expensive, as in a town, that kind of sur
condenser which is called an **evaporative condenser,** may be u
It consists of a number of tubes for the exhaust steam, their out
surfaces being exposed to the atmosphere ; a small circulating pu
being employed to keep them wet on the outside. It is not o
used for engines indicating more than 100 horse-power, because
outsides of the pipes give off white clouds of condensed vapour w
may be thought to be a nuisance.

In electric lighting stations and other places where large po
is needed, and therefore the increased economy due to condensa
is important, and in places where a large supply of condensing w
cannot be cheaply obtained, this kind of condenser becomes imp
ant. Ordinary surface condensers need 70 lbs. of water per po
of steam. Where there is large space for cooling, the water
an ordinary surface condenser may be used over and over again,
such space is expensive in cities. Now, evaporative conden
giving 24″ to 26″ of vacuum need water supply in amount only al
$\frac{3}{4}$ of the weight of steam condensed. The surface must obvio
be larger than in an ordinary surface condenser. Care must be ta
that the condensing water trickles from the hotter to the co
parts. Leakage must be carefully prevented, and so joints mus
good and accessible, and for another reason they must be access
because the trickling water deposits from 10 to 40 oz. of s
matter per square foot per annum and the pipes must be clea
Horizontal tubes are found to be more effective than vertical,

ake up more space. Various contrivances have been invented e a fairly even supply of trickling water everywhere. Artificial


Fig. 138a — The Injector

entilation is found to greatly (50 per cent. or more) increase the g effects. Sometimes the fan is only used when the heavy are on.

A **jet condenser** is like an injector. A central jet of injection water is surrounded by a nozzle for exhaust steam, and the receiving pipe gradually expands towards the hot well. The steam condenses and passes with the injection water to the hot well, no air pump being needed.

**95. The Injector.**—Steam from a boiler enters the injector at *S*, Fig. 138 *a*, and as it enters a place of low pressure at the end of the nozzle, it acquires a velocity which may be greater than 1,300 feet per second. There is a partial vacuum at *D*, and water flows towards it from a tank through *W*. Imagine the space *D* filled with water; the steam mingles with this water, condensing and heating the water, and the mingled stream passes across the place of low pressure *G* into *A* with a sufficiently great momentum to overcome the pressure of the boiler, which it enters past a check valve *V* and an ordinary controlled valve as well.

The tank may be below, or on the level of, or above the injector.

As the steam handle is gradually turned first a small quantity of steam enters from *S* driving air before it, creating a partial vacuum at *D*, filling the spaces with water, and the condensed steam water and air escape by the overflow to *O F*. The valve admitting water through *W* is now opened. As the steam valve is more opened a greater rush of steam takes place, and the water has enough momentum to open the valve and enter the boiler; there is now a partial vacuum in the chamber *G*, and hence it is thought good to have a valve in the overflow pipe to prevent air entering with water into the boiler; water can always escape through *O F*. If the engine is non-condensing I approve of allowing air to enter the boiler, as it prevents condensation in the engine cylinder, but it produces very bad effects in the condenser of a condensing engine. It is evident from the figure that we can control the flow of steam and water; when we diminish the water supply it is fed into the boiler at a higher temperature, and if this is too high the water may boil near *M* and the action be spoilt. As the lift from the tank is greater, there is more chance of trouble, and it is seldom that the lift is more than 20 feet. There are various arrangements in use for automatically adjusting the proportion of the water and steam areas at the nozzles to suit changing boiler pressures.

We shall see in Art. 381 that as the velocity with which the water reaches *M* is greater, the efficiency is greatly increased. Now, coming from a tank on the level of or below the injector, it is not possible for the water to have a great velocity. Hence there are injectors of double action. The water is first forced into a chamber, where its pressure is about 20 lbs. per square inch above

of the atmosphere, and by a second jet of steam it is then forced the boiler. In the injector of the future this principle will ably be greatly amplified, for it is quite easy to have a central copic system of steam nozzles from which the steam emerges ually, at first with the slowly moving and later with the more kly moving water.

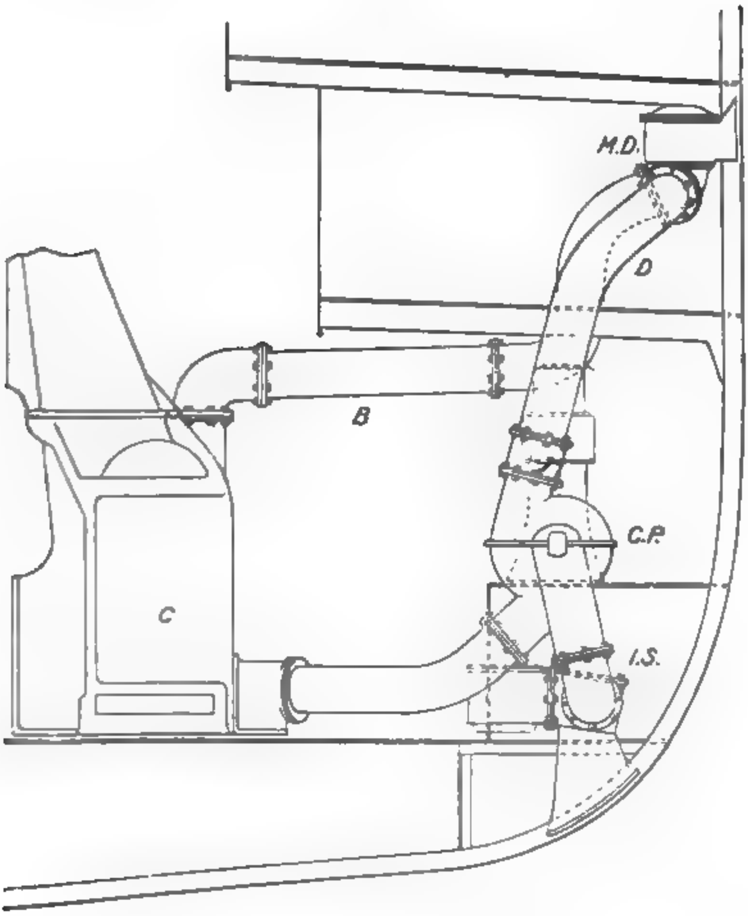



FIG. 130.—MARINE ENGINE CIRCULATING PUMP AND CONDENSER, &c.

his shows the gun metal circulating pump C P. (usually there are two) driven by an independent e, drawing sea-water through the inlet suction valve I.S., driving it through the tubes of C the nser, and by the discharge pipe B, through the main delivery valve M D into the sea again Both nd M.D. are screw-down stop valves worked by hand. In case of need C.P. will draw water from lge instead of the sea, and discharge directly through D instead of the condenser. In this case is actual lift and it is found that twice the speed is needed to deliver about half as much water. therwise the possible discharge is enormous, in some ships 1,000 to 1,500 tons of water per hour. ndependent engine driving the pumps is placed well above the bilge, and takes steam either from ain boilers or auxiliary boilers, and exhausts either to main or auxiliary condensers, or is non-nsing.

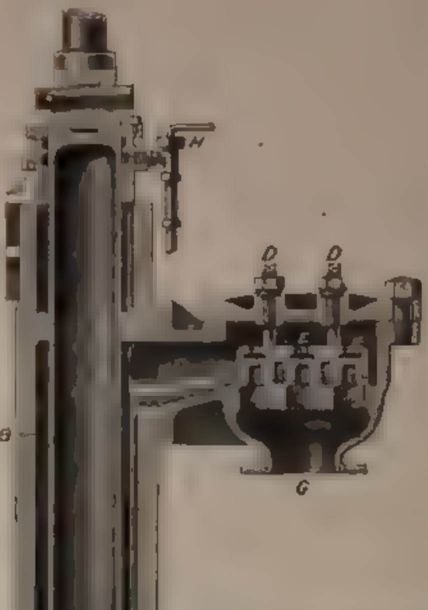


FIG. 140. — FEED PUMP FOR MARINE ENGINE

In marine engines water is pumped [illegible] to the [illegible] feed tank of galvanized steel plate large enough for from [illegible] feed at full power [illegible] to escape from the water. There [illegible] of these [illegible] in each [illegible] usually connected by a pipe to the other. There are two overflow pipes. One at the reservoir [illegible] from the bottom of the tank arranged so that [illegible] the surface [illegible]. The other takes [illegible] water from the surface [illegible] fitted. The gauges showing the [illegible] of water [illegible] the tanks must be visible from the starting platform. There are [illegible] in the tanks. The [illegible] bottom [illegible] and boiler rooms [illegible] feed tanks [illegible] large ships. These [illegible] fresh water from distilling apparatus. Of late it has [illegible] the [illegible] discharge from the air pump into a [illegible] tank and to pump the water from this through an oil [illegible] to the feed [illegible].

Apparatus for distilling fresh from sea water is to be found [illegible]. This supplies water for [illegible] and also for waste of steam. There are many varieties but they all use the [illegible] passing boiler steam through tubes to heat sea water [illegible] the tubes, the resulting steam [illegible]. They differ from one another in the case with which the scale may be removed.

FIG. 141. — AIR VESSEL FOR F[illegible] MARINE ENGINE

The pump [illegible] through [illegible] the [illegible] boilers [illegible] which [illegible]. When [illegible] air vessel and [illegible] water escapes by way of the [illegible] through J to the hot well.

# CHAPTER X.

## FLY-WHEEL AND GOVERNOR.

**96.** It is really on the fly-wheel that we depend for the prevention of sudden changes in speed. The governor is too leisurely.

The mass of a fly-wheel is mainly in its rim, and it is usual to neglect the mass of the arms and boss in calculations. The weight of the fly-wheel in pounds divided by 32·2 gives the mass in engineers' units. Half the mass multiplied by the square of what we may call the average velocity of the rim in feet per second is the kinetic energy stored up in the wheel. If $W$ is the weight of a fly-wheel in pounds, if $R$ is the average radius of the rim in feet, when the wheel makes $n$ revolutions per minute, it is easy to show that the energy stored up in it is $WR^2n^2/5874$ foot-pounds.

The following exercises will bring home to students the value of the fly-wheel :—

1. The rim of a cast iron fly-wheel has a rectangular section $12'' \times 10''$. Its average radius is 5 feet, what is its weight? Its volume is $12'' \times 10'' \times 2\pi \times 60$ or 45,200 cubic inches; its weight about 11,760 lbs. and

$$WR^2 \div 5874 = 50{\cdot}1.$$

If this wheel makes 100 revolutions per minute its kinetic energy is 501,000 foot-pounds. If it makes $n$ revolutions per minute its kinetic energy is $50{\cdot}1 \times n^2$.

Hence, in changing from any speed to another, we can calculate the energy that it will store or unstore.

2. An engine with the above fly-wheel gives out on the average 120 horse-power at 100 revolutions per minute. Therefore the energy given out in one revolution is $120 \times 33{,}000 \div 100$ or 39,600 foot-pounds. Now let us suppose that the fly-wheel is called upon to store the whole of the energy which would be supplied in half a

revolution, because perhaps the governor is too sluggish, what is the highest speed?

The wheel had $50{\cdot}1 \times 100^2$ or 501,000 foot-pounds already; we give to it $39{,}600 \times 0{\cdot}5$ or 19,800 foot-pounds. So that its higher store is 520,800, and this is 50·1 times the square of the new speed. Divide therefore by 50·1 and extract the square root, and we find 104 revolutions per minute as the highest speed.

A large fly-wheel is usually built up of many pieces carefully fitted, keyed and bolted together; an example is given in Fig. 144 its rim arranged with grooves for rope-driving. It only differs in its rim from a common form, which is a spur-wheel which would drive a mortise wheel. In America, engineers often use a wrought iron fly-wheel which may be run at much higher speeds than a cast iron wheel. Sometimes the power is taken from the fly-wheel by a belt; but in England this is never done on large engines. The Americans are beginning to imitate the much superior English method of direct driving.

When an engine has to drive a single machine, such as a dynamo machine, it is now quite usual to couple the crank shaft directly unto the shaft of the dynamo; indeed engine and dynamo are placed on one bed, and the four sets of brasses are bored out at one time so that they may be exactly in line. When this can be done there is a very distinct saving in power.

**97.** Fig. 142 shows the modern form of the **Watt Governor, loaded** as it now usually is. $AB$ is kept rotating, being geared from the crank shaft. When the speed is steady, the centrifugal forces of the balls just balance their own weights and the great additional weight [illegible] and the weights of any other parts of the gear. Should the speed increase, there is increased centrifugal force [illegible] the [illegible] lifting the neck at $B$ [illegible] that [illegible] [illegible] the throttle valve. [illegible] is [illegible]. If the speed is [illegible] steam [illegible] the steam [illegible]

[illegible]

[illegible] **[illegible] Governor** [illegible] The [illegible]

fastened to the cap at $A$ and makes it rotate. The balls $W$ are at the ends of the bell crank levers $WFH$. When the speed increases, the centrifugal force of the balls causes them to lift the sleeve against the downward push of the spiral spring $S$; the lifting of the sleeve throttles the steam or in some other way diminishes the work done by the steam in the cylinder. There is usually an adjustment of the force in the spring which is easily made if the top of the cap is removed. By means of this adjustment we can make the governor

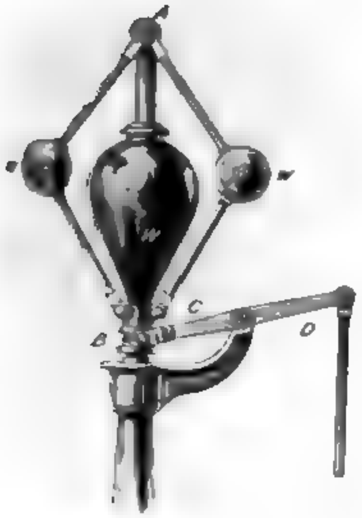



FIG. 142.—LOADED WATT GOVERNOR.

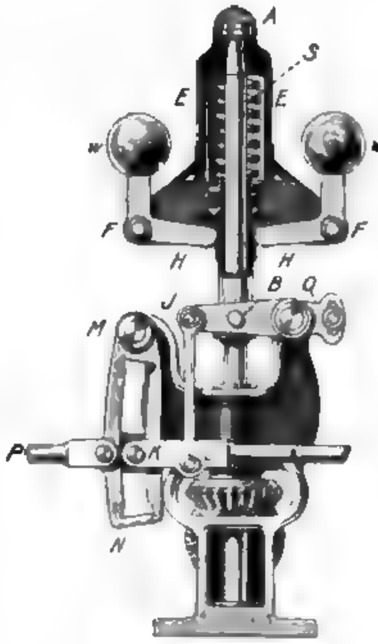


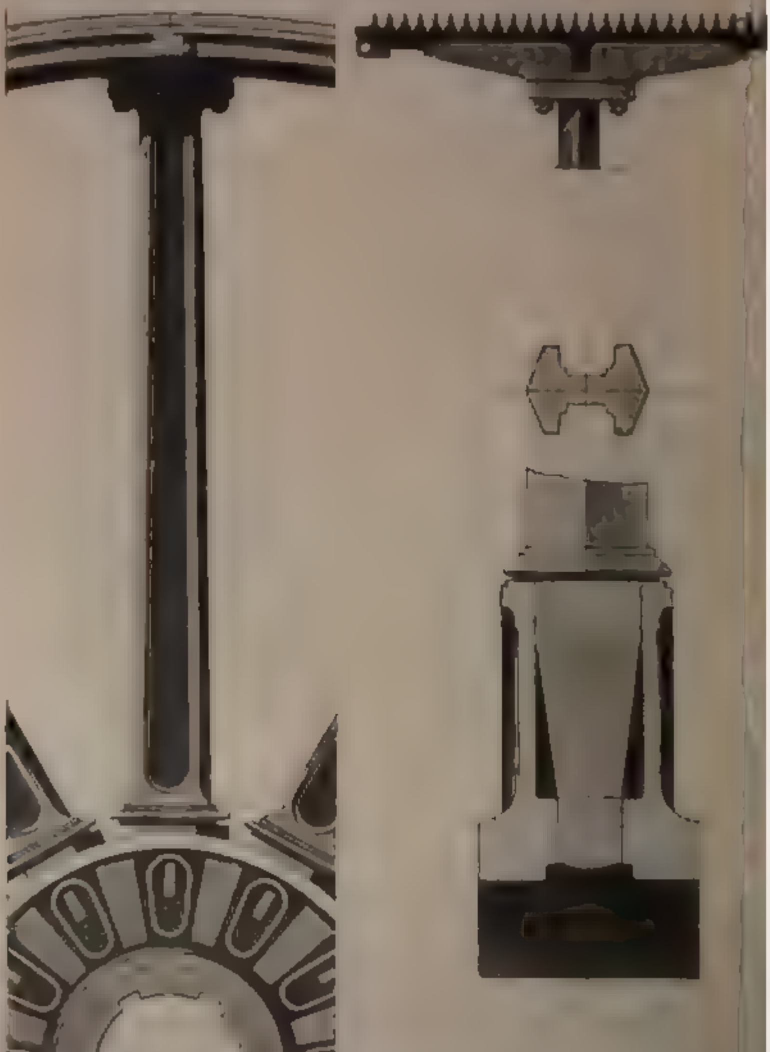

FIG. 143.—HARTNELL GOVERNOR, REGULATING THE CUT OFF.

very much more sensitive than that of Fig. 142. That is, suppose the normal speed of the engine to be 100 revolutions per minute; if the speed increases to $100\frac{1}{2}$, or diminishes to $99\frac{1}{2}$, we may find that the balls fly out very far or come in near one another very much.

When we try to make a governor too sensitive and quick we may cause the engine to *hunt*. That is, the balls may fly out so much for a very small increase in speed that steam is shut off too much; the speed decreases, the balls fly too near together and too much steam is admitted, and so the speed is continually fluctuating. This hunting action cannot be thoroughly understood unless one has

studied vibratory motion generally. Solid friction sometimes
it worse, fluid friction as of a dash pot greatly destroys it.

The governor can only produce effects during the admis-



[illegible]

steam to the cylinder, consequently for the prevention of
changes of speed we must depend upon the inertia of the fly-

**98. To study any centrifugal governor,** Figs. 142

for example, what we have to do is to find the equal forces $F$, Fig. 146, which (if the balls were not rotating) would just keep the balls in that particular position. We can calculate this (except what is due to friction) if we know the weights and shape of all the parts. It is an excellent exercise for students to find this force experimentally.

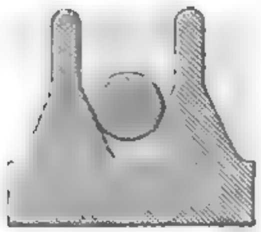


FIG. 145.—GROOVE FOR ROPE DRIVING.

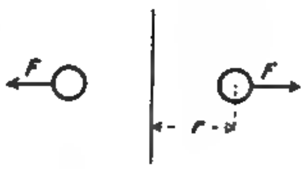



FIG. 146.

Thus in a particular governor the weight of each of whose balls was 6 lbs., the distance which I call $r$, Fig. 146, was carefully measured at the same time that the two equal forces $F$ were exerted horizontally out from the axis by means of two spring balances. The first set of readings was taken when the balls were being overcome and were pulled out from the axis farther and farther; the second set when the balls were moving inwards and overcoming the spring balances.

| | Values of $r$ in feet. | Values of $F$ in pounds experimentally found. | Speed at which the centrifugal force is just $F$. |
|---|---|---|---|
| balls pulled out | ·3 | 54 | 297 |
| | ·4 | 73 | 299 |
| | ·5 | 96 | 306 |
| balls going in | ·5 | 92 | 300 |
| | ·4 | 70 | 293 |
| | ·3 | 52 | 291 |

It is essential that even a beginner should understand clearly that centrifugal forces must be just equal to the values of $F$ when the balls are just going out or in for their various positions. The numbers in the third column are the speeds at which these centrifugal forces would be produced, and they are easily calculated[1]

[1] It is easy to show that a body of $w$ lb. making $n$ revolutions per minute if its centre revolves in a circle of radius $r$ feet, has a centrifugal force in pounds, of the amount $wrn^2 \div 2937$. Let the student take this up as an easy exercise. Hence if the centrifugal force is equal to $F$, $n = \sqrt{2936\, F/wr}$. In our case $w = 6$ lbs., and it

Observe the calculated speeds. We see that if the speed is 29 the balls will still tend to fall nearer even when $r$ is so little as ·3 if the speed is 306 centrifugal force will just be able to cause th balls to move out beyond $r = \cdot 5$. In fact, for all conditions of thing for this range of motio whether centrifugal force i being overcome or is ove coming, the limits of spee are 291 and 306.

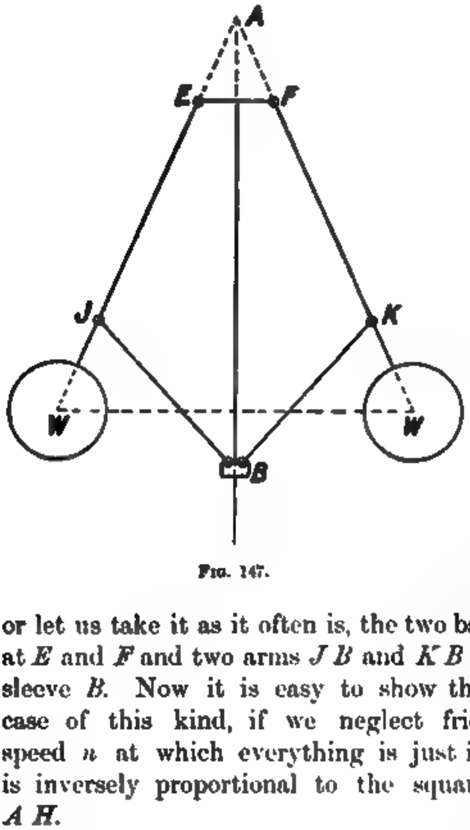


FIG. 147.

The experimental num bers are, however, a littl misleading, because the fri tion of the mechanism i always very much less whe the engine is running tha what it is in such an exper ment.

**99.** I have said that w can *calculate* such a set o numbers as are given i column 2, and therefore th speeds of column 3. Thu for example, in the pendu lum governor of Fig. 14 or let us take it as it often is, the two balls, Fig. 147, hung from pin at $E$ and $F$ and two arms $JB$ and $KB$ lifting the sleeve $B$. Now it is easy to show that in any case of this kind, if we neglect friction, the speed $n$ at which everything is just in balance is inversely proportional to the square root of $AH$.

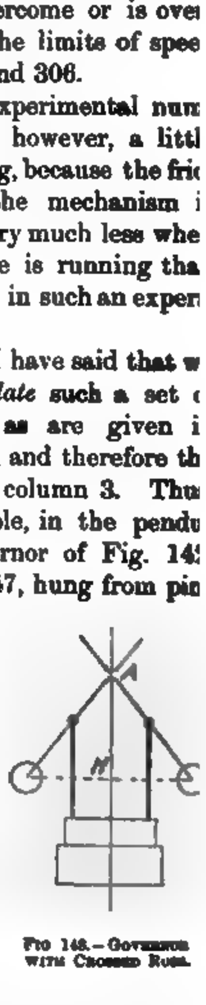


FIG 148.—GOVERNOR WITH CROSSED RODS.

The point $A$ is found by producing the arms $WF$ and $WE$ to meet the axis, and $H$ is on the same level as the centres of the balls. But if the balls fly out a little, $A$ falls and $H$ rises, and hence for a double reason the distance $AH$ diminishes. Hence for quite a small range of

is easy to see that $n = 22{\cdot}1\sqrt{F/r}$. This is the formula from which the speeds i column 3 have been calculated. It is to be noticed that if $F$ and $r$ are plotted o squared paper, any straight line drawn through the origin as it passes through poin where $F/r$ is constant will represent a particular speed. If the slope of the $F$ curve is greater than that of a radial line there, it means stability.

motion there is considerable change of speed. It is much better to let $E$ and $F$ be close to the axis, or even to be in the axis as shown in Fig. 142. When a more sensitive governor is desired the arms are sometimes crossed as shown in Fig. 148. In this case when the balls go out, $H$ rises but $A$ rises also, and there may be as little change as we please in the speed, for quite different positions of the balls. Indeed, it is evident that we may go beyond the limit and have a governor the balls in which go further out as the speed is lessened.

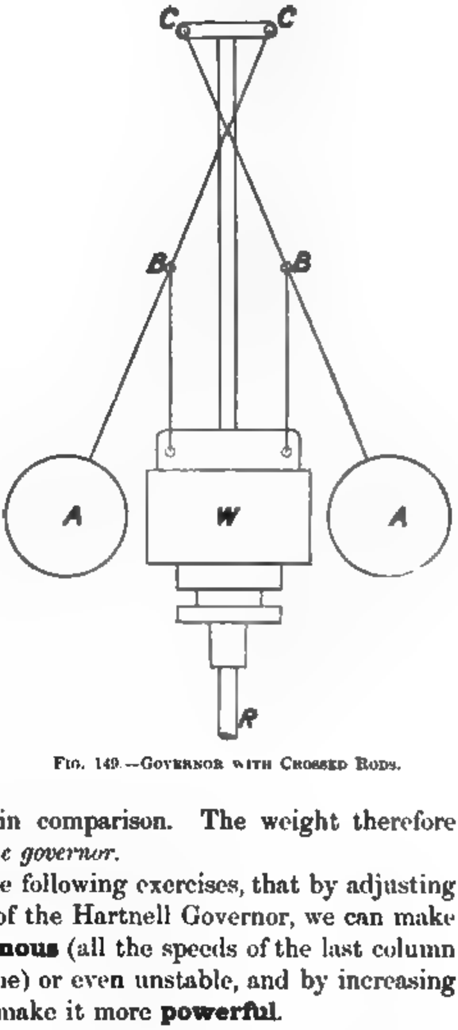


FIG. 149.—GOVERNOR WITH CROSSED RODS.

In the Watt or Pendulum Governor of Figs. 142 or 147, if there were no friction, there would be no virtue in the load $W$. $W$ is useful because it is necessary with it to have the centrifugal and resisting forces ever so much greater, and therefore the forces of friction in the gear which must be moved, become quite inconsiderable in comparison. The weight therefore gives what we call *power to the governor.*

It is easy to show, as in the following exercises, that by adjusting the initial push in the spring of the Hartnell Governor, we can make it more or less nearly **isochronous** (all the speeds of the last column of the table, page 169, the same) or even unstable, and by increasing the stiffness of the spring we make it more **powerful**.

**100.** *Loaded Watt Governor.* EXERCISE. In Fig. 142 the pins above and below being supposed to be in the axis and the arms each of length $l$, the distance of each $w$ from the axis being $r$; $W \pm f$ being the axial load, including much besides friction: it is easy to see that

$$F = (W + w \pm f)\frac{r}{\sqrt{l^2 - r^2}}$$

A student will find it very interesting to take numerical examples, plotting $F$ and $r$ and dealing with the curve or with the numbers as described above.

EXERCISE. Take $w = 3$ lbs., $f = 1$ lb., $l = 1$ foot, find the limiting speeds, if the limiting positions of the balls are $r = 0{\cdot}45$, $r = 0{\cdot}55$ feet. Let this be done when the loads $W$ are 0, 10, 50 and 100 lbs. The answers are given in the table. I also give the fluctuation of speed which is the range of speed divided by the average speed.

| Values of $W$. | 0 | 10 | 50 | 100 | Unloaded frictionless governor. |
|---|---|---|---|---|---|
| Highest speed . . . | 68·45 | 128·1 | 251·7 | 349·2 | 59·30 |
| Lowest speed . . . | 46·82 | 114·8 | 238·7 | 334·4 | 57·35 |
| Fractional fluctuation of speed . . . . | 0·3753 | 0·1095 | 0·0530 | 0·0433 | 0·0334 |

As we see that $n = \frac{60}{2\pi}\sqrt{\frac{g}{h}}\sqrt{\frac{W + w \pm f}{w}}$ where $h$ stands for $\sqrt{l^2 - r^2}$ we had better calculate the first part $\frac{60}{2\pi}\sqrt{\frac{g}{h}}$ at once; this gives evidently the limiting speeds for an unloaded frictionless governor, or 59·30 and 57·35 revolutions per minute; these multiplied by $\sqrt{\frac{W+4}{3}}$ and $\sqrt{\frac{W+2}{3}}$ give the limiting speeds of the loaded governor with friction.

EXERCISE. Prove that the constant load $W$ on the Watt Governor is better than the load produced by a spring.

To do this it is only necessary to remember that with a spring, $W$ will become greater as $r$ increases; hence in calculating the numbers of such a table as the above, take $W$ a certain amount too much for each of the higher speeds, this will evidently produce a greater fluctuation.

When the balls are connected with the weight in a *more complicated* fashion it is easy to arrange that the action is as if $W$ diminished when $r$ increases, and in this case it is easy to approach isochronism or even instability. There are several governors on this principle.

**101.** EXERCISE. Assuming for ease of calculation that in the *Hartnell Governor*, $w$ (the whole mass may be supposed to be at $w$) moves out not in the arc of a circle, but horizontally, show that we can get any amount of power and sensitiveness, stability or instability.

It is evident that $F = a + br \pm f$ where the constant $a$ depends upon the amount of tightening up of the spring and the weight of the gear; $b$ is proportional to the stiffness of the spring, and $f$ represents friction. Hence

$$n = \sqrt{\frac{2936}{w}\left(\frac{a \pm f}{r} + b\right)}$$

Take $w = 3$, $f = 1$, and find the greatest and least speeds if the greatest and least values of $r$ are 0·45 and 0·55.

I take two cases, a stiff spring, $b = 200$, and a weak spring, $b = 10$; I take also various amounts of tightening up. Increasing $a$ means increasing the initial compression of the spring. It will be noticed that if $a$ is 0, it means

that the push on the stiff spring when in the innermost position of the balls, or $r = 0{\cdot}45$ is 90 lbs., and in the case of the weak spring 4·5 lbs.

Algebraically, neglecting friction, it is evident that

$$\frac{dn}{dr} = -\frac{1468\,a}{wn\;r^2}$$

so that for isochronism $a$ must be 0, and for stability $a$ must be negative. But when there is friction, such tables of numbers as these, easily worked out even by elementary students, ought to be studied—

| | For stiff Spring, $b = 200$. | | | | | | |
|---|---|---|---|---|---|---|---|
| Values of $a$ . . . | 20 | 1 | $-1$ | $-2$ | $-10$ | $-20$ | $-80$ |
| Highest speed . . | 482·9 | 446·5 | 442·5 | 440·4 | 424·0 | 402·5 | 235·0 |
| Lowest speed . . | 487·0 | 442·5 | 437·5 | 435·0 | 414·6 | 387·4 | 139·9 |
| Fractional fluctuation of speed . | $-0{\cdot}008$ beyond stability | ·0090 | ·0116 | ·0123 | ·0231 | 0·382 | ·507 |

| | For weak Spring, $b = 10$. | | | | | |
|---|---|---|---|---|---|---|
| Values of $a$ . . | 3 | 2 | 1 | $-1$ | $-2$ | $-3$ |
| Highest speed . . | 130·0 | 123·0 | 115·5 | 98·9 | 89·5 | 79·0 |
| Lowest speed . . | 118·9 | 109·4 | 98·9 | 73·8 | 57·1 | 33·0 |
| Fractional fluctuation of speed . | ·0892 | ·1170 | ·1584 | ·290 | ·443 | ·821 |

**102.** The static theory of governors which I have given must suffice for my readers for the present. A satisfactory general dynamic theory does not yet exist, although there are elaborate French and German treatises on the subject, and yet it seems to me that if a scientific engineer were to study the matter he would not find it difficult to create a satisfactory theory. It would deal with the solution of two differential equations.

1. The statement that (keeping to the letters of Art. 100) if $t$ is time, and if $2w'/g$ is the whole effective inertia of balls and gear when the balls move out radially, and if $2c\frac{dr}{dt}$ is a fluid frictional resistance.

$$\frac{w}{g}ra^2 - F = \frac{w'}{g}\frac{d^2r}{dt^2} + c\frac{dr}{dt} \quad \dots\dots\dots\dots \quad (1)$$

2. At the angular velocity $a$ suppose that there would just be equilibrium, if each ball were at the axial distance $r - x$, the actual distance being $r$. Let the method of regulation be such that there is a torque acting upon the engine, which is, say, $2\beta\psi(x)$. As a simple case we might take this as proportional to $x$, say $2\beta x$. Let the whole momentum of the engine be imagined gathered in a fly-wheel on the spindle of the governor, of moment of inertia $2I$. Then

$$\frac{d}{dt}\left\{\left(I - \frac{w}{g}r^2\right)a\right\} + \beta\psi(x)r = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The solutions of these equations are easy enough for the governors of Figs. 142 and 143. I have sometimes given them to students, but in truth the practical problem has too little in common with this. In the first place part of this suits only a steam turbine, to which one time for regulation is the same as another. Secondly, it is only in electromotors that we have the right to assume that when a regulating device is moved the regulation begins almost immediately. In truth we want $2\beta\psi(x)$ to be a function in which there is a time lag. Thus $x$ is some function of the time; let $\psi(x)$ be called $\phi$ $(t)$, then an approximate solution would be obtained by taking the quickening torque to be, not $2\beta\phi$ $(t)$, but $2\beta\phi(t - m)$ where $m$ is a constant, the amount of time by which the actual regulation lags behind the motion of the governor balls.

I made an attempt myself some time ago to form a theory on these lines, but I had not leisure to finish it, nor can I now recall any useful part of it to my memory.

**103.** The balls of even the most powerful governor must alter in position if the gear is to be altered, and it is evident that it cannot maintain an absolutely constant speed. For very perfect governing we let a governor with the very smallest motion of its parts command some other agency to shift the gear. Such a **relay governor** may command the movement of great sluice valves of water wheels; it acts as if by pulling the trigger of a gun, or like Von Moltke of the German Army. A common plan is to let it shift the valve which admits steam to an auxiliary steam engine which really does the work.

**104.** If the governor, instead of throttling the steam, were **to lift and lower a link** of the Stephenson link motion, it would govern the engine in quite a different way. This method is very seldom employed.

But what is very often done is to let the governor affect in some way the point of cut off. To explain how this may be done I will first describe a slide valve which has an **independent cut off valve** on the back of it. In Fig. 150, $HLD$ is an ordinary slide worked in the ordinary way by a single eccentric or by a link motion. It is the part from $D$ to $H$ which is exactly like a simple slide, but the valve is made larger so that instead of terminating at $D$ and $H$, $D$ and $H$ are merely two openings in a larger casting. Notice, however, that $DH$ has outside lap and inside lap as before, and so

long as steam is allowed to exist at $D$ and $H$ and exhaust at $L$, this is an ordinary slide valve. The eccentric to move it is usually arranged to cut off at about $\frac{3}{4}$ of the stroke. When a link motion drives it, the motion is never used in intermediate gear, it is always either in full forward or full back gear.

As a matter of fact, we rely upon $HLD$ only for admission, release, and compression. The edges $X$ and $Y$ may cut off in a sense, but it is shutting the stable door after the steed is stolen; they only cut off the port $A$ or $C$ from the steam spaces $D$ or $H$, but in truth $D$ or

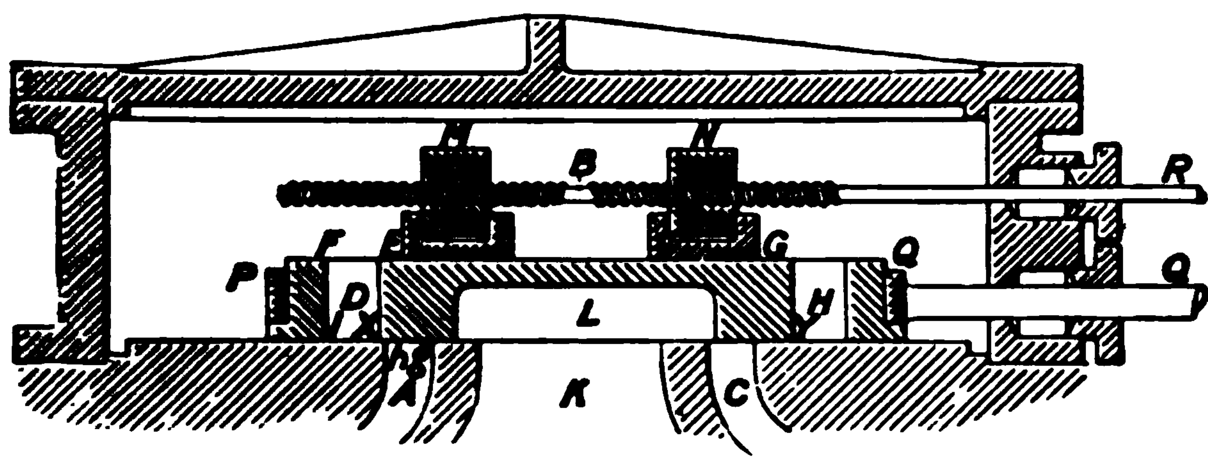



FIG. 150.—INDEPENDENT CUT OFF VALVE.

$H$ has had its own steam cut off previously by the block $ME$ or the block $NG$. The blocks $ME$ and $NG$ are worked by a second spindle, $BR$, driven by an eccentric of its own.

If only one main eccentric works $PQ$, the main valve, the cut off eccentric is about 90° ahead of the main eccentric. This is the best angle. But if $PQ$ is worked by a reversible link motion, the cut off eccentric is symmetrically placed relatively to the fore and back main eccentrics.

I do not think that this motion can be understood by beginners unless there is a model.

With a model one interesting exercise is to show how the cut off alters when we alter the distance from $E$ to $G$. The rod $B$ has right and left-handed screw threads, so that if it is turned, as it may be by the engine driver or by the action of a governor, the cut off is altered.

Another exercise, more important, is to show how the cut off alters when we alter the travel of the cut off valve. This is the usual way in which the governor varies the cut off, and in Fig. 143 I show how a Hartnell Governor lifts the rod $JL$ and so lifts the valve rod $KL$, the block $K$ sliding in the slot of a link $MN$. $M$ is the fixed point of the link and $P$ is the cut off eccentric rod, and hence when $K$ is lifted the travel of the cut off valve is lessened.

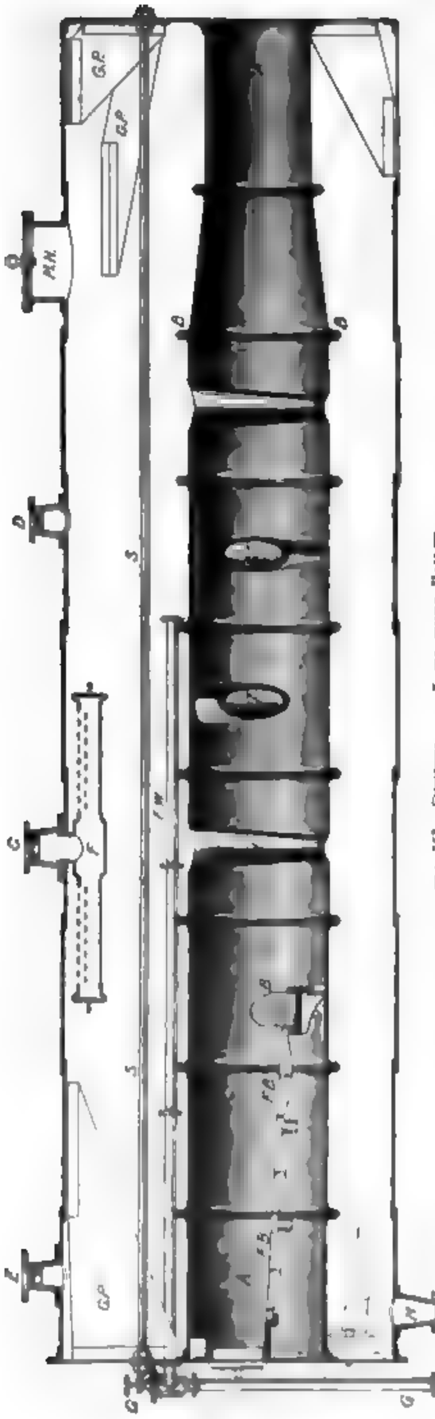



FIG. 153.—SECTION OF A LANCASHIRE BOILER.

from two to three t
water per hour. Al
of it are easily access

**The shell** is for
plates of iron or stee
single or double r
joints; a single plat
20 feet long, 3½ fee
forms one of the
shown in Fig. 151.
cylindric vessel is tv
likely to burst sidev
endwise (even neg
the extra endlong st
due to flues and sta
straight side seams
break joint along the
are much more st
made than the c
seams. Thus if th
seams are double
butt-joints, with two
ing plates (see Fig
the circular seams are
or double riveted la
like Figs. 156 or 15
side seam is exposed
flue gases.

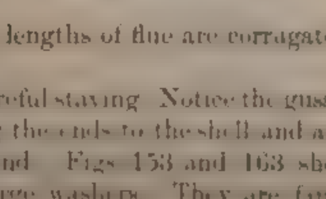
The holes for th
are bored out of the f
plates (which ar
turned up on their
and the flues are f
either by stiff angle i
shown in Fig. 151,
flanged ends. **The**
formed of lengths o
welded up so that t
no visible straight
notice that even th
**loway water tube**
are welded into t
rings without visibl

...at advantage, not merely because there is less fear of ... also because riveted joints in any flue are apt to get ...er the grate we are particularly anxious to avoid seams

...ig. 152 Fig. 153

...Lancashire boiler showing gussets G.P. joining of flues to ends, two stays and Galloway Tubes.

...ces. The ends of the rings are flanged and riveted ...h a ring of plate between which is good for caulking ...). These flanged joints stiffen the flue against a crumpling ... kind of collapse and ...y much better than the ...gs 159, 160, and 161. ... it is well that the ...rings in the two flues ...e close together, as the



... 154.—Fire-bars.

Fig. 155.—Double Riveted Butt-Joint, Two Cover Plates.

...dy small. Sometimes the lengths of flue are corrugated ...

...at parts of boilers need careful staying. Notice the gusset ...gs 151 and 152 fastening the ends to the shell and also ...stay bolts from end to end. Figs 153 and 163 show ...lts are fastened with large washers. They are fairly ... and 14 inches above the flue. All so necessary as

these long bolts may be, some engineers think that they ought not be used, as they unduly prevent bulging of the ends. In the figu the gusset pieces come down too closely on the stays, giving t

FIG. 156.—SINGLE RIVETED LAP-JOINT.

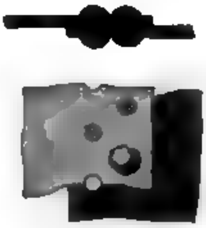


FIG. 157.—DOUBLE RIVETED LAP-JOINT.

much stiffness. The ends are usually $\frac{5}{8}$ths thick for a worki pressure of 100 lbs. per square inch. The ends ought not to thicker than is actually necessary for strength, because it is go that they should yield easily. The ends of the shell are in t halves welded together, turned up on the edge, bored out for the flu

Probably the most important thing to consider in boilers is t effect of **unequal temperatures** in the various parts. Soon after t

FIG. 158.

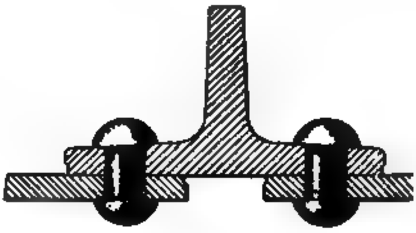


FIG. 159.

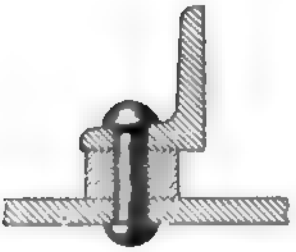


FIG. 160.

FIG. 161.

fires in a Lancashire boiler are lighted, the front end will be foun to bulge or *breathe*, as it is called, as much as $\frac{1}{8}$th of an inch, al the flues are found to *hog* or rise in the middle as much as 0·5 inch.

EXERCISE. In a 30-foot boiler the flues are at 200° C., the shell is at 100° C. What is the difference in the amounts of expansion from 0° C. if both are free to expand?

*Answer.* ·86" − ·43" = ·43".

It is therefore very important in designing any boiler, to arrange that any part may become larger or smaller without unduly stressing

FIG. 162. CORRUGATED FLUES.

itself or the other parts. It is for this reason that many makers say that 30 feet is the maximum length for a Lancashire boiler. Note that to have an external angle iron at the front allows more spring. We cannot have one at the back, as furnace gases would hurt it. Modern boilers are distinguished by possessing this **thermal springiness**; corrugated flues, flanging of plates in general, and in particular the flanging of the flue rings of Fig. 151. The bent tubes of the Thornycroft boiler (Fig. 209) conduce to springiness. Of course we prevent unequal heating as much as possible. For example, note how the cool feed water enters by the long pipe *F W* (Fig. 151), as it does also in the marine boiler, so that it cannot produce local rapid cooling of any part of the boiler. The thermal straining of the marine boiler of Fig. 205 shows itself most by the leakage of the tubes in the combustion chamber under forced draught.[1]

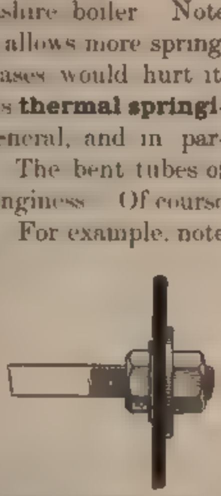


FIG. 163. END OF LONGITUDINAL STAY.

The theory of strength of a shell really depends upon the pulling force being uniformly distributed round any plane section that may be imagined. When we make a hole and especially when we make a large hole (this is why we like all fittings to have separate mouth pieces), care must be taken to so strengthen the plate round the hole, that it may be able to resist the quite different sort of forces introduced because we have a mouth-piece for some kind of fitting, instead of

[1] When the tightness at an iron joint depends on squeezing, a red heat produces an annealing action, and the elastic pressure is apt to disappear; hence tubes leak.

a continuous piece of boiler plate. Fig. 165 will show the sort of precautions taken. A single row of rivets may suffice when an opening is small, but a double row is necessary when the opening is large.

**109. The dome,** sometimes wrongly used on Lancashire boilers (because it is expensive, weakens the shell, tends to leakage, and is

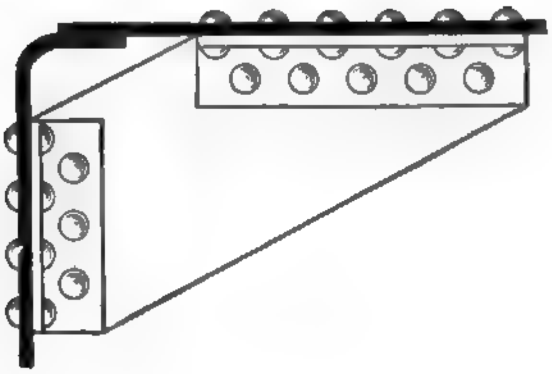


FIG. 164.—GUSSET PIECE.

unnecessary, or unhandy when the boiler is carried or is being turned round on its seat to be mended), as well as on locomotive boilers, needs special care. Some makers do not make a large hole, but merely perforate the plate underneath the dome with many holes.

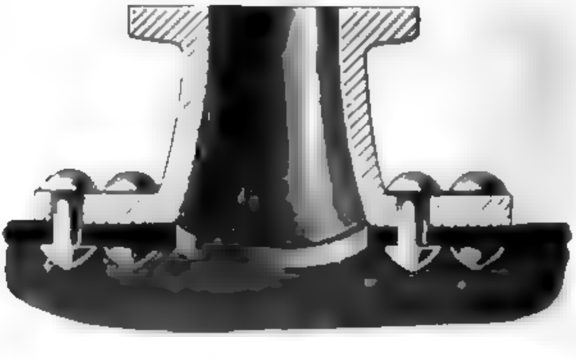


FIG. 165.—SEATING BLOCK.

To attach any fitting we must have suitable **fitting or seating blocks** like Fig. 165, permanently riveted to the shell, and bolt the safety valves, stop valve, man-hole door, &c., to them on truly planed faces. Such seating blocks are never now of cast iron, nor indeed of malleable cast iron, for although this lends itself to the riveting process, and is sufficiently malleable for the purpose, we can now obtain

forgings or steel castings, which are much stronger. Also we find there is less tendency to leakage, and leakage leads to corrosion. A man-hole fitting of the most approved design is shown in Fig. 166. The boiler, Fig. 151, has no dome. *F* is a horizontal pipe which perforated along its upper surface, and dry steam may be drawn away through the **stop-valve** attached to *C*. The water is in ebullition so the steam space has much spray in it, and domes or other contrivances are adopted so that steam may be drawn off without priming at some place where there is almost no spray. Priming means the carrying off of water with the steam. The steam pipes, however well covered, will allow some more steam to condense, and hence a **separator** like Fig. 3 is interposed before the steam gets to the cylinder. A pound of high pressure steam is produced with less radiation than one of low pressure steam, because it occupies a smaller volume. Priming is not only excessively wasteful of energy, but it may cause fracture in the cylinder. In boilers powerful for their size priming leads to unexpected shortness of water. It is produced when there is high **water in even a well-arranged** boiler if there is too sudden a demand for steam with rapid combustion, and especially if there is much scum on the surface of the water. The only immediate remedy is to check the demand for steam, check the fires, and blow off scum if necessary. When priming is less serious and as it is very troublesome to measure the amount of it, it is usual to drain the cylinder. What is called superheating is in many cases only the removal by heat of the wetness of the steam.

Fig. 166.—Man-hole Door.

Fig. [illegible] ... with Double Seat Valve, ... sometimes used as a Stop on ... Valve.

**110. The main steam pipe** like the feed pipe common to a number of boilers and connecting them with the engine ought not to be straight so that there may be elastic yielding to expansion and

contraction. This is better than having an expansion joint or expansion diaphragms. Parallel U joints are used greatly in electric light stations. The stop valve of each boiler admits steam to the main pipe through a junction piece, which ought to drain down to the main pipe, else it may become filled with condensed water when its boiler is not working. Condensed water produces *water hammer* effects which may cause fractures in pipes.

Figs. 167 to 171 show forms of stop valve which may be used on the fitting $C$ in taking steam from the pipe $F$, Fig. 151. The valve is adjusted in position by the handwheel, the screw, and nut. Notice that the nut, which is often out of sight, is much better in sight on a sort of bridge. The stop valve, Fig. 168*a*, used in marine boilers, and the regulator, Fig. 64, used in locomotives, ought also to be studied. The double beat or equilibrium stop valve of Figs. 167, 170 and 171 require no explanation. There is very little force required to open it or to close it.

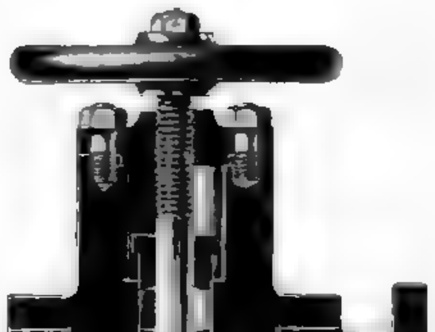


Fig. 168.

$M H$, Fig. 151, is the manhole, to allow a man to get inside the boiler to clean it. The mouth is one forging and is riveted to the shell with a double row of rivets as in Fig. 166. The boiler is given a "hang" of an inch or two to the front end to ensure complete drainage, and $M$ is the mudhole (also with a strong mouth-piece, external or internal) placed at the front so that the boiler may be completely emptied.

Fittings that are frequently in use are attached to the front of the boiler. **The feed** is admitted at $F W$, Fig. 172, at 4 inches above the level of the furnace crowns, so that should the feed valve leak the boiler water cannot be syphoned away; the feed drops from the dispersing pipe $F W$, Figs. 151 or 173, 12 feet long, perforated for the last 4 feet, in such a way that there is not much local cooling.

The scum tap $SP$, Fig. 172, discharges from the sediment catcher. Two **glass gauges,** $GG$, Figs. 172, 174, 175, show the height of the

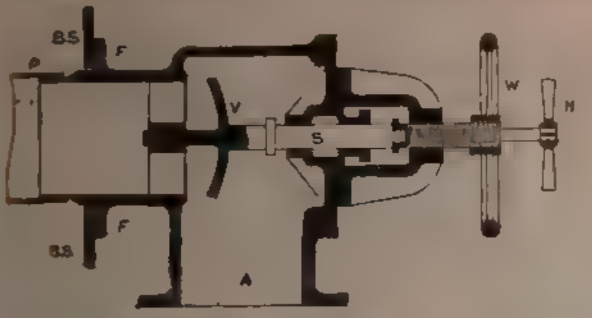



Fig. 168a.—Marine Boiler Stop Valve.

P is the horizontal steam pipe inside boiler, usually in two branches with many holes in the upper surface, taking steam without priming. The handle H is merely to turn the valve V on its seat. The handwheel W closes the valve and shuts off the boiler from the others. When as shown the valve keeps open only so long as the boiler pressure exceeds that in the pipes, it will shut if the boiler is receiving steam from the pipes. Note the stuffing-box. There are stop valves of the same kind on the supply to auxiliary engines.

water. There are many forms in the market. When open above and below the water level is visible in the glass tube. The tube ought to be easily replaceable when broken. The plugs $A$, $B$ allow of a wire entering to clean the passages. The stand-pipe $P$ is of gun metal, sometimes it is not used. The lowest tap, $C$, allows of blowing off. In modern boilers all cocks are packed inside with asbestos. In marine boilers the three cocks may be opened from the stokehold floor. Usually three common taps (**test cocks**) are also provided (sometimes on the standpipe, usually on the boiler shell), one above, one below, and another just about at the usual water level. Much judgment is necessary as to the water level in a marine boiler when a vessel has a list to one side and also on a locomotive on a steep incline. $PG$, Fig. 172, is a **Bourdon pressure gauge** shown also in Fig. 176. Sometimes two are used on each boiler. By turning the handle the steam pressure is applied to the tube $EDC$ whose section is shown at $A$. Such a tube tends to straighten itself

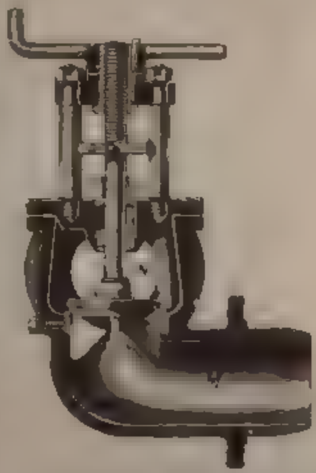
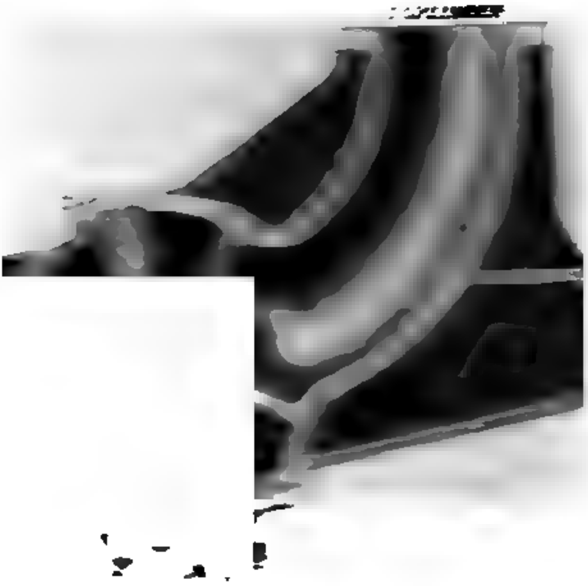
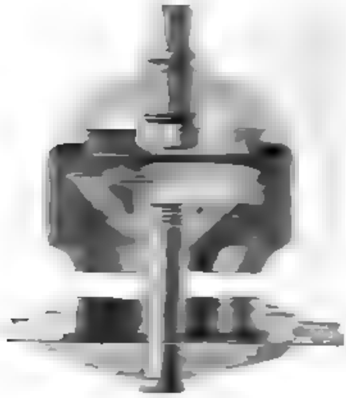


Fig. 169.





is nearl
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Fig 172 Front of Lancashire Boiler

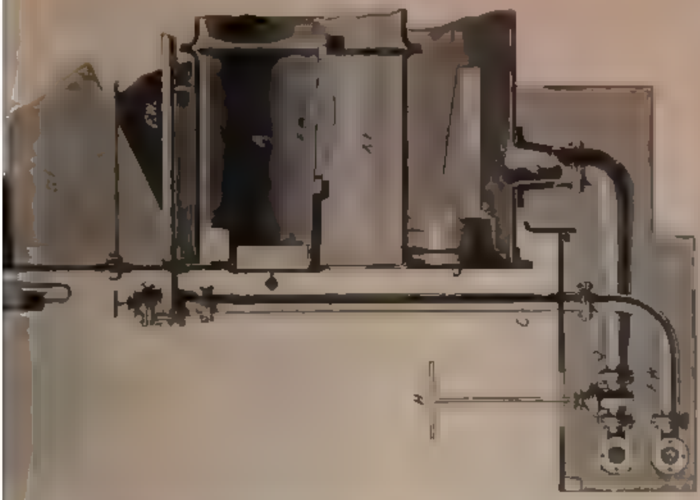


Fig 173

because this allows it to become larger in volume, and in do so its closed free end *E* pulls a link, and by the spur sector a

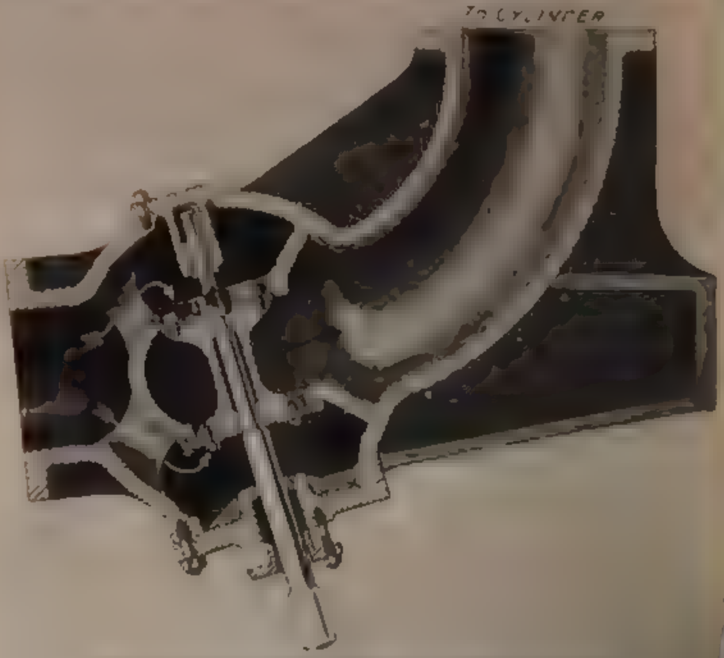



FIG. 170.—MARINE ENGINE REGULATOR. A DOUBLE BEAT VALVE.

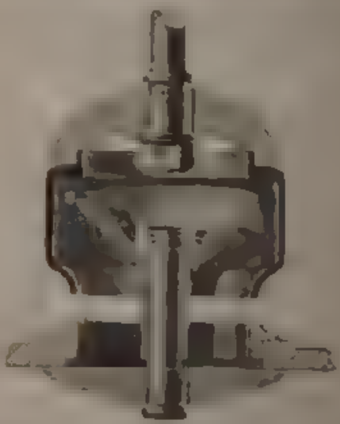


FIG. 171.—DOUBLE BEAT VALVE.
Sometimes used as a stop or regulator valve.

pinion turns the pointer, whose angular motion is nearly pro- tional to the pressure (above atmosphere). Such gauges o

Fig. 173.

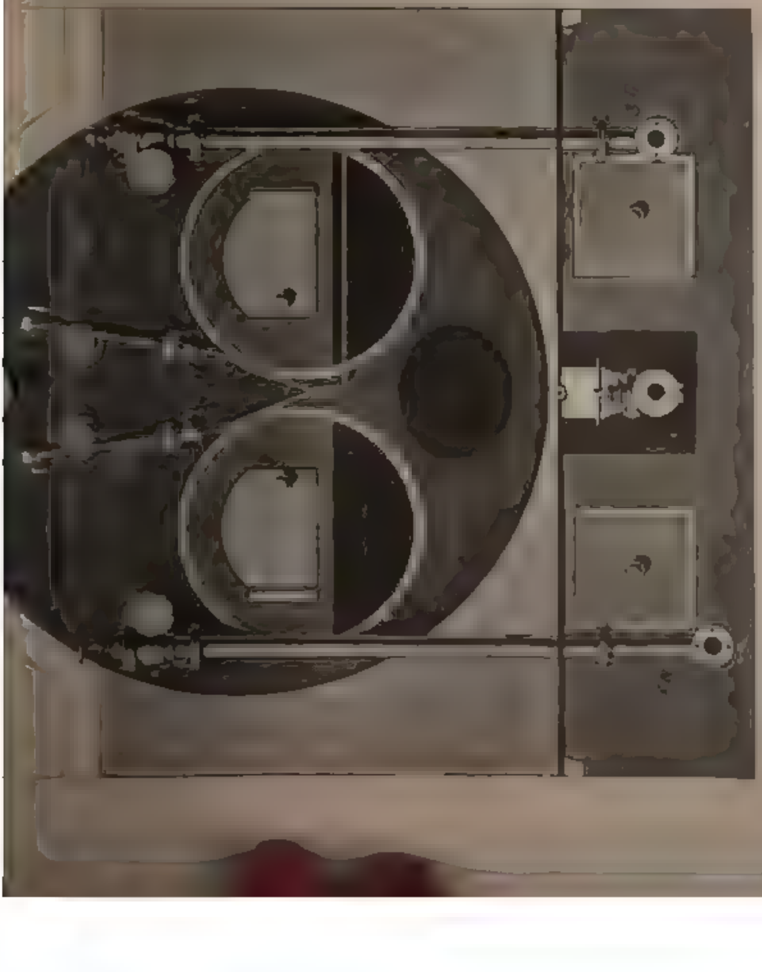


Fig. 172.

Front of Lancashire Boiler

*Note* that with all these fittings at the front of the boiler the stoker, without climbing any ladder, sees the height of the water, and the pressure of steam; his blow-out tap is handy, behind him is his coal and his damper balance. If he has been properly encouraged, he is really a skilled workman and he keeps the boiler-room perfectly tidy-looking, the floor clean, no evidence of leaking water, the brass and other beading on the furnace mounting parts bright.

[illegible]

The flooring plates ought not to butt up against the boiler; they ought to be easily lifted so that the hearth-pit may be open all along a range of boilers. In it is the main feed pipe and the discharge pipe for blowing out scum. The pit may be 3 feet wide, 2½ deep. The flue doors open into it. The brickwork is shown in Fig. 173, set back 6" in front to be clear of the angle iron. The front wall is recessed round the blow out elbow pipe, leaving it free in case of settlement.

**119. The best covering** for a stationary boiler is an arch of brickwork with a 2 inch clearance from the shell. This space may [illegible] or other non-conducting material. [illegible] fittings exposing the [illegible]

[illegible]

The **low water safety valve** needs frequent testing and an examination every time the boiler is cleaned. It is fitted to $D$, Fig. 151. The valve $2\frac{1}{2}''$ diameter, is loaded directly by a spindle with a weight, also by another weight and negatively by a float through a lever. When the water is too low the weight of the float is greater and causes less pull on the valve spindle, and the valve lifts and gives an alarm. The valve $V$ if it lifts, lifts another valve $V'$ of 5″ diameter, but $V'$ may lift independently of $V$, being a lever loaded safety valve. It is important that even a skilled workman may not have it in his power to tamper with safety valves.

Fig. 182 shows a **lever safety valve,** well known to everybody. Note that the seat is flat and very narrow.

EXERCISE. The valve has an area of about 5 square inches. The horizontal distances are $CD = 3''$, $DG = 10''$ ($G$ is above the centre of gravity of the lever and the lever weighs 6 lbs.). $E$ is above the centre of gravity of the weight which is 60 lbs. The valve, &c., weigh 7 lbs. Find the distance $DE$ if the valve is to lift at 120 lbs. per square inch above atmosphere. Repeat the calculation for the pressures 110, 100, 90, &c.

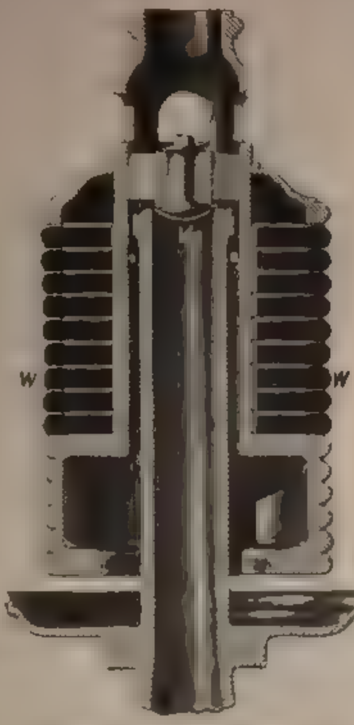



Fig. 181.—Deadweight Safety Valve.

*Answer.* 28·65, 26·15, 23·65, 21·15, &c. Hence the marks showing the positions of $E$ on such a lever if it is graduated are $2\frac{1}{2}$ inches apart for every 10 lbs. difference in pressure.

Weights, whether direct or through levers, are replaced by springs when for locomotive and marine safety valves. Now **when a safety valve opens** and steam is escaping the total force exerted by the steam may be greater or may be less than when the valve was closed, depending upon the shape of it. It does not seem to be sufficiently known that by properly shaping the under surface of a valve and especially by extending it beyond its seat, it is easy to get

greater lifting force when the valve is open. Some engineers have for long been applying this principle. Generally the lifting force is less if the valve is open. For example, even in weighted safety valves it has been found that when set to lift at 60 lbs. per square inch, even twice the lifting pressure was needed to keep the valve sufficiently open for the escape of steam. This was probably too small a valve for the size of boiler. It is evident that a number of small valves must be better than one large one because there is more opening for the same lift.[1] In well-proportioned dead weight safety valves it is usually expected that if a pressure of 60 lbs. per square inch opens the valve, a pressure of 70 lbs. will keep it sufficiently open for the escape of all the steam produced. It would be better if the load diminished as the valve opened more and more.

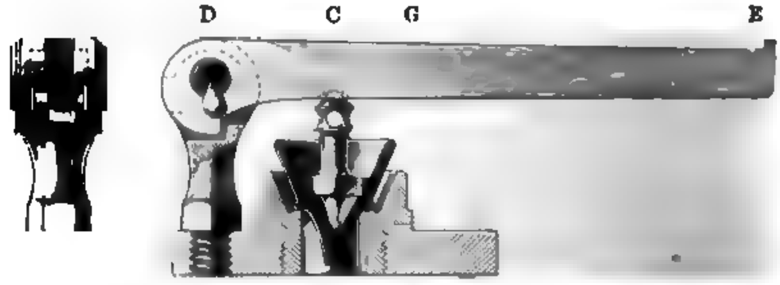



FIG. 182. LEVER SAFETY VALVE.

Unfortunately, when a spring is used, the more the valve opens the greater is the force exerted by the spring, so that the evil is intensified.

Much ingenuity has been displayed in remedying this defect, but in marine boilers reliance is usually placed upon largeness of valves, and using two or three on one valve box so as to get sufficient opening with small lift; also upon the use of so long a spring that a small amount (about $\frac{1}{8}$th of an inch at most) of extra compression produces but little extra force in the valve lift.

Thus the springs are usually compressed axially by an amount equal to [illegible] of a valve when it is closed: the extra force is [illegible] the extra compression. Notice in Fig. 183 [illegible] compressing the springs, and how the com[illegible] by the nuts. Steam escapes into the

[1] [illegible] edge area is $\pi Dl$. If we have two d[illegible] and the sum of their edge areas is [illegible]

waste steam pipe which goes up alongside the funnel. Also notice that the valves may be lifted by the lever $L$ independently, either from the deck of a vessel or the stokehold.

It has been found that a $1\frac{1}{2}$-inch pipe will discharge steam from the most powerful locomotive boiler as fast as it can be generated.

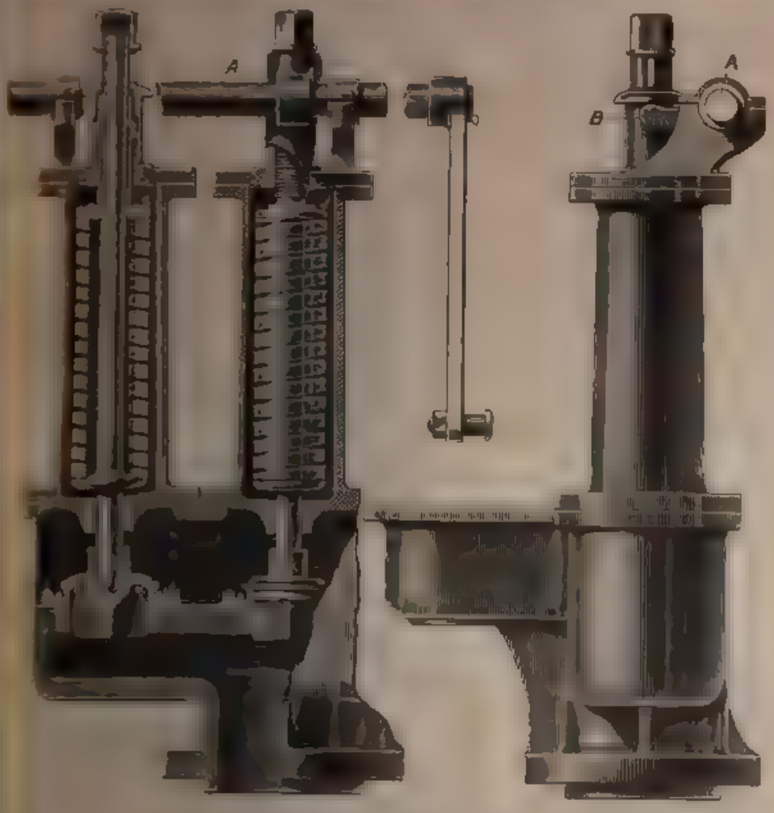



FIG. 183 MARINE SAFETY VALVE.

The kinds of safety valve used on locomotives are the spring lever and the Ramsbottom. In the first we have an ordinary lever safety valve loaded by a spring instead of a weight.

The **Ramsbottom** arrangement is shown in Fig. 184. The two valves at $A$ and $B$ tend to lift equally against the force of the spring. For if $A$ lifts before $B$ the load on $B$ slightly diminishes and the load on $A$ increases, because the point $C$ is at a level below the point $A$. The piece $A$ $B$ is lengthened to enable the driver to try the valves. He is able to diminish the load on either valve, but

not to increase it. In this arrangement there is no compensati the increased pull of the spring as the valves open.

The "Naylor" contrivance for altering the leverage whe valve opens was one of the first methods adopted, and the pri on which it acts is that of subsequent forms of which the

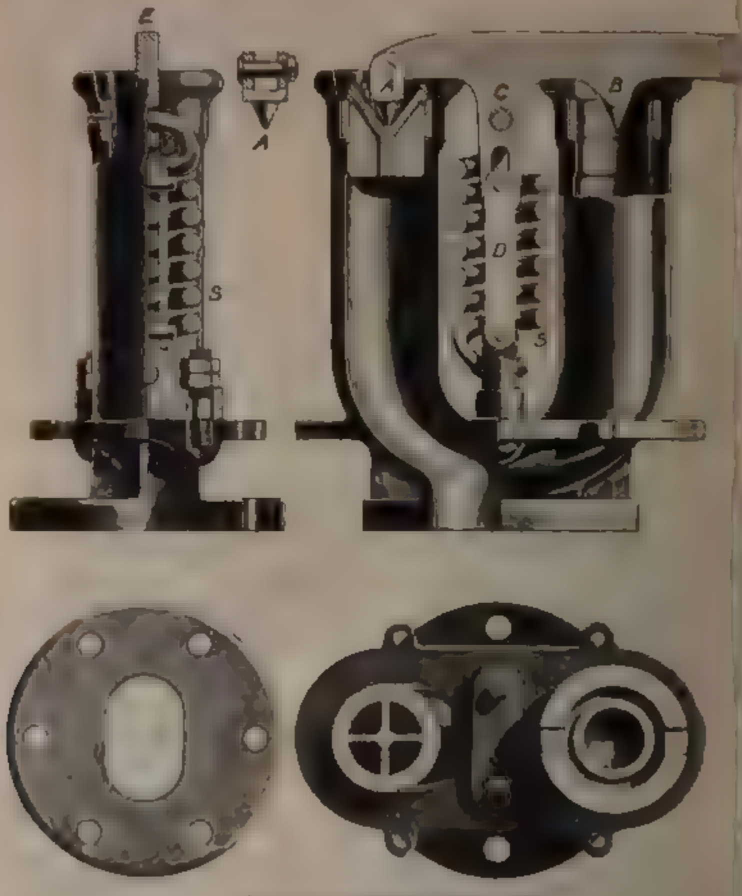



FIG. 184.—RAMSBOTTOM SAFETY VALVE.

many A spring kept the valve pressed down upon its seat t a bent lever and when the valve opened the leverage of the diminished on account of the shape of the lever and therefo tendency of the spring to keep the valve closed did not get although the pull in the spring itself might be greater; inde tendency to keep the valve closed might be lessened as the

pened, depending on the exact lengths and shapes of the two parts of the lever.

This same principle of compensation is used in many other applications of springs—when it is thought necessary to diminish the influence of a spring, as it is more strained. I have myself used the idea in the construction of measuring instruments.

In practice it is found that with ordinary care **regularly inspected** factory boilers almost never burst. Ordinary care involves: 1. Attention to water gauges (never let water level sink out of sight, and often try the cocks), and blow off cocks (sediment in elbow pipes before starting engine, and scum before stopping to be cleared off); never empty boiler when steam is up. 2. Never raise steam hurriedly: in a Lancashire boiler six hours are often given to gradual heating from cold condition. 3. Clean monthly or oftener, removing scale when soft, that is, as the cool boiler gradually empties of water, remove scale about water level. Sweep plates and flues every three months. Leakages ought to be stopped at once to avoid corrosion. Fusible plugs cleaned both on fire and water sides once a month, and the fusible metal renewed once a year. All cocks ought to be examined once a month. 4. Ease and test safety valves and low water alarms every day and never overload. Beware of condensed water before opening a stop valve and open gradually. 5. Use no unknown chemicals for the prevention of scale. 6. At every opportunity raise objections to the admission of oil with the feed water. If oil must be used in the engine cylinder (and it need not be) let it be filtered out of the feed-water.

## CHAPTER XII.

### STRENGTH OF BOILERS.

**113. Strength of Thin Shells.**—In thin-shelled vessels, such as boilers and pipes, subjected to fluid pressure $p$ inside, we assume that the tensile stress $f$ is the same throughout the thickness; so that if $a$ is the area of *metal* cut through at any *plane* section of the boiler, $af$ is the resistance of the metal to the bursting of the boiler at that section. The force tending to cause bursting is $Ap$ if $A$ is the *whole* area of this plane section of the boiler. Hence the law of strength is

$$af = Ap \quad \ldots \ldots \ldots \quad (1).$$

(I.) Thus in a **spherical thin boiler** of diameter $d$ and thickness $t$, if we consider a plane diametrical section, $A$ is $\frac{\pi}{4}d^2$ and $a$ is $\pi dt$, and hence (1) becomes $\pi dt f = \frac{\pi}{4}d^2 p$, or

$$p = 4\, tf / d \quad \ldots \ldots \ldots \quad (2).$$

It is easy to show that there is more tendency to burst at a diametrical section than any other.

(II.) In **a thin tube** of diameter $d$ and thickness $t$

1. Consider a section at right angles to the axis; $A$ is $\frac{\pi}{4}d^2$ and $a$ is $\pi dt$, and hence we get the same rule as for a spherical shell.

(2). Consider a section through the axis and imagine the boiler so long that the strength of the ends may be neglected. If $l$ is the length, $A$ is $ld$ and $a$ is $2lt$, and (1) leads to

$$p = 2\, tf / d \quad \ldots \ldots \ldots \quad (3).$$

Hence *the tendency to burst laterally is twice as great as the tendency to burst endwise.* Also if we study in the same way the tendency to burst at any other section we find that (3) gives the least bursting pressure, and so we use it in calculations.

*Note.*—To prove that the force tending to cause bursting at a plane section of area $A$ is $p\,A$. Let $D\,E$, Fig. 185*a*, be a thin boiler, inside which there is the uniform pressure $p$. The pressure is always greater at greater depths in any fluid because of its weight, but I shall neglect this. The fluid forces are everywhere normal to the shell; what is the resultant of all the forces acting on the part $B\,F\,C$? Now these forces are exactly the same on $B\,F\,C$, Fig. 185*b*. But in Fig. 185*b* the whole boiler consists of the part $B\,F\,C$ and a plane rigid plate $B\,C$, on which the forces are all parallel, so that we can find their resultant. The resultant force on $B\,C$ is its area $A$ multiplied

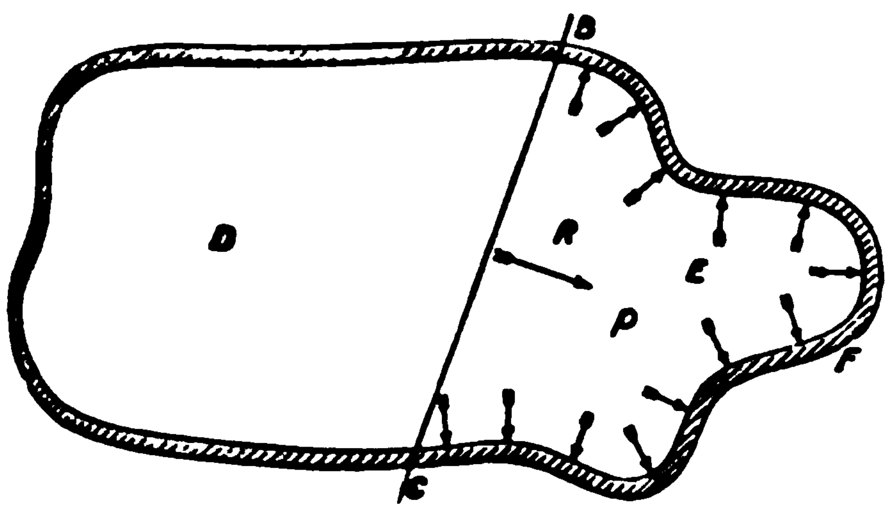


FIG. 185*a*.

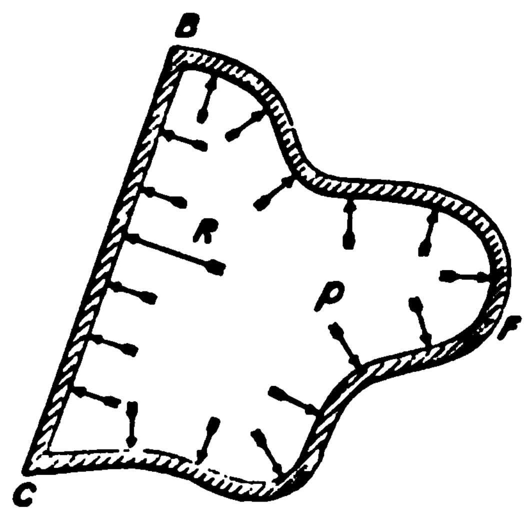


FIG. 185*b*.

by $p$, and we know that this must be equal and opposite to the resultant force on $B\,F\,C$. The principle used in this proof is the fundamental principle of mechanics; Newton's great law (sometimes called three laws) of motion is perfectly easy to understand, and, when understood, applicable to the solution of most complex questions.

If the boiler (Fig. 185*b*) were placed on a truck with frictionless wheels there would be no more tendency to move on a level road (or on any road if we neglect weight) when there is great pressure inside than when there is little. The force due to pressure on any one little portion of the surface balances the forces on all the rest of the surface. Hence it is that if we make a hole there is a want of balance, and our truck will tend to move. When we make a hole anywhere the pressure is no longer the same everywhere because the fluid is in motion, and hence we can only calculate the unbalanced force by knowing the momentum which leaves the vessel per second.

**114.** *Storage Capacity of Cylindric Vessels.*—The volume of the cylinder being $v$, and the safe pressure $p$, we may take $vp$ as proportional to the energy which may be stored. If the diameter is $d$, and thickness $t$, and length $l$, the volume is $v = \frac{\pi}{4}d^2l$. The safe pressure is $p = 2tf/d$. The weight of the metal is $W = \pi dtlw$, if $w$ is the weight of unit volume of the material. The surface of the vessel is $S = \pi dl$. In all cases we neglect the ends. The storage capacity for energy per unit weight of vessel is $\frac{\pi}{4}d^2l\frac{2tf}{d} \div \pi dtlw$ or $f/2w$, so we see that it is independent of the diameter. In tubes of water-tube boilers, in which the surface ought to be great, we want surface $\div vp$ to be great. This is $2/tf$ or $4/pd$. Hence the thinner the tubes are, and if the pressure is fixed, the smaller they are, the more surface they have as compared with their storage capacity for energy; for somewhat similar reasons we need small thin tubes in surface condensers. In cases where energy is stored in hot water and steam (see Art. 123) the rate of loss of energy is proportional to the surface, and so we require thick boilers of large diameter. The best shape, if otherwise convenient, is obviously the spherical shape. Questions of cost, convenience, and danger, modify these general results in their applications.

**115.** Fig. 186 shows some **forms of rivets** before and after the making of the heads. Figs. 155–7 show some joints.

FIG. 186.—FORMS OF RIVETS.

Fig. 187 shows the various ways in which we may imagine a strip of plate or the rivet which corresponds to it to break (1) the rivet breaking in single shear, (5) in double shear.

The diameter of a rivet hole is settled, for plates that are punched, by a variety of considerations, which lead to the rule ($t$ being the thickness of the plate) $d = 1.2\sqrt{t}$. The pitch or spacing of the rivets is settled by the consideration that we may imagine each rivet to correspond to a strip of plate of width $w$ and thickness $t$. When rivets are in double shear $w$ will evidently be just twice what it is in single shear.

In single shear the shearing resistance of the rivet is $\frac{\pi}{4}d^2f'$

the tearing resistance of the strip of width $w$ is $wtf$ if $f$ and $f^1$ are the resistances of the material to tension and shearing. If these are equal, we find $w = \frac{\pi}{4} d^2 f^1/tf$. Draw round each rivet a circle of diameter $d + w$ and let lines come dividing the plate up into strips of the breadth $w$, so that we allot a strip of plate to each rivet. There is more interest in scheming out the proportions of riveted joints in this way than in working common puzzles.

**116. The strength of the joint** ought evidently to be the fraction $\frac{p-d}{p}$ of the unhurt plate, if $p$ is the pitch of a row of rivets; or calling $p-d$ by the letter $A$; $\frac{A}{A+d}$ expresses the relative strength. Students know that when we have only guiding notions like the

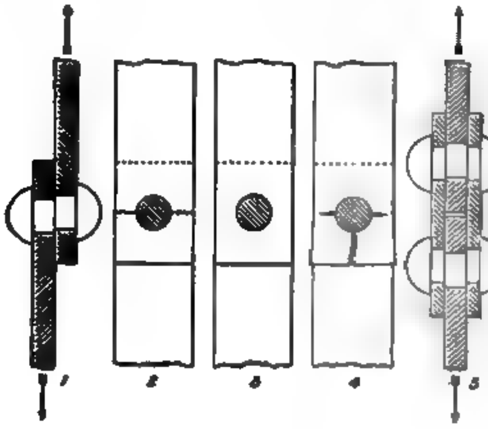


FIG. 187

above we must resort to experiment, and actual measurement shows that instead of $A$ in the numerator of the above fraction we ought to take $kA$ where $k$ is some number. By means of the above kind of theory and the results of numerous experiments made up to the present time by experienced men, the author has been led to the following easy rule for the strength of well-riveted joints. Hydraulic riveting is almost always better than that done by hand. Indeed, steel riveting is hardly ever done by hand because of the greater probability of overheating rivets.

If $t$ is the thickness of the plate, the diameter of each rivet-hole is $d = 1{\cdot}2\sqrt{t}$, the pitch $p = A + d$, and the strength of the

joint $= \dfrac{kA}{A+d} \times$ strength of the unhurt plate, where $k$ is given in the following table:—

| | Iron plates. | Steel plates. |
|---|---|---|
| Single riveted, drilled holes . . | ·88 | 1·0 |
| ,, ,, punched ,, . . | ·77 | 0·9 |
| Double ,, drilled ,, . . | ·95 | 1·06 |
| ,, ,, punched ,, . . | ·85 | 1·0 |
| Treble ,, drilled ,, . . | — | 1·08 |

and $A$ is given in the following table:—

| | | Iron plates and iron rivets. | | Steel plates and steel rivets. | |
|---|---|---|---|---|---|
| | | Drilled holes. | Punched holes. | Drilled holes. | Punched holes. |
| Lap joint or butt joint with one covering plate | Single riveted | 1·20 | 1·47 | 0·9 | 1·08 |
| | Double ,, | 2·22 | 2·66 | 1·7 | 1·93 |
| | Treble ,, | 3·23 | — | 2·5 | — |

All these values of $A$ are to be doubled for butt joints with two covering plates. The distance of a hole from the edge of the plate must not be less than $d$, and when only half-inch rivets are used there is an additional quarter of an inch.

The friction between the plates caused by the contraction of rivets in cooling gives additional strength, which is usually neglected because it is of unknown amount. **Caulking** (inside and outside all joints) is performed by a blunt-edged tool which indents the metal of the edge of one plate into the other; a fullering tool produces a more uniform tight contact of the overlapping parts. Caulking, especially if done with too sharp a tool, may hurt the plate; in any case it alters the surface, and this may induce "grooving."

**A punched hole** is to be called **a drilled hole** if the plate has been annealed or if the hole has been rhymered out after punching. All drilled holes must be slightly counter-sunk on the outer side and all burrs removed. The old careless, senseless **boiler-shop methods** led to non-agreement of holes when they came together, and only about 5 per cent. of the holes being really true to one another, a violent drifting process was resorted to. Modern methods are carefully scientific, so that even much rhymering is not needed. We now use drilling machines, hydraulic riveters, edge planing machines, &c.,

and all good work is done to templates. Angle irons are greatly dispensed with, the edges of plates being flanged. Great care is taken as to details, such as whether rivets in certain places ought or ought not to have countersunk heads. Flanging, dishing, and rolling processes are done quickly by large tools at one heating of the plates instead of being done by hand in many heats, and this adds greatly to the strength of boilers, and what is as important, our knowledge of that strength.

**117.** The working value of $f$ for **copper** in Art. 113 ought not to be taken greater than 2,400 lbs. per square inch for steam pipes. Copper is used for steam pipes because it is easily worked cold, but indeed steel is now being generally used instead of copper.

Copper for fire box plates (generally $\frac{1}{2}$ inch thick) or short stays or rivets has a tensile strength of about 16 tons per square inch, and elongates about 25 per cent. before fracture. Small holes are drilled into such stays from the ends, so that fracture may be detected by leakage. Alloys of copper change so greatly in their strength qualities as to be unreliable at 350° F. or 400° F., whereas pure copper can be relied upon up to 800° F., as, indeed, iron and mild steel may be, although they are all rather weaker than at ordinary temperatures. The malleability of copper and its endurance of furnace heat without surface deterioration cause many engineers to prefer it in furnaces and tubes to iron or steel.

In **cast-iron pipes** and in steam engine cylinders, it has to be remembered that the difficulty in getting castings which are of the same thickness everywhere, and the allowance that must be made for tendency to cross-breaking when the pipes are handled, as well as the great allowance in cylinders for stiffness and the difficulty of casting and boring out, cause such calculations as might be suggested by the formula (3) of Art. 113, to be somewhat useless. Thus it will usually be found that, whereas a large cast-iron water pipe is not much thicker than the above formula would lead to (taking the working $f$ as not greater than 3,000 for cast iron), because it is usually carefully moulded in loam, yet a thin cast-iron pipe has often an average thickness twice as great as the formula would lead to, and we never attempt to cast a nine-foot length of pipe of less than $\frac{3}{8}$th inch thick.

**118.** The law of strength of a strut is exactly the same as that of a tie bar if artificial means are provided for preventing bending For the same reason the law (3), Art. 113, gives the **strength of a flue to resist** collapse, the working compressive stress which the material will stand being $f$ lb. per square inch, the diameter $d$ inches, and the thickness $t$ inches; but this is on condition that

all tendency to buckling is artificially prevented by using rings like those shown in Figs. 158–161.

The flues of Fig. 151 are built up of rings (each ring being a plate bent and welded upon itself) flanged at the ends as shown. The flanged joints give sufficient stiffness for resisting buckling, and the Galloway tubes help in this. Figs. 162, 197, 205 show corrugated flues, the corrugations producing the same effect in resisting buckling. The thickness of any of these flues is to be taken as the total section in an axial length $l$, divided by $l$. We have as yet no exact knowledge of the behaviour of thin tubes under external pressure. There is a theory, but it can be of but little use to the engineer until it has been tested by experiment; it leads to the result that if a tube of diameter $d$ and thickness $t$ is prevented from collapse by rings, the distance between the rings divided by $\sqrt{dt}$ must not exceed a certain limit. Assuming the theory to be correct, we do not know yet what the limit is. In strengthening the flues of Lancashire boilers, the distance between the rings is usually $10\sqrt{dt}$. The working value of $f$ for flues is in practice taken as only 2 tons per square inch, first because of doubtfulness as to possible buckling, second because of oxidation and other deterioration due to the flame, third because steel and iron at 600° F. cannot be depended on for a greater strength or ductility than half their strength when cold, and above this temperature there is a further great lowering in strength and increase of brittleness. **Steel used for boilers** has about 28 tons per square inch tensile strength with an elongation of 25 per cent. in the direction of rolling, the breaking stress being 6 per cent. less and the elongation 20 per cent. less in the cross direction. The following composition is recommended. Carbon ·16 to ·18 per cent., silicon ·01 to ·018 per cent., sulphur ·03 to ·05 per cent., phosphorus ·02 to ·04 per cent., manganese ·25 to ·48 per cent. The plates must be clean looking, and must be annealed after shearing. The maker's name ought to be on every plate; every plate while in a boiler shop has a number for identification, and its strength and other qualities are known. Test strips heated and cooled in water at 80° F. should bend to a circle of internal diameter only three times the thickness. Rivet steel ought to have less than ·15 per cent. of carbon and ·04 per cent. of phosphorus, and ought to show no flaw when a straight strip is doubled back upon itself cold. The time spent in straightening plates is greatly lessened by the use of multiple roller straightening machines

**119. Exercises. Strength of Cylindric Shells, and Flues and Pipes.**—The strength of a thin tube is given by

$$p=2\,tf/d$$

where $p$ is the difference of pressure inside and outside in pounds per square inch, $t$ the thickness (or effective thickness if the tube is corrugated or has strengthening rings), $d$ the average diameter, $f$ the tensile (or compressive in the case of flues), stress on the material in pounds per square inch. If $p$ is the working gauge pressure, $f$ in tension may be taken as 5 tons per square inch for iron, and 7 for mild steel: $f$ in compression is usually taken as only 2 tons per square inch. The weakening of a plate produced by a riveted joint is known from Art. 116.

EXERCISE 1. A boiler 7 feet diameter is $\frac{5}{8}$th inch thick, what safe working pressure will it stand if the safe working tensile stress of the material is 5 tons per square inch? Assume that the longitudinal seams have a strength only 60 per cent. of that of the plate itself. That is, take the safe stress to be 60 per cent. of $5 \times 2{,}240$ or 6,720 lbs. per square inch, so that

safe gauge pressure $= 6{,}720 \times \frac{5}{8} \div 42 = 100$ lbs. per square inch.

2. What must be the thickness of the flue of this boiler if its diameter is 2′ 9″, and if the welded joint in it is assumed to stand a working crushing stress of 2 tons per square inch. *Answer.* $\frac{3}{8}$ of an inch.

3. The marine boiler shell, Fig. 206, is 16 feet diameter, and withstands a gauge pressure of 150 lbs. per square inch; if the thickness is $1\frac{1}{4}$ inch, what is $f$? *Answer.* 9,600.

4. The corrugated flue of Fig. 206 is 4 feet average diameter, the length of metal is 1·3 times the axial length, the metal is $\frac{5}{8}$ inch thick, the working gauge pressure is 150, what is $f$? *Answer.* 7,400.

5. The steam vessel of a water tube boiler is 30 inches in diameter, thickness $\frac{3}{8}$ inch, pressure 200 lbs. per square inch, find $f$. *Answer.* 8,000.

6. Each of the tubes of a boiler is 1·5 inches in diameter, and 0·25 inch thick; if $f$ is 8,000 find $p$. *Answer.* 2,600 lbs. per square inch.

EXERCISE 2. A boiler like Fig. 151 intended for 100 lbs. per square inch (gauge) is usually of steel $\frac{1}{2}''$ to $\frac{9}{16}''$ thick in its 7 foot shell, the straight seams being double riveted butt joints with two covering plates, its 33″ flues being $\frac{3}{8}''$ to $\frac{1}{2}''$ thick. Neglecting the extra virtual thickness due to the joints in the flues, what are the greatest stresses in the metal taking the smaller thicknesses?

*Answer.* $f = \dfrac{pd}{2t}$ for both shell and flue.

$f = \dfrac{100 \times 84}{2 \times \frac{1}{2}}$ or 8,400 lbs. per square inch in the shell; but as the

joint is 0·85 of the strength of the unhurt plate (see Art. 116), w
take the greatest stress in the plate at the joint as 8400 ÷
or about 10,000 lb. per square inch.

$f = \frac{100 \times 33}{2 \times \frac{3}{8}}$ or 4,400 lbs. per square inch compressive str

the flue. Such a boiler is usually only tested hydraulically to 15
per square inch.

**120.** The **flat parts of boilers** need staying. Figs. 151
163, 164 show the gusset plates and end to end stays in co

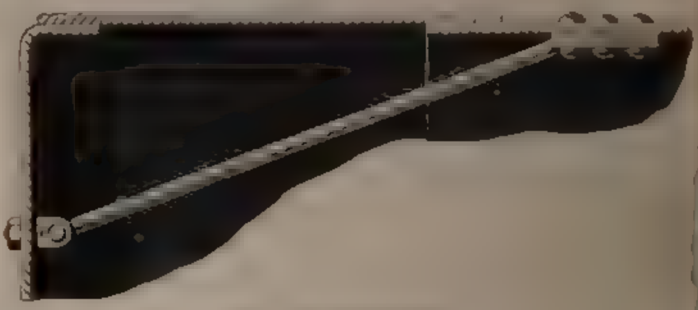


FIG. 188.—DIAGONAL STAY.

use. Fig. 188 is a diagonal stay which may take the place of a g
In flat parts near together, stud stays riveted or with nuts o
shapes shown in Fig. 189, are used. Thus in Fig. 202 copper is
in the 3-inch water space between a locomotive fire-box a

STUD STAY. STUD STAY. STUD STAY. END OF STAY B

FIG. 189.

shell, the stays, 4 inches apart, are screwed into the plates, th
allowed to project $\frac{3}{8}$ inch and then riveted over. The end of
stay bar may have a pin joint, as in Fig. 189. In multit
boilers, stay-bars, as in Fig. 151, may be used in the steam spa
many of the tubes are screwed into the shell tube plate

The ordinary tubes are merely expanded at their ends tube plates as in Fig. 192. Fig. 193 shows the Admiralty ften used to protect the joint from ce flame. In Fig. 190, the fastening y-tube is more elaborate, there being and internal nuts.

FIG. 190.—STAY-TUBE.

194 shows one way in which numerous s or girders support the flat top of ace of a locomotive, or of the com- chamber of a marine boiler. They slung at their middles to the shell. es greater freedom for expansion of of the fire box before the shell gets heated. It is getting to use another method, supporting the flat plate from steel

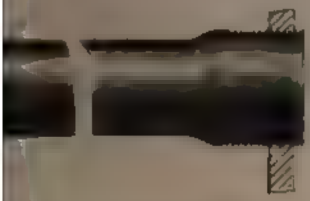


FIG. 191.—STAY-TUBE.

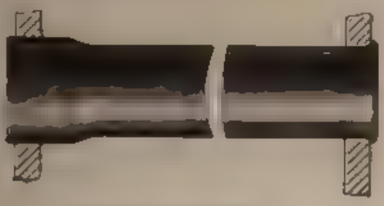


FIG. 192.—PLAIN TUBE.

193.—ADMIRALTY FERRULE.

FIG. 194.—DOG OR GIRDER STAY.

gs on the outer shell by means of numerous stay-bars. This etter circulation of the water.

*Flat Plates*

he theory of the strength of a flat plate has not yet been put in a simple It will be found in Thomson and Tait's *Natural Philosophy*. The results theory agree with such careful experiments as have been made.

(1) For a circular plate of thickness $t$ and radius $r$, *supported* all round edge with a normal load of $p$ lb. per square inch, if $f$ is the greatest stress the material—

$$f = 5r^2p/6t^2.$$

(2) If the circular plate is *fixed* all round its edge

$$f = 2r^2p/3t^2.$$

(3) A square plate of side $s$ fixed at the edges

$$f = s^2p/4t^2.$$

(4) A rectangular plate of length $l$ and breadth $b$ fixed round the edges

$$f = l^4b^2p/2t^2(l^4 + b^4).$$

(5) A round plate supported at the edge, with a load $W$ applied at a circu ference of radius $r_0$

$$f = \frac{W}{\pi t^2}\left(\frac{4}{3}\log\frac{r}{r_0} + 1\right).$$

(6) For stays in square formation, distance asunder $s$, each stay has a lo $ps^2$, and the greatest stress in the plate of thickness $t$ is

$$f = 2s^2p/qt^2.$$

Lloyd's and other associations have formulated elaborate practical rules the strength of curved and flat parts of boilers and stays, based on the formu I have given, Arts. 113-120. These will be found in the manuals written boiler-makers.

**121. Grooving and Corrosion.**—Even zinc, if pure, in dilu sulphuric acid is not acted upon chemically. But if a piece of a other metal, such as copper, is also in the liquid and the meta touch anywhere, the zinc is acted upon rapidly. Two kinds of met are needed as well as an electrolytic liquid, and the metals mu touch, else corrosion will not take place. The better conductor t liquid is, and the more different in certain qualities the metals a the more rapid is the action. One of the metals is almost entire protected, the other being acted upon. Now in ordinary zinc the are impurities and physical differences, and consequently we ha rapid corrosion when it is in an electrolytic liquid such as dilu sulphuric acid. When iron touches water, although the wat may be very free from salts and therefore rather non-conducti electrically, yet in time we find corrosion, and especially near t water level.

Where the metal is sometimes wet, sometimes dry, very sma surface differences in the metal are sufficient to allow of the form tion of deep grooves due to corrosion. Probably the fretting of t surface of the metal, due to the plates being bent and unbent near t more rigid angle iron, in the breathing of the boiler, causes sufficie difference of surface to start the action. It is usual to make t inside surface of a boiler more uniform by sponging it all over wi a weak solution of salammoniac. Hanging lumps of zinc inside

oiler, either lying against the plates or attached metallically, very aterially prevents corrosion of the iron, the zinc being eaten way. From 200 to 600 lbs. of zinc are sometimes consumed per nnum in the boilers of a large vessel. Air free water produces uch less corrosion. Vegetable and animal oils decompose in boilers nd produce corrosion because of acidity.

It is because of this electro-chemical action that any trace of acidity in lubricating oil does so much harm between brasses and journals. If water finds its way to the place where a gun-metal liner touches the steel of a propeller shaft, it causes rapid corrosion.

Making every part of a boiler more elastic greatly prevents such fretting of the metal anywhere as may lead to grooving and pitting. This is another reason why the spaces between flues, and flues and shell, ought to be as much as possible; it is for this reason that some makers prefer five to four gusset stays.

**122.** *Straining of a Boiler.*—Parts of a boiler are continually altering in temperature in different ways. Thus, in a Lancashire boiler, after the fire is lighted a flue "hogs," rising in the middle, or rather nearer the furnace, as much as $\frac{3}{8}''$ or $\frac{1}{2}''$, although it bulges out the flat ends, perhaps $\frac{1}{8}''$. It is well to leave a flue free to hog and not to try to restrain it with stays.

Exercise. A Lancashire boiler is 35 feet long, the flue has an average temperature of 500° F. when the shell is only at 100° F.; what would be the relative change in length if it were not prevented?

*Answer.* By Art. 171 a difference of 400 degrees produces a fractional change of length $400 \times \cdot 000009$ or $0 \cdot 0036$ in iron, so that in 35 feet there is a difference of $35 \times 12 \times \cdot 0036$ or $1 \cdot 5$ inches.

Exercise. To shorten an iron tube 35 feet long, by the amount of 1 inch, what must be the compressive stress?

*Answer.* The compressive strain is $1 \div (35 \times 12)$, and as Young's modulus of elasticity for iron is about $3 \times 10^6$, the compressive stress being $3 \times 10^6$ multiplied by the strain, the answer is $3 \times 10^6 \div (35 \times 12)$ or 7,140 lbs. per square inch.

Exercise. If the flue is 33 inches in diameter, $\frac{1}{2}$ inch thick; if the heat tends to make it $1\frac{1}{2}$ inches longer, and although it bulges out the ends of the boiler and hogs, it only gets $\frac{1}{2}$ inch longer, what is the total pushing force in the flue?

*Answer.* By last example the stress is 7,140 lbs. per square inch; the section of metal is $33\pi \times \frac{1}{2}$ or $51 \cdot 86$ square inches, so that the total push is 370,280 lbs. or 165 tons.

**123. Boiler Accumulator.** EXERCISE 1. A vessel contains $w_1$ lb. of water at 406° F. under a pressure of 265 lbs. per square inch. How much steam must be taken away (dry at 347° F. through a reducing valve) for the temperature to become 347° F., the pressure being 130 lbs. per square inch?

*Answer.* If $w_1$ lb. of water at 406° F. has as much energy as $w_2$ lb. of water, and $x$ lb. of steam at 347° F. ($w_2 + x$ being equal to $w_1$), measuring heat from 347° F.

$$w_1(406 - 347) = x \times 869,$$

as 869 is the latent heat of steam at 347° F. Hence $x = \frac{59}{869} w_1$, or $14\frac{3}{4}$ lbs. of water falling from 406° F. to 347° F. will yield one pound of steam.

EXERCISE 2. If 20 lbs. of steam per hour at 130 lbs. per square inch will develop 1 horse-power, what is the storage capacity of a vessel, 30 feet long, 15 feet diameter, containing water at 265 lbs. per square inch, allowed to fall to 130 lbs. per square inch?

*Answer.* By the table, Art. 180, we see that 1 cubic foot of such water weighs 54 lbs., so that we have $\frac{\pi}{4} 15^2 \times 30 \times 54$, or 286,270 lbs. of water stored. Dividing by $14\frac{3}{4}$ we find that the supply of steam may be 19,470 lbs., dividing by 20 we get a supply of 973 horse-power-hours.

EXERCISE 3. An electric light station has many small steam engines, each coupled to a dynamo machine; some of these are stopped or started, as the load varies. They all take steam at 130 lbs. per square inch through reducing valves from a reservoir, and give out 1 electrical horse-power for 25 lbs. of steam. The reservoir contains water never higher than 406° F., never lower than 347° F., and this water is kept constantly circulating by means of a centrifugal pump between the reservoir and a number of boilers, using steadily half a ton of coal per hour. Three-fourths of the total heat of the coal is given to the water, which enters at 62° F., the coal being such that its total heat per pound is 15,000 heat units.

In 24 hours the water receives—

$$24 \times \tfrac{1}{2} \times 2240 \times 15{,}000 \times \tfrac{3}{4} \text{ or } 3 \times 10^8 \text{ heat units.}$$

A pound of steam at 347° F., the feed being at 62° F., needs 1,157 units, and hence if the engines had a perfectly constant load, they would give out $3 \times 10^8 \div (1{,}159 \times 25)$ horse-power-hours in the 24 hours, or 435 horse-power.

EXERCISE 4. Now suppose that there is such a load factor that there is a maximum supply at the rate of 1,740 electrical horse-power, and in fact that for eight successive hours the power given out is greater than 435, the average of the excess power being 510, so that in fact there must be a store of 510 × 8, or 4,080 horse-power-hours. In this rough calculation we may neglect the fact that the steam if taken away at a higher pressure through a reducing valve, is probably superheated instead of being just dry as assumed above, and we may assume that for every $14\frac{3}{4}$ lbs. of water stored we can produce 1 lb. of steam, or for every 25 × $14\frac{3}{4}$ or $367\frac{1}{2}$ lbs. of water stored we can produce 1 electrical horse-power-hour. We therefore need to store 4,080 × $367\frac{1}{2}$, or $1{\cdot}50 \times 10^6$ lb. of water at 406° F. At this temperature a cubic foot of water weighs 54 lbs., and therefore we need a reservoir of 27,200 cubic feet, neglecting the volume of the heated tubes. This reservoir if cylindric might consist of four cylinders, 40 feet high and 15 feet in diameter. The cost of such a reservoir with the necessary brickwork, &c., would probably be £2,400. Assuming interest, maintenance, depreciation, rent, &c., as 10 per cent. on the cost, we find £240 per year.

# CHAPTER XIII.

## HEATING ARRANGEMENTS OF BOILERS.

**124.** The fireplace, 6 feet long, Fig. 151, consists of a front dead plate and sets of fire bars resting on wrought iron or steel bearers, and the support of the fire-brick bridge *B* riveted across the flue. Notice the spaces between the bars, Fig. 154, to allow of air entering from the ashpit. The door is double or sometimes treble with air between, so that the outer part may remain cool. **The clever stoker** knows that it is by regulating the air coming through the ventilators in the door, as well as by the ashpit, that he may obtain perfect combustion and no smoke, even with the most bituminous coals. The careless stoker can only obtain good combustion with Welsh coals. With good stoking the same results are obtained with Newcastle or Cheshire coals as with Welsh. Here is the best method with non-Welsh coals. Suppose fresh coal is needed, the red-hot stuff is pushed forward till it is thicker near the bridge; the fresh coal is put on near the dead plate and the door closed, air coming in. The coal begins to coke (this is called the *coking* system, and is better than the *spreading* system of feeding a furnace, except for very small coal); it gives off its gaseous hydrocarbons, which, passing over the white-hot part and also by meeting the hot air which has come from the ashpit through the grate, and also by its own combustion, reaches a high temperature. Now **for perfect combustion** of the gases we have merely to recollect that

1. There must be at least a sufficient quantity of air.
2. The air and gases must be well mixed.
3. The mixture must be at a high temperature.

If any of these conditions is not fulfilled there is an escape of unburnt gases. If these unburnt gases are hydrocarbons and if they are suddenly cooled, they become decomposed and form smoke or soot. Impinging on a cold solid surface, some of these hydrocarbons deposit a very hard kind of soot difficult to remove.

In the Lancashire boiler we depend upon the mixing that goes on above and behind the fire bridge as well as above the fire, and this is why we call the space behind the bridge a combustion chamber. It is fatal to good economy to attempt to cool the gases much until they are well mixed, and in Fig. 151 the first Galloway tube is perhaps too close to the bridge. And yet although it cools the gases, it also helps to mix them. More space is needed for more **bituminous coal.**

We do not like to rely altogether upon the air coming up through the grate, and it is necessary to think a little about what happens to such air. Suppose air to come up through a thick mass of white hot coke; first its oxygen combines with carbon to form carbonic acid $CO_2$; later this carbonic acid dissociates into carbonic oxide CO and oxygen; this oxygen again takes up carbon to form more carbonic acid. If the fuel is thick enough no doubt there are more changes—but the result is this, that escaping from the top of the coke, we have carbonic oxide and carbonic acid and the nitrogen of the air. Students must have seen such CO burning with a blue flame over a thick coke fire. That such carbonic oxide may not go off unconsumed, **air** must be admitted **by the door.** Now in the Lancashire boiler we do not like thin fires, but even when thickest much of the oxygen which comes through the grate will probably not form either CO or $CO_2$, and air through the fire door is not so necessary (although we always take care to open the ventilator of the door about a minute after a fresh firing) as it is in the locomotive and other boilers using thick fires. In these there is probably little free oxygen after passage through the fire; hence both for the sake of the CO and also of the hydrocarbons, air must be admitted through the door. The space above the grate in a locomotive is the only combustion chamber, and it ought to be large. In some cases of Lancashire and marine boilers, advantage is found in admitting air through passages behind the fire bridge.

In chimney draught, or when jets of steam produce draught in the uptake of locomotives or marine boilers, the entering air can only be heated by the inner part of the hot fire door or the hot ashpit, but when the forced draught consists in blowing air in through orifices above the grate and also into the ashpit, fire door and ashpit door being well closed, it is possible to heat this air by the gases in the uptake as it comes through pipes. In this case very perfect combustion is obtainable. Note that in no case can a stoker, however careful, obtain good combustion unless he can command just **as much draught as is necessary.** With chimney draught he performs his regulation by lifting or lowering the damper, which

is hung from a chain passing over pulleys to the balance weight, which is within easy reach of the stoker. The density of the furnace gases is an indication of the amount of carbonic acid present, and this is an indication of the amount of air supplied per pound of fuel. The **Dasymeter** makes an automatic record of this and ought to be used as commonly as a pressure gauge. It acts by the apparent change of weight of a hollow globe in a box through which a small supply of filtered furnace gases passes.

The opening of the fire door admits too much cold air (usually checked by the damper beforehand), and yet it is certain that frequent small supplies of coal are far better than infrequent large supplies. Indeed, the feeding of the fire ought to be continuous, and the conditions of draught, &c., ought to keep constant. Hence for the most perfect combustion we depend upon mechanical stoking, which keeps admitting fresh fuel all the time, the coal as it gets coked and more and more burnt, finding its way towards the bridge, where the ash and clinker drop. Indeed, in small boilers of great power it is almost absolutely necessary that all the operations, feeding with water and fuel, and regulating draught, &c., should be automatically and continuously performed.

**125.** The combustion chamber is filled with white hot flame, and as the gases travel towards *A* they give up most of their heat to the boiler. Usually about half the total heat given to the boiler is given up by **radiation from the fire** and the hot gases in the furnace and combustion chamber of a Lancashire boiler. The rest of the heating surface seems to take up heat by mere contact with the hot gases, and hence it is that the Galloway tubes prove to be useful, because the gases strike upon them and the eddying and mixing motion causes a continual renewal of hot gases near the metal, and the water circulates easily through the tubes. The seatings of six boilers are shown in Fig. 197. A fire-brick wall makes the stuff pass down and underneath the bottom nearly to the front of each boiler; there it divides into two streams, passing up and along the sides of the boiler by passages, which unite again in the passage going to the chimney. An iron door or damper passes usually down through a slit, supported by a chain going over pulleys to the front of the boiler, where there is a counterweight. The boiler rests on the seating blocks of fire-brick, made of special shape. Some men let the gases pass along the side flues before the bottom, and it may be more economical, but the other is on the whole better because there is less unequal heating of the boiler. What the actual temperatures are, everywhere, I do not know, for although

I know of many published measurements, I know of none yet made with accurate instruments.

EXERCISE. If half the heat of fuel is radiated in the furnace, and the other half is carried off by gases. If the gases are 20 lbs. per pound of fuel, and the calorific power of the fuel is 14,500 Fah. units; neglecting the fact that there is vapour present, and that there is almost certainly dissociation, find the temperature of the gases leaving the furnace if their specific heat is 0·24.

*Answer.* 7,250 ÷ (20 × ·24) or 1,510 Fah. degrees above ordinary temperature.

It is said that thick copper wire lying on the brightest fuel in any boiler furnace does not melt. Probably therefore the temperature never reaches the melting point of copper. Copper wire will of course rapidly disappear, because of oxidation, &c. The temperature near the chimney is often about that of melting lead. There is no doubt a great advantage in letting the two flues unite in one, just behind the fire bridge, as in the usual hand firing, if the furnaces are fired alternately, the mixing is most conducive to good combustion. The best large stationary boiler known to me is shown in Fig. 196, and may be called **a multitubular boiler.** Here when the mixing of the gases has occurred in *C C*, they pass through a great number of tubes, which take away their heat far more rapidly than it is taken in any Lancashire boiler, than which this occupies less space for the same power. Space must, however, be left behind *A* for the cleaning of the tubes. The best results are obtained with two furnaces meeting in the combustion chamber *C C*, fired alternately.

An economiser (Figs. 195 or 197) or **feed-water heater** consists of a number of vertical iron pipes (sixty for a single boiler with three-quarters of the heating surface of the boiler, say 600 square feet), through which the feed-water passes, their sooty outsides are kept constantly scraped, and they are placed in the passage between the boiler and the chimney. It is found that the use of an economiser adds from 10 to 15 per cent. to the amount of steam evaporated by a Lancashire boiler. Water may be raised to 240° F. It causes great gain in economy, and lessens the straining of the boiler, due to local cooling. It does not benefit a multitubular boiler so much, because the flues of this boiler are already very efficient. In this, as in many other cases, the extra contrivance, such as a feed-water heater, owes its value to the uneconomical nature of the contrivances which it supplements.

As much as 33 per cent. better results are obtained over the ordinary hand-stoking by the use of **mechanical stokers,** but it is

ly in the case of steady loads on engines, and therefore on boilers, at they are used. Vicar's stoker has a hopper, which has to be

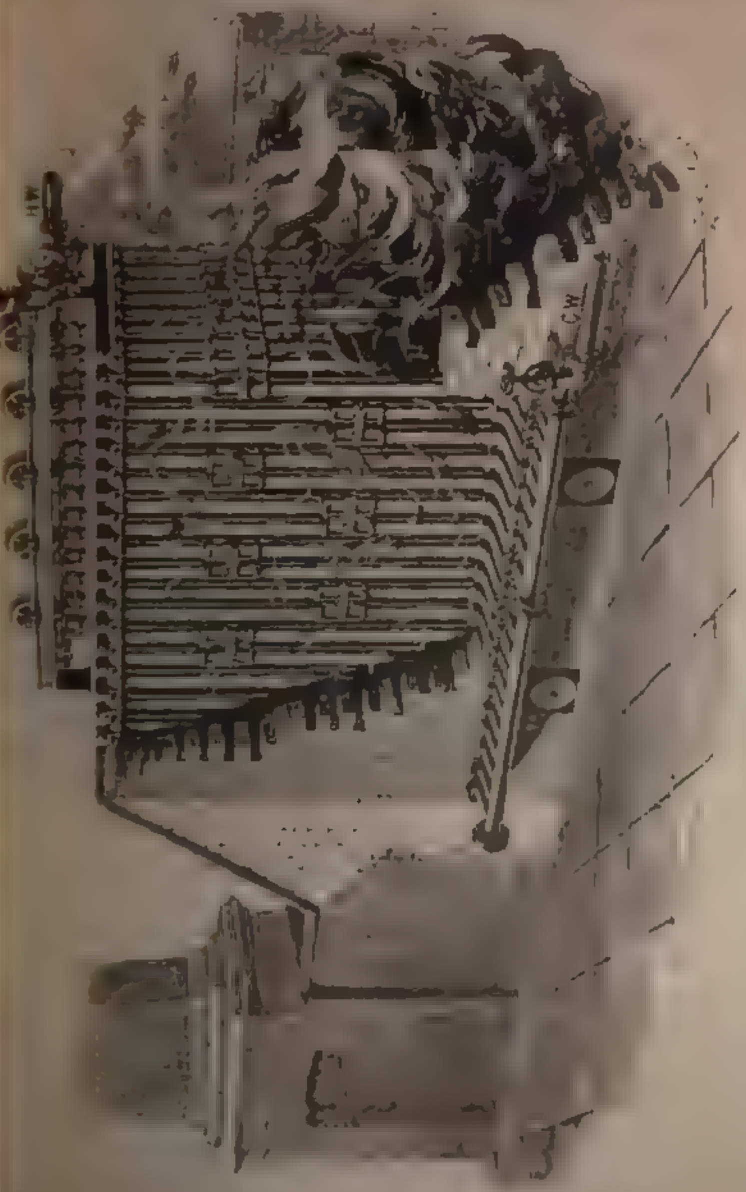



FIG. 103. ECONOMISER OR FEED WATER HEATER.

led with fuel, and the fuel falls into small boxes; a slowly rotating aft drives plungers forcing coal from the boxes on to the dead

plate, and also gives a reciprocating motion to the fire bars, so that the coal is carried towards the bridge, where it falls into the ashpit. Henderson's form breaks up the coal coming from the hopper; it falls on fans, which spread it on the bars. Half the bars rise and fall, the others have a reciprocating horizontal motion.

EXERCISE. A Lancashire boiler 27 feet long, 7 feet diameter, shell $\frac{7}{16}$th of an inch thick, flues 33 inches diameter, $\frac{3}{8}$ of an inch thick, ends $\frac{5}{8}$ of an inch thick, what is its approximate weight?

*Answer.* Neglecting, overlapping, &c.

Each end $\{84^2 - 2(33)^2\} \times \cdot 7854 \times \frac{5}{8}$ or 2,393 cubic inches of metal, or 4,790 for both.

$$\text{Shell } 84\pi \times 27 \times 12 \times \frac{7}{16} = 37{,}400 \text{ cubic inches.}$$

$$\text{Flues } 2 \times 33\pi \times \frac{3}{8} \times 27 \times 12 \text{ or } 25{,}200 \text{ cubic inches.}$$

Total 67,400 cubic inches, and taking ·28 lbs. to the cubic inch, the weight is 18,760 lbs., or 8·43 tons. Now the actual weight will

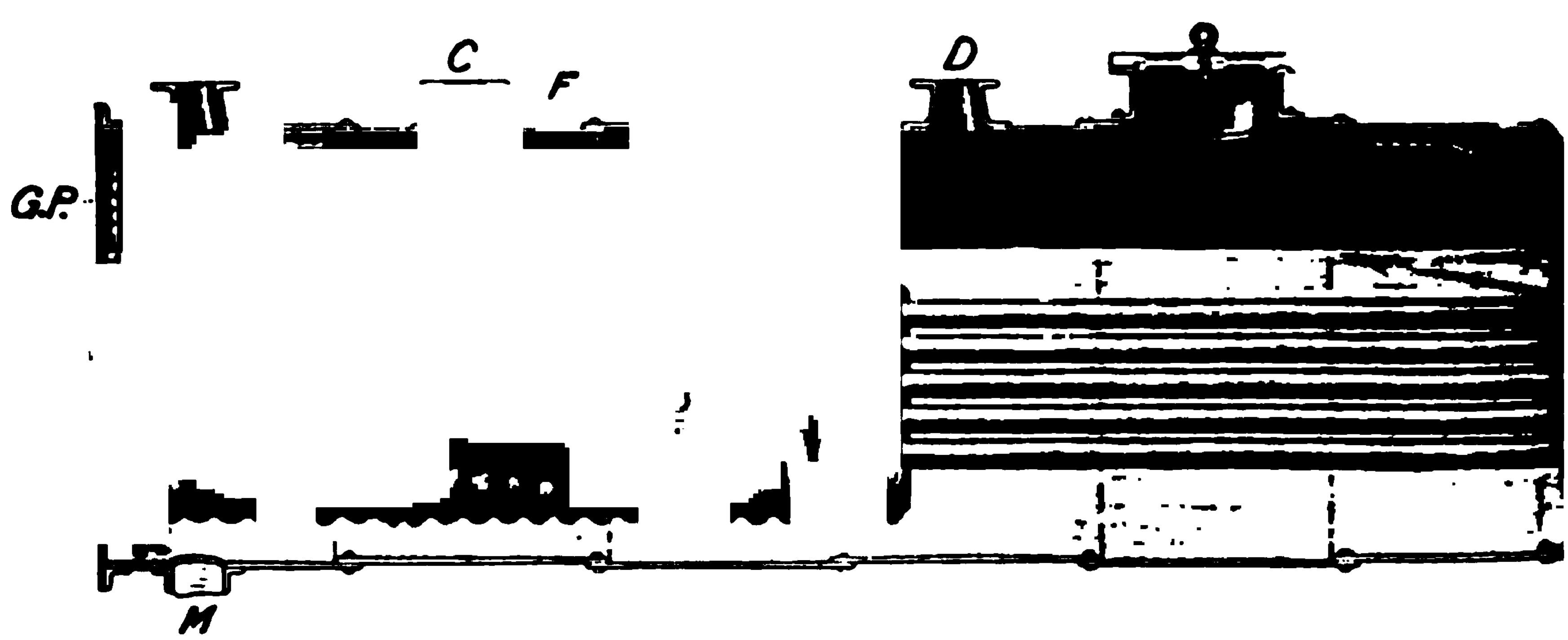
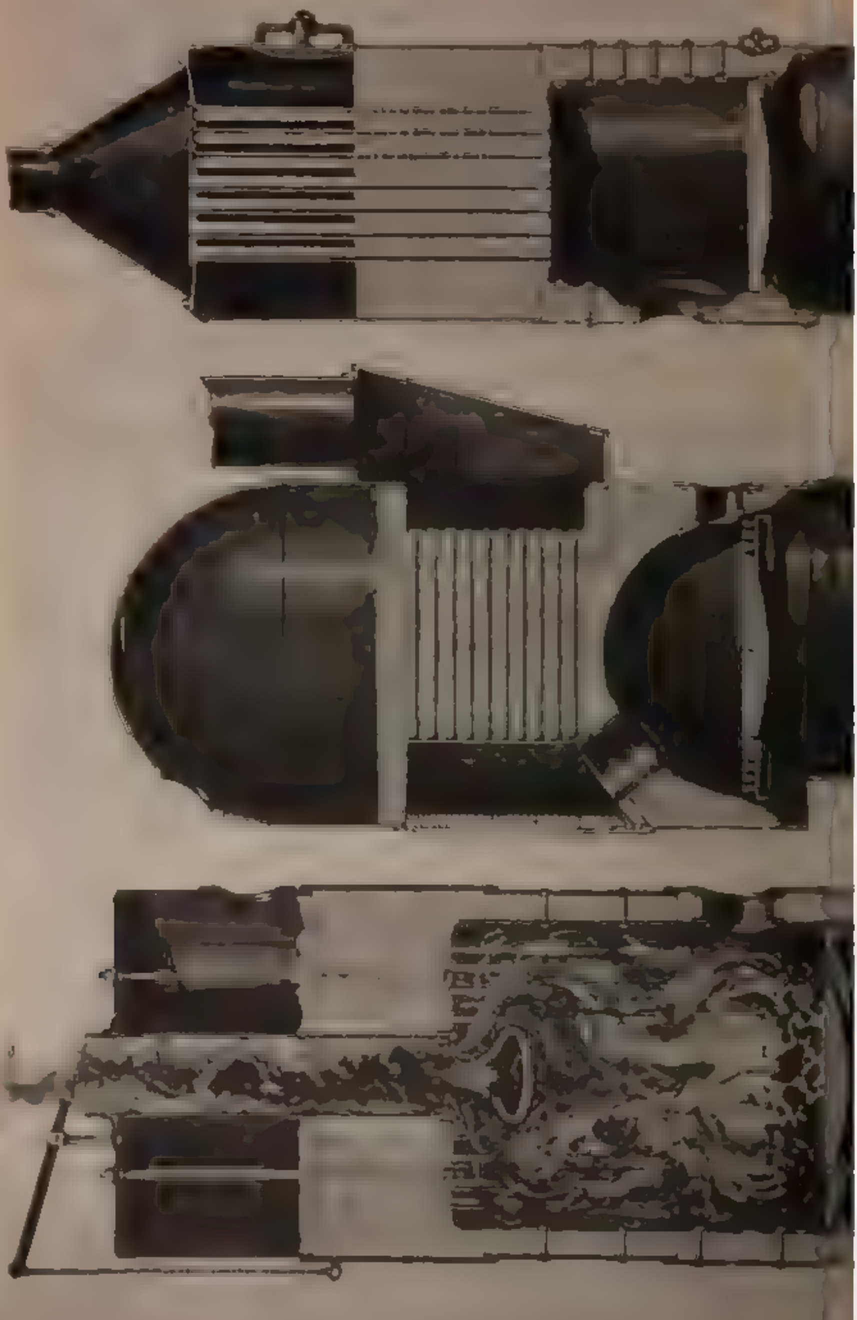



FIG. 196.—MULTITUBULAR BOILER (STATIONARY).

be about 12 tons, together with 3½ tons of fittings, and this gives a fairly correct notion of the usual allowance to be made for flanges, angle irons, &c., in rough calculations.

If the student will make measurements he will find that the total heating surface on the external shell is about 370 square feet; flues, 450 square feet + water tubes 30 square feet; altogether say 870 square feet; economizer say 600 square feet. The grate is about 33 square feet in area, so that there is 26 square feet of heating surface (with economizer 45) per square foot of grate.

Such a boiler will usually burn 12 to 18 tons of coal per week of hours, or 15 to 22 lbs. of coal per hour per square foot of grate (a



## CHAPTER XIV.

### BOILERS (*continued*).

**126. THE vertical boilers** shown in Figs. 198–200 are easy nderstand. Fig. 201 shows a "Field" water-tube which pro- downwards into a fireplace, and is surrounded by flame. A ical tube closed at the end with water in it, unded by flame, will get nearly red hot and suddenly much of the water becomes steam sively. The interior tube allows the most circulation to take place, and these field tubes ite wonderful for quick evaporation.

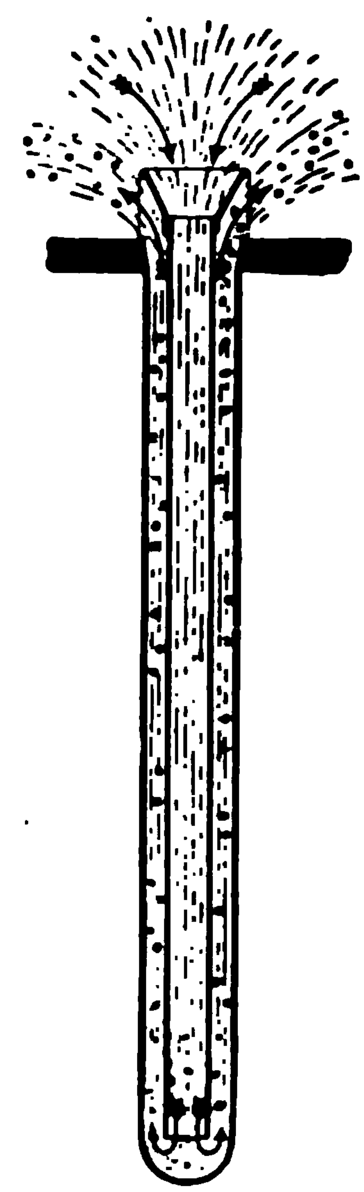


FIG. 201.—FIELD TUBE.

the locomotive boiler the usual pressures to 200 lbs. per square inch absolute. 02 shows the fire box, whose top and sides ally copper ½ inch thick) are in one piece, the plate *T P*, ⅞ inch thick and the back fire box *B F P* being connected by flanges to the rest. s enclosed by its ½ inch steel casing which a shoulder plate joining it to the steel ½ inch el formed of three iron or steel plates called back, the middle, and the front plate. The plate is fastened to the ¾ inch smoke box tube *S T P* by a circular angle iron. About 200, 1½ inch (10 W. G. thick) brass flue tubes convey hot gases from furnace to smoke box *S B* and the ney. Rivets usually ¾ inch. Circular joints lap but sometimes butt. The straight joints always butt two covering plates.

he holes in both the tube plates are larger than the tubes, which assed through from the smoke box end of the barrel and then nded and made steam tight with a tube expander, ferrules being

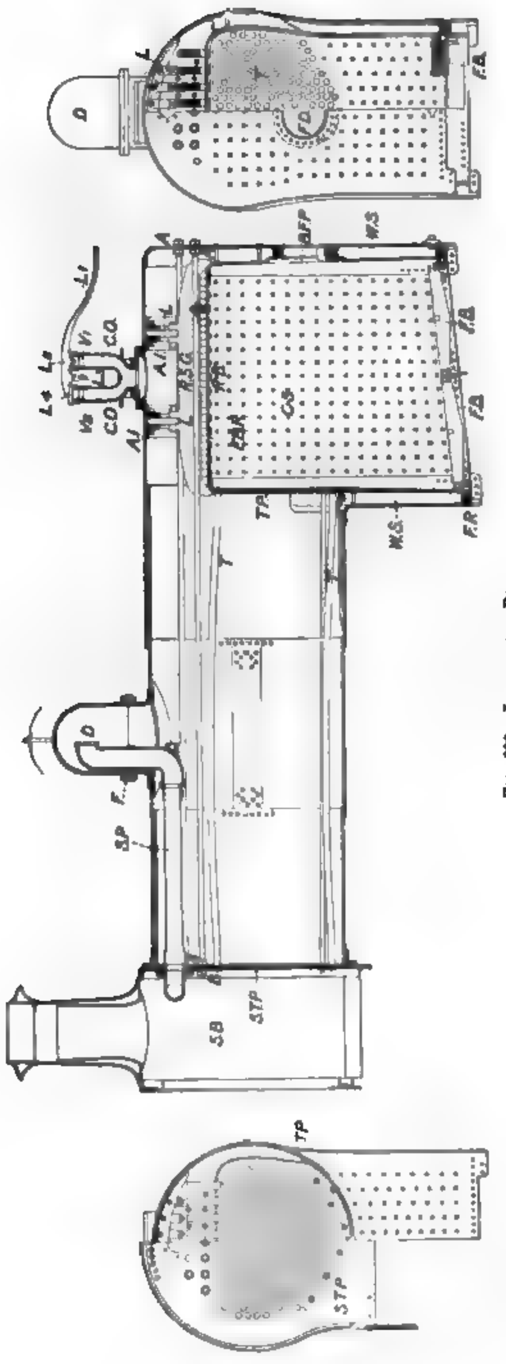



FIG. 202.—LOCOMOTIVE BOILER.

No manhole in bottom of barrel in this specimen. About a dozen $1\frac{1}{4}$ inch mud holes are made in the shell, with brass plugs. Bituminous coal in burning needs more space above the fuel. A fire-brick arch allows better mixing of gases, and prevents solid particles entering the tubes. Usually the bottom tubes have too little draught. Small tubes much more efficient, but apt to get choked. Tubes wear greatly by attrition of small cinders and coal.

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stays also
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fitting *F* r

on. Notice the shapes of the fire bars and how they are carried by bolts through the foundation ring *FR*. The air space is from $\frac{1}{8}$ to $\frac{1}{4}$ of the whole grate area. A wrought iron rectangular ashpan is bolted to *FR*. It has a damper in front (sometimes one at the back also), a hinged door worked by a notched rod from the foot plate. There is usually a fire brick arch nearly across the fire box to deflect the flame and so mix the gases better. It was the use of this brick arch which first enabled coal to be burnt instead of coke in locomotives. There is also usually a deflector plate inside the fire hole to deflect the cold air downwards when the door opens. As this obstructs radiation it is not so good as having a door opening inwards which itself acts as a deflector plate.

The regulator for admitting steam through the steampipe *SP* to the valve chest is shown in Fig. 64.

The heating surface of a locomotive is usually 750 times the area of one of the pistons; the grate area is usually 10 times the area of one of the pistons. The tube heating surface is usually 10 times the heating surface of the fire box.

EXERCISE. One piston 16 inches diameter, what is its area? What is the customary total heating surface, tube surface, &c.?

*Answer.* Piston 201 square inches; grate 14 square feet; heating 1,047; tube surface 951; fire box heating surface 95. If the tubes are $1\frac{1}{2}$ inches in diameter inside and 10 feet long how many of them are there?

*Answer.* Each tube has an area of 3·93 square feet, so that there are about 265 of them.

**High cylindric marine boilers** are from 11 to 17 feet in diameter, and are either double or single ended. Fig. 203 is single ended, 9 to 10 feet long, and Fig. 205–6 is double ended, 17 to 18 feet long, being like two single-ended boilers set back to back. There is greater economy of weight and space and heat radiation. In men-of-war there may be an advantage in having more boilers quite distinct.

Fig. 205 is one of four marine boilers. The shell is cylindric with corrugated furnaces. The straight joints are treble riveted butt, with two covering plates, breaking joint. The ring joints are double riveted lap. Usually there are two or three combustion chambers, not always the same in number as the furnaces. The uptakes meet at the base of the funnel, with a damper in each; indeed there is usually a damper for each combustion chamber for greater ease in cleaning the separate furnaces. The furnaces are from 36 to 45 inches in diameter, 78 inches long, grate 6 to 7 feet long in two or three lengths of steel fire bars (Fig. 206). There is always an ash tray because of the

corrugations in the furnaces, and it is usual to keep a little water in it. The furnace tubes are kept 4 to 5 inches apart both at heights and hollows of the corrugations. The ends flanged are $\frac{3}{4}$ to $\frac{7}{8}$ inch thick. The front one is in three pieces. The central piece is the front tube plate; the lower, flanged out at the holes, carries the furnaces. The combustion chambers are of flat plates curved and flanged $\frac{1}{2}$ to $\frac{9}{16}$ inch thick, well stayed.

**The tubes** are still sometimes of brass but almost always of drawn steel $\frac{1}{8}$ inch thick, $2\frac{1}{4}$ to 3 inches internal diameter. They are a good fit for the holes in the tube plates and a tube expander is

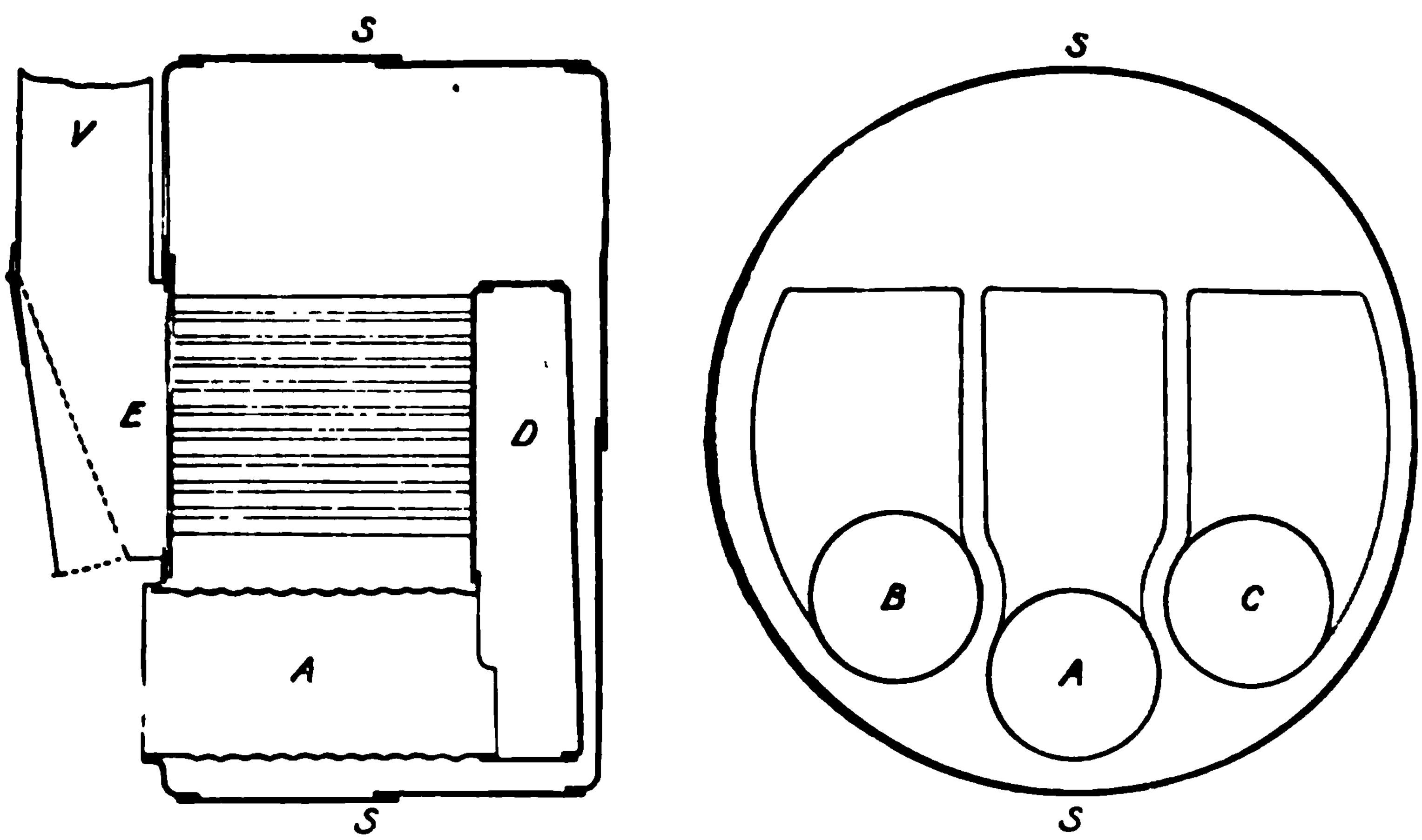


FIG. 208.—SINGLE ENDED MARINE BOILER, THREE FURNACES, THREE COMBUSTION CHAMBERS.

used. The holes are a little larger at the smoke box end to facilitate insertion and withdrawal. The tubes are usually about 1 inch apart on their outsides. Notice the large number of tubes that are stay tubes marked blacker than the rest in Fig. 205. (Many people object altogether to the use of stay tubes, which indeed are seldom used in locomotives.) The Servé tube has internal ribs for the better abstraction of heat; it is of twice the usual weight and cannot be more efficient than a small tube with great draught.

Notice the end to end stay bars 2 or $2\frac{1}{4}$ inches diameter, the holes in the plates not screwed.

Already there are single-ended boilers of 13 feet diameter, whose cylindric part is $1\frac{3}{8}$ inches thick 10 feet long, in two plates each 11 feet broad, with one welded joint; the other joint, being welded at its end parts only, the rest of it treble riveted. The flanges are internal and on the cylindric part, each of the end plates being in one piece

It is difficult to convey larger plates than these by rail. Very large flanging and welding machinery has thus given great simplicity and strength of construction.

**Exercise.** A marine boiler shell is 16 feet 3 inches diameter, $1\frac{1}{2}$ inches thick ($1\frac{1}{2}$ inches thickness has been exceeded in the mercantile marine), for a working gauge pressure of 170 lbs. The furnaces are 43 inches diameter and $\frac{5}{8}$ inch thick. Neglecting the increase in *effective*

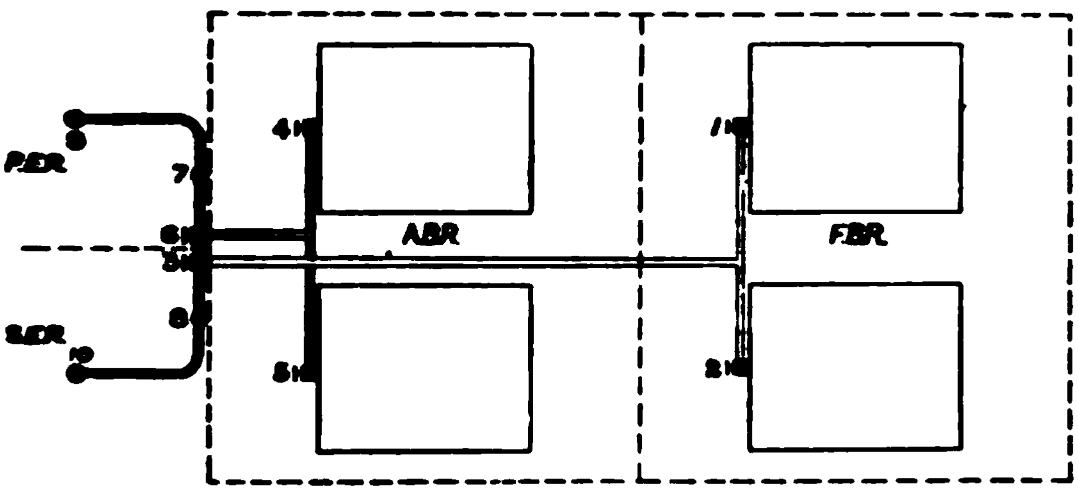


Fig. 204.—Marine Engine Steam Pipes.

thickness due to the corrugations, what are the working stresses? *Answer.* Shell $f = \dfrac{195 \times 170}{2 \times 1\frac{1}{2}} = 11{,}050$, and as the joint is $\dfrac{1{\cdot}08 \times 5}{5 + 1{\cdot}45}$, or $\cdot 837$ of the strength of the unhurt plate, the answer is $11{,}050 \div \cdot 837$, or 6 tons per square inch tensile stress in the joints of the shell.

In the furnace tube $f = \dfrac{43 \times 170}{2 \times \frac{5}{8}}$, or 5,848 lbs. per square inch.

The working and test pressures of a marine boiler are usually engraved on a brass plate fixed to the front of the boiler. It is usual to provide two Bourdon pressure gauges; one scale goes to 15 or 20 lbs. above the working pressure, the other to the highest pressure used in testing the boiler hydraulically.

To show the general nature of the **steam pipe connections**[1] in the Navy, in Fig. 204, the dotted lines are bulkheads, and I assume that there are four double-ended boilers and twin screw engines; 1 and 2 are the stop valves of the boiler in the forward boiler room *F B R*, giving steam to their main pipe, which goes to the starboard engine-room bulkhead stop valve 3. The stop valves 4 and 5 in the after boiler room *A B R* give steam to their main, which goes to the port engine-room bulkhead stop valve 6. There is a thwart-ship main pipe on which are the stop valves 3 and 6 just mentioned; also valves 7 and 8 (to shut off either engine) which may be closed either by hand in the engine-room or from outside; and the regulator valves of the engines 9 and 10. Seven and 8 are held open against the pressure,

[1] Electric light station engineers are evolving important schemes of steampipe arrangement.

so that they may be easily closed, and small pass valves are prov[...] to ease their opening. Sometimes there is another valve prov[...] between 3 and 6.

It is most important that the **water level** should be kept [...] in all the boilers. There is ample feeding power, and in [...] emergency all the feed may be given to one boiler, and we pr[...]

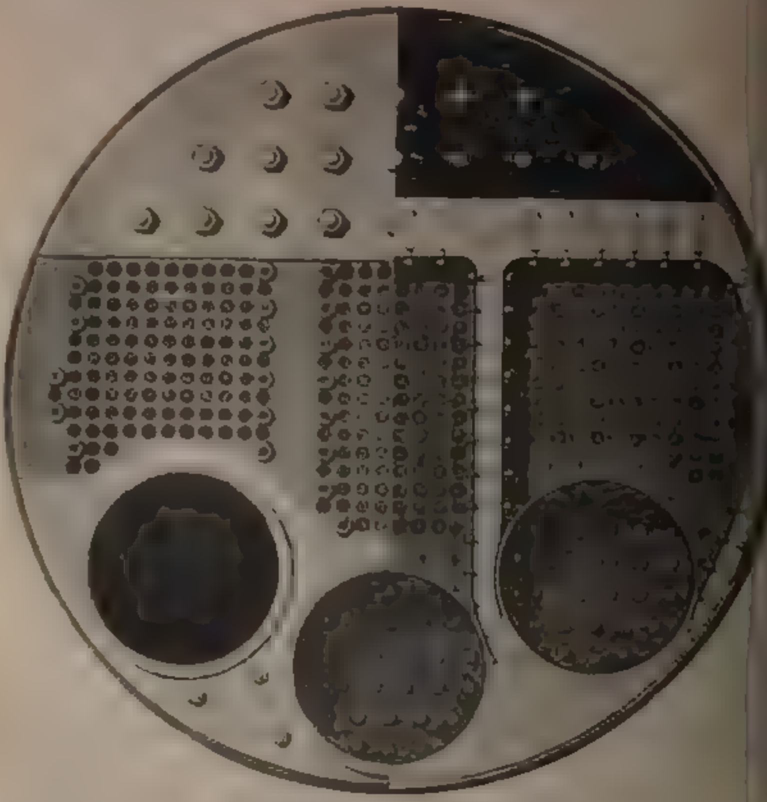

FIG. 26[illegible]. DOUBLE-ENDED MARINE BOILER.

that there may be a great increase in the speed of the main [...] pump and besides this there is an auxiliary feed pump also. I[f], in spite of this, the water level gets lower, the stop valve must be cl[osed], the safety valve opened and the fires drawn.

Unless there is time given to prepare so that there may [be a] good reserve of steam by throttling &c., it is difficult to main[tain] constant pressure when the speed of the ship alters. It is po[ssible] now to blow off without noise by the *silent blow off* or stop valv[e]

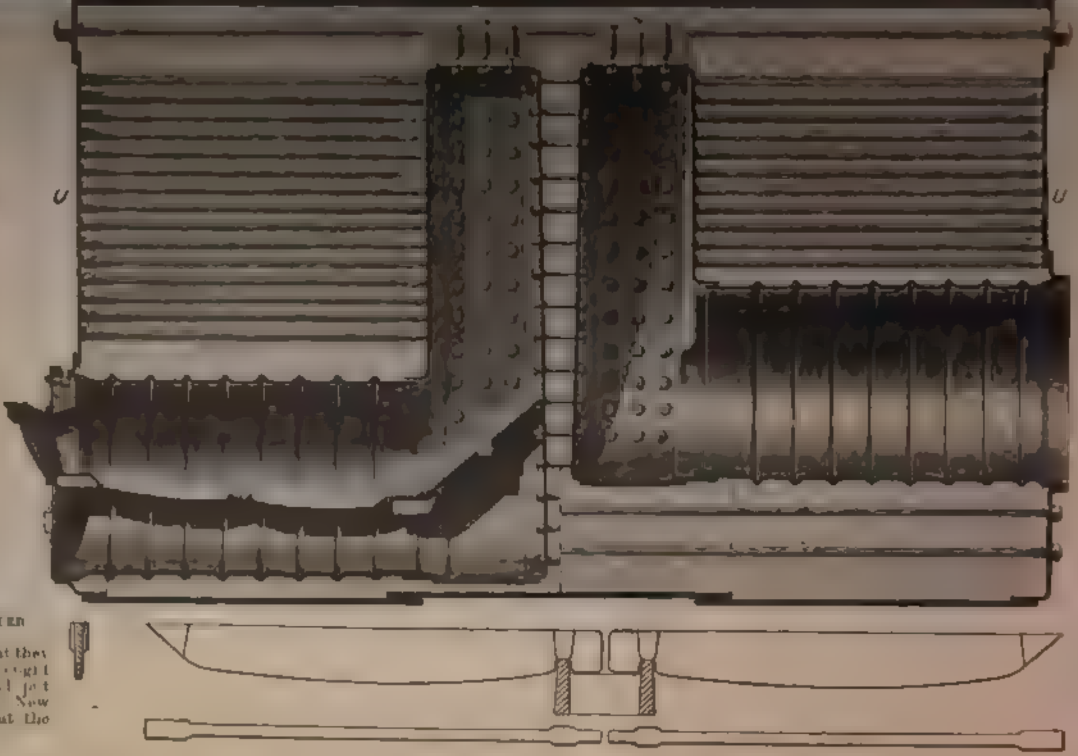

FIG. 20[illegible]. DOUBLE ENDED MARINE BOILER

FIG. 206. MARINE BOILER FIRE BARS.

[illegible] so that they sag, bend [illegible]. They ought to be [illegible] taken out and put back again [illegible] order. New bars ought to be placed at the back.

the main steam pipe, which lets steam directly into the condense
thus saving feed-water. Care must be taken in doing this graduall
so as not to damage the condenser tubes.

It is a good exercise for students starting at the feed tan
to describe **how the water stuff travels** in a marine engin
Feed tank at 100° F.—feed pump suction pipe, suction valv
increased pressure, delivery valve with branch to boiler-feed valv
feed pipe inside boiler. Great hea
**received through heating surfac**
from furnace and flues, becom
steam at 370° F. and 170 lbs. pre
sure, passes through stop valv
nearly dry, main steam pipe, bulk
head valve, stop valve, regulatin
valve getting a little wet, valv
chest of H.P. engine, H.P. cylinde
condensing on entrance a good dea
doing work on piston, expandin
and evaporating a little, exhaust a
larger volume and smaller pressur
and evaporating all that was con
densed as it passes into first re
ceiver, valve chest of intermediat
cylinder condensing as it ente
doing work on piston, expandin
and evaporating a little, exhaus
at larger volume and smaller pres
sure and evaporating all that wa
condensed as it passes into secon
receiver—valve chest of L.P. cylin
der, condensing on entrance to L.
cylinder, doing work on piston, ex
panding and evaporating a littl
**exhaust** at larger volume and smaller pressure, evaporating all tha
was condensed at first as it passes by exhaust pipe to condenser,—
suction pipe, foot, bucket and delivery valve—discharge pipe to fee
**tank.**

FIG. 207.—MARINE BOILER FIRE DOOR

All the sulphate of lime coming in with feed-water is insoluble a
290° F. and **deposits** as a close fitting scale. Common salt is solub
and magnesium sulphate although insoluble falls as a soft depos
Besides it is removable as carbonate of lime is removable (by previou
boiling). Sea water contains $3\frac{1}{2}$ lbs. of sulphate of lime per ton.

**ISE.** Engines of 12,000 I.H.P. use 17 lbs. of steam per hour power; what is the weight of feed-water per day?

r. $\dfrac{12,000 \times 17 \times 24}{2,240}$, or 2,186 tons.

se the feed-water to have 5 per cent. of sea water per cycle it because of leakage, what is the amount of deposit of f lime in the boilers per day? *Answer.* Each ton of feed 175 lb., or there is a total deposit of 383 lbs. per day.

total heating surface is 50,000 square feet, and if the of lime is deposited uniformly over it, and if its specific 2·6, what thickness will be deposited in three months?

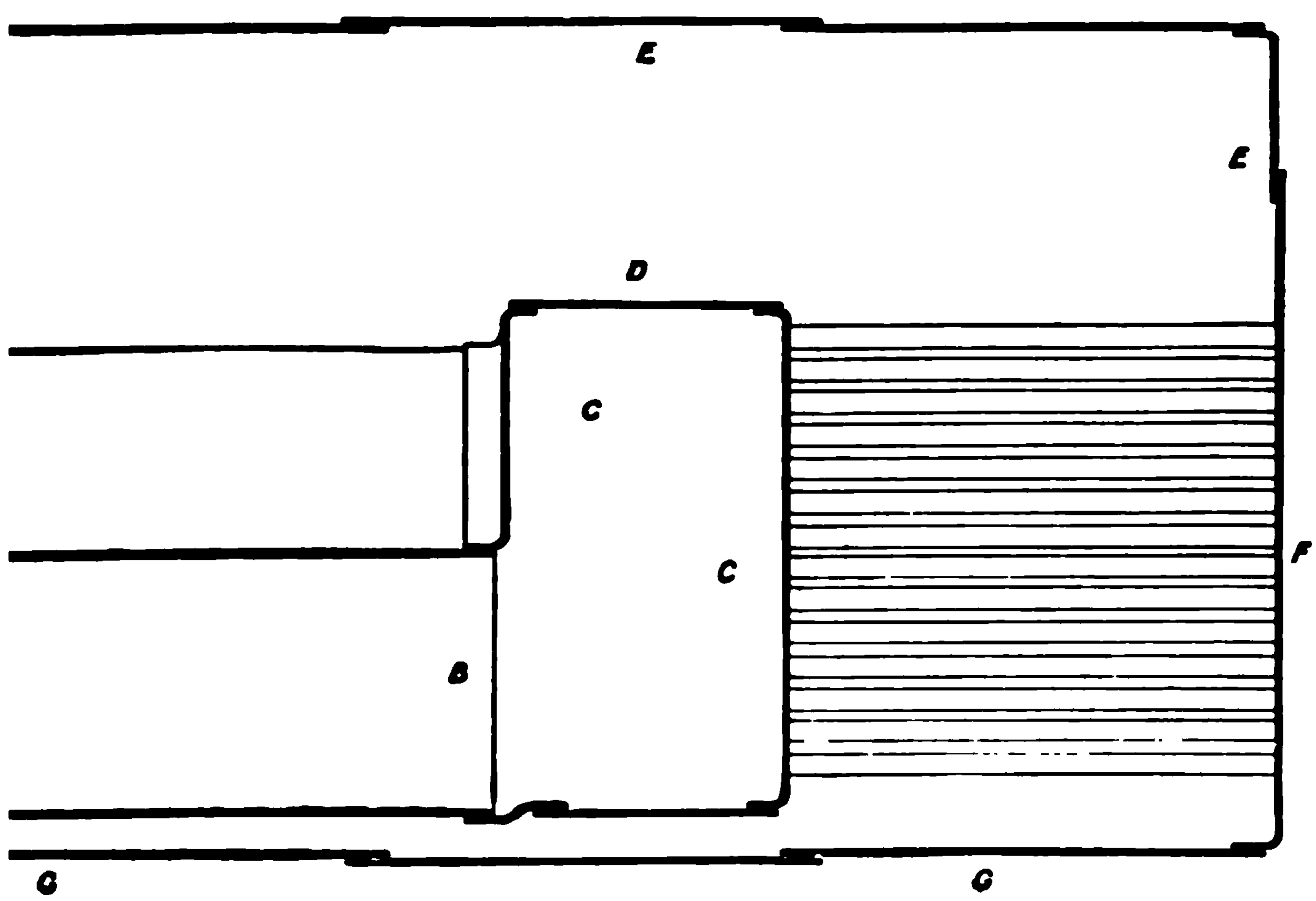


FIG. 208.—LOW CYLINDRIC MARINE BOILER.

The volume per day is 383 ÷ (62·3 × 2·6) or 2·364 cubic ness in feet per day, 2·364 ÷ 50,000, and thickness in inches s is ·0516 or a little more than $\frac{1}{20}$th of an inch.

08 is a low form of cylindric marine boiler, (7 to 9 feet with two furnaces, 10 feet with three furnaces, 17 to 18 feet om employed except in small vessels. The stay bars from stion chamber to the shell, *D* to *E*, make the interior less The heating surface is about 30 times the grate. About coal are used per hour per square foot of grate, or 26 if the aught is helped. The weight of boiler, including water, is 1¼ cwts. per indicated horse-power.

**r tube boilers** are mostly used in cases where space is

limited, as in ships, electric light and other stations in cities. They are now used in the very largest ships. They give the same heating surface with less weight both of boiler and of water contained by it; the pressures are high and safety great; their weight per horse-power is about 40 lbs. as against 130 lbs. in cylindric boilers. There are no other boilers capable of producing so much steam per hour which have so **little reserve power**; and it almost seems as if we were nearing the time when boilers will only contain as much water as will supply their engines with steam for a few seconds. At present steam is raised in them in twenty to thirty minutes without undue straining. These boilers are almost all now fitted with floats which open equilibrium feed valves automatically, to keep the water level nearly constant. The arrangements must be very frictionless. In considering gauge glasses, &c., it is to be remembered that there are considerable differences of pressures between different parts of these boilers. Hence, when evaporation stops in the *non-drowned* types the water level falls. Impure water is specially troublesome. A reducing valve is often relied upon to steady and dry the supply from these boilers.

In ordinary boilers there would be a very much more rapid generation of steam if centrifugal pumps or other **stirring arrangements** were worked inside. The water tube boiler gives probably the very best circulation that we are likely to see due to mere natural changes of density produced by heat. This matter has become much more important since the use of surface condensers has caused boiler water to be greatly free from air. (See Art. 354.)

These boilers are seen at their worst when supplied with unsuitable water. In towns they are supplied with fresh water continually, when supplying non-condensing engines. This water is passed through a feed-water heater, called a **water purifier**, arranged so as to be easily cleaned of sediment.

All town water has from 10 to 100 grains of solid matter per gallon.

EXERCISE. Engine of 100 indicated horse-power using 20 lbs. of water per hour per horse-power. How much solid matter is deposited by the water per month (10 hours per day)? *Answer.* 50 to 500 lbs.

Filters are used to remove the mud, or sometimes mere settlement suffices. As for the salts: the carbonates of lime and magnesia are only soluble if carbonic acid is present, so that if this is removed, either by boiling or by the addition of lime-water or soda the salts are deposited. As for the sulphates of lime and magnesia at a very

temperature they are insoluble and will **deposit**. But the tion of carbonate of lime will also cause them to deposit as a e powder, which may be removed by filtering.

Ir. Thornycroft was probably the first to introduce the water boiler with rapid circulation of the water in small tubes, the e and hot gases playing round their outsides. The water in his l curved pipes is partly water, partly steam; the mixed mass rapidly, being very light compared with the more compact mass ater in his down-comer pipes. In this way the water is always lating in a way which has been examined through thick glass on his top horizontal steam chamber, and he has measured the unt of water circulating by means of a gauge notch inside. The of the small tubes could be seen spurting out water intermently, and there is a complete circulation of 105 lbs. of water for y 1 lb. of steam generated. There is still some discussion as e relative values of the **Thornycroft system**—tubes opening re water line—and the drowned tube system. Thornycroft ns greater safety, more certain and more rapid circulation, better king with bad water and better efficiency, and more power the weight. The curved tubes bend easily without straining boiler.

In one form of Thornycroft boiler the furnace fuel does not ate heat directly to the water tubes: the furnace has a firebrick ering: the products of combustion are well mixed before they are itted to the tube space.

In the **Yarrow** boiler, Fig. 211, the tubes are straight, they enter steam-chamber below the water level. In the Belleville boiler 4 inch or 5 inch tubes are straight, joined to elbow pieces or ction boxes by screwed joints, making zig-zag paths of small e from a low small water chamber to an upper steam chamber over ordinary grate. All is enclosed, except the steam-chamber. The l admitted to the steam-chamber mixes there with rising water, both descend through a non-return valve to a quiet sediment ector before being used. The sediment is blown out periodically. ne lime put into the feed tank causes the oil to deposit also.[1] The es may be examined by opening doors on the front: there is an omatic feed control, a float in a stand pipe controlling the feed ulation valve.

**The Babcock and Wilcox**, Fig. 212, is much employed in tric light stations.

EXERCISE.—If the proper working pressure for a tube 16 feet

[1] These boilers are particularly affected by a list of the vessel to one side.

diameter and $1\frac{1}{2}$ inches thick, weakened by no joint, is 200 lbs. per square inch (above atmos.), what is the proper working pressure for a tube 1 inch diameter $\frac{1}{16}$ inch thick of the same material?

*Answer.* 1,600 lbs. per square inch.

EXERCISE. A vertical pipe of length $l$, has many thin copper tubes lying inside it, nearly touching; water is driven through the space round the tubes, along the pipe, at the velocity $v_1$; hot gases from a furnace are driven through the tubes in the opposite direction at the velocity of $v_2$. If the heat given to the water is proportional to $l\sqrt{\frac{v_1 v_2}{m_1 m_2}}$, where $m_1$ and $m_2$ are the hydraulic mean depths of the gas and water spaces, prove that (if the thicknesses of the tubes are proportional to their diameters) if the diameters of the tubes and pipe are halved, keeping the same number of tubes and same arrangement of them, and if the same quantities of water and gas are drawn through, the amount of heat given up is the same, if the length is only one-eighth of what it was before.

For if $d$ is the diameter of a tube, $m_1$ is proportional to $d$, so that $m_1$ is halved; also it is easy to see that $m_2$ is halved; also the velocities are inversely proportional to the areas, so that $v_1$ is four times as great, and so is $v_2$. If $x$ is the new length, then

$$x\sqrt{\frac{4v_1,\ 4v_2}{\frac{1}{2}m_1,\ \frac{1}{2}m_2}} = l\sqrt{\frac{v_1 v_2}{m_1 m_2}},\ \text{or } x\sqrt{64} = l,\ \text{or } x = \frac{1}{8}l.$$

I have no proof that the above rule truly holds, but I have no doubt that some such rule holds. If it does, the application of it ought to lead to great reforms in boiler construction. See Chap. XXXIII.

**127. Draught.** If students work the following exercises they will possess the small amount of knowledge that seems in anybody's possession on the subject of **chimney** draught.

EXERCISE 1. The weight of a cubic foot of air at atmospheric pressure and 32° F. is ·0807, what is the weight of a cubic foot at 62° F.; at 552° F.? *Answer.* ·0761 lb., ·0393 lb.

EXERCISE 2. A column of air at 552° F., 1 square foot in section, and $h$ feet high, how much less is it in weight than a column of equal height at 62° F.? *Answer.* $h$ (·0761 − ·0393), or ·0368 $h$.

EXERCISE 3. What height of chimney will produce a draught equal to the pressure of 1 inch of water, if its average internal temperature is 552° F. and the temperature of the atmosphere is 62° F.? A square foot 1 inch high of water is $\frac{1}{12}$ of a cubic foot, and weighs 62·3 ÷ 12, or 5·2 lbs. This must be the difference in weight of a column of hot air inside the chimney and a column of the same height of cold air: taking the answer of Exercise 2, if $h$ is height of chimney in feet,

$$\cdot 0368\, h = 5{\cdot}2, \text{ or } h = 140 \text{ feet nearly.}$$

This answer ought to be remembered by all engineers.

The stuff in the chimney is a little denser than air; the

mperature is perhaps less or more than 552° F., the outside air ay be different from 62° F. Nevertheless, a chimney of the height 140 feet will produce a draught of about 1 inch of water if the ow of gas is slow. When the gas flows fast, the draught diminishes ecause of the friction in the chimney itself.

This draught is needed to overcome the **frictional resistance** to e passage of air, (1) through the coals on the grate; the more these re scrubbed by the air the more rapid being the combustion of what ay be called the fixed carbon. Indeed, this scrubbing conduces to ss air being needed per pound of coal. The frictional resistance in he fire is probably the greatest of the frictional resistances in a oiler which has a thick fire; (2) round corners and obstructions in he flues; (3) along all the more regular parts of the flues and himney. This is probably the smaller of the three terms whereas t ought to be much the greatest in a well-arranged boiler. It is roportional to the whole surface of flues and chimney, and is nversely proportional to their average cross section. Indeed, it is sual to say, what comes to the same thing, that it is proportional to the length, divided by $m$ the **hydraulic mean depth** (cross section of any channel conveying fluid divided by perimeter touched by the fluid is called the hydraulic mean depth). It will be found that almost everything that makes friction great in flues conduces also, and for much the same reasons, to better combustion and the more rapid transmission of the heat to the water. If the velocity of air through a boiler is doubled the friction is quadrupled, and so the draught must be four times as great. And if produced by a chimney we saw that the draught is proportional to its height. Nevertheless, when a boiler is intended to burn twice as much coal per hour on every square foot of grate, although the velocity of air is to be twice as great and the draught necessary is four times as great, it is usual to assume that the height of the chimney need only be twice as great The subject, like all connected with it relating to friction of air in passages, has not yet been carefully studied. The height of a low chimney is usually fixed, not by calculation of the draught, but by the sanitary requirements of the neighbourhood.

The area of cross section of a brick chimney flue is usually taken to be this fraction of the whole grate area of the boiler or boilers $\frac{cw}{\sqrt{H}}$, where $H$ = height of chimney in feet, $w$ = weight of coal per square foot of grate per hour. $c$ is 0·1 for one Lancashire boiler, ·06 for six boilers, ·065 for twelve boilers.

The average height of a steamer's chimney is 70 feet above the

grate; its section is usually $\frac{1}{3}$ to $\frac{1}{6}$ of the total firegrate area. In locomotives and portable engines $\frac{1}{12}$.

More rapid rates of firing than 30 lbs. per hour per square foot of grate need **forced draught**. Nobody who has noticed the demoralisation of a good stoker when he is firing quickly and has no command of sufficient draught will attempt to have a consumption of 35 lbs. per square foot with natural draught. With good draught and thick fires (never less than 10 inches thick after or 7 inches before stoking) we use less air and have higher temperatures.

In locomotives the forced draught is produced by the exhaust steam puffing up the chimney. In marine boilers the steam must all be returned to the boiler, and a surface condenser must be used, because the use of sea water in the boilers was always troublesome, even when low pressures were used; but with the high pressures now in use, sea water would deposit all its sulphates of magnesium and lime in the boiler. Hence a steam blast cannot be used. Indeed, in all cases natural draught is relied upon in the ordinary working, and forced draught is only used in emergencies. And yet when using the natural draught of the chimney we find differences due to the weather, so that **fans blowing air** into the boiler room (a certain amount of care, but not too much, being taken to close all vents except through the furnaces) produce a wonderful improvement. The supply to fans is always through cowls on the upper deck. The name "forced draught" is more usually applied to the case in which the stokeholds alone are made air-tight, and air is pumped into them so that the draught obtainable is 1 to $1\frac{1}{2}$ inches of water in cruisers and 2 inches in battleships. Entrance to these stokeholds is through **air locks** (that is, two air-tight doors with a space between them). These are open if the forced draught is not on, and other openings are then also made.

Indeed, the fans are usually kept going all the time, and when the draught produced by them is only $\frac{1}{2}$ an inch of water it is really used as "natural draught" on the trials of a ship's engines. From 25 to 70 per cent. is said to be the increase of development of steam with fairly good combustion, producible (with good fires) by from 1 to 2 inches of forced draught.

Under natural draught in Lancashire boilers we find that we obtain the best results when twice the absolutely necessary quantity of air is admitted. Unless we admit this excess, the thicker parts of the fire get too little air. Under the fan-helped natural draught in marine boilers, about 50 or 60 per cent. of excess air is admitted, and under forced draught less than 50 per cent. of excess is admitted.

With the strong forced draught and thick fires of locomotive boilers good combustion is obtained with much less than 50 per cent. of excess air.

In **Howden's system** of forced draught, air driven by a fan passes through tubes in the uptake, and so is heated; it is admitted into the ashpit and over the grate, both spaces being air-tight, producing a draught of $\frac{3}{8}$ to 1 inch of water.

There is another system, of drawing the gases through a fan before they get into the chimney.

EXERCISE 1. If grate area is 160 square feet, 45 lbs. coal per square foot per hour, 200 cubic feet of air at 60° F. and atmospheric pressure, per lb. of coal. Find the useful work done by a fan if the draught produced by the fan is 1 inch of water pressure (the draught due to chimney is in addition to this).

*Answer.* 1 inch water pressure is $\frac{62 \cdot 3}{12}$, or 5·2 lbs. per square foot, and hence the work done per hour is $5 \cdot 2 \times 200 \times 45 \times 160$, or $7 \cdot 5 \times 10^6$ foot-pounds. The useful power is therefore 3·8 horse-power.

EXERCISE 2. If the useful power of the fan is 20 per cent. of the indicated power of the engine driving it, what is the indicated power?

*Answer.* 19 horse-power.

EXERCISE 3. The engine driving the fan consumes 30 lbs. of steam per hour per indicated horse-power, and the above boilers develop 10·2 lbs. of steam per pound of coal, what fraction of the total supply of steam is spent in driving the fan?

*Answer.* The total evaporation is $10 \cdot 2 \times 45 \times 160$, or 73,440 lb. per hour. The fan uses $19 \times 30$, or 570 lbs. per hour; the answer is therefore 0·0078, or 0·78 per cent.

EXERCISE 4. In the above boilers the heating surface is 45 times the grate area, and if boilers and their engines produce 1 indicated horse-power for 2 lbs. of coal per hour; at the above rate as they use $45 \times 160$, or 7,200 lbs. of coal per hour, the indicated horse-power is 3,600. This is 22·5 horse-power per square foot of grate, or 1 horse-power for every 2 square feet of heating surface.

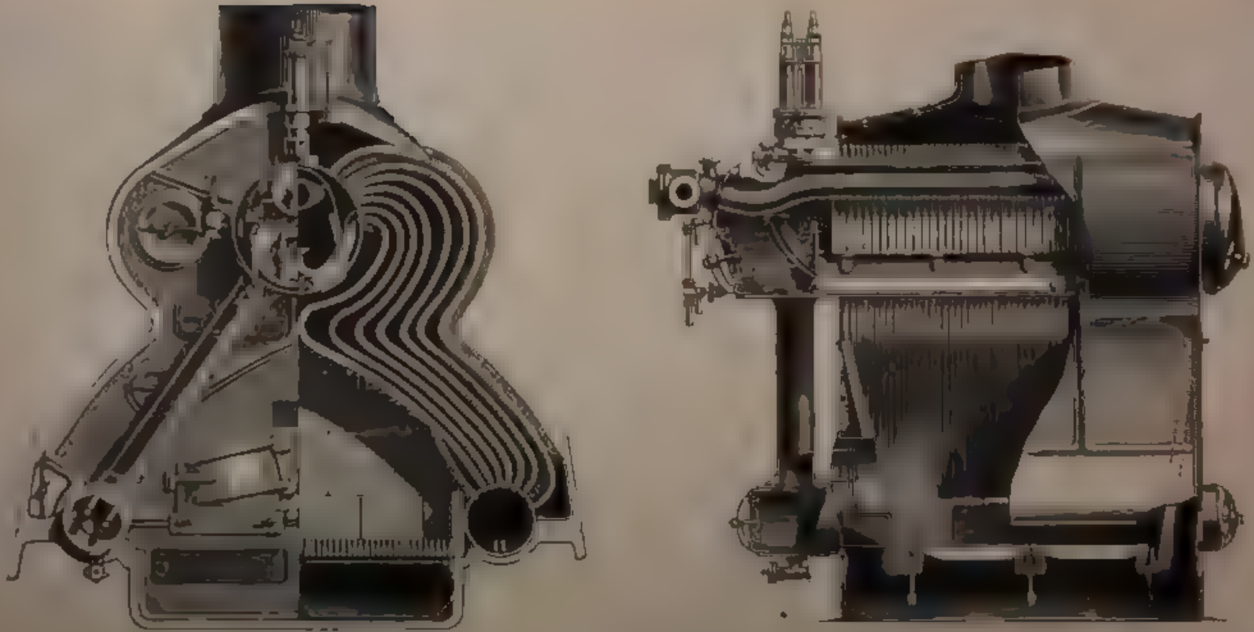

FIG. [illegible]. THORNYCROFT BOILER, "SPEEDY" TYPE. ONE FURNACE.

The inner row of [illegible] inch, [illegible] inch thick, tubes are so close at the top as to make an impervious roof and the outer row forms a similar wall except near the top, where the gases escape to the funnel. The water comes down by the two large tubes outside furnace at front end. See baffle plates in steam chamber. [illegible]

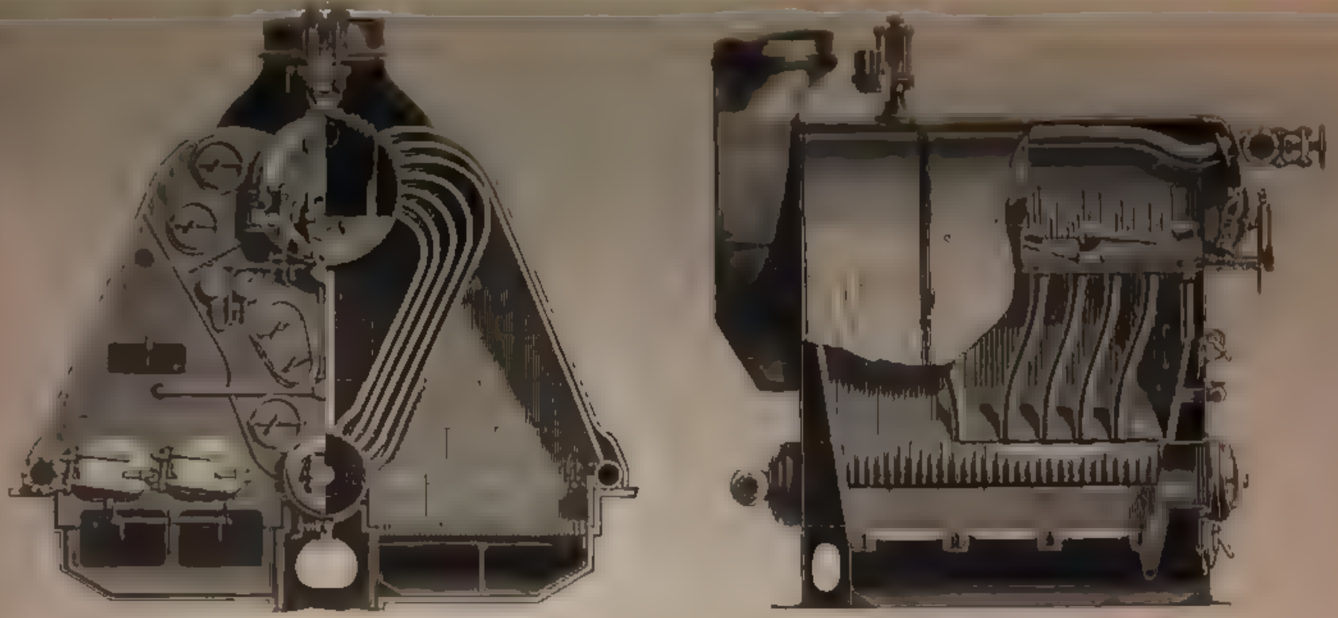

FIG. 210. THORNYCROFT BOILER "DARING" TYPE.

This type designed of greater height, less length, more accessibility. There are two furnaces. One large water cylinder below with two much smaller, the uptake is quite inside. The gases enter the inner rows near the bottom, and after rising, enter the uptake at the top. The down water tubes are in the central uptake and therefore get some heat. The baffle plate in the steam chamber is a series of gratings. The tubes are ·104 inch thick.

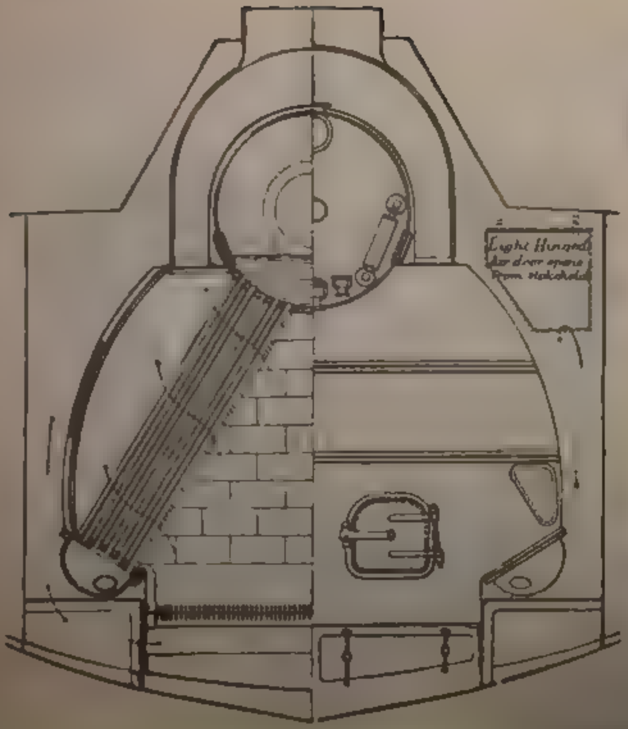


Fig. 111.—Yarrow Boiler.

[illegible]

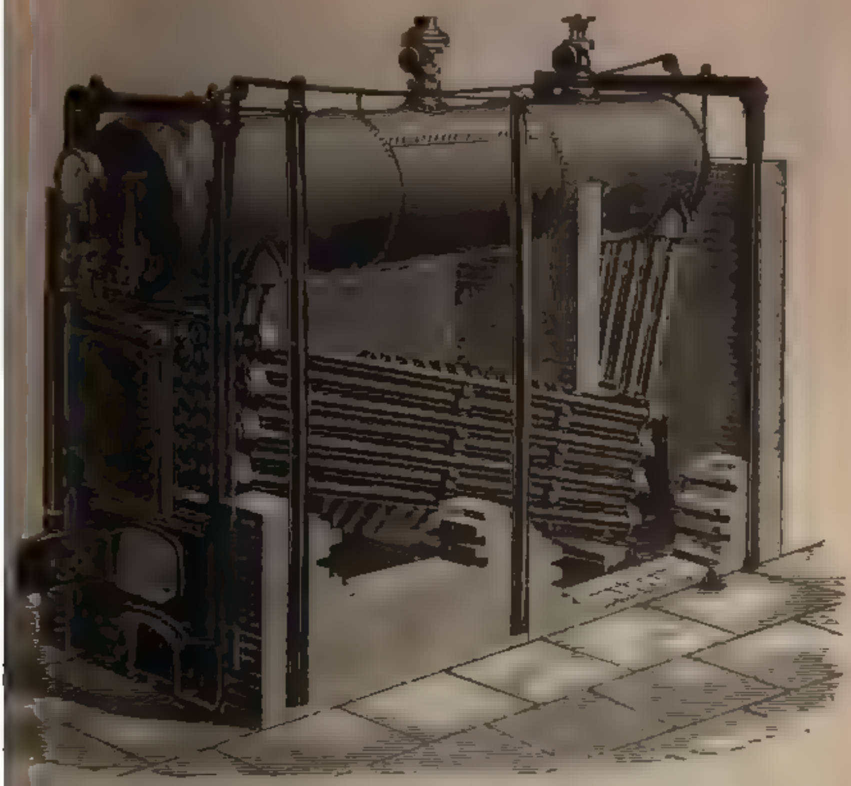

FIG. 212.—BABCOCK AND WILCOX

...straight sloping tubes connect water boxes each with a cover for examination and repair and ...h water pipes to the top drum which is half filled with steam. The nearly vertical water pipes ... in front, where there is an upward flow, and long behind. The boxes are also connected ...ly, and sometimes additional circulating pipes are added.

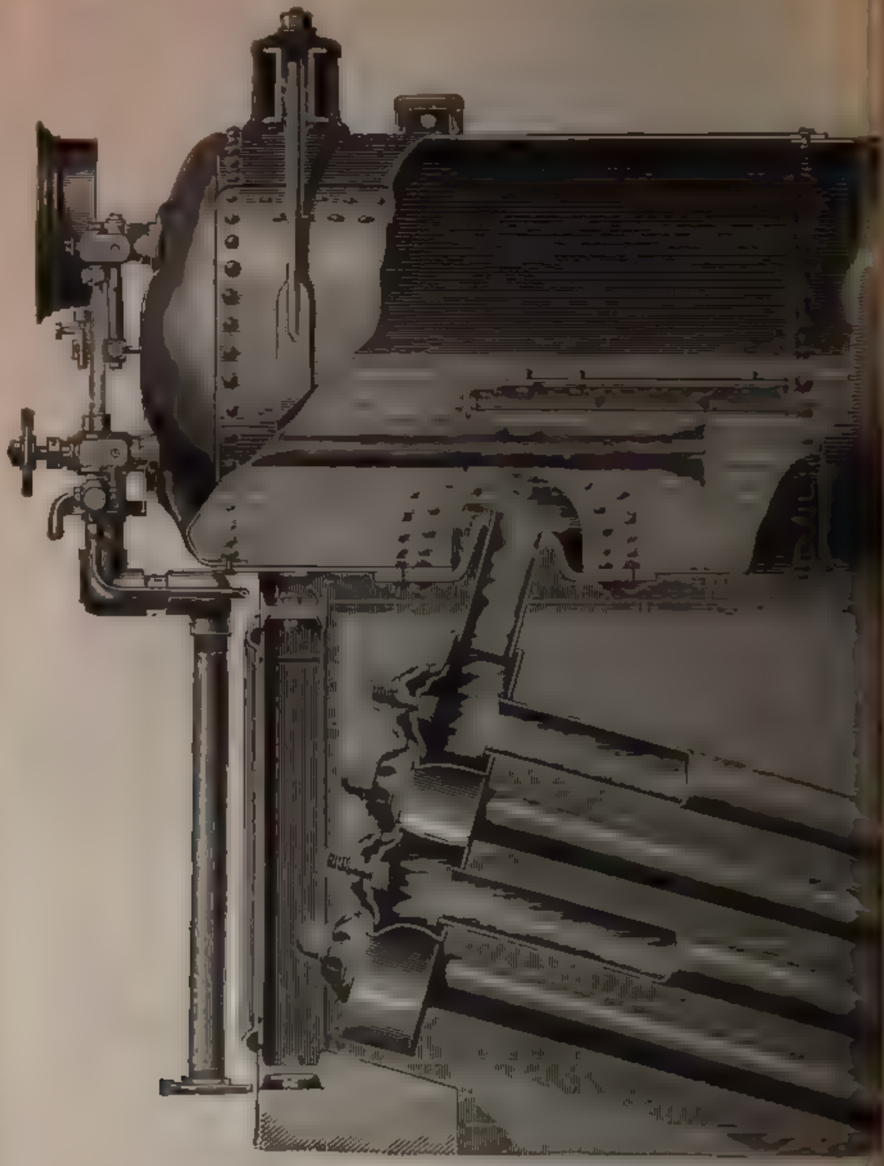

FIG 213.—BABCOCK AND WILCOX.

## CHAPTER XV.

### NUMERICAL CALCULATION.

**128.** EVEN the beginner in this subject must know not merely how to multiply and divide numbers, he must be able to work by logarithms. Let him therefore practise multiplication and division and extraction of roots, &c., in this way, at once. He must know the ordinary symbols of arithmetic and algebra, such as $+$, $-$, $\times$, $\sqrt{\ }$, $\sqrt[3]{\ }$, &c. Also what $a^2$ or $a^3$ mean. But he must also practise the calculation of $a^b$ where $a$ and $b$ are any numbers whatever. Let him, therefore, at once, work the following exercises, *by logarithms*.

EXERCISE 1. Calculate $a \times b$, which is sometimes written $ab$ or $a.b$; also calculate $a \div b$, which may be written $a : b$ or $\frac{a}{b}$ or $a/b$ when $a = 1323$ and $b = 24{\cdot}32$. *Answers.* 32175, 54·40.

Again when $a = 17{\cdot}56$ and $b = 143{\cdot}5$. *Answers.* 2520, 0·1224.

Again, when $a = 0{\cdot}5642$ and $b = 0{\cdot}2471$. *Answers.* 0·1394, 2·283.

Show that the following statements of the standard taken as the pressure of one atmosphere agree. 14·70 lbs. per square inch; 2116·3 lbs. per square foot; 29·92 inches of mercury at 0° C.; 760 mm. of mercury at 0° C.; 1033 kilos per square mètre; 33·9 feet of pure water at 0° C.; 33·04 feet of sea-water at 0° C.

For a certain purpose it is necessary to measure two distances in inches, to multiply the numbers and to extract the square root. Find the answer when the distances are 2·34 and 1·56 inches. Find the answers also when the distances are 2·33 and 1·55; 2·35 and 1·57; 2·35 and 1·55.

*Answers.* 1·9106, 1·9004, 1·9208, 1·9085.

If the student will suppose the method of measurement of the

distances to be such that an error of ·01 of an inch was possible, he will see that only three figures, say 1·91, ought to be given in the answer.

In a leading newspaper a few days ago I saw the indicated horse-power of a marine engine quoted as 3562·74 horse-power. Well, it is very probable that this measurement is in error at least 5 per cent. That is, the person who made the measurements and calculations is not sure whether the answer might not be 3,700 or 3,400, and yet he pretends that his last figure has a meaning. I am sorry to say that many misleading figures of this kind are published in the best books written on the steam engine.

I often notice that even careful experimenters have been using thermometers such that errors of one degree are quite probable, and yet they will state results of observation and calculation to six significant figures. The very best English thermometers cannot be relied upon in the most experienced hands to the tenth of a degree Fahrenheit if ranges of from 20° F. to 212° F. have been observed.

A teacher ought to manufacture a great many exercises in multiplication and division to make his pupils familiar with logarithms, and not until they are so, ought he to proceed to the following.

EXERCISE 2. Calculate $a^b$. That is, the number $a$ raised to the power indicated by $b$.

Find the logarithm of $a$, multiply it by $b$, and this is the logarithm of the answer.

Let $a=20{\cdot}52$ and $b=2$. *Answer.* 421·1.
$a=1{\cdot}564$ and $b=1\frac{1}{2}$. *Answer.* 1.956.
$a=0{\cdot}5728$ and $b=3$. *Answer.* 0·1879.
$a=60{\cdot}71$ and $b=\frac{1}{3}$. *Answer.* 3·930.

Note here that to multiply by $\frac{1}{3}$ means that we are to divide by 3.

$a=0{\cdot}2415$ and $b=\frac{1}{3}$. *Answer.* 0·6227.
$a=1{\cdot}671$ and $b=2$. *Answer.* 0·3581.
$a=5014$ and $b=3\frac{1}{2}$. *Answer.* $1{\cdot}12\times10^{-13}$.

Pupils must be well drilled upon the fact that $a^{-b}$ means $1\div a^b$, and that $a^b\times a^c=a^{b+c}$.

EXERCISE 3. Work out the values of $M=(sr^{-1}-r^{-s})/(s-1)$. When $s=0{\cdot}8$ and $r$ has the values 1·333, 1·5, 2, 3, 5, 8, 12, 20. The answers are given at page 286.

EXERCISE 4. Work out the values of $M$ in Exercise 3 when $s=1{\cdot}2$. The answers are given at page 286.

EXERCISE 5. It is said that the numbers headed $\theta$, $p$, $u$, $H$ and $l$

n the tables Art. 180 are nearly connected by the laws given in (1), 2), (5). (9), &c. Take a few of the numbers in the table and make he calculations, and state the apparent inaccuracy per cent. There no better kind of exercise, for it ought to be well understood that to rm a good acquaintance with the table means more than the one-ird part of our study of the steam engine. Hence, when a student actises the use of a slide-rule or book of 4-figure logarithms, he ight to practise on these numbers. Such a table also gives rise to e best kind of exercise work on squared paper.

**Exercise 6.** Let the student practise finding **rates of increase.** hus, if he takes numbers at random from columns $\theta$ and $H$ of the ble, say these :—

| $\theta$° F. | $H$. | $\delta\theta$. | $\delta H$. | $\delta H/\delta\theta$. |
|---|---|---|---|---|
| 230 | 1152·1 | 9 | 2·7 | 0·3 |
| 239 | 1154·8 | 9 | 2·8 | 0·31 |
| 248 | 1157·6 | 9 | 2·7 | 0·3 |
| 257 | 1160·3 | | | |

He ought in this way to practise the finding of $dp/d\theta$ (this is qual to $dp/dt$ if $t$ is the absolute temperature) and others; using quared paper, perhaps, to help him to find an exact set of values.

Exercise 7. In the following case how ought one to proceed? From a table of values of $p$ and $\theta$ to find $dp/d\theta$ for 105° C. with the greatest accuracy possible, $p$ being pressure of saturated steam in pounds per square foot, and $\theta$ temperature.

| $\theta$° C. | $p$. | $\delta p$. | | |
|---|---|---|---|---|
| 90 | 1463 | 302 | | |
| 95 | 1765 | 351 | 49 | |
| 100 | 2116 | **408** | **57** | **8** |
| 105 | 2524 | **470** | 62 | 5 |
| 110 | 2994 | 540 | **70** | 8 |
| 115 | 3534 | 618 | 78 | **8** |
| 120 | 4152 | | | |

One-fifth of 408 is evidently too small, one-fifth of 470 is too great; a little thought will show that the average of these is not correct either. There is a rule, deduced by an application of Taylor's Theorem, which can be employed in such cases. Note the figures in clarendon type; it will be found that the true $dp/d\theta$ for $\theta = 105°$ C. is given very accurately by the series :—

$$\tfrac{1}{5}\{\tfrac{1}{2}(408 + 470) - \tfrac{1}{16}(70 - 57) + \tfrac{1}{36}(8 + 8)\} = 87\cdot59.$$

**Proof.** If the quantities sloping down to the right be called $d_1$, $d_2$, $d_3$,

&c., and those sloping upwards to the right be called $e_1$, $e_2$, $e_3$, &c., then, representing the value of $p$ when $\theta$ is 105° C. as $f(\theta)$, we have :—

$$d_1 = f(\theta + h) - f(\theta) = hf' + \frac{h^2}{\lfloor 2}f'' + \frac{h^3}{\lfloor 3}f''' + \&c.$$

$$e_1 = f(\theta) - f(\theta - h) = hf' - \frac{h^2}{\lfloor 2}f'' + \frac{h^3}{\lfloor 2}f''' - \&c.$$

Therefore $$d_1 + e_1 = 2hf' + 2\frac{h^3}{\lfloor 3}f''' + \&c.$$

By further application of Taylor we obtain $d_2$ and $e_2$; $d_3$ and $e_3$, and neglecting terms with the 7th and higher powers of $h$, we are able to express $f'''$ and $f^v$ in terms of $d_2 - e_2$ and $d_3 + e_3$, and so obtain

$$hf' = \tfrac{3}{4}(d_1 + e_1) - \tfrac{3}{20}(d_2 - e_2) + \tfrac{1}{60}(d_3 + e_3).$$

In the same way we find

$$h^2f'' = 1{\cdot}209\,(d_1 - e_1) - 0{\cdot}1045\,(d_2 + e_2) + 0{\cdot}0098\,(d_3 + e_3).$$

In particular cases we can find $\frac{dp}{d\theta}$ with great accuracy even from only two terms if we know a good empirical formula. Thus, for example, we know that with not very great, but with some accuracy, the pressure and temperature of steam are connected by the law $\theta = a + bp^{1/5}$, if $\theta$ is the temperature Centigrade or Fahrenheit. Hence if we only get

$$p = 2524 \text{ for } \theta = 105° \text{ C.}, \; p = 2994 \text{ for } \theta = 110.$$

Extracting the fifth roots of these two pressures

$$105 = a + 4{\cdot}790\,b \qquad 110 = a + 4{\cdot}958\,b.$$

Solving, we find $b = 29{\cdot}77$, $a = -37{\cdot}6$.

$$\text{Now } \frac{d\theta}{dp} = \tfrac{1}{5}bp^{-\frac{4}{5}}, \text{ or } \frac{dp}{d\theta} = \frac{5}{b}p^{\frac{4}{5}},$$

so that when $p = 2524$, $\frac{dp}{d\theta} = 88$.

I often ask a large class of students to work out many of the values of $\frac{dp}{d\theta}$ in the table Art. 180, and to show the answers in a curve on squared paper.

Exercise 8. Assuming from Exercise 7 that $\frac{dp}{dt}$ for $\theta = 105°$ C. is 87·68, find $u$ the volume of a cubic foot of saturated steam from the formula

$$l = \frac{dp}{dt}(u - v_w)t$$

where $l$ is the latent heat of 1 lb. of this kind of steam in mechanical units or 740,710 foot-pounds; $t$ is the absolute temperature, or $\theta + 273{\cdot}7$, and $v_w$ is the volume of one pound of water which is nearly negligable. *Answer.* 22·31.

Exercise 9. A student is supposed to know that $yx^n = a$ is really the same as $\log. y + n \log. x = \log. a$, any kind of logarithms being used, and he ought to practise calculations requiring this knowledge.

For example: let us suppose that some kind of stuff follows the law $pv^{1{\cdot}13} = a$ where $p$ is pressure in pounds per square inch and $v$ is

volume in cubic feet. If $p=100$ lbs. per square inch and $v=1$ cubic foot, find $a$. *Answer.* 100.

Now if $v$ becomes 1·5, using the same value of $a$, find $p$.

*Answer.* 63·24.

Again, if $v$ becomes 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, in each case find the corresponding value of $p$.

See if your answers are as shown in the third column of the first table of Art. 156.

Repeat the above work when $pv^{0.9}=a$, taking $p=100$ and $v=1$ to start with, and compare your answers with the figures of Art. 156.

EXAMPLE. It is said that if $p$ is the pressure of saturated steam in pounds per square inch and $u$ is the volume (in cubic feet) of a pound of steam, then there is a rule which is very nearly true,

$$pu^{1.0646}=479.$$

Take some of the values of $p$ in the table Art. 180 and calculate values of $u$ for the purpose of this exercise, and also notice to what extent the formula does really represent the relation between $p$ and $u$.

EXERCISE 10. Mr. D. Baxandall and Mr. Lister find that the numbers in the last columns of Table II. Art. 180, may be calculated by the simple formulæ

$$w=5{\cdot}84+\frac{443}{p+50}$$

where $w$ is the weight of dry saturated steam per horse-power per hour, of pressure $p$ lb. per square inch, which would be used by a perfect condensing engine using the Rankine cycle (see Art. 214); and

$$w=8{\cdot}28+\frac{1077}{p+4}$$

where $w$ is the weight of dry saturated steam per horse-power of pressure $p$ lb. per square inch, per hour, which would be used by a perfect non-condensing engine using the Rankine cycle.

Test the accuracy of these formulæ for the following values of $p$ by comparing with Table II. of Art. 180.

| Condensing. | | | Non-Condensing. | | |
|---|---|---|---|---|---|
| $p$. | $w$ by above formula. | $w$ in table. | $p$. | $w$ by above formula. | $w$ in table. |
| 50 | 10·27 | 10·33 | 50 | 28·04 | 28·4 |
| 110 | 8·61 | 8·60 | 110 | 17·73 | 17·65 |
| 170 | 7·85 | 7·86 | 170 | 14·36 | 14·50 |
| 280 | 7·18 | 7·16 | 180 | 12·07 | 12·05 |

EXERCISE 11. If $p_1v_1^{1.13}=p_2v_2^{1.13}$ and if $\frac{v_2}{v_1}$ be called $r$. If 6 lbs. per square inch, find $r$ for the following values of $p_1$.

| $p_1$ . . . . | 250 | 200 | 150 | 100 | 50 |
|---|---|---|---|---|---|
| $r$ . . . . . | 27·13 | 22·27 | 17·26 | 12·06 | 6·53 |

We have here found the ratio of cut off which enables pressure $p_1$, to become 6 at the end of the expansion.

EXERCISE 12. If $p = a(\theta + b)^5$, and if we have given following values

| $\theta°$ C. . . . | 130 | 135 | 140 |
|---|---|---|---|
| $p$ . . . . | 39·25 | 45·49 | 52·52 |

Find $\theta$ when $p = 45$. Also find $\frac{dp}{d\theta}$, which is $5p/(\theta + b)$.

*Answer.* $\theta = 134°{\cdot}66$ $\quad \frac{dp}{d\theta} = 1{\cdot}31$

The student will find that the above formula, although enough for interpolation purposes, is not an accurate general fon connecting $p$ and $\theta$ for saturated steam.

EXERCISE 13. In proving the reasonableness of the Willans la steam engines, I use in Art. 161 the approximate formula

$$\frac{1}{u} = {\cdot}0171 + {\cdot}0021\, p.$$

where $u$ is the volume in cubic feet of 1 lb. of steam and $p$ is pressure in pounds per square inch.

Take the following values of $p$, calculate $u$, and compare wi as given in table.

| $p$. | $u$ by the above formula. | Real $u$ |
|---|---|---|
| 80 | 5·40 | 5·37 |
| 120 | 3·71 | 3·67 |
| 140 | 3·21 | 3·18 |
| 180 | 2·53 | 2·51 |
| 220 | 2·09 | 2·09 |
| 280 | 1·65 | 1·63 |

**129**. The *common* logarithm of a number $n$ may be and oft written as log. $n$, but if we wish to let readers be quite sure th

s on the common system, which suits our decimal system, or is to the base 10 as we say, then we write it as $\log._{10} n$.

Mathematical men use **Napierian** (mechanical engineers sometimes call them hyperbolic) logarithms to the base $e$, as they are called, where $e$ is a well-known number 2·7183. Thus $\log. n$ is read "The Napierian logarithm of the number $n$." In mathematical work generally, $\log. n$ always means the Napierian logarithm, the being left out. To convert *common* into Napierian logarithms, multiply by 2·3026.

The Napierian logarithm is very useful to the engineer, and so we have given a table at page 288.

EXERCISE 1. Using a table of common logarithms calculate the Napierian logarithms of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 16, 20. The answers are given at page 288.

EXERCISE 2. Work out the values of $(1 + \log_e r)/r$ when $r$ has the values 1, 1·333, 1·5, 2, 3, 5, 12, 20. The answers are given in the fourth column, page 286.

EXERCISE 3. If $\frac{p_0 v_0}{t_0} = R$, find $R$ when $v_0 = 12{\cdot}39$, $t_0 = 493$ (corresponding to 32° F.) and $p_0 = 2{,}116$.

*Answer.* $R = 53{\cdot}2$.

EXERCISE 4. If $v = 3$, $t = 500$, find $p$ if $\frac{pv}{t} = 53{\cdot}2$.

*Answer.* $p = 8{,}865$.

EXERCISE 5. If $K = {\cdot}2375$, $k = {\cdot}1688$, and if $v$, $t$ and $p$ have their values in the last exercise, calculate $\phi$, the entropy of a pound of air, in the following ways:—

$$\phi = k \log.\frac{p}{p_0} + K \log.\frac{v}{v_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$= K \log.\frac{t}{t_0} - R \log.\frac{p}{p_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$= k \log.\frac{t}{t_0} + R \log.\frac{v}{v_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The logarithms are Napierian.

*Answer.* $-$ 0·0411 in all three cases.

EXERCISE 6. The numbers headed $\phi_w$ (the entropy of 1 lb. of water) are very nearly equal to $\log_e \frac{t}{t_0}$.

This would be exactly right, only that the specific heat of water is not constant. $t$ is any absolute temperature, and $t_0$ is the

absolute temperature corresponding to 0° C. or 32° F. Calcul a few values.

EXERCISE 7. The numbers headed $\phi_s$ (the **entropy** of a pou of steam) are calculated by adding to $\phi_w$, the latent heat $l$ divi by the absolute temperature $t$. Calculate a few of them.

## 130. Exercises in Mensuration.

(1) A cylinder 18 inches diameter, 30 inches long, what is volume in cubic feet?

*Answer.* 4·41.

(2) A cubic foot of water at ordinary temperatures weighs 62·3 l A gallon contains 10 lbs. of water. There are two pints in a qu and four quarts in a gallon. The clearance spaces in the cylinder a steam engine are filled with water and emptied; the water measured and found to be 13·2 and 15·6 pints. What are t volumes of the clearance spaces?

*Answer.* 457 and 533 cubic inches, or ·26 and ·31 cubic feet.

(3) If the answer to Exercise 1 is the volume of the working stro of the same cylinder, compare the volumes of clearances and worki stroke.

*Answer.* 6 per cent and 7 per cent.

(4) A pound of stuff, partly steam and partly water, at a pressure 69·21 lbs. per square inch, fills a vessel whose volume is 5·2 cubic fe Neglecting the volume of water, what is the weight of the porti which is steam? In the table, page 320, you will find the volume one pound of this kind of steam. *Answer.* 0·843 lb.

(5) If the vessel of the last question gets larger, its volume becomi 9·8 cubic feet, and we find that the pressure is 33·71 lbs. per squ inch, what are now the weights of steam and water present? *Answ* 0·809 lb. steam, 0·191 lb. water.

In the above two questions we neglected the volume of the wat We had ·157 and ·191 lb. of water respectively in the two cases, a these must be very nearly the correct amounts, however carefully t calculation had been made. Taking water at 62·3 lbs. per cubic f the volumes are ·0025 and ·0031 cubic feet, obviously small enou to be neglected in comparison with the volumes of steam in stea engine calculations.

(6) A volume of 7,620 cubic inches is represented on a diagra to scale by a distance of 8·6 inches, what distance will represe 457 cubic inches? What volumes will be represented by 3·34, 5· 8·39, 0·65 inches?

*Answer.* 0·52 inches; 1·715, 2·865, 4·323, and 0·3334 cubic feet.

The above answers are all used in Chap. V.

(7) A locomotive travels at 50 miles per hour; how many revolutions per minute are made by one of its wheels, 6 feet diameter, assuming no slip?

*Answer.* 233·3 revolutions per minute.

(8) A screw propeller makes 150 revolutions per minute, its slip is 3 per cent., the ship travels at 15 knots. What is the pitch of the screw?

*Answer.* 10·43 feet.

(9) A cylinder is 15 inches in diameter. What is the area of its cross-section in square inches and in square feet? The crank is 14 inches. What is the working volume in cubic feet? It takes exactly a gallon of water to fill the clearance space. What is its volume? Express the clearance as a fraction of the working volume.

*Answers.* 176·715 square inches. 2·8634 cubic feet. 0·16 cubic foot. 0·0561.

(10) The length of the indicator diagram from the cylinder of (3) parallel to the atmospheric line is 2·8 inches; what distance will, to the same scale, represent the clearance?

*Answer.* ·157 inch.

(11) A boiler has 300 tubes 8 feet long, 3 inches diameter inside. What is the total cross-sectional area? What is the area of tube-heating surface?

*Answer.* 2,122 square inches. Heating surface = 1,880 square feet.

(12) The hydraulic mean depth $m$ of a pipe or channel is its cross sectional area, divided by its perimeter touched by the fluid; in the case of a pipe running full of water, or of a pipe in which gas is flowing, this is the whole perimeter. What is the hydraulic mean depth of one of the above tubes?

*Answer.* The area is $\frac{\pi}{4}\times 3^2$; the perimeter is $\pi\times 3$; $m=\frac{3}{4}$ inch.

(13) Find the volume and weight of the rim of a cast iron wheel of square section, outside and inside radii 20 feet and 18 feet 6 inches.

*Answer.* Volume = 272 cubic feet. Weight = 54·5 tons.

(14) When the piston of (3) has passed through one-third of its stroke, what is the volume behind it?

*Answer.* 1·50 cubic feet.

(15) In the back stroke the piston of (9) is one-tenth of its stroke from the end when cushioning is taking place, what is the volume?

*Answer.* 0·446 cubic foot.

(16) In the engine of (3) if the crank shaft makes 200 revolutions per neglecting angularity of the connecting rod, find the velocity of the when it has travelled over these fractions of its stroke, 0·2, 0·4, 0·6, 0·8

*Answer*. 19·44, 23·8, 23·8, 19·44 feet per second.

(17) When the piston of (3) has travelled over 0·4 of its stroke, at wh (cubic feet per second) is steam coming in through the port? (neglect t that some stuff already in expands). If the port opening is 8″ × ½″ wha velocity through it of the entering steam?

*Answer*. 29·2 cubic feet per second.

**131.** Students are supposed to know how to find the **area** **volumes** of regular figures, and to find the **weights** of objec calculation. Exercises will be found in many books, or they easily be manufactured by teachers. It is necessary here, how to speak of the **area of irregular figures**. Thus to find the a Fig. 214. Every student ought to practise the use of the plani in finding areas. Simpson's rule will be found in all book

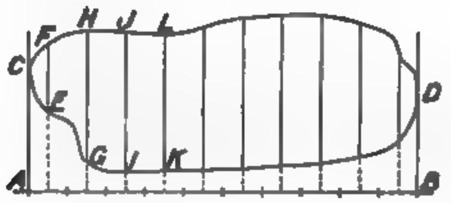


FIG. 214.

mensuration, but the following simpler rule is so much use engineers that the beginner ought to get well accustomed to it Let any direction be called the direction of the length of the —we shall take it to be horizontal. Draw two parallel lines $C$ $D B$ touching the figure at its extreme ends, and let the distance which is at right angles to both, be called the length of the fi Divide $A B$ into any convenient number of equal parts. Let us ten equal parts. Draw a perpendicular at the middle of each and measure $E F$, $G H$, $I J$, &c., the parts intercepted by the fi We should call these the ten equidistant breadths of the fi Add these together and divide by ten, and we have the **ave breadth** of the figure. This average breadth multiplied by the area of the figure approximately.

EXERCISE 1. If the ten equidistant measured breadths are 0·92, 1·16, 1·27, 1·25, 1·27, 1·24, 1·18, 1·15, 0·55 inches, and i length $A B$ is 3·24 inches, adding the breadths we have 10·2 dividing by ten we find 1·02 the average breadth. The ar

1·02 or 3·30 square inches. Notice that it is not well to give any figures in an answer in engineering.

If the area of a figure is 25·06 square inches and its length is es, what is its average breadth? *Answer.* 2·78 inches.

In the **indicator diagram** shown in Fig. 77, we take $A_1 A_2$ to length. It is 2·50 inches. The area of the figure is found to 6 square inches, by means of a planimeter. Find the average h. *Answer.* 4·56 ÷ 2·5 = 1·82 inches.

The ten equidistant breadths of Fig. 77 are 1·87, 2·15, 2·46, 2·30, 1·35, 1·20, 0·92, 0·85, 0·74 inches, what is the average h? *Answer.* 1·63 inches.

In the last exercise, if the breadth of the diagram represents essure of steam to such a scale that 1 inch represents 40 pounds ssure per square inch, what is the average pressure shown on agram? *Answer.* 65·2.

In Art. 32 you will find numbers which will enable you to any of the hypothetical indicator diagrams there described. their average pressures graphically, and see how nearly you to the answers given in page 76.

The **pull on a tramcar** varies quite gradually in the following find the average pull. The instrument with which the observer ured the pull was really a spring balance, which you may call a nometer if you please. Its vibrations were damped by a dash-

| $s$ | $P$ |
|---|---|
| 0 | 800 |
| 10 | 750 |
| 13 | 720 |
| 15 | 710 |
| 20 | 690 |
| 26 | 650 |
| 30 | 630 |
| 34 | 610 |
| 40 | 600 |
| 45 | 540 |
| 51 | 530 |
| 60 | 570 |
| 70 | 620 |
| 80 | 670 |

se it would have been a little difficult to read it. $s$ is the ce in feet, which the car had travelled through from a parti- place when the reading of each pull $P$ (in pounds) was made he average pull. If all the readings were at equidistant points

there would not be any great error in taking the average of all the numbers $P$ by merely adding them up and dividing by 14. But the distances $s$ must be plotted horizontally (to scale) on a sheet of paper, and ordinates to represent $P$ must be raised so as to get a curve which shows how $P$ varies. I have drawn a curve through the ends of the above ordinate and find that the average value of $P$ seems to be about 635 lbs., but of course any such answer is only approximately correct.

If a student does the following exercise, and meditates a good deal on his answers, it will be time well spent. In the table on p. 247 insert the time in seconds at which the tramcar reached each of the places at which the reading was taken. We count time from the first observation, the fourteen observed times were (let us suppose), 0, 0·2, 2·5, 3·0, 3·7, 4·1, 4·8, 5·2, 6·2, 6·5, 7·0, 8·3, 9·7, 11·2 seconds. Now find the **time average** of the force. That is, plot ($P$) and the time, and draw a curve and find its average breadth. I find the answer to be 640 lbs.

Now why is the time average different from the *space* average found above? I put this suggestive question here in a note because I do not wish a student to think it an essential part of our elementary treatment of the steam engine; but every student of applied mechanics will find the speculation an important one.

# CHAPTER XVI.

## ENERGY ACCOUNT.

**132.** In Chap. III. I assume that my reader knows how to make calculations concerning the **doing of work**; these belong to the more elementary subject of applied mechanics. Average force (pounds) in the direction of motion, multiplied by the distance (feet) moved through, is work done (foot-pounds). Many exercises ought to be dealt with where work is done against and by gravity or done against friction, or done in order that some equivalent energy may be stored. **Power is rate of doing work.** The power which an agent must exert in many operations must be well known through many numerical calculations. The operations about which a student's mind must be stored with exact figures are:—Traction or the pulling of railway trains, tramcars and all sorts of carriages on different kinds of roads with different kind of wheel tyres; the power which must be exerted in the propulsion of ships of different tonnage and shape, at different speeds; the waste of power by friction in such operations as the pumping of water, the creation of electric energy and its transmission and reconversion; the power needed to drive workshop tools of all kinds, and the machines used in all kinds of manufacture. English manufacturers are now beginning to copy the more sensible or scientific methods of their rivals in Germany and America, and like Dorothea in the story they are "learning what everything costs" and not only what everything costs in money but in money's worth. The mathematics of this subject of energy is the simple mathematics of the housekeeper and the butcher and the baker. There is still much measurement to be done with dynamometers, but the wonderful improvements which have been effected in steam engine manufacture in the last twenty years, due altogether to the good (energy) account keeping of electrical people who know exactly what they want and whether they get what they want, already enable us to say that a beginning has been made. There are at least two firms

of steam engine makers who have given up the slipshod and shiftless methods of working of the past; and the recent strike has done this much good that English manufacturers were forced to travel and now blame themselves a little for their own shortcomings.

An English cook specifies "pinches" of salt, "handfuls" of flour, "small amounts" of other things. An English engineer was often more hopelessly vague and kept no account of coal, steam, indicated power, actual power, power wasted in transmission, power given to a machine. Machines had to be driven and the engineer drove them, and his client was satisfied because like Toddy he only wanted "to see the wheels go wound."

As I have said, the subject is supposed to be dealt with under the head "applied mechanics." Nevertheless, as I shall refer to these figures, I have placed in the following sheets some scraps of information that I usually carry about in my head. **The answers to the following exercises** are always in the mind of a practical engineer.

Again, I have found it convenient to assume here that a student knows something of heat and other forms of energy, the heat required to produce a pound of steam, &c. although I do not enter upon the subject of heat and steam regularly until I get to Chap. XVIII. I have done this illogical looking thing because any course that one can take in teaching a subject must be illogical and the best course is usually the one that seems most illogical.

### **Units of Energy** *used Commercially.*

1 horse-power hour = 1.421 centigrade heat units = 2.558 Fahrenheit heat units = 1.980.000 foot-pounds.

1 000 gallons of water at a pressure difference of 750 lbs. per square inch convey 17.300.000 foot-pounds.

1 Board of Trade electrical unit = 1 000 watt hours = 2.654.000 foot-pounds.

1 centigrade heat unit [illegible] probably the most suitable value if we deal with Regnault's numbers = 1 [illegible] foot pounds. See Art. 177.

1 Fahrenheit heat unit [illegible] 774 foot-pounds.

The standard unit of evaporation which is the latent heat of [illegible] pressure = 536 centigrade heat units = [illegible] foot-pounds = the evaporation [illegible]

[illegible] = [illegible] centigrade = [illegible] units = about [illegible] evaporation units = about [illegible]

[illegible] horse-power [illegible] watts [illegible] unit of power = 1,000 watts.

Calorific energy of 1 lb. petroleum, about 17,000,000 foot-pounds.

Calorific energy per pound of town refuse actually obtained in a destructor (after deducting the large amount of energy wasted in steam-jets) = 900,000 foot-pounds.

Calorific energy of 1 cubic foot average coal gas = 530,000 foot-pounds.

Calorific energy of 1 cubic foot of Dowson gas (1 lb. of anthracite produces about 70 cubic feet) = 125,000 foot-pounds.

**133.** Exercises. Calculate the **efficiencies** of the following engines, using the above figures. The power is actually given out by the engines.

Small engine with varying load using 20·9 lbs. of coal per horse-power hour. *Answer*. ·00790.

Small steam engine using 8 lbs. of coal per horse-power hour. *Answer*. ·0206.

Lenoir gas engine using 105 cubic feet of coal gas per horse-power hour. *Answer*. ·0356.

Oil engine (varying load) using 2·5 lbs. petroleum per horse-power hour. *Answer*. ·0457.

Hugon gas engine using 70 cubic feet of coal gas per horse-power hour. *Answer*. ·0536.

Large good condensing steam engine using 2 lbs. of coal per horse-power hour. *Answer*. ·0825.

Oil engine (constant load) using 1·0 lb. petroleum per horse-power hour. *Answer*. ·1165.

Gas engine (using Dowson gas) using 1·4 lb. of coal per horse-power hour. *Answer*. ·1179.

Modern gas engine using 26 cubic feet of coal gas per horse-power hour. *Answer*. ·1436.

Modern gas engine using 97 cubic feet of Dowson gas per horse-power hour. *Answer*. ·1633.

The Diesel oil engine is said to use only 0·56 lb. of kerosene per brake horse-power hour. *Answer*. ·21.

*Exercises.*

Change into horse-power the rate of conversion of chemical energy burning in the following cases :—

1 lb. of kerosene per hour. *Answer*. 8·48.

1 lb. of coal per hour. *Answer*. 6·06.

1 cubic foot of coal gas per hour. *Answer*. 0·286.

1 cubic foot of Dowson gas per hour. *Answer*. 0·0631.

*Exercises.*

The student will for himself find information to enab!e him check or correct the following figures.

Roughly, the cost in pence of a horse-power hour in London various agents may be taken to be:—Labourer carrying things ladders 150; labourer lifting weights by rope and pulley 100; labou using winch or capstan 50; horse in whin gin 8; horse on a wagg 4; electric power (at 8d. per unit) 6; hydraulic power in London 18 pence per 1,000 gallons 700 lbs. pressure) $2\frac{1}{2}$; small gas engine oil engine or good small steam-engine of about 15 horse-power steady work, including cost of attendance 1; large gas engine ( 1 horse-power for a year, 10 hours a day (Sunday rest) is 3,130 hor power hours. At one penny per horse-power hour this is about £ per annum. The power in a cotton factory costs about $\frac{1}{4}d.$ per hor power hour.

EXERCISE. If a company sells electric power, 24 hours a d every day, and charges £5 per horse-power per annum, how much this per horse-power hour? *Answer.* $\frac{1}{8}$ of a penny. How much it per electrical unit? *Answer.* 0·15 of a penny.

The following prices per horse-power hour are for the fuel alo where the fuel is cheap in England; large steam engine in stea work 0·13, gas engine using Dowson gas 0·08.

**134.** *Engines of any size from* 10 *to* 250 *maximum indicated horse-power.*

Take $I_1$ as the highest indicated power, $I$ as any indicated power, t usually, if $B$ is brake, or actual horse-power given out,

$$B = \cdot 95I - \frac{1}{10}I_1,$$

If $K$ = cost of engine, boilers, fittings, buildings, &c., in pounds sterling

$$K = 100 + 20I_1,$$

If $C$ = coal used in pounds per hour

$$C = 34 + 1\cdot 8I.$$

Petty stores per annum (of 3,000 hours) $P = 2 + \cdot 25I$ in pounds sterling.

,, ,, (1,000 hours) $P = 1\cdot 2 + \cdot 15I.$

Labour in pounds sterling for a year (of 1,000 hours) $L = 24 + \cdot 6I.$

,, ,, ,, (of 3,000 hours) $L = 40 + I.$

For electric lighting in London at present (1898) the total indicated hor power supplied is

| | | |
|---|---|---|
| 50,000 | horse-power from | high speed engines. |
| 20,000 | ,, | low speed vertical. |
| 5,000 | ,, | low speed horizontal. |
| 5,000 | ,, | special engines. |

The tendency for the time seems to be to return to a low speed verti marine type of 2,000 or less power. The best usual results from high sp engines are 1 indicated horse-power hour for 16 to 18 lbs. of steam of 170 lbs.

square inch absolute, the electrical power being 82 to 86 per cent. of the indicated power. How much steam do these figures give per electrical unit? *Answers.* 26 to 28 lb.

The boilers using Welsh coal, at $\frac{3}{4}$ of their full load, evaporate about $8\frac{1}{2}$ lbs. of water per pound of coal; what do the above figures give in pounds of coal per unit? *Answers.* 3·1 to 3·3.

The average of all the daily load factors for a year is from 10 to 20 per cent. The *Load Factor* means the ratio of the average power supplied to the maximum power. It was Mr. Crompton who first drew attention to the great importance of the load factor on economy. One of the companies whose average daily load factor is 15 per cent., uses 5·6 lbs. of coal per unit generated throughout the year. This is the best result yet obtained; this is 3 lbs. of coal per hour per average indicated horse-power. What is the ratio between electrical and indicated horse-power? *Answer.* 72 per cent.

The cost of coal at that station is found to be $\frac{1}{2}d.$ per electrical unit. How much is this per ton? *Answer.* 16*s.* 8*d.*

The lowest coal bill for any company using alternating current is 6 lbs. of coal per indicated horse-power hour. Wages in both systems cost about $\frac{3}{8}d.$ per unit.

I dwell at some length, see Art. 149 particularly, on the effect of a light load on an engine in diminishing its economy. In boilers the great loss of economy due to light load on a central station is mainly because of the great waste in banking up the fires and putting the boilers in steam again, also because loss of heat goes on all the time.

**195.** As an evidence of the progress of science I venture to publish here a copy of one weekly report made by the resident engineer (Mr. O. B. Smith) of the Hove Electric Lighting Co., Ltd. Mr. Crompton has been kind enough to get me permission to publish it, and he thinks with me that it is good for young engineers to see how accounts are now being kept at an electric light station. I have before me a copy of the daily log sheet, giving the measurements made every fifteen minutes throughout the day, separate logs being kept for each department of the station; but I refrain from publishing this.

## REPORT No. 261.

| Date. | − and + Generated. Volts. | Amps. | Units. | Units. | − Generated. Volts. | Amps. | Units. | + Ger Volts. | An |
|---|---|---|---|---|---|---|---|---|---|
| Nov. 12 . . . . . . . . | 245<br>265 | 4480<br>1706 | 1090<br>452 | 1542 | | | | | |
| ,, 13 . . . . . . . . | 244<br>268 | 4637<br>1766 | 1128<br>473 | 1601 | | | | | |
| ,, 14 . . . . . . . . | 241<br>268 | 3198<br>959 | 770<br>280 | 1050 | | | | | |
| ,, 15 . . . . . . . . | 246<br>270 | 4944<br>1244 | 1211<br>336 | 1547 | No | out | of | balance | lo |
| ,, 16 . . . . . . . . | 245<br>268 | 4883<br>1461 | 1195<br>392 | 1587 | | | | | |
| ,, 17 . . . . . . | 244<br>270 | 4789<br>1487 | 1165<br>401 | 1566 | | | | | |
| ,, 18 . . . . . . . . | 246<br>270 | 4931<br>1218 | 1210<br>329 | 1539 | | | | | |
| Totals . . . . . | — | — | — | 10432 | | | | | |

*Engines and Dynamos—Hours running and Output in Kilowatt Hours.*

| Date. | A, 60 Kw. hrs. | Kw.hrs. | B, 60 Kw. hrs. | Kw.hrs. | C, 60 Kw. hrs. | Kw.hrs. | D, 90 Kw. hrs. | Kw.hrs. | E, 154 Kw. hrs. | Kw.hrs. | | | To Out Kw |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nov. 12 . . . | 9 | 540 | 5 | 300 | 4½ | 270 | 7 | 630 | — | — | | | 1: |
| ,, 13 . . . | 9½ | 570 | 6½ | 390 | 3¼ | 195 | — | — | 5¼ | 808 | | | 1( |
| ,, 14 . . . | 6½ | 390 | 5¼ | 315 | — | — | 6 | 540 | — | — | | | 1( |
| ,, 15 . . . | 6½ | 390 | 7¼ | 435 | 5¾ | 345 | 7 | 630 | — | — | | | 1: |
| ,, 16 . . | 3¾ | 225 | 8¼ | 495 | 7¼ | 435 | 5½ | 495 | 1¼ | 192 | | | 1: |
| ,, 17 . . . | 8½ | 510 | 7¾ | 465 | 4¾ | 285 | 7 | 630 | | | | | 1! |
| ,, 18 . . . | 5¾ | 345 | 1¼ | 75 | 6½ | 390 | 1¾ | 157 | 6 | 924 | | | 1! |
| Totals . . | 49½ | 2970 | 41¼ | 2475 | 32 | 1920 | 34¼ | 3082 | 12½ | 1924 | | | 10( |

| | £ s. d. | pence. | Pence Delivered t consumers. |
|---|---|---|---|
| 53,312 lbs. of coal @ 19/4 per ton . . . . . . . . . . . . . | 23 0 1 | 5521 | ·619 |
| ,, ,, ,, . . . . . . . . . . . . | — | — | — |
| 45,900 galls. of water @ 10*d.* per 1,000 gallons . . . . . . . | 1 18 3 | 459 | ·052 |
| Petty stores . . . . . . . . . . . . . . . . . . . . . . . | 1 9 5 | 353 | ·039 |
| Wages . . . . . . . . . . . . . . . . . . . . . . . . . . | 7 13 0 | 1836 | ·206 |
| Superintendence . . . . . . . . . . . . . . . . . . . . . | 3 0 0 | 720 | ·081 |
| | £37 0 9 | 8889 | ·997 |

Highest load during the week . . . . . . . . 2070 amps.
,, ,, corresponding week last year . 1645 ,,
Increase . . . . . . . . . . . 25·8 %.

Units delivered to consumers . . .
,, corresponding week last year .
Increase . . . . . . . . . . .

Load factor . . . . . . . . . . . . $\frac{10432}{12371}$ = 84·5 %.

Battery.
− = 85 %.

Effic
+8(

Units delivered to consumers . . . . . . . . . 8910
,, for motors . . . . . . . . . . . . 157

Total delivered . . . . . . . . . . 9067

Units generated . . . . . . . . . . . . . .
,, delivered . . . . . . . . . . . .
Units unaccounted . . . . . . . . . .

Units unaccounted, 13·1 %.

5·1 lbs. of coal per unit generated
4·4 galls. water ,, ,,

### Week Ending Nov. 18, 1897.

| Delivered. | | + Delivered. | | | Motors. | | − Battery. | | + Battery. | |
|---|---|---|---|---|---|---|---|---|---|---|
| Amps. | Units. | Volts. | Amps. | Units. | − | + | Discharge. | Charge. | Discharge. | Charge. |
| 6096 | 670 | 110 | 6228 | 685 | 1 | 6 | 690 | 825 | 680 | 735 |
| 6267 | 690 | ,, | 6379 | 701 | — | 10 | 670 | 814 | 690 | 715 |
| 4057 | 446 | ,, | 4127 | 454 | 9 | 5 | 520 | 715 | 440 | 539 |
| 6175 | 679 | ,, | 6135 | 674 | 11 | 22 | 745 | 820 | 690 | 790 |
| 6270 | 690 | ,, | 6272 | 690 | 14 | 21 | 680 | 790 | 680 | 825 |
| 6186 | 680 | ,, | 6173 | 679 | 10 | 19 | 750 | 890 | 750 | 935 |
| 6000 | 660 | ,, | 6088 | 669 | 11 | 18 | 760 | 913 | 690 | 814 |
| — | 4515 | — | — | 4552 | 56 | 101 | 4815 | 5767 | 4620 | 5353 |

| Boiler hours under Steam. | | | | Condenser. | Coal @ | | Coal @ 19/4. | | Water. | Waste. | Oils. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | Machine. | Cylinder. | Castor. |
| 1 | 2 | 3 | 4 | hrs. | Tons. | cwts. | Tons. | cwts. | Gallons. | lbs. | pints. | pints. | pints. |
| 7 | 10½ | | | Not in use. | | | 3 | 10 | 6700 | 2 | 4 | 2½ | ½ |
| 11 | 7¾ | | | | | | 3 | 12 | 6900 | — | 4 | 1 | 1½ |
| 7¼ | 6½ | | | | | | 2 | 2 | 4700 | — | 4 | 1 | — |
| 11 | 7 | | | | | | 3 | 10 | 5900 | 2 | 6 | 3 | ½ |
| 6¾ | 10½ | | | | | | 3 | 14 | 7400 | 1 | 2 | 1 | 1½ |
| 11 | 7 | | | | | | 3 | 16 | 7100 | 1 | 2 | 2 | ½ |
| 7¾ | 2 | | 7½ | | | | 3 | 12 | 7200 | — | 2 | — | 1½ |
| 61¾ | 51¼ | | | | | | 23 | 16 | 45900 | 6 | 24 | 10½ | 6 |

| Sundries. | s. | d. |
|---|---|---|
| and 12 lbs. soda . . . . . . . . . . | 1 | 0 |
| r rings . . . . . . . . . . . . | 0 | 8 |
| in . . . . . . . . . . . . . . | 0 | 3 |
| . . . . . . . . . . . . . . . | 0 | 3 |
| r cloth . . . . . . . . . . . . . | 0 | 6 |
| Carried forward . . . . . . . . | 2 | 8 |

| Petty Stores. | £ | s. | d. |
|---|---|---|---|
| 24 pts. machine oil @ 2/- per gall. . . . . . . | | 6 | 0 |
| 10½ ,, cylinder ,, 4/- ,, . . . . . . | | 5 | 3 |
| 6 ,, castor ,, 2/4 ,, . . . . . . | | 1 | 9 |
| 6 lbs. waste @ 3*d*. per lb. . . . . . . . . | | 1 | 6 |
| 7 cwts. firewood @ 1/6 per cwt. . . . . . . | | 10 | 6 |
| 4 gauge glasses . . . . . . . . . . . . | | 1 | 9 |
| Separator ,, . . . . . . . . . . . . . | | — | |
| Incandescence lamps . . . . . . . . . . . | | — | |
| Sundries (brought forward) . . . . . . . . | | 2 | 8 |
| | £1 | 9 | 5 |

### eek Ending Nov. 19, 1896.

| | £ | s. | d. | pence per unit delivered. |
|---|---|---|---|---|
| . . . . . . . . . | 19 | 3 | 9 | ·671 |
| . . . . . . . . . | 1 | 7 | 10 | ·048 |
| . . . . . . . . . | 1 | 3 | 3 | ·041 |
| . . . . . . . . . | 7 | 13 | 6 | ·269 |
| ndence . . . . | 3 | 0 | 0 | ·005 |
| | £32 | 8 | 4 | 1·034 |

57 lbs. coal per unit generated.
16 galls. water ,, ,,

Note. This is a copy of the working for the corresponding week last year.

**136.** Five years ago there was no known case of a **central electrical** station burning less than 7 lbs. of coal per hour per Board of Trade electrical unit sold during the year, although in particular months the consumption was as low as 4 lbs. Many stations burned 12. Yet on a steady best load the engines and dynamos burnt only about 2·8 lbs. of coal per unit delivered by the station. The difference is mainly due to **variation of load** (see art. 149). Small engines, whose maximum indicated powers on full load were 12, 45, and 60, have been found by trial to burn 36, 18, and 8½ lbs. of coal, respectively per hour per indicated horse-power in their ordinary small factory use under altering loads in Birmingham.

Leaving out very exceptional tests, **the most favourable results** in test trials of steam per hour, per indicated horse-power, may be said to be 20 in non-condensing and 13½ in condensing engines. Assume 9 lbs. of steam per pound of coal and we have the most favourable coal results per indicated horse-power as 2·2 and 1·5.

Exercise. Assume a mechanical efficiency of 85 per cent. in condensing and of 90 per cent. in non-condensing engines, and calculate the figures usually supposed to be the most favourable yet found in ordinary testing. *Answers.* 2·47 and 1·77 lbs. of coal per brake horse-power hour.

As a matter of fact I may say that almost all our figures concerning actual or brake power are speculative, there are few numbers to be relied upon. I always feel doubtful of the accuracy of indicated power measurements, and very doubtful indeed if the speed is higher than 300 revolutions per minute (see Art. 47). In the following table of the best results from special trials of engines, N means non-condensing, C means condensing, 1, 2, or 3 mean single, compound, or triple expansion, S means that super-heated steam was used. J means jacketed. Under $w$, I give the numbers from the table, Art. 180, the number of pounds of saturated steam of the boiler pressure which a perfect steam engine (condensing or non-condensing) would use for 1 horse-power hour on the Rankine cycle.

When the pounds of coal are marked thus *, it means that only the steam was measured, and it is assumed that 1 lb. of coal would have produced 9 lbs. of steam. In most cases this means 10·7 lbs. from and at 212° F.

I have not included Prof. Ewing's results from a **Parsons' Steam Turbine** of 135 electrical horse-power; its consumption was 21·2 lbs. of steam (say 2·35 lbs. of coal) per electrical horse-power-hour, which corresponds with a reciprocating engine using 16 lbs. of steam (or 1·8 lbs. of coal) per indicated horse-power-hour (see also Figs. 56 and 57.)

**Steam Turbine** is said to have used 19·73 lbs. of steam, and of coal per hour, per brake horse-power, a truly wonderful

*Dow* Steam Turbine, the velocity of the circumference of its ing 9 miles per minute (25,000 revolutions per minute) is ve used 55 lbs. of steam (at 85 lbs. per square inch) per horse-ur. The steamship *Ohio*, 2,100 indicated horse-power, is said ised on her trial trip in 1887, only 1·23 lbs. of coal per indi-se-power hour. (See also Art. 221.)

SPECIAL TRIALS OF ENGINES.

| | Character. | Indicated horse-power | Piston speed in feet per minute | Boiler pressure (absolute). | Steam in lbs. per indicated horse-power hour. | Coal in lbs. per indicated horse-power hour | x lbs. steam per horse-power hour Rankine Cycle. |
|---|---|---|---|---|---|---|---|
| able . . . . . . . . | N 1 | 5 | 263 | 76 | 65 | 6·5 | 21·6 |
| l . . . . . . . . . | N 1 | 10 | 238 | 75 | 31·8 | 3·5* | 21·8 |
| . . . . . . . . . . | N 1 | 34 | 406 | 137 | 26·0 | 2·9* | 16·3 |
| . . . . . . . . . . | N 1 S | 78 | 335 | 62 | 28·5 | 3·2* | 24·6 |
| . . . . . . . . . . | N 1 S | 134 | 606 | 111 | 22·0 | 2·2 | 17·5 |
| n . . . . . . . . . | N 2 J | 84 | 499 | 132 | 25·3 | 2·8* | 16·2 |
| . . . . . . . . . . | N 2 | 40 | 401 | 180 | 19·2 | 2·1* | 14·2 |
| . . . . . . . . . . | N 3 | 39 | 400 | 187 | 18·5 | 2·1* | 14·0 |
| . . . . . . . . . . | C 1 | 33 | 380 | 85 | 22·2 | 2·46* | 9·10 |
| . . . . . . . . . . | C 1 J | 166 | 606 | 111 | 19·4 | 1·9 | 8·58 |
| . . . . . . . . . . | C 1 J | 284 | 372 | 102 | 18·4 | 2·0* | 8·74 |
| nping . . . . . | C 1 J | 120 | 240 | 60 | 21·3 | 2·5* | 9·87 |
| table . . . . . . . . | C 2 J | 6 | — | 116 | 35·7 | 4·1 | 8·49 |
| . . . . . . . . . . | C 2 | 25 | 404 | 187 | 14·7 | 1·6* | 7·72 |
| Mill . . . . . . . . | C 2 | 888 | 442 | 102 | 17·8 | 1·78 | 8·73 |
| . . . . . . . . . . | C 2 | 247 | 493 | 100 | 13·35 | 1·5* | 8·77 |
| *ille de Douvres* . . . | C 2 | 2977 | 442 | 121 | 20·77 | 2·32 | 8·42 |
| *ujiyama* . . . . | C 2 | 371 | 306 | 72 | 21·17 | 2·66 | 9·46 |
| mping . . . . . . | C 2 J | 252 | 237 | 114 | 13·9 | 1·5* | 8·53 |
| . . . . . . . . . . | C 3 | 30 | 379 | 185 | 13·02 | 1·4* | 7·73 |
| . . . . . . . . . . | C 3 | 360 | - | 160 | 11·70 | 1·3* | 7·96 |
| ngine, *Iowa* . . . . | C 3 J | 645 | 397 | 180 | 13·35 | 1·46* | 7·77 |
| gton pumping . . . | C 3 | 260 | 164 | 95 | 14·10 | 1·66 | 8·88 |
| ping (American) . . | C 3 J | 574 | 203 | 130 | 11·68 | 1·39 | 8·23 |

**137.** The non-condensing trials of the **Willans Engine** gave the following results. Calculate the efficiency in every case, taking as the standard a perfect non-condensing engine, Rankine Cycle, see Table II., Art. 180, using the same kind of steam.

POUNDS OF STEAM PER INDICATED HORSE-POWER HOUR.

| Pressure (absolute). | Perfect non-condensing engine. Rankine cycle. | Measured results. | | | Best results. | Percentage efficiency of perfect non-condensing engine. |
|---|---|---|---|---|---|---|
| | | Single cylinder. | Compound. | Triple. | | |
| 40 | 35·5 | 42·5 | — | — | 42·5 | 83·5 |
| 70 | 22·7 | 30·8 | — | — | 30·8 | 73·5 |
| 90 | 19·62 | 27·7 | 24·25 | — | 24·25 | 80·9 |
| 110 | 17·65 | 26·0 | 22·0 | — | 22 | 80·1 |
| 140 | 15·75 | — | 20 | — | 20 | 78·6 |
| 150 | 15·28 | — | 19·5 | 19·8 | 19·5 | 78.3 |
| 160 | 14·88 | — | 19·2 | 19·0 | 19·0 | 78.4 |

EXERCISE. A good **locomotive** using steam of 160 lbs. pressure, uses 42 lbs. of steam per hour per horse-power; show that its efficiency is 0·354 as compared with a perfect non-condensing engine.

**138.** There are various ways of stating **Engine Performance**. One horse-power hour is 1,421 Centigrade heat units, or 2,558 Fahrenheit heat units. **It is usual** to rest content with "The engine uses 18 lbs. of steam per hour per indicated horse-power." Or "The engine uses 2 lbs. of coal per hour per indicated horse-power." If the steam is 100 lbs. steam (164° C.), and the feed-water was at 20° C. the heat given to produce 18 lbs. of steam was $18 \times (606{\cdot}5 + {\cdot}305 \times 164 - 20)$, or 11,457 units. Taking 8,300 units of heat as developable per pound of the coal, we have the following more correct ways of stating the performance.

I. Engine and boiler 2 lbs. of coal per indicated horse-power hour, or 16,600 heat units (C.) per indicated horse-power hour, or 277 units (C.) per minute per horse-power. An efficiency of 1,421 ÷ 16,600, or [illegible] per cent.

II. [illegible] 18 lbs. of steam per indicated horse-power hour, or 11,457 [illegible] horse-power hour, or 191 units (C.) [illegible] An efficiency of 1,421 ÷ 11,457, or 0·124, [illegible]

III. [illegible] engine, working between the [illegible] C. would have an efficiency [illegible] engine has $\cdot 124 \div \frac{124}{438}$ or 438

per cent. of the efficiency of the perfect heat engine, using the same temperatures.

IV. Engine. An engine using the Rankine Cycle (see Art. 214) between 164° C. and 20° C. would consume 7·6 lbs. of steam per horse-power hour. The efficiency ratio of our engine is 7·6 ÷ 18, or 0·42.

V. The above engine had a surface condenser, supplied with 918 lbs. of water per hour per indicated horse-power; this water entered at 15° C. and left at 25° C.; the condensed water left at 40° C., or 20° above that of the feed; therefore the heat rejected per hour per indicated horse-power was—

$$918 \times 10 = 9180 \text{ by condensing water,}$$
$$18 \times 20 = 360 \text{ by condensed water,}$$
The heat utilised is 1421.

So that we can account for the amount 10,961. Now the heat supplied, as we saw in II., was 11,457, and there is 496, or 4·3 per cent., to be accounted for by radiation and leakage.

It is very usual to merely measure the heat going off by the condensing water and to call it the whole rejected heat, and as we have 1,421 utilised we may say—

$$\text{Efficiency} = \frac{1421}{1421 + 9180} = \cdot134.$$

A more correct plan is to take in the 360 also, and say—

$$\text{Efficiency} = \frac{1421}{1421 + 9180 + 360} = \cdot129.$$

This is all that can be done if we only measure by the condenser water, unless we estimate the heat lost by radiation to be, say, 5 per cent. of what we measure from the condenser: this would give us 10,017 wasted altogether, so that the closer estimate would be—

$$\text{Efficiency} = \frac{1421}{1421 + 10017} = 0\cdot124, \text{ or } 12\cdot4 \text{ per cent.}$$

It will be observed that if we calculate from the feed-water or steam supplied, there is a doubt as to the wetness of the steam; and if we calculate from the condenser, there is doubt as to the amount of radiation and leakage.

It is quite a usual thing to say: let $h$ be the Fahrenheit heat

units gained by the condensing water per minute per indicated horse-power, then the efficiency $= \frac{42 \cdot 63}{h + 42 \cdot 63}$.

A very perceptible saving is effected when the **feed-water** of a main engine is heated up to near the boiler temperature by the exhaust steam of auxiliary engines, such as are often used now to work air and feed and circulating pumps.

We cannot see our way to much improvement in the non-condensing steam engine; the performance often approaches 90 per cent. of what is theoretically possible, as may be seen in the table, Art. 136. This is by no means the case in condensing engines, but it does not seem practicable to expand steam to the low pressures which might give better results. There is a chance for a **binary vapour** engine; using steam at the higher temperatures and petroleum or ether for the lower temperatures. (See also the note Art. 214.) Cutting off the toe of the diagram as Mr. Willans called it, that is, releasing steam at a much higher pressure than the exhaust pressure, is a serious loss to put up with; but a good practical remedy is not yet known to us.

**139.** We may with a fair amount of accuracy say that a **large gas engine** burning Dowson gas uses at full load 85 cubic feet per effective horse-power hour. If the anthracite costs 25*s.* per ton, and we charge 15 per cent. per annum for interest and depreciation on total cost (we may take the total cost to be the same as that of a steam engine, boiler, &c., of the same power), then we find cost per hour in pence $= 5\frac{1}{2} + \cdot 45$ I.

Since 1877 there has been a sale of 31,000 Otto engines in England and 16,000 in Germany, with a total brake power of 508,000 horses. The consumption of coal-gas used to be about 30 cubic feet of gas per hour per brake horse-power; now in a special trial it has been found to be as low as 14. There is much more improvement possible. There are now single-cylinder engines of 140 and double-cylinder engines of 220 horse-power. Even now, however, power from a 20-horse engine worked by coal-gas costs more than from a steam engine. In some electric-light stations working arc and incandescent lights the total expenditure in coal and coke in producing Dowson gas was only 3 lbs. per electrical unit for the first half of the year 1897. In a special test of two engines at Leyton during 5 hours, October, 1897, the following results were obtained: Output, 319 electrical units; anthracite per hour per indicated horse-power, 0·846 lb.; per brake horse-power, 0·975 lb.; per electrical horse-power, 1·152 lb.; per electrical unit, 1·543 lb.; coke per electrical unit, 0·225 lb. Total fuel per unit, 1·768 lb. I find that at Leyton the average total fuel per unit

enerated from January to October of 1897 was 2·55 lbs. I do not iow the load factor at Leyton, and therefore cannot compare this erage with the 5·6 lbs. of coal per unit, which is the best yet hieved with steam engines having a load factor of 15 per cent. steady running on full load there is no doubt that the **gas engine ing Dowson gas** is already consuming not much more than half : coal per brake horse-power that is consumed in the largest and st steam engines, and that the cost of repairs and attendance is y much less than with steam engines.

**140.** The usually accepted figure for the result of burning 1 lb. **town refuse** in the production of steam is 1 lb. of steam (nett, er deducting the steam used in the furnace) at 140 lbs. per iare inch produced from feed-water, at 60° F. If an engine at l load uses 23 lbs. of this steam per electrical unit (1,000 watt urs). If a cell burns 0·25 tons of refuse per hour for 24 hours a y at a cost of $13\frac{1}{4}$ pence per ton (the labour part of this cost is d. per ton) including everything, what is the cost of steam per ectrical unit, and what is the number of units produced per cell in hours? *Answers.* 0·136 pence: 584 units.

There are several places in the British Islands where the fee mple of **water-power** of about 600 total horse-power with the cessary land may be bought for £6,000. The cost of utilising this wer with turbines and dynamos, giving out usefully 70 per cent. it, would be £5,000. Taking 10 per cent. of the total cost to present wages, repairs, rates and taxes, depreciation, and interest, hat is the yearly profit if one halfpenny per electric unit is paid, e load being full for 24 hours a day for 313 days in the ar?

10 per cent. of £11,000 is £1,100 per year, the power given out ing 70 per cent. of 600, or 420 horse-power. One horse-power is 6 watts, and 1,000 watts for 1 hour is called a unit.

$$0 \times \frac{746}{1000} \times 24 \times 313 = 2{\cdot}353 \times 10^6 \text{ electrical units per year.}$$

viding these halfpence by 480 we find £4,902 per annum to be id for the energy, and so the profit is £3,802 per annum.

**141.** The student will work exercises on **traction** such as he l find in a book on Applied Mechanics. The following figures y be remembered.

The resistance in pounds per ton of a **moving train** (including gine) on the level is found roughly by adding two to one quarter the speed in miles per hour. This is for speeds greater than miles per hour. At less speeds there is a different law which for

some trains and permanent ways may be indicated by the following figures:—

| | | | | | |
|---|---|---|---|---|---|
| Speed in miles per hour . . . | 0 | $1\frac{1}{2}$ | 2 | 5 | 10 |
| Resistance in pounds per ton . | 20 | 10 | 7 | 5 | 6 |

A curved line adds 12 per cent. to the resistance on the average English railway.

In the best locomotives *on special trials* the best performances are 25 to 30 lbs. of steam (pressure 165 lbs. per square inch) per hour per indicated horse-power. In ordinary use the consumption is over 40 lbs.

EXERCISE. The average resistance of an **express train** on an English railway, as measured on the draw-bar between engine and train, is, say, 18 lbs. per ton at 45 miles per hour; the weight of the train (not including the engine) is 180 tons, what is the actual power?

*Answer.* 389 horse-power.

It has been found that in such a case the power actually exerted on the train is only 45 per cent. (in short express trains it is about 40 per cent., in slow goods trains as much as 75 per cent.) of the indicated power of the engine: what is the indicated power?

*Answer.* 864.

The consumption of steam is 40 lbs. per hour per indicated horse-power. How much feed-water must be provided for one-hour's run, neglecting leakage?

*Answer.* 15·4 tons.

If each pound of coal evaporates $8\frac{1}{2}$ lbs. of water, what weight of coal is used per hour?

*Answer.* 1·8 tons.

An American train is usually only two-thirds of the length of an English train for the same weight. For the same speeds the draw-bar force of traction of the English train being (in certain experiments) 6 lbs. per ton was only $3\frac{1}{2}$ lbs. per ton on the American, and yet the American road was not so good. The superiority was due to the construction of the American cars.

Wherever roller bearings have been tried they have greatly reduced the friction: the starting pull on a railway vehicle is sometimes as low as 3 lbs. per ton.

There seems to be no **electrical accumulator** which can be relied upon to discharge more than 7 watt-hours per pound of its total

eight, during a 5 hours' discharge. It is seldom that one finds a ublished statement on this subject which can be relied upon. In eculative calculation it is better to take only 5 watt-hours, or 15 orse-power hours per ton, the average rate of discharge being 3 horse-ower per ton. But **in tramcar work** the discharge is sometimes hree times this rate, and I shall take 9 horse-power per ton for $\frac{2}{3}$ hours.

A tramcar when supplied with electrical power by trolley wire r by accumulators receives just about twice as much electrical ower as the mechanical power actually utilised in propulsion. That s, the average power received may be calculated on an average ractive force of 60 lbs. per ton (at 8 to 10 miles per hour), instead of he 30 lbs. per ton, which it probably is on the average. It is not afe to take a better figure than this for the efficiency when one onsiders any new project. It is to be understood that this tractive force is not what would be measured in a trial at uniform speed. It is proportional to the average power divided by the average speed. The average power is greatly increased by stopping and starting, kinetic energy being created to be soon destroyed.

EXERCISE. A car to take 52 passengers worked by accumulators weighs with its electro-motor and gearing and fittings 7 tons empty, 10 tons fully loaded. Taking the tractive force to be 30 lbs. per ton at 10 miles per hour; taking the electrical power to be twice the useful, what is the weight of the accumulators? What is the electrical power? What would it be if it were supplied by a trolley wire? Take the discharge as 9 horse-power per ton.

Let $x$ tons be the weight of accumulators. The tractive force is $(10+x)\,30$, and the electrical power is $\dfrac{(10+x)\,60 \times 10 \times 5280}{33000 \times 60}$ or $1{\cdot}6(10+x)$. But it is 9 horse-power per ton of accumulators, or $9x$, so that $9x = 1{\cdot}6(10+x)$ or $x = 2{\cdot}16$ tons. We need 2·16 tons of accumulators discharging at the rate of 19·5 electrical horse-power. If supplied by a trolley wire only, 16 electrical horse-power is wanted. In fact, with accumulators, if $W$ is the weight of the loaded car in tons $x = 0{\cdot}216\ W$, and the electrical power is $1{\cdot}95\ W$, whereas by trolley wire the power is $1{\cdot}6\ W$.

The tractive force on a tramcar was measured as 30·5 lbs. per on. A similar car with roller bearings on the same road needed 5 lbs. per ton. The starting force for a tramcar is diminished by 0 to 60 per cent. by the use of roller bearings, and the general aving may be put down as 30 per cent.

The tractive force of a bicycle or any vehicle with **inflated tyres**

on a concrete road seems to be 30 lbs. per ton at 6 miles per hour about 40 on wood pavement, and 40 to 60 lbs. per ton at 12 miles per hour on a good macadamised road, slightly wet; in heavy mud at 5 miles per hour, as much as 146 lbs. per ton has been registered.

These were towing forces; in **self-propulsion** it is understood that the resistances are considerably greater. The resistance per ton of a locomotive is considerably greater than that of the train. Speculative calculations ought to be based on the highest figures. The towing tractive force for **iron-tyred** passenger carriages on London roads when muddy seems to vary from 22 lbs. per ton (asphalte), 30 to 40 lbs. per ton (wood), 50 to 60 lbs. per ton macadam, to perhaps as much as 80 lbs. per ton on macadam new. A committee of the Society of Arts some time ago found 101 lbs. per ton on ordinary macadam, and 44·5 on macadam gravelled.

EXERCISE. It is found that the accumulators of the **electric cabs** (with pneumatic tyres) in London give out at the rate of 3 horse-power on smooth wooden roads, when going at 7 miles per hour, and 5 horse-power on macadamised roads; this is the average power up and down the London street gradients. Take 4 horse-power as the average. What is the average actual tractive force if only half the electrical power is utilised?

*Answer.* 107 lbs.

I do not know the actual weight of the cab, but take it that the cab and motor and gearing and fittings weigh 22 cwt., and that the accumulators gave out 5 watt hours per pound on a 5 hours' run. What is the total weight of the cab, and what is the average tractive force per ton?

*Answer.* $4 \times 5 \times 746 \div 5$, or 2,984 lbs. of accumulators, and 2,464 lbs. of vehicle, or 2·433 tons. The average actual tractive force is 44 lbs. per ton.

On the City of London Electric Railway, the weight of a train and locomotive and passengers being 36 tons, 5·4 electrical units were supplied in a journey of 5,550 yards, taking 15 minutes (including stopping). Check the following figures. Assume useful tractive power to be half the electrical; speed, 12·6 miles per hour; we find 0·047 electrical units per ton mile; tractive force, 12·0 lbs. per ton.

On the Montreal tramways, at an average speed of $7\frac{1}{2}$ miles per hour (total load about 10 tons), 0·26 electrical units are used per ton mile. Assume the useful tractive power to be half the electrical, and find the average tractive force. *Answer.* 76 lbs. per ton.

It will be seen that this Montreal figure is much greater than the figure taken by me as more usual; but it is a figure taken often

by some of the most experienced of my friends, and we have so much inexact knowledge, that it is quite possible for this to be a better guide than the other.

**142. Ship Propulsion.** Up to the highest speeds of commercial ships we may assume without great error that, for vessels not dissimilar in form and character, and going at the usual speeds, the indicated horse-power is $I = D^{\frac{2}{3}}v^3 \div a$, when $D$ is the displacement in tons, and $v$ is the speed in knots, and $a$ is constant, which for many classes of vessel may be taken as not very different from 240.

EXERCISE 1. If a vessel of 1,720 tons moves at 10 knots when its indicated horse-power is 655, what is the value of $a$ in such a class of vessel? *Answer.* 219.

A vessel of the same class of 2,300 tons moves at 15 knots, what is the power? *Answer.* 2,680.

EXERCISE 2. Taking $a$ as 240, a vessel of 6,000 tons going at 22 knots, what coal will it consume in a passage of 3,000 nautical miles, neglecting the effect of its lightening, if it uses 2 lbs. of coal per hour per indicated horse-power?

*Answer.* If an engine uses $c$ lb. of coal per hour per indicated horse-power, the whole weight of coal consumed on a passage of $s$ nautical miles, is $\frac{c}{a} sD^{\frac{2}{3}} v^2$ pounds. In this case it is 1,784 tons.

EXERCISE 3. For students after they read Art. 156. For a marine engine we have the rough and ready rule, "The speed $v$ is proportional to the square root of the absolute boiler pressure and the amount of admission of steam." Show that if $p_3$ be taken as 11 per cent. of the boiler pressure $p_1$, the rough and ready rule is fairly true.

EXERCISE 4. Two boats of the same shape were driven, one by jet propulsion, the other by twin screws. The following results were obtained:—

Jet, 12·6 knots, with 167 I.H.P.: screw, 17·3 knots, 170 I.H.P. Compare the efficiencies, if the displacements were as 100 to 65. *Answer.* As 0·51 to 1.

EXERCISE 5. During eleven sea voyages the average figures for R.M.S.S. *Britannic* (450 feet long) were:—

$D = 8,500$ tons, speed 15 knots, 4,900 indicated horse-power; show that $a = 287$.

EXERCISE 6. H.M.S. *Iris* has $D = 3,290$, speed 18·6 knots, $I = 7,714$; show that $a = 184$.

A torpedo boat $D = 29·73$ tons, speed 22 knots, $I = 460$; show that $a = 222$.

EXERCISE 7. A ship whose displacement at starting is 6,000 tons,

uses 5 tons of coal per hour, producing 1 indicated horse-power fo an amount of coal per hour which gradually increases and may be expressed as $c = 2 + \frac{t}{500}$. The value of $a$ diminishes according to the law $a = 240 - \frac{1}{5}t$, where $t$ is the time in hours from starting. How far has she gone, and at what speed is she going when her displacement is 4,000 tons? As $5t_1 = 6{,}000 - 4{,}000$, $t_1 = 400$ hours, where $t_1$ is the total time taken.

$$\text{As } 11{,}200 = \frac{2 + \frac{1}{500}t}{240 - \frac{1}{5}t}(6{,}000 - 5t)^{\frac{2}{3}}v^3$$

$v = 13{\cdot}09 \times \left(\frac{1{,}200 - t}{2 + \frac{1}{500}t}\right)^{\frac{1}{3}}(6{,}000 - 5t)^{-\frac{2}{9}}$. Calculating $v$ for many values of $t$, and using squared paper, it is easy to integrate it. I find the required $v = 13{\cdot}64$, distance 5,850 nautical miles.

In addition it is worth while giving some fairly accurate results for large steam ships.

DIMENSIONS OF TYPICAL BRITISH AND GERMAN ATLANTIC LINERS.

| | Length between perpendiculars. | Ratio length to beam. | Length to depth. | Displacement. Tons. | Indicated power. | Speed on trial. |
|---|---|---|---|---|---|---|
| *Britannic* (1874) | 455 | 10·111 | 12·640 | 8,500 | 5,500 | 16 |
| *Alaska* (1881) | 500 | 10 | 12·607 | — | 10,500 | 18 |
| *Umbria* (1884) | 500 | 8·772 | 12·500 | 10,500 | 14,321 | 20·18 |
| *Latin* (1887) | 448 | 9·174 | 12·274 | 7,700 | 8,900 | 17·8 |
| *Paris* (1888) | 527·6 | 8·373 | 12·910 | 13,000 | 20,600 | 21·8 |
| *Augusta-Victoria* (1888) | 460 | 8·288 | 11·795 | 9,500 | 12,500 | 19·5 |
| *Teutonic* (1890) | 565 | 9·826 | 13·425 | 12,000 | 13,680 | 19·15 |
| *Havel* (1890) | 463 | 9 | 12·346 | 9,195 | 11,500 | 19·5 |
| *Fürst-Bismarck* (1891) | 502·6 | 8·777 | 13·224 | 10,200 | 14,000 | 20 |
| *Campania* (1893) | 600 | 9·231 | 14·457 | 17,000 | 30,000 | 22 |
| *Kaiser Wilhelm der Grosse* (1898) | 625 | 9·46 | 14·544 | 20,000 | 27,000 | 22½ ? |
| *New Hamburg American* liner (1899) | 662·9 | 9·89 | 15·06 | 23,000 | 33,000 | 23 ? |
| *Oceanic* (1899) | 685 | 10 | 14 | 25,000 | 25,000 | 22 ? |

EXERCISE 8. The engines of a ship when running steadily at lower power are regulated rather by the lowering of the boiler pressu than by keeping the links permanently shifted and are found to u 2·13 tons of coal per hour when the ship goes at 15 knots and 1·] tons when the ship goes at 10 knots; what coal does she use at ] knots? As power is proportional to $v^3$, if $C$ is the weight of co burnt per hour, we know that $C$ is a linear function of $I$ (tl Willans rule, Art. 148), and therefore

$$C = a + \beta v^3$$

where $a$ and $\beta$ are constants and $v$ is the speed in knots.

Applying the above figures we find $C=0·78+·0004v^3$. Hence at 12 knots $C=1·471$ tons per hour.

EXERCISE 9. The above ship is to make a passage with the greatest economy possible; what is the best speed?

If $s$ is the passage in miles, the time taken is $\frac{s}{v}$ and the total coal consumed is $C\frac{s}{v}$, so that it is proportional to $\frac{0·78}{v}+·0004v^2$.

This is a minimum when $\frac{-0·78}{v^2}+·0008v=0$, or $v^3=975$ or about 10 knots.

EXERCISE 10. If economy of coal is not all-important; suppose that the loss of every hour is valued at the worth of 0·6 ton of coal, what is the best speed?

The total loss per hour is now to be taken as represented in tons of coal. $0·6+0·78+·0004\,v^3$, or $1·38+·0004\,v^3$.

The total loss in the voyage is proportional to

$$\frac{1·38}{v}+·0004v^2.$$

And this is a minimum when $v^3=1725$, or $v$ almost exactly 12 knots

It is worth while trying what the number representing the total loss in the voyage amounts to at other speeds, and I show it in the table. We see that to use a slightly different speed than the best is not very harmful.

| $v$ knots. | Number proportional to total loss in voyage. | $v$ knots. | Number proportional to total loss in voyage. |
|---|---|---|---|
| 10 | ·178 | 13 | ·174 |
| 11 | ·174 | 14 | ·177 |
| 12 | ·173 | | |

EXERCISE 11. If $a$ is 240, find the speeds of ships of from 1,000 to 10,000 tons when their displacements in tons are numerically equal to their horse-power. Write your answers in a table for easy reference.

For auxiliary engines the consumption of coal is about 10 tons per day in first class line-of-battle ships and 8 to $3\frac{1}{2}$ tons in cruisers.

**143.**[1] **The resistance to the motion of a ship** is considered to be made up of two parts. 1. **The skin friction** in pounds $=fAV^n$, where $V$ is the speed in knots, $n$ is 1·83 for varnished or painted wooden models or clean iron ships, $A$ is the wetted area in square feet, $f$ is ·009 for ships of over 200 feet long, and ·012, ·0106, ·0096 for ship lengths of 8, 20 and 50 feet. At speeds of 6

[1] Some numbers valuable to students will be found in Sir Wm. White's British Association Address, 1899.

to 8 knots in ordinary vessels this skin resistance is about 80 or 90 per cent. of the whole; at high speeds it is about half the whole.

2. **A residuary resistance** due to the fact that **eddies** (the smaller part) and **waves** are produced. Eddy resistance is thought not to be more than 8 per cent. of the skin resistance even at high speeds. It is mainly caused by bluntness of the stern of a vessel. In two perfectly similar ships, similarly loaded, of lengths $l$ and $L$, at speeds $v$ and $v\sqrt{L/l}$, which are said to be the *corresponding speeds*, the residuary resistances are proportional to $l^3$ and $L^3$.

The skin resistances $S_1$ and $s_1$ of the ship and its model can be calculated from Froude's numbers given above. Hence if $R$ is the resistance in pounds of a ship $L$ feet long, $A$ its wetted area in square feet, $V$ its speed in knots, and if $r$ and $l$ are the resistance and length of a model which is exactly similar and of similar draught when the model is drawn at the corresponding speed $v$ knots, where $V : v :: \sqrt{L} : \sqrt{l}$, prove that it follows from the above that

$$R = \frac{L^3}{l^3} r - 0{\cdot}009 A V^{1{\cdot}83} \left\{ 1{\cdot}2 \left( \frac{L}{l} \right)^{0{\cdot}085} - 1 \right\}$$

if the ship is more than 150 feet long and the model is from 8 to 30 feet long. Instead of 1·2, which suits a model of 20 feet long, we really ought to use 1·33 if the model is 8 feet long and 1·11 if the model is 30 feet long.

*Example.* Before building a vessel 400 feet long of wetted surface 26,000 square feet, we wish to know $R$, its resistance, at $V = 12$ knots. A model is made ten feet long, it is drawn at a speed of $12 \div \sqrt{40}$ or 1·9 knots in the tank, and its resistance $r$ is found to be 0·9 lb. We find $R$ to be 39,720 lbs.

Prove that $R$ in pounds $\times$ $V$ in knots $\div$ 307 $=$ utilised horse-power. In this case we find 1,550 horse-power. The indicated power will probably be more than 3,000.

The vagueness of our knowledge as to the probable loss of power by friction makes any attempt to calculate $R$ for the above purpose rather useless and the better use of the tank would therefore seem to be in helping to improve a particular class of vessel.

The following great **simplification** has recently been tried by **Colonel English**. Suppose an existing vessel to be run at various speeds and its indicated horse-power noted. Now assume that the effective horse-power at any speed is the same fraction of the indicated, than we take [illegible] existing ship—say one-half. Find the resistance of the existing ship at the speed $V_1$. We wish to know the resistance of the new ship at the speed $V_2$. We only need to

mpare the wave and eddy resistances, which we shall call $W_1$ and $W_2$. **Make two models,** one of the existing and one of the ship ing designed. Let the values of $V$, $D$, $S$, $L$, $W$ for the two ships d the two models be indicated by capital and small letters, the isting ship and its model having the affixes $_1$.

$S$ is skin friction; $D$ is displacement, which in similar ships is roportional to the cubes of the lengths.

Let $v_1 = V_1\left(\frac{d_1}{D_1}\right)^{\frac{1}{6}}$, $v_2 = V_2\left(\frac{d_2}{D_2}\right)^{\frac{1}{6}}$, and let $v_1 = v_2$; that is, make the econd model of such a size that $V_2$ and $v_2$, as well as $V_1$ and $v_1$, are "corresponding speeds," and yet that the speeds of the two models shall be the same. In fact $\frac{d_2}{d_1} = \frac{D_2}{D_1}\left(\frac{V_1}{V_2}\right)^6$. Now let the two models be towed from the two arms of a lever whose fulcrum may be adjusted and the **ratio of the resistances**, $n$, may be measured. Note that we need only find this ratio—a much easier thing to do than to find either resistance. Show that the total resistance of the new ship is—

$$S_2 + \left(\frac{V_2}{V_1}\right)^6 \left\{ nW_1 + \frac{D_1}{d_1}(ns_1 - s_2) \right\}.$$

Mr. Froude's estimate of the disposal of the whole indicated power of a marine steam engine was :—

Friction of engine, 26 per cent.; power wasted in driving air, feed and other pumps, 7; loss of power due to slip of screw, 9·1; friction of screw, 3·8; loss due to the greater resistance of a vessel when the propeller is working than when the vessel is towed, 15·5; power really effective in propelling the vessel, 38·7.

It is usually stated (on what experimental authority I do not know) that in modern ships the effective horse-power is 53 per cent. of what is indicated. In the first edition of this book, after a long description of tests of propellers made in France, I stated that a well-arranged propeller utilised $\frac{2}{3}$rds of the work actually given out by the engine. The mechanical efficiency of a good modern engine is ·85, and $\frac{2}{3}$ of ·85 is 56 per cent. Froude's idea was that the useful power was the fraction $\frac{38 \cdot 7}{66}$ of the useful power of the engine: this would give 50 per cent. as the probable ratio of useful propelling power to indicated power in modern steam engines.

Students will do well to keep the following figures in mind.

Exercise. **In 1845** a ship with a total machinery and coal load of 500 tons (besides its cargo and hull load of 1,000 tons more) going at 8 knots, its indicated power being 335, used 1 ton of coal per hour,

what was the amount of coal used per hour per indicated horse-power? *Answer.* 6·7. The total weight of boilers, engines, and other machinery was 120 tons, leaving 380 tons available for coal. What was the **indicated horse-power per ton** of machinery? *Answer.* 2·8.

What was the locomotive performance? *Answer.* $D^{\frac{2}{3}}v^3 \div I$ is nearly 200, where $D$ is 1,500 tons, $v$ is 8 knots, $I$ is 335.

The ship could run at full speed $380 \div 24$, or 15·8 days, without coaling, a distance of $15·8 \times 24 \times 8$ or 3,040 nautical miles.

**Now in 1898** a ship with the same loading to run at 10 knots, its indicated power being 524, has a locomotive performance of 250. The total weight of its machinery is 50 tons, leaving a weight of 450 tons available for coal. What is the indicated power per ton? *Answer.* 10·5. It uses 10 tons of coals in the 24 hours. This is at the rate of $1\frac{3}{4}$ lb. of coal per hour per indicated horse-power. It can run for 45 days at full speed without fresh coal, a distance of 10,800 nautical miles.

With forced draught the power per ton of machinery is 12 in battleships, 30 in torpedo catchers.

**144. Brake and Indicated Power.** The actual horse-power delivered from the crank shaft of a steam engine (usually called the brake-horse-power) is less than the indicated power, because of friction. In mechanical laboratories it is almost always found that when we give power $I$ to any machine and receive power $B$ from that machine, there is some such law as—

$$B = aI - b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $a$ and $b$ are constants.

In my book on Applied Mechanics I have considered this matter carefully, describing the methods of measuring mechanical power when it is being transmitted through belts or along shafts, and also when it is consumed by a brake for the purpose of measurement. When we test steam, gas, oil, electric, hydraulic, or other motors, we usually consume all the power given out: but whether we consume it or not we are in the habit of calling it the actual or brake-horse-power. The total horse-power given to the engine by the steam pressing on the piston is $I$ the indicated power. The following [illegible] obtained ought to be plotted on squared paper [illegible] ought to try for himself if there are [illegible]

[illegible] condensing . . . . . . (2)

[illegible] non-condensing . . . . (3)

The engine, when working as a condensing engine, was supposed to be at full power at the highest load shown in the table. When working as a non-condensing engine the highest figure is supposed to be its full power; in this case the pumps were not working, and presumably this is what made the difference in the character of the laws.

| Condensing. | | | | Non-Condensing. | | | |
|---|---|---|---|---|---|---|---|
| *I.* | *B.* | *B/I.* | *F.* | *I.* | *B.* | *B/I.* | *F.* |
| 50·5 | 40 | ·80 | 10·5 | 42·5 | 35 | ·82 | 7·5 |
| 38·5 | 30 | ·78 | 8·5 | 31 | 25 | ·81 | 6 |
| 29 | 20 | ·69 | 9 | 24 | 17·5 | ·73 | 6·5 |
| 17 | 10 | ·58 | 7 | 15·5 | 10 | ·65 | 5·5 |
| 8 | 0 | 0 | 8 | 5·5 | 0 | 0 | 5·5 |

If we denote by $F$ the power lost by friction, it is evidently greater at greater loads. Possibly there is some such law as—

$$F = \frac{1}{20} I + 10, \text{ in the condensing trials.}$$

$$F = \frac{1}{20} I + 5, \text{ in the non-condensing trials.}$$

The frictional loss is therefore by no means merely proportional to the indicated or brake-power, and we always find from our tests of engines that if we for speculative purposes assume the frictional loss constant for all loads, we are not greatly in error. This is really to assume that $c$ in (1) is unity. It is the great dead load of all the parts of the machinery, the flywheel, for example, which causes this result. Also, at the same speed the loss by friction due to mere inertia of the parts of the engine must be much the same for all loads. In well-made condensing engines we may take the loss as about 20 per cent. of the indicated power at full load, and in non-condensing engines as about 15 per cent. For the largest engines we may perhaps subtract five from each of these figures. A certain triple expansion engine has given 122 indicated and 107 actual—a mechanical efficiency of 88 per cent. There is usually more loss by friction in single cylinder engines than in double or triple.

If the frictional loss were really constant, it would be completely represented by taking a **constant back pressure as representing**

**friction.** Thus in an engine of the size described in the exercises of Art. 35, I find that if a back pressure of about 14 lbs. per square inch for a condensing engine and 10 lbs. for a non-condensing engine be added to the usual back pressures 3 and 17 of the indicator diagrams, we may speak of the calculated work or power as *actual* or *brake* work or power, instead of *indicated.* Hence the remarks made in Art. 37.

For speculative calculation the following back pressures may be fairly well taken as representing the effect of friction in well-made engines. $p_1$ is supposed to be the initial pressure of the steam used when the engine works with its greatest load. These numbers ought only to be used in academic problems. I know of engines whose friction is represented by back pressures of only about half these.

| Condensing. | | Non-Condensing. | |
|---|---|---|---|
| Greatest $p_1$. | Back pressure to take as representing friction at any load of the engine. | Greatest $p_1$. | Back pressure to take as representing friction at any load of the engine. |
| 50 | 10 | 50 | 5 |
| 75 | 11 | 75 | 8 |
| 100 | 13 | 100 | 10 |
| 150 | 15 | 150 | 13 |
| 200 | 18 | | |

In making such calculations, the results of which are only to be employed in rapid speculative (but not altogether misleading) calculation, let $p_3$, the indicated back pressure, be taken as 3 in condensing and 17 in non-condensing engines.

It is only in calculations like those of Art. 35 that I venture to use back pressure as representing friction. It is quite a common practice in finding the power necessary to drive some machine to take the indicated power of the engine when driving and when not driving the machine, and take the difference as representing the power given to the machine. This method may have its practical value, but it does not measure the power given to the machine, unless we assume the same loss by friction in engine and shafting in both cases.

We have very few actual power tests of large steam engines. Captain Sankey published a set from a Willans' engine capable of

developing 150 indicated horse-power (*Proc. Inst. C. E.*, 1893, discussion on Mr. Willans' Paper). Measuring from the published diagram I find

| I. H.-P. | B. H. P. |
|---|---|
| 104·5 | 94·5 |
| 49 | 38 |
| 12·5 | 0 |

find that $B = 1{\cdot}03\,I - 13$, or $F = 13 - 0{\cdot}03\,I$.

Here we find less power spent in friction at a large load than at small one. It is in contradiction to the sort of law found in all machines which I have ever examined, and shows how important such trials on large engines might be. It gives a very good excuse for the common practice of assuming that a constant back pressure may represent the friction of an engine. It will be noticed that at the highest load published, which is only $\frac{2}{3}$ of the full load of the engine, the mechanical efficiency is over 90 per cent. At the average load the friction would seem to be represented by a back pressure of only $4\frac{1}{2}$ lbs. to the square inch on the low pressure piston.

**Exercise.** The following measurements were made on a compound condensing engine; find the law connecting $I$ and $B$: also check $B/I$.

| $I$ | 288 | 223 | 136 |
|---|---|---|---|
| $B$ | 249 | 189 | 108 |
| $B/I$ | ·86 | ·85 | ·80 |

*Answer.* $B = {\cdot}922\,I - 17$.

**Exercise.** The high pressure cylinder of the above engine was used alone as a condensing engine, and the following results were obtained: find the law and check $B/I$.

| $I$ | 153 | 109 | 55 |
|---|---|---|---|
| $B$ | 128 | 88 | 38 |
| $B/I$ | ·84 | ·81 | ·69 |

*Answer.* $B = 0{\cdot}93\,I - 13$.

Now produce a few more columns of numbers and study thei
Give $W \div I$ and $C \div I$. Give $W \div B$ and $C \div B$. Give $W \div E$ ai $C \div E$. Also give $W \div C$. Observe that practical engineers u occasionally every one of these methods of stating the performan of their plant.

Students may compare the above results with the followir average measurements made at an **electric supply station** usin several engines and boilers in 1891:—

| | *I.* | *E.* | *W.* | *C.* |
|---|---|---|---|---|
| Average for 7 hours, 11 a.m. to 6 p.m. . . . | 80·3 | 57·1 | 3268 | 552 |
| Average for 6 hours, 6 p.m. to midnight . . | 227·7 | 163·2 | 7122 | 742 |
| Average for 11 hours, midnight to 11 a.m. . | 37 | 23·64 | 2143 | 232 |
| Twenty-four hours, 11 a.m. to 11 a.m. . . . | 97·3 | 68·3 | 3718 | 453 |

Here it will be found that although the load was varying, eve when the averages for the 24 hours are taken with the others w have linear laws between $I$, $E$ and $W$, $W = 1150 + 26{\cdot}25\ I$, an $E = {\cdot}72\ I - 2$. But $C$ does not follow a linear law with the other The reason lies in the fact that a spare boiler was used during par of the time, and there is consequently a greater consumption of fue than if one or two boilers had been used the whole time. Since w have considered fuel consumption in the above exercises, it may no be out of place to introduce here some figures from the testing of **water-tube boiler.**

| Steam per hour from and at 100° C. per lb. of coal. | Coal per square foot of grate per hour. *f.* | Water evaporated per square foot of total boiler heating surface per hour. This is not reduced to 100° C. | *w* |
|---|---|---|---|
| 13·40 | 7·74 | 1·24 | 103 |
| 12·48 | 18·6 | 3·20 | 233 |
| 12·00 | 29·8 | 4·70 | 357 |
| 10·29 | 66·8 | 8·50 | 686 |

$w$ = steam per hour per square foot of grate, $f$ = fuel per hou

per square foot of grate. Plotting $w$ and $f$ on squared paper, we find a fair approach to a linear law,

$$w = 45 + 9{\cdot}78 f$$

$$\text{or} \qquad \frac{w}{f} = \frac{45}{f} + 9{\cdot}78$$

Evidently also, the total steam per hour is a linear function of total coal per hour.

**146.** Work the following EXERCISES :—

1. In a spinning and weaving factory suppose each spinning frame to need 1 actual horse-power for every sixty spindles in it, and that each loom needs 2 horse-power to be actually supplied. What is the actual horse-power to be supplied in the following cases? Check the numbers in the table.

2. Suppose a steam engine to have the law

$$B = 0{\cdot}95\ I - 52$$

where $I$ is the indicated and $B$ the brake horse-power, and that it drives a dynamo which feeds motors which give out mechanical power $P$, such that $P$ is $0{\cdot}90\ B$.

Find the indicated horse-power when driving the following loads. Check my answers.

3. The above steam engine drives ordinary shafting which delivers power, $P$, to the spinning frames and looms, the friction being such that,

$$P = {\cdot}93\ B - 160.$$

Find the indicated horse-power when driving the following loads and so check my answers.

| Spindles. | Looms. | Actual horse-power needed. | Indicated horse-power, electrical driving. | Indicated horse-power by shaft driving. |
|---|---|---|---|---|
| 12,000 | 95 | 390 | 511 | 692 |
| 6,000 | 48 | 196 | 283 | 459 |
| 3,000 | 24 | 98 | 169 | 347 |

**147.** Two engines each with the law $W = 370 + 21{\cdot}6\ B$, where $W$ is weight of steam per hour and $B$ is brake power, are required to give out 70 brake horse-power. If $x$ is the brake power given out by the first, and $70 - x$ by the second, find $x$ that the total expenditure of steam may be a minimum.

*Answer*. The expenditure is $370 + 21{\cdot}6\,x + 370 + 21{\cdot}6\ (70 - x) = 740 + 1{,}510 = 2{,}250$ lbs. per hour.

It is therefore of no consequence what proportion of the lo comes from each if both must work. Of course if one alone c do all the work it only uses 1,882 lbs. per hour.

It is evident that if there are many engines, the best arrangeme at any time is for all that are working (but one) to be worki at full power, one at less than full power, the others at rest.

**148.** The following results of the numerous tests made by M **Willans** on his condensing central valve engine (see Art. 236), whi he used as a simple or a compound or as a triple expansion engine, a interesting. In every case he found that the plotted points represei ing $W$ and $I$ lay in a straight line, $r$ and $n$ being constant and variable. $W$ is water per hour, $I$ indicated horse-power, $r$ the to ratio of expansion (intended by the valve setting; in the tab published by Mr. Willans the true values of $r$ are given as measur on the diagram taking clearance into account). I find that, usi the following values of $r$, all the compound trials fairly well satis the law:—

$$W = \beta + a I$$

$$\text{Where } \beta = 40 + \cdot 0058\,(n - 100)\,(r + 3\cdot 4)$$

$$a = 12\cdot 34 + \frac{n}{10\,r} - 0\cdot 0105\,n$$

CONDENSING TRIALS.

| $n$. | $r$. | Value of $W$ in terms of $I$. | Highest and lowest values of $I$ in the trial. | |
|---|---|---|---|---|
| 400 | 2 | $70 + 23\cdot 4\,I$ | 31·6 and 9·1 | Simple. |
| 400 | 3·45 | $90 + 20\,I$ | 33·2 and 6·9 | Simple. |
| 400 | 2 | $29 + 23\cdot 8\,I$ | 33·6 and 11·8 | Simple; steam much wire drawn before admission. |
| 400 | 4·8 | $54 + 15\cdot 3\,I$ | 40 and 11 | Compound. |
| 300 | 4·8 | $49 + 14\cdot 7\,I$ | 31 and 7·6 | Compound. |
| 200 | 4·8 | $45 + 15\cdot 1\,I$ | 20 and 5·3 | Compound. |
| 100 | 4·8 | $27\cdot 5 + 16\cdot 1\,I$ | 9 and 3 | Compound. |
| 400 | 10 | $62 + 12\cdot 8\,I$ | 33 and 12 | Compound. |
| 400 | 15·5 | $75 + 11\cdot 5\,I$ | 27·5 and 13 | Compound. |
| 300 | 15·5 | $60 + 12\cdot 2\,I$ | 20 and 10·6 | Compound. |
| 200 | 15·5 | $50 + 13\cdot 2\,I$ | 13·5 and 6 | Compound. |
| 400 | 12·3 | $37\cdot 5 + 11\cdot 5\,I$ | 29·5 and 8·3 | Triple. |
| 300 | 12·3 | $37\cdot 5 + 11\cdot 4\,I$ | 23 and 6·7 | Triple. |
| 400 | 20·6 | $41 + 10\cdot 9\,I$ | 22 and 9 | Triple. |

**149. In an electric light central station** it was found that when a steady load was maintained for 12 hours, all the engines and boilers at their full load, the total electric energy given out was 4,600 units (a unit is 1,000 watt hours, and one horse-power is 746 watts) and the total coal consumed was 6 tons. In regular working during each month of 720 hours, 44,200 units (on the average for a year) were given out for a consumption of 138 tons. Assume a linear law connecting coal and power and that it holds for average power as well as steady power (see my *Applied Mechanics*, Art. 77). What is the load factor of the station? *Answer.* $\frac{44200 \times 12}{4600 \times 720} = 0{\cdot}16$ or 16 per cent. What is the law connecting pounds of coal per hour $C$, and watts given out $P$? *Answer.* We have $P = \frac{4600 \times 1000}{12}$ or 383,000 watts for $C = \frac{6 \times 2240}{12}$ or 1,120 lbs. of coal per hour and $P = \frac{44200 \times 1000}{720}$ or 61,400 watts for $C = \frac{138 \times 2240}{720}$ or 430 lbs. per hour, and if there is a linear law it is easy to see that it is $C = 298 + {\cdot}00215\ P$

EXERCISE. If the power factor sank to 10 per cent., or rose to 20 or 30 per cent. find the coal per unit. *Answer.* The full power is $\frac{4600 \times 1000}{12}$ or 383,000 watts. The above percentages would give 38,300, 76,700, 115,000 watts, as the average powers. Applying these in the formula we get the coal consumed. The other numbers in the following table are easily found. One unit means 1,000 watt hours.

| | Power in watts. $P$. | lbs. of coal per hour. $C$. | lbs. of coal per hour per unit. |
|---|---|---|---|
| Full load . . . . | 383,000 | 1120 | 2·9 |
| Load factor 10% . | 38,300 | 380 | 9·93 |
| ,, ,, 16% . | 61,400 | 430 | 7·00 |
| ,, ,, 20% . | 76,700 | 463 | 6·03 |
| ,, ,, 30% . | 115,000 | 546 | 4·74 |

Since the above figures were given, all the steam-pipe arrangements have been simplified. More steam separators have been introduced. More precautions taken in regard to priming and leakage, and chimney draught has been greatly increased. The total output of the station has been increased, but there is about the same load factor, 16 per cent., as before. The full power of the station is now

520,000 watts, with an expenditure of 1,352 lbs. of coal per h
and at an average load of 83,000 watts, the coal is 482 lbs. per h
Work out a table like the above one for the reformed conditions.

| | Power in watts. $P$. | lbs. of coal per hour. $C$. | lbs. of coal per unit. |
|---|---|---|---|
| Full load . . . . | 520,000 | 1352 | 2·6 |
| Load factor 10% . | 52,000 | 420 | 8·1 |
| ,, ,, 16% . | 83,000 | 482 | 5·8 |
| ,, ,, 20% . | 104,000 | 524 | 5·0 |
| ,, ,, 30% . | 156,000 | 626 | 4·0 |

The law is now $C = 316 + \cdot 002\, P$.

**150.** In an **electric light** station the load varies greatly, the change of load is so gradual that we can shut down one eng and boiler after another in an installation of many units; one eng only need be on light load at any time and we can gradually decre the pressure in its boiler. Thus (except for the loss of heat due the boilers) the engines are working nearly always under their t conditions and the losses are mainly due to the boilers. In **electric traction** station or factory where the load is often chang greatly and quickly, in a few minutes or seconds, it is evident t we must keep the pressure in the boiler or boilers nearly const unless the boilers are of very small capacity. Assuming any ordin kind of boiler the pressure is nearly constant. The engine ought be most efficient when working at its average load.

In the following case the law is not linear.

Exercise. The specification of an engine for an electric tract station, after a clause stating that three-quarters of the whole l might be thrown off or on suddenly without a greater fluctuation speed than 5 per cent. above or below the normal speed, went on say that at 30 per cent. of the full load not more than 25·3 lbs. of ste (at 165 lbs. absolute per sq. inch) was to be used per actual hor power hour. At full load, or 400 actual horse-power, the consumpt was not to be more than 16·5 lbs. of steam per horse-power ho Now if the engine satisfied these conditions exactly, and was govern by the cut off, it would in all probability be working most eco mically when giving out 280 brake horse-power, using 15·73 lbs. steam per hour per actual horse-power.

Suppose these three points given: draw approximately the cu showing steam per hour and actual horse-power. Suppose

electrical power to be given out at a varying rate, which for the sake of simplicity I shall take to be shown by the sine law. Electrical power $= 250 + 150 \sin qt$; and that the electrical power is 90 per cent. of the actual power, find the average weight of steam used per hour per electrical horse-power. *Answer.* 19·38.

Taking it that $8\frac{1}{2}$ lbs. of such steam is evaporated by 1 lb. of coal (this is the figure usually taken as true in London), what is the average amount of coal per hour per electrical horse-power? *Answer.* 2·28.

The average electrical power is 250. If this were steadily given out, the consumption of steam would be 16·2 lbs. per hour per electrical horse-power; whereas the two answers would be the same if the Willans law were true.

**151.** The tests at the small electric lighting station at Leyton in 1897 showed the following results, for $E$, the electrical horse-power, and $C$, the coal per hour for one **gas engine** and dynamo. Assuming a linear law, and that 45 electrical horse-power was the full load, find the efficiency with load factors of 40, 60, 80, and 100 per cent. on this

| $E$ | $C$ |
|---|---|
| 44 | 49 |
| 29 | 41 |

one engine and dynamo. (For experimental results on a whole station with many such engines, we have still to wait: they would, of course, show higher efficiency than these.)

The numbers give the law—

$$C = \frac{8}{15} E + 25{\cdot}5$$

We calculate the following values of $C$ from the assumed values of $E$.

| | | | | |
|---|---|---|---|---|
| $E$ | 45 | 36 | 27 | 18 |
| $C$ | 49·5 | 44·7 | 39·9 | 35·1 |
| $C/E$ | 1·1 | 1·24 | 1·48 | 1·95 |
| $C$ per unit | 1·47 | 1·66 | 1·98 | 2·61 |

**152.** When, as in most cases of **hydraulic work,** change of load means change of speed, there is quite a different connection than a linear law between useful and indicated power. In working with an engine, and pump, and accumulator at Marseilles, the energy given to the accumulator being called useful power, it was found that the frictional loss of engine and pump was 20 per cent. at slow speeds, and 30 per cent. at high speeds. In the lifting machines used, the useful work was 21 per cent. of the indicated work of the engine, or 44 per cent. of that of the pressure water. In a hoist with variable load, the useful work was 15 per cent. of the pressure water energy with a load of half a ton, and 60 per cent. with a load of 2 tons. This is of course mainly due to the dead weight of the cradle. The practical efficiency of any general system of hydraulic supply in towns is probably less than 50 per cent. for useful work ÷ indicated work.

**153.** Exercise. If $H$ horse-power is supplied at one end of a line of pipes in a system of **hydraulic transmission,** the useful power coming out of the other end is—

$$U = H - aH^3,$$

where for a straight line $a = \cdot 00374\, l / p^3 d^5$, where $l$ = length of pipe in feet, $d$ the diameter in feet, and $p$ the pressure in lbs. per square inch at entrance.

If $l$ = 10,000 feet; pipes 6 inches diameter, or $d = 0{\cdot}5$, $p = 700$ lbs. per square inch. If the useful power $H$ of the pump is—

$$H = 0{\cdot}7I - 25,$$

and if

$$C = 1{\cdot}25I + 225,$$

where $I$ is the indicated power of the engine and $C$ is the coal used per hour; find $U$ for the following values of $H$, and also $C$, and calculate the efficiency in the form $U/I$. Plot $C$ and $U$ on squared paper.

Here $U = H - 3{\cdot}49 \times 10^{-6} H^3$.

| | | | | |
|---|---|---|---|---|
| [illegible] | [illegible] | [illegible] | [illegible] | 4·[illegible] |
| [illegible] | [illegible] | [illegible] | [illegible] | 3·[illegible] |
| [illegible] | [illegible] | [illegible] | [illegible] | 3·[illegible] |
| [illegible] | [illegible] | [illegible] | [illegible] | 3·[illegible] |
| [illegible] | 404 | 340 | 296 | 3·44 |

It is noticeable here that the economy does not greatly alter when he useful load alters very much.

**154.** Exercise. If $H$ is the horse-power given to an **electric onductor,** the useful power given out is—

$$U = H - aH^2,$$

ere $a = 746R/v^2$, $R$ ohms being resistance of the mains, and $v$ the tential difference at the receiving end. Take $R = 0{\cdot}64$ (this is the istance of about 4 miles of copper rod of one quarter of a square h in section). Take $v = 1{,}000$ volts.

Then $a = 746 \times {\cdot}64/1000^2 = 4{\cdot}77 \times 10^{-4}$.

If the above values of $H$ be taken and the same formulæ for $C$ d $H$, we get another table interesting to compare with the above e.

It is easy to frame many other exercises showing how the economy a system alters when the useful load is altered.

**155. Mechanical Transmission.** In transmitting power rough contrivances in which there is approximately the solid iction law, as through successive machines of the same kind, or ire ropes and pulleys, &c., if we take the system as a continuous ne; on the length $\delta l$ let there be a loss of horse-power $\delta P$, and t $-\frac{dP}{dl} = a(P + b)$, where $a$ and $b$ are constants.

The rate of loss $ab$ would exist if no power $P$ were being ransmitted, being due to the weight of parts of the transmission mechanism; due to the bending of ropes or belts, &c. Solving this, and letting $U$ be the useful power transmitted to the listance $l$,

$$U = H - (H + b)(1 - e^{-al}) \quad . \quad . \quad .$$

r if $H - U$ is called $F$, the power lost,

$$F = (H + b)(1 - e^{-al}).$$

Exercise. Taking $l$ to mean the number of the usual spans in certain line of wire rope transmission, I find that $a = {\cdot}03$, $b = 60$, that if we take $l = 12$ spans, that is, there is transmission for a stance of about 3,000 feet, we find $e^{-al} = 0{\cdot}6977$, which I shall ll ${\cdot}7$.

$$U = H - (H + 60) \times {\cdot}3 = {\cdot}7H - 18.$$

whereas if there is transmission for 24 spans, $e^{-al} = \cdot 487$, which shall call ·5, and we have—

$$U = \cdot 5\ H - 30.$$

It is interesting to imagine the above engine working one of these two systems, and then the other, and finding in each case the cost per useful horse-power delivered when it varies as in the other cases.

At Schaffhausen the average life of a steel rope is only 11 months and the loss due to this is £2 per year per transmitted horse-power or 35 per cent. of the gross income from power. Hence at Schaffhausen electric methods of transmission are about to be adopted (1899), the rope method being discarded.

## CHAPTER XVII.

### THE HYPOTHETICAL DIAGRAM.

**156.** IN Art. 32 I used the rough and ready rule of expansion $pv$ constant. If a gas such as air is kept *at constant temperature* when it expands, it follows very nearly the law, $pv$ constant. But the stuff we deal with is steam with water present, and even if it were air, it is not by any means at constant temperature. Indeed it is an astonishing thing that the rule, $pv$ constant, should be so nearly true, and yet I have heard men speak of this law as "the theoretical law of expansion." What meaning can they attach to the word *theoretical?*

EXERCISE. Calculate the numbers in the second, third and fourth columns of the following table. When found, plot $v$ and $p$ on squared paper, or in some other way try to get an idea of the sort of departure we may sometimes expect from our rough and ready rule.

The pressures in the third column are calculated according to the formulæ $pv^{1.130}$ constant, and in the fourth column $pv^{0.9}$ constant.

| Volume. | Pressure according to our roughly correct rule. | Pressure in a badly clothed cylinder, piston leaking. | Pressure in a steam-jacketed cylinder. |
|---|---|---|---|
| 1 | 100 | 100 | 100 |
| $1\frac{1}{2}$ | 66·7 | 63·2 | 69·4 |
| 2 | 50 | 45·7 | 53·6 |
| $2\frac{1}{2}$ | 40 | 35·5 | 43·8 |
| 3 | 33·3 | 28·9 | 37·2 |
| $3\frac{1}{2}$ | 28·6 | 24·3 | 32·4 |
| 4 | 25 | 20·9 | 28·7 |

In hypothetical calculations I use $\theta$ for temperature, $p$ for pressure (absolute) in pounds per square inch, $u$ for the volume in cubic feet of one pound of steam, $v$ for volume in cubic feet in general, $r$ for

the ratio of cut off; we cut off at $\frac{1}{r}$th of the stroke. I use affi letters, 1 to indicate admission, 2 to indicate the end of e sion, 3 the exhaust. Thus $p_2$ means the pressure at the end o expansion, $u_1$ means the volume of 1 lb. of steam at the initial pre as given in the table, Art. 180.

In Art. 33 I asked the student to find graphically the forward pressure during admission and expansion to the end o stroke, the back pressure being taken as 0. I call this $p_m$, **effective pressure** being $p_e = p_m - p_3$, if $p_3$ is the back pre Instead of taking so much trouble, the student might have foun answer as—

$$p_m = p_1 \frac{1 + log_e r}{r} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But if the law of expansion is $pv^s$ constant,

$$p_m = p_1 \frac{sr^{-1} - r^{-s}}{s - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

EXERCISE. Comparison between the rules for the following v of $s$. It is a pity that one formula like (2) will not serve us f values of $s$. But there is one case in which (2) is of no use t namely the most common case, where $s = 1$; let the student tr himself. He ought to calculate every one of the following num ~~taking $p_1 = 1$.~~

VALUES OF $p_m$ FOR THE FOLLOWING VALUES OF $s$

| $r$ | 0·8 | 0·9 | 1·0 | 1·0646 | 1·1111 | 1·135 | 1·2 |
|---|---|---|---|---|---|---|---|
| 1·333 | ·972 | ·970 | ·965 | ·964 | ·961 | ·960 | ·959 |
| 1·5 | ·948 | ·941 | ·937 | ·934 | ·931 | ·930 | ·926 |
| 2 | ·872 | ·859 | ·846 | ·838 | ·833 | ·830 | ·823 |
| 3 | ·743 | ·721 | ·700 | ·687 | ·678 | ·674 | ·662 |
| 5 | ·580 | ·549 | ·522 | ·505 | ·496 | ·489 | ·475 |
| 8 | ·447 | ·414 | ·385 | ·369 | ·356 | ·352 | ·337 |
| 12 | ·351 | ·318 | ·290 | ·274 | ·265 | ·259 | ·246 |
| 20 | ·257 | ·225 | ·200 | ·186 | ·177 | ·173 | ·162 |

*Proofs of the above Rules.*

I. The student who knows a little calculus—surely it ought to be tau mere beginners—knows that when a fluid of volume $v$ and pressure $p$ inc in volume by the very small amount $\delta v$, the work done by it is $p \,.\, \delta v$. If, fluid at $v_1$ and $p_1$ increases in volume to $v_2$, and if its law of expansion is $pv$ the total work done is

$$\int_{v_1}^{v_2} p \; dv, \text{ or } p_1 v_1 \int_{v_1}^{v_2} \frac{1}{v} dv, \text{ or } p_1 v_1 \log_e \frac{v_2}{v_1}.$$

If the pressure $p_1$ kept constant when the volume was increasing from 0 to $v_1$ the work done was $v_1 p_1$. If the back pressure is $p_3$ being constant, when the volume diminishes from $v_2$ to 0 the negative work done by the fluid is $p_3 v_2$ and hence the total amount in all is, if we indicate $v_2/v_1$ by the letter $r$,

$$p_1 v_1 + p_1 v_1 \log_e r - p_3 v_2.$$

Now if this is to be the same as the work done under a constant effective pressure $p_e$ from volume 0 to volume $v_2$ and no back pressure, it ought to be equal to $p_e v_2$. Putting it equal and dividing by $v_2$, we find

$$p_e = p_1\left(\frac{1 + \log_e r}{r}\right) - p_3 \ldots\ldots (1)$$

II. If $pv^s$ remains constant during expansion and $s$ is not 1.
In the above proof,

$\int_{v_1}^{v_2} p \,.\, dv$ is $p_1 v_1^s \int_{v_1}^{v_2} v^{-s} \,.\, dv$, or $p_1 v_1^s (v_2^{1-s} - v_1^{1-s})/(1 - s)$, and hence

$$p_e = p_1(sr^{-1} - r^{-s})/(s - 1) - p_3 \,.\,. (2)$$

I nearly always use the first rule, but if I want to be more general I use $p_e = p_1 R - p_3$, where $R$ stands for any of the numbers in the above table.

In hypothetical calculations nearly everybody uses the rough and ready rule $pr$ constant. To help in calculating $p_m$, and indeed for other reasons, I give the table on the following page.

If the area of the piston is $A$ square inches, $l$ the length of the stroke in feet (twice the length of the crank), the steam supplied for one stroke is $\frac{A}{144} \frac{l}{r}$ cubic feet, or $\frac{Al}{144 r v_1}$ lbs. The work done in one stroke is $p_e Al$, and hence the **work done per cubic foot of steam** is, if expansion is according to the law $pv$ constant,

$$144\{p_1(1 + \log. r) - p_3 r\}$$

It is easy to show that this is a maximum for given values of $p_1$ and $p_3$ when $r = p_1/p_3$.

The student must bear in mind that we are dealing with the hypothetical diagram. It is usually found that wire drawing, cushioning, and the effects of clearance, cause the real $p_e$ of an indicator diagram to be smaller than our hypothetical $p_e$ by, roughly, the fraction $cr$ of itself, where $c$ is the clearance as a fraction of the whole volume. Mr. Willans generally tabulated the ratio of his real $p_e$ to the hypothetical $p_e$, and he called this ratio his *plant efficiency*, a name of which I do not approve. The plant efficiency would probably have been about 97 per cent. or more, only for clearance. He usually found it less than 90 per cent., often much less.

**157.** *Important Exercises on* **Regulation and Economy.**—The student will in the following cases (Art. 158) calculate $p_e$ or

### Table of Napierian Logarithms.

**The Napierian or Hyperbolic Logarithm of a number may be obtained from the ordinary Logarithm of the number by multiplying by 2·3026.**

| n | log. n | n | log. n | n | log. n | n | log. n | n | log. n |
|---|---|---|---|---|---|---|---|---|---|
| 1·05 | ·049 | 3·05 | 1·115 | 5·05 | 1·619 | 7·05 | 1·953 | 9·05 | 2·203 |
| 1·1 | ·095 | 3·1 | 1·131 | 5·1 | 1·629 | 7·1 | 1·960 | 9·1 | 2·208 |
| 1·15 | ·140 | 3·15 | 1·147 | 5·15 | 1·639 | 7·15 | 1·967 | 9·15 | 2·214 |
| 1·2 | ·182 | 3·2 | 1·163 | 5·2 | 1·649 | 7·2 | 1·974 | 9·2 | 2·219 |
| 1·25 | ·223 | 3·25 | 1·179 | 5·25 | 1·658 | 7·25 | 1·981 | 9·25 | 2·225 |
| 1·3 | ·262 | 3·3 | 1·194 | 5·3 | 1·668 | 7·3 | 1·988 | 9·3 | 2·230 |
| 1·35 | ·300 | 3·35 | 1·209 | 5·35 | 1·677 | 7·35 | 1·995 | 9·35 | 2·235 |
| 1·4 | ·336 | 3·4 | 1·224 | 5·4 | 1·686 | 7·4 | 2·001 | 9·4 | 2·241 |
| 1·45 | ·372 | 3·45 | 1·238 | 5·45 | 1·696 | 7·45 | 2·008 | 9·45 | 2·246 |
| 1·5 | ·405 | 3·5 | 1·253 | 5·5 | 1·705 | 7·5 | 2·015 | 9·5 | 2·251 |
| 1·55 | ·438 | 3·55 | 1·267 | 5·55 | 1·714 | 7·55 | 2·022 | 9·55 | 2·257 |
| 1·6 | ·470 | 3·6 | 1·281 | 5·6 | 1·723 | 7·6 | 2·028 | 9·6 | 2·262 |
| 1·65 | ·500 | 3·65 | 1·295 | 5·65 | 1·732 | 7·65 | 2·035 | 9·65 | 2·267 |
| 1·7 | ·531 | 3·7 | 1·308 | 5·7 | 1·740 | 7·7 | 2·041 | 9·7 | 2·272 |
| 1·75 | ·560 | 3·75 | 1·322 | 5·75 | 1·749 | 7·75 | 2·048 | 9·75 | 2·277 |
| 1·8 | ·588 | 3·8 | 1·335 | 5·8 | 1·758 | 7·8 | 2·054 | 9·8 | 2·282 |
| 1·85 | ·615 | 3·85 | 1·348 | 5·85 | 1·766 | 7·85 | 2·061 | 9·85 | 2·287 |
| 1·9 | ·642 | 3·9 | 1·361 | 5·9 | 1·775 | 7·9 | 2·067 | 9·9 | 2·293 |
| 1·95 | ·668 | 3·95 | 1·374 | 5·95 | 1·783 | 7·95 | 2·073 | 9·95 | 2·298 |
| 2·0 | ·693 | 4·0 | 1·386 | 6·0 | 1·792 | 8·0 | 2·079 | 10·0 | 2·303 |
| 2·05 | ·718 | 4·05 | 1·399 | 6·05 | 1·800 | 8·05 | 2·086 | 15 | 2·708 |
| 2·1 | ·742 | 4·1 | 1·411 | 6·1 | 1·808 | 8·1 | 2·092 | 20 | 2·996 |
| 2·15 | ·765 | 4·15 | 1·423 | 6·15 | 1·816 | 8·15 | 2·098 | 25 | 3·219 |
| 2·2 | ·788 | 4·2 | 1·435 | 6·2 | 1·824 | 8·2 | 2·104 | 30 | 3·401 |
| 2·25 | ·811 | 4·25 | 1·447 | 6·25 | 1·833 | 8·25 | 2·110 | 35 | 3·555 |
| 2·3 | ·833 | 4·3 | 1·459 | 6·3 | 1·841 | 8·3 | 2·116 | 40 | 3·689 |
| 2·35 | ·854 | 4·35 | 1·470 | 6·35 | 1·848 | 8·35 | 2·122 | 45 | 3·807 |
| 2·4 | ·875 | 4·4 | 1·482 | 6·4 | 1·856 | 8·4 | 2·128 | 50 | 3·912 |
| 2·45 | ·896 | 4·45 | 1·493 | 6·45 | 1·864 | 8·45 | 2·134 | 55 | 4·007 |
| 2·5 | ·916 | 4·5 | 1·504 | 6·5 | 1·872 | 8·5 | 2·140 | 60 | 4·094 |
| 2·55 | ·936 | 4·55 | 1·515 | 6·55 | 1·879 | 8·55 | 2·146 | 65 | 4·174 |
| 2·6 | ·956 | 4·6 | 1·526 | 6·6 | 1·887 | 8·6 | 2·152 | 70 | 4·248 |
| 2·65 | ·975 | 4·65 | 1·537 | 6·65 | 1·895 | 8·65 | 2·158 | 75 | 4·317 |
| 2·7 | ·993 | 4·7 | 1·548 | 6·7 | 1·902 | 8·7 | 2·163 | 80 | 4·382 |
| 2·75 | 1·012 | 4·75 | 1·558 | 6·75 | 1·910 | 8·75 | 2·169 | 85 | 4·443 |
| 2·8 | 1·030 | 4·8 | 1·569 | 6·8 | 1·917 | 8·8 | 2·175 | 90 | 4·500 |
| 2·85 | 1·047 | 4·85 | 1·579 | 6·85 | 1·924 | 8·85 | 2·180 | 95 | 4·504 |
| 2·9 | 1·065 | 4·9 | 1·589 | 6·9 | 1·931 | 8·9 | 2·186 | 100 | 4·605 |
| 2·95 | 1·082 | 4·95 | 1·599 | 6·95 | 1·939 | 8·95 | 2·192 | 1,000 | 6·908 |
| 3·0 | 1·099 | 5·0 | 1·609 | 7·0 | 1·946 | 9·0 | 2·197 | 10,000 | 9·210 |

$p_1 \frac{1 + \log_e r}{r} - p_3$ and the work done per stroke, multiplying by the number of strokes per minute and dividing by 33,000, to get the hypothetical horse-power. He will also calculate the weight of steam indicated per stroke (neglecting clearance) $Al/144ru_1$, and from this the weight per hour.

**ng Water.** In working out the following important exer- **non-condensing engine,** the student will assume that which is not indicated, that is, which is *missing* because sation in the cylinder or through leakage past valve or to be found by the following rule:—

$$y = \frac{\text{Missing steam}}{\text{Indicated steam}} = 15\frac{1+r}{d\sqrt{n^1}} \quad . \quad . \quad . \quad . \quad (1)$$

the ratio of cut off, $n^1$ is the number of strokes per minute, iameter of the cylinder in inches. Instead of 15 we might small a number as five in a well-jacketed, well-drained f good construction with four double beat valves, and we e as great a number as 30 or even more in badly drained keted engines with slide valves.

not concerned just now with a **condensing engine,** y say that instead of (1) I am in the habit of using $p_1$ being the initial pressure)—

$$y = \frac{\text{Missing steam}}{\text{Indicated steam}} = \frac{120\,(1+r)}{d\sqrt{n^1 p_1}} \quad . \quad . \quad . \quad (2)$$

ic problems on condensing engines. Instead of 120 I use s small as 50 or as great as 300 or even more.

**Non-Condensing Engine.** $n$ revolutions per minute $=2n$ strokes per minute in the following work, the ing double acting; piston 12 inches diameter; crank 1 at $l = 2$, back pressure $p_3 = 17$ lbs. per square inch. Take 'able I., Art. 180. Calculate $I$ the indicated horse-power, unds the weight of steam used per hour. In Table II., I give the weight per hour of each kind of initial ded by a *perfect* non-condensing engine per horse-power. up for each initial pressure and multiply by each horse- get $W^1$, which may be compared with $W$.

tudent will plot $W$ and $I$ for all the cases on one sheet of aper. He will note that $W$ is a linear function of $I$ in two s, and not in the other two (see Fig. 215). He ought to on separate sheets of squared paper also, plotting $W^1$ in Fig. 215) in each such case.

pressure $p_1$ altering. 100 revolutions per minute, $r = 3$.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| . | 100 | 90 | 80 | 70 | 60 | 50 | 40 | 30 |
| . | 2200 | 1950 | 1743 | 1530 | 1350 | 1174 | 900 | 700 |
| . | 71·9 | 62·3 | 52·9 | 43·4 | 33·9 | 24·4 | 14·9 | 5·42 |
| . . | 1330 | 1220 | 1107 | 985 | 855 | 695 | — | — |

II. Cut off altering. $p_1 = 75$, $n = 100$.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $r$ | 6 | 5 | 4 | $3\frac{1}{2}$ | 3 | $2\frac{1}{2}$ | 2 |
| $W$ | 1030 | 1155 | 1340 | 1475 | 1650 | 1900 | 2270 |
| I.H.P. | 24·2 | 30·5 | 37·9 | 41·8 | 48·0 | 54·8 | 62·8 |
| $W_1$ | 530 | 665 | 830 | 910 | 1050 | 1200 | 1370 |

III. Speed altering. $p_1 = 85$, $r = 3\frac{1}{2}$.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 |
| $W$ | 947 | 1090 | 1230 | 1370 | 1510 | 1650 | 1780 | 1910 | 2040 | 2170 |
| I.H.P. | 25·9 | 31·1 | 36·3 | 41·5 | 46·7 | 51·8 | 57·0 | 62·3 | 67·2 | 72·6 |
| $W_1$ | 525 | 630 | 735 | 840 | 945 | 1050 | 1150 | 1260 | 1360 | 1470 |

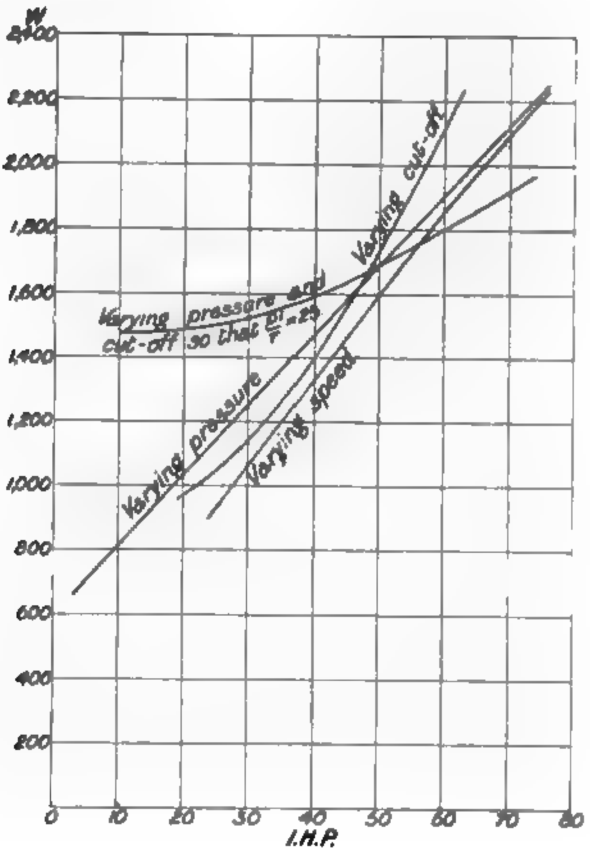


FIG. 215.

IV. Speed constant 100. The pressure and cut off altering at the same time so as to keep $p_1 \div r = 25$.

| | | | | | |
|---|---|---|---|---|---|
| $p_1$ | 25 | 50 | 75 | 100 | 125 |
| $r$ | 1 | 2 | 3 | 4 | 5 |
| $W$ | 1489 | 1537 | 1596 | 1675 | 1756 |
| I.H.P. | 10·8 | 34·4 | 48·2 | 57·9 | 64·8 |
| $W_1$ | — | 970 | 1050 | 1070 | 1080 |

The student will note that if any point in any of these diagrams joined to 0 the origin, the *slope* (or tangent of the angle of inclinon) of the line represents water per hour per indicated horse-power. r any engine it always gets great with the smaller loads. It is only the case of varying cut-off that there is a particular load giving ximum efficiency, and it is for this reason that whereas for centraltion electric lighting work where the load alters slowly, many small gines are recommended, working most of them at full power; for ctric traction central-station work where the load is constantly alterg, only one or two engines are recommended, governing by the cut-off.

In a triple expansion engine even at full load there is much exnsion; hence cutting off much earlier in the stroke will reduce the wer without much gain of economy; in fact, there is a considerable nge of load possible with much the same economy.

In a single cylinder engine, governing by the cut-off, at its reatest load there is a late cut-off; at small load, a very early cutff; hence there is a very much greater gain in economy with less oad than in compound or triple expansion engines. In all cases here is a great advantage in regulating by the cut-off, but it is more noticeable in single cylinder engines.

**159. Condensing Engine.** Sizes as in the last exercise, $p_3 = 3$.

$$y = \frac{\text{missing steam}}{\text{indicated steam}} = \frac{120\,(1+r)}{d\sqrt{np_1}}$$

If $p_1$ varies from 100 to 20, if $r = 3\frac{1}{2}$, $n = 100$, calculate $W$ and $I$.

$$\frac{1+\log_e r}{r} = \cdot 644, \quad \frac{120\,(1+r)}{d\sqrt{np_1}} = \frac{4\cdot 5}{\sqrt{p_1}}.$$

$p_i = \cdot 644\,p_1 - 3$, so that $I = \cdot 874\,p_1 - 4$. Also steam $W$ used per hour in pounds is $\dfrac{5333}{u_1}\left(1 + \dfrac{4\cdot 5}{\sqrt{p_1}}\right)$

| $p_1$ | W lbs. | I. |
|---|---|---|
| 100 | 1775 | 83·4 |
| 80 | 1492 | 66·0 |
| 60 | 1199 | 48·4 |
| 40 | 887 | 31·0 |
| 20 | 542 | 13·5 |

**160.** We see various reasons for thinking that the following mparison is not altogether fair; but it is not altogether unfair. nyhow, it is worth making.

Compare results from the above engine taken as a condensing and as a non-condensing engine; but instead of taking indicated power, which would be too unfair to the non-condensing engine, let us take actual or brake horse-power, assuming

$$B = 0{\cdot}95\, I - 12 \text{ in the condensing engine,}$$
$$B = 0{\cdot}95\, I - 7 \text{ in the non-condensing engine.}$$

| $p_1$ | Condensing. $r = 2\frac{1}{2}$. | | | Non-condensing. $r = 2$. | | |
|---|---|---|---|---|---|---|
| | B. | W. | W/B. | B. | W. | W/B. |
| 100 | 67·2 | 1775 | 26·4 | 61·3 | 2200 | 35·9 |
| 80 | 50·7 | 1492 | 29·5 | 43·3 | 1743 | 40·2 |
| 60 | 34·0 | 1199 | 35·2 | 25·2 | 1350 | 53·6 |
| 40 | 17·5 | 887 | 50·7 | 7·2 | 900 | 125 |
| 20 | 1·0 | 542 | — | — | — | — |

In pages 257–8 I give a number of characteristic results of engine tests. In each case the engine may be taken as working under its most favourable conditions.

In each case I compare the result with that of a *perfect* steam engine using the same kind of steam. It is right to distinguish between perfect non-condensing and condensing engines, because there are a great number of cases where a supply of water cannot be obtained for condensation purposes.

**161. The Willans' Rule.** The calculations of Art. 158 for non-condensing engines lead to a linear law connecting indicated water per hour and indicated horse power, if $e$ is constant. We see the reason from the following algebra:—

$p_m = p_1 R - p_3$ where $R$ stands for $\dfrac{1 + \log. r}{r}$ or the other function of $r$ given in Art. 158.

$n$ being strokes per minute, $A$ area of piston in square inches, $l$ being length of the stroke in feet, $v_1$ being volume in cubic feet of 1 lb. of the steam initially. If the actual total steam is $e$ times the indicated steam, $I$ being the indicated horse power, and $W$ lb. the weight of steam per hour—

$$I = (p_1 R - p_3) \frac{Aln}{33000} \quad \ldots \ldots \ldots \ldots (1)$$

$$W = \frac{60\, Aln\, e}{[illegible]} \quad \ldots \ldots \ldots \ldots (2)$$

Now [illegible] I find that with small error

$$[illegible] \quad \ldots \ldots \ldots \ldots (3)$$

The student ought to try this as an exercise. He will find that [illegible] more than [illegible] per cent. of error for any pressure between [illegible] and [illegible] lb. per square inch. See Ex. [illegible] Art. [illegible]

Inserting this value of $\frac{1}{u_1}$, in the expression for $W$, and using, instead of $p_1$, its value from the first equation

$$p_1 = \frac{1}{R}\left(\frac{33000I}{Aln} + p_3\right) \quad \ldots \ldots \ldots \ldots \quad (4)$$

we find

$$W = \frac{29z}{nR}\left\{\cdot 00003\, Alnp_3\left(1 + 8\cdot 14\frac{R}{p_3}\right) + I\right\} \quad \ldots \ldots \quad (5)$$

This is of the form

$$W = \beta + aI \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (6)$$

I usually take $z$ to be 1 + our old $y$ of Art. 159 + $c^1r$ where $c^1$ is the clearance volume as a fraction of the working volume of the cylinder. It is evident that if $y$ is of the shape (1) of Art. 159, or if it has any shape independent of $p_1$, we have a reason for the Willans' rule. In condensing engines $z$ certainly seems to depend upon $p_1$; if we had an exact law it would be worth while using it in the above work, although the elimination of $p_1$ might not be so easy as before.

It is obvious that the indicated steam per hour is a linear function of $I$, if $r$ is constant, whether clearance is neglected or not. In Mr. Willans' non-condensing trials we see in Art. 235 that $y$ is not a function of $p_1$ and therefore the whole water per hour $W$ is a linear function of $I$, which is the Willans' rule. I have found by careful trial that the missing steam in Mr. Willans' condensing trials is only in one or two cases approximately a linear function of $I$, and as the indicated water is such a function, the whole cannot be. Of course, the Willans' rule is only an approximation to the truth; but when the missing water is small in amount, the discrepance is small, as the above algebra makes obvious. I would here warn students of the danger of assuming an empirical law to be true much beyond the limits of the experiments on which it is based. I have read mischievous discussions as to the *meaning* of the Willans' rule *when $I$ is negative!*

**162. Exercises on Clearance.** To see what is the effect of clearance the student cannot do better than work one or more exercises like the following. In an actual indicator diagram, we have cushioning. The actual weight of the steam present just before admission ought to be found; and the volume of an equal weight at the initial pressure ought to be subtracted from the volume of the clearance space itself to get the clearance which has the same amount of evil effect that we find in these exercises. But, indeed, this is a small matter, and there are other small matters which I might refer to, but there is no use in trying to get a hypothetical indicator diagram which shall represent the general case better than ours of Art. 156.

When the piston passes through $\frac{1}{r}$-th of its stroke let cut-off take place. Let $c$ be the clearance volume in terms of linear displacement of piston. Steam is really cut off at $1/r^1$th of its final volume, if

$$r^1 = (l + c)\Big/\left(\frac{l}{r} + c\right) = r(l + c)/(l + cr).$$

If $p_m$ is the mean forward pressure

$$p_1\frac{1 + \log. r^1}{r^1}(l + c) - p_1c = p_m l$$

$$p_e = p_1\left\{\frac{1 + \log. r^1}{r^1}\left(1 + \frac{c}{l}\right) - \frac{c}{l}\right\} - p_3 \quad \ldots \ldots \quad (1)$$

The work done in one stroke is $p_eAl$.

The volume of steam used in one stroke is $\frac{Al}{144r}\left(1 + \frac{c}{l}r\right)$

Thus, taking $p_3 = 17$, $A = 112$ sq. in., $l = 2$ feet, $n = 100$, clearance 8 per cent., or $c/l = \cdot 08$, the student ought to find for many values of the cut-off, the indicated horse-power, $I$, and the weight, $W$ lbs., of steam per hour—First, when $p_1 = 200$; second, when $p_1 = 150$; third, when $p_1 = 100$; fourth, when $p_1 = 75$; fifth, when $p_1 = 50$. For each case he ought to plot the curve connecting $I$ and $W$ with and without clearance. These tables of numbers will enable him also for a particular value of $r$, and letting $p_1$ alter, to plot $I$ and $W$. Such curves carefully studied will give much useful information. We are neglecting all missing steam. Here is a sample table, $p_1 = 100$ :—

| $r$. | No clearance. | | Clearance 8 per cent. | |
|---|---|---|---|---|
| | $I$. | $W$. | $I$. | $W$. |
| 6 | 40 | 715 | 49 | 1060 |
| 5 | 48 | 860 | 56 | 1200 |
| 4 | 58 | 1070 | 64 | 1420 |
| 3·5 | 64 | 1230 | 70 | 1580 |
| 3 | 72 | 1430 | 76 | 1770 |
| 2·5 | 81 | 1720 | 84 | 2060 |
| 2 | 92 | 2140 | 94 | 2480 |
| 1·5 | 104 | 2860 | 105 | 3200 |

EXERCISE.—In a triple expansion engine where the low pressure cylinder is nine times the volume of the "high," and where the clearance in the "high" is ·15 of its volume, what is the true fraction to take for clearance in comparison with a one-cylinder engine? *Answer.* ·0167.

The effect of clearance is probably sufficiently well illustrated, unless when $r$ is large, by the simpler assumption that the clearance volume of steam is quite wasted, doing no work.

Thus, $p_m = p_1$ [illegible] $1 + \log$ [illegible] $- p_3$,

and the volume of steam used per stroke is

$$\frac{Al}{144r}\left(1 + \frac{c}{l}r\right)$$

I usually employ $y$ to mean $\frac{\text{missing steam}}{\text{indicated steam}}$. It is evident that in hypo[illegible] the effect of clearance is very nearly (except when $r$ is [illegible] being the clearance volume as a fraction of [illegible] the steam expands.

**163. The Best Cut-off.** In putting before beginners the con[illegible] of expansion. I represent both [illegible] back pressure. It is quite easy to [illegible] being the total back pressure, [illegible] when this value of the expansion [illegible] of steam is $144u_1 p_1 \log. \frac{p_1}{p_3}$. [illegible] figures, such as a beginner can

rk out for himself as in Chap. III., shows easily the great inherent rantages of using a small back pressure, and, consequently, of using idensation. It is easy now to understand my remarks in Art. 38. the case of that engine, I took a back pressure of 10 lbs. per are inch to represent the effect of the missing water when one culates work per pound or cubic foot of steam. As we then used am of 100 lbs. per square inch, initial pressure, we ought to use following values of $r$:—

CONDENSING ENGINE.

| To get the most | per pound of | $r$ ought to be |
|---|---|---|
| Indicated work | Indicated steam | $100 \div 3 = 33.$ |
| Brake work | Indicated steam | $100 \div 17 = 6.$ |
| Brake work | Actual steam | $100 \div 27 = 3{\cdot}7.$ |

NON-CONDENSING ENGINE.

| To get the most | per pound of | $r$ ought to be |
|---|---|---|
| Indicated work | Indicated steam | $100 \div 17 = 6.$ |
| Brake work | Indicated steam | $100 \div 27 = 3{\cdot}7.$ |
| Brake work | Actual steam | $100 \div 37 = 2{\cdot}7.$ |

If such exercises as those of Chap. III. are worked, and the results efully studied, the beginner will learn a lesson which ought to be ressed almost more than any other on Steam Engine Engineers. virtue of great expansion seems obvious to everybody at first it. But it is evident that even if we only consider indicated er and indicated steam, we ought not to let expansion continue the pressure falls below the back pressure $p_3$. It will be found this will just not occur if $r = p_1 / p_3$. When we consider e power and indicated steam, we ought not to let expansion inue till the pressure falls below $p_3 + f$, if $f$ is the frictional pressure, that is, $r = p_1 / (p_3 + f)$. When we consider ated power and actual steam, we ought not to let expansion inue till the pressure falls below $p_3 + c$, if $c$ is the number which present condensation enters into the calculation as if it were a pressure. That is $r = p_1 / (p_3 + c)$.

When we consider actual power and actual steam,—and although nstantly forget it, this is more important than any of the others ought not to let expansion continue till the pressure falls below $f + c$, that is $r = p_1 / (p_3 + f + c.)$

In all cases we must cut off later if we want true economy rather than if we merely consider indicated power.

In fact, in the above example, to cut off very early, say at $\frac{1}{10}$rd of the stroke, is to the man who uses his mathematics in a foolish way, to get wonderful economy: we now see that we get the best results if we cut off at from $\frac{1}{3}$rd to $\frac{1}{4}$th of the stroke if the engine is condensing, and at from $\frac{1}{2}$ to $\frac{1}{3}$rd of the stroke if the engine is non-condensing.

There are many other matters forgotten by these men, who speak of their absurd notions as 'theory,' and so get true theory into disrepute—the most important is this;—even if by cutting off very early we did get greater work per pound of steam, this is only one kind of economy. There are other kinds to be considered, for instance, the interest and depreciation on the cost of a large engine, which one is using at much less than its full power: this is another consideration to make us cut off still later in the stroke.

**164.** I want to show now that it is not necessary to assume a constant back pressure as representing the friction of an engine when we calculate the best $r$; it is practically as easy to do the work when we take any usual linear law (Art. 144) as connecting $B$ and $I$. I shall take the most general case.

If the useful power is to be delivered at the end of a long line of shafting, and especially if it is such that there is nearly as much friction whatever be the power transmitted, this friction of the shafting may be represented by a back pressure, and if we desire to get maximum useful power per pound of steam we must use a later cut-off in consequence.

Generally, if useful power $U = aI - b$ where $I$ is the indicated power, $a$ being less than unity [illegible]

[illegible]

ck pressure to use as representing friction is then 18·4. And we have the
.

| To get the most | per pound of | $r$ ought to be |
|---|---|---|
| icated work | Indicated steam | $100 \div 17 \quad = 5{\cdot}88$ |
| icated work | Actual steam | $100 \div 27 \quad = 3{\cdot}70$ |
| ful work | Indicated steam | $100 \div 35{\cdot}4 = 2{\cdot}82$ |
| ful work | Actual steam | $100 \div 45{\cdot}4 = 2{\cdot}20$ |

e see, in fact, that if $x$ is any quantity such as "useful power," or rical power," or "yards of stuff woven" per hour, or any other which is r function of $I$ so that $x = aI - b$, where $a$ and $b$ are constants; and if $p_3$ is k pressure of the indicator diagram, and $p_1$ the initial pressure, and if $\beta$ is $\times \dfrac{b}{2uAln}$, the cut-off which will give maximum $x$ per pound of *indicated* is given by

$$r = p_1 \div (p_3 + \beta).$$

r pound of actual steam $r = p_1/(p_3 + c + \beta)$.

i5. In calculating the work done per pound of steam, my excuse for using a pressure term ($c$) to represent condensation is this, that it represents facts not much more unfairly than any other method, and it lends itself at an easy way of putting before a beginner the evil effects of attempting too much expansion. Even the student who takes up the subject in the correct way of Art. 168 will be glad to use this $c$ in thinking about al problems.

ras led to its use in the following way. The late Mr. Willans, as the of his great observation and experience, arrived at this rule for his own idensing engines, whether single cylinder or compound or triple expansion; indicated work done per actual cubic foot of steam is greatest when

Mr. Willans gave a theory to explain the reasonableness of this rule, was not correct in my opinion. The following way of looking at the is, I think, reasonable.

$$p_e \text{ being } p_1 \frac{1 + \log. r}{r} - p_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

g initial pressure, $p_3$ the indicated back pressure, say 17 lbs. per square inch n-condensing engine, $r$ the ratio of cut-off; the work done in one stroke th $l$ feet, piston $A$ square inches in area is $p_eAl$; the cubic feet of steam d to do this work being $\frac{l}{r}\frac{A}{144}$, we have $w$ the indicated work done per oot of steam as

$$w = 144rp_e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

ecause of condensation, we get less than this amount. Let the amount t lacking per cubic foot of steam be $x$ and write $p_e$ as in (1), and

$$w = 144p_1(1 + \log. r) - 144\, rp_3 - x \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

To make $w$ a maximum by obtaining the best value of $r$ we put $\frac{dw}{dr} = 0$, or

$$0 = \frac{144p_1}{r} - 144p_3 - \frac{dx}{dr} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

But if the practical rule is correct this will agree with $r = \frac{p_1}{25}$ and inserting this value of $r$ in (4) we are led to

$$144 \times 25 - 144p_3 - \frac{dx}{dr} = 0$$

$$\text{or } \frac{dx}{dr} = 144\,(25 - p_3)$$

or as $p_3 = 17$, $\frac{dx}{dr} = 1152$, $x = 1152r$ + constant. That is, the lacking work per cubic foot of steam is a linear function of $r$, the ratio of cut-off, a rule which cannot be said to be contradicted by experimental facts if we say that it can only apply within reasonable limits.

If there is no condensation, that is, if $x$ is 0, (4) gives us the rule $r = p_1/p_3$, and it is obvious that our new rule is exactly as if instead of the ordinary back pressure $p_3 = 17$, we had an additional back pressure of 8 lbs. per square inch.

**166.** Since, then, my idea is in agreement with the practical rule adopted by Willans in his single-acting engine, it is worth while to see if it agrees with actual experiments on condensation in cylinders. I found that it did agree very well indeed with the results of Messrs. **Gateley and Kletch**, which I tried first because I thought them much more to be relied upon than any others ever made on the cylinder of a double-acting engine.

Thus taking condensation to be represented by a back pressure $c$ we have, if $w$ = work done per cubic foot of steam as above

$$w = 144\{p_1(1 + \log. r) - r(p_3 + c)\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But what an experimenter usually measures is $x$, the part of a whole cubic foot of steam which is missing at cut-off.

From this point of view, not using $c$,

$$w = (1 - x)144\{p_1(1 + \log. r) - p_3 r\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Putting (1) and (2) equal to one another we get, if $p_e = p_1 \frac{1 + \log. r}{r} - p_3$,

$$c = p_e x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

To test therefore the worth of my assumption, we must try under what circumstances we may consider $p_e x$ to be constant.

Gateley and Kletch (in 1884) testing an engine with a single unjacketed cylinder and Corliss' valve gear, $d$ (the diameter of cylinder) being 18 inches; $l$ (or twice the length of the crank) being 3·5 feet, obtained the following results. There was not much variation of speed.

On plotting the values of $p_e x$ for the engine when used as a condensing engine, on squared paper with $p_1$ as the abscissa, I found so fair an approximation to a straight line that I am convinced that there is almost no better way of representing these results than to take $c$ as

$c = 0·19p_1$ when condensing.

$\phantom{c =}$ $0·18p_1$ when non-condensing.

Thus, therefore, to subject my notion to a rather severe test, I have calculated $p_e x \div p_1$.

| $p_1$ | $p_2$ | $r$ | $n$ | $p_e$ | $x$ | $c$ or $p_e x$. | $\frac{p_e x}{p_1}$ |
|---|---|---|---|---|---|---|---|
| 61·5 | 4·2 | 1·70 | 34 | 51·20 | ·227 | 11·6 | ·19 |
| 68·3 | 3·9 | 2·26 | 34 | 50·94 | ·271 | 13·8 | ·20 |
| 62·1 | 4·5 | 3·03 | 34 | 38·72 | ·339 | 13·1 | ·21 |
| 49·1 | 3·7 | 7·63 | 34 | 15·82 | ·501 | 7·9 | ·16 |
| 78·8 | 3·2 | 4·81 | 35 | 38·92 | ·352 | 13·7 | ·18 |
| 66·9 | 3·8 | 4·85 | 35 | 31·77 | ·478 | 15·2 | ·23 |
| 53·2 | 3·2 | 4·10 | 36 | 28·08 | ·369 | 10·4 | ·20 |
| 39·8 | 3·6 | 4·76 | 34 | 17·82 | ·414 | 7·4 | ·19 |
| 28·7 | 3·5 | 4·13 | 34 | 11·76 | ·412 | 4·9 | ·18 |
| 65·4 | 14·7 | 2·43 | 34 | 36·09 | ·109 | 3·9 | ·06 |
| 50·4 | 14·8 | 2·38 | 34 | 24·71 | ·235 | 5·8 | ·11 |
| 40·5 | 14·9 | 2·49 | 34 | 16·20 | ·159 | 2·6 | ·06 |
| 28·4 | 14·8 | 2·15 | 33 | 8·51 | ·273 | 2·3 | ·08 |

Any one accustomed to deal with such experimental results will say that the discrepancies in $p_e x/p_1$ from constancy are surprisingly small.

**Messrs. Emery and Loring** in their famous experiments in 1874–5 found that the best value of the cut-off was given by $r = 1 + \frac{p_1}{22}$. It will be found that for all values of $p_1$ above 35, this really corresponds to our using in the above method of calculation, a total back pressure

$$12\tfrac{1}{2} + \frac{1}{16}p_1.$$

**167.** As I have already said, **Mr. Willans** used a practical rule for best expansion in non-condensing engines (single, compound, and triple), which really comes to using a total back pressure of 25. I am sorry to say that I cannot test the rule by his non-condensing experiments, as in very few of them did he let $p_1$ and $r$ vary independently. His single-cylinder results would point to using a $r$ whose value is $42\frac{p_1 - 25}{d\sqrt{n}}$ (where $d = 14''$), only that we may just as well write $1050\frac{r-1}{d\sqrt{n}}$, since he varied $r$ as well as $p_1$ and this last rule would upset my scheme. All his compound non-condensing results might point to some such rule as $r \propto \frac{p_1 r}{n}$.

I have not tried his condensing results, and I mention these facts here merely to warn a student that although the idea of a back pressure (independent of $r$) as representing for some purposes the effect of condensation and leakage, is exceedingly valuable when one is showing beginners the limitations in the value of much expansion; yet it is not sufficiently well established for us to use it for much more than this at present.

**158.** When we thought that the missing water might be regarded as if it were represented by a back pressure $c$, we saw that **the best cut-off** was given by

$$r = \frac{p_1}{p_3 + c},$$

and the maximum work that could be done per pound of steam was

$$144 u_1 p_1 \log. \frac{p_1}{p_3 + c}.$$

It seems more correct, and for some kinds of engine it must, I think, be more correct to calculate from the value of $y$, where

$$y = \frac{\text{missing steam}}{\text{indicated steam}}$$

If the total steam is $z$ times the indicated steam, the work done per pound of steam is

$$144\, u_1\{p_1(1 + \log. r) - p_3 r\} \div z \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

This is a maximum when

$$z(p_1\frac{1}{r} - p_3) = \{p_1(1 + \log. r) - p_3 r\}\frac{dz}{dr} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

Now let $z = a + \beta r$, say, where $a$ and $\beta$ may be functions of $p_1$ and $a$, and we find

$$p_3/p_1 = \frac{1}{r} - \frac{\beta}{a}\log. r \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

For given values of $p_1$ and $p_3$, $a$ and $\beta$, it is easy to find by trial the best value of $r$. If this best value be used it is easy to see by inserting this value of $p_3/p_1$ in the work expression, that the work per pound $= \dfrac{144\, u_1 p_1}{a} \log. r$.

I. When there is no missing water the work per pound is $144\, u_1 p_1 \log. r$ The value of $r$ being $p_1/p_3$.

II. Non-condensing engine, probably $z = 1 + br$. The work per pound is $144\, u_1 p_1 \log. r$, and the value of $r$ is given by

$$\frac{p_3}{p_1} = \frac{1}{r} - b \log. r.$$

This may be applied to the case where there is no condensed or leaking water, but there is a clearance volume, which is the fraction $c^1$ of the working volume of the cylinder, and we simply use $c^1$ instead of $b$.

If, again, $y$ of Art. 157 is $\dfrac{Cr}{\sqrt{n}}$ we merely use $\dfrac{C}{\sqrt{n}} + c^1$ for $b$.

III. Condensing engine $z = 1 + \dfrac{br - a}{p_1^{\frac{1}{2}}}$ so that $a = 1 - ap_1^{-\frac{1}{2}}$

$$\text{Work per pound} = \frac{144\, u_1 p_1 \log. r}{1 - ap_1^{-\frac{1}{2}}}$$

and the best value of $r$ is given by

$$\frac{p_3}{p_1} = \frac{1}{r} + \frac{br}{\sqrt{p_1} - a}.$$

EXERCISE. In a non-condensing engine of a certain size at a certain speed, $z = 1\frac{1}{8} + \frac{1}{8}r$. Find $r$ to give the best actual work per pound of steam, letting the friction of the engine be represented by a back pressure. Take $p_3 = 27$, $p_1 = 100$, find the best value of $r$, and the work per pound when this best value is used. For the best $r$,

$$\frac{1}{r} - \frac{1}{9}\log. r - \frac{27}{100} = 0.$$

Taking various values of $r$, and calculating the value of this expression, and using squared paper, I find that $r = 2{\cdot}646$ satisfies it nearly, and the work per pound is 128 $u_1 p_1$ log. 2·646 or 124·5 $u_1 p_1$, or 54200 foot-pounds. Now if there were no condensed water the best value of $r$ would be 3·703, and the work per pound of steam would be 82,000 foot-pounds.

By drawing a curve showing $\frac{1}{r} - \frac{1}{9}\log. r$ for various values of $r$ on squared paper, it is easy to find those values of $r$ which give to this the value $27/p$, and so we find the following other most economical values of the cut-off. I tabulate also $r^1$ as giving *indicated* work most economically, and it is interesting to compare them both with the Willans' rule. This shows why Case IV., p. 290, was so uneconomical with light loads.

| $p_1$ . . . | 200 | 150 | 100 | 75 | 50 |
|---|---|---|---|---|---|
| $r$ . . . | 3·59 | 3·20 | 2·65 | 2·21 | 1·68 |
| $r^1$ . . . | 4·13 | 3·80 | 3·28 | 2·88 | 2·28 |
| Willans' $r$ | 8 | 6 | 4 | 3 | 2 |

## CHAPTER XVIII.

### TEMPERATURE AND HEAT.

**169.** IN the first part of this work I have usually expressed temperature on the Fahrenheit scale because all practical engineers use that scale. I am sorry that this should be so, as *scientific* men both of the English and other races, calculate in and think according to the Centigrade scale. To the average practical engineer, this is of no consequence because he cares nothing about Physical Science, and as he never calculates (except in that sense in which any shopkeeper may be said to calculate) he rather welcomes artificial obstructions to calculation, and it is astonishing how much obstruction is caused by that obnoxious 32°.

To the one or two engineers who are interested in science it does not matter either, because they use both scales readily; but it is of enormous consequence to young men trained scientifically, because even a small thing like this will gradually create a disinclination to keep up their acquaintance with physical science.

It is imperative that the young engineer should think in the scale which he practically uses, but the disadvantages of the use of either scale by itself are so great that during the writing of this book, all temperature measurements have been altered from one scale to the other four times. I have therefore come to the conclusion that in general heat problems I will use either scale indifferently and in practical steam engine problems I will incline rather to the use of Fahrenheit. I would use only the Fahrenheit scale if it were not that I want no readers who are ignorant of chemistry and physics, and they must have used only the Centigrade scale when studying those subjects. I am glad to say that I know of no science class

text book on these subjects in which the Fahrenheit scale is sed.

Steam from water boiling under atmospheric pressure is at a mperature called 100° C. or 212° F. The temperature of melting e is called 0 °C. or 32° F. These two points being marked on a ercurial thermometer (and every beginner ought to make a ermometer for himself and graduate it and compare it at various mperatures with a good standard one), the volume between is vided into 180 equal Fahrenheit or 100 Centigrade degrees. Hence is that if $f$ is a Fahrenheit reading and $c$ is a Centigrade reading r the same temperature

$$\frac{f-32}{180}=\frac{c}{100}$$

If this equation is remembered there is no difficulty in changing once from one kind of reading to another. It may be written

$$c=\frac{5}{9}(f-32) \text{ or } f=32+\frac{9}{5}c$$

Or, in words, 32° F. and 0° C. correspond; 212° F. and 100° C. rrespond. Therefore subtract 32 from the Fahrenheit reading, ultiply by 5, and divide by 9, and we have Centigrade reading. Or ultiply Centigrade reading by 9, divide by 5, add 32, and we have 'ahrenheit reading.

Any change in a reading Fahrenheit multiplied by 5 and divided y 9 gives the same change in the reading Centigrade.

To get absolute temperature Fahrenheit, add 460·7 to the rdinary reading. To get **absolute temperature** Centigrade, add 73·7 to the ordinary reading.

1. Convert the following readings to Fahrenheit. At atmospheric ressure mercury freezes − 39·4° C., ice melts 0° C., greatest density f water 4° C., blood heat 36·6° C., water boils 100° C., red heat 26° C., cast iron melts 1530° C.

*Answers.* − 38·9° F., 32° F., 39·2° F., 97·9° F., 212° F., 969° F., 786° F.

2. What is the C. equivalent to a difference of temperature of 15 the F. scale? *Answer.* 8·33.°

Change the following readings:—Polished steel is of a deep blue lour at 580° F., pale straw colour at 460° F.; sea water freezes at ° F. *Answers.* 304·5° C., 237·75° C., −2·2° C.

Change the following Centigrade temperatures and quantities heat (see Art. 175) to the Fahrenheit scale.

| | Boiling points at atmospheric pressure. | Latent heat of vapour, above atmospheric pressure. Centigrade heat units. | Ratio of volume vapour to volu of liquid per pou |
|---|---|---|---|
| Sulphur | 444·5° C. | — | — |
| Mercury | 350 | 62 | — |
| Oil of turpentine | 159·3 | 74 | 193 |
| Water | 100 | 536 | 1696 |
| Alcohol | 77·9 | 202 | 528 |
| Bromine | 58 | 45·6 | — |
| Bisulphide of carbon | 46·2 | 86·7 | — |
| Ether | 34·9 | 90·4 | 298 |

**170.** The **latent heat** of fusion of ice is 79 Centigrade heat u or 142 Fahrenheit or British Thermal Units. = B.T.U. I would give a table of the latent heats of fu of some other substances, but inasmuch as good authorities give q different numbers it seems on the whole better to leave them altogether. Rankine gives 500 as the latent heat of fusion of and Box gives 26·6. Rankine gives 148 for spermaceti, Box 4 M. Person gives the empirical formula for substances generally

$$L = (K_2 - K_1)\,(\theta°\text{C} + 160)$$
$$\text{or } L = (K_2 - K_1)\,(\theta°\text{F} + 256)$$

where $K_1$ and $K_2$ are the specific heats in the solid and liquid st and $\theta$ the temperature of fusion at atmospheric pressure, $L$, the lat heat of fusion. Regnault's latent heat of steam is in Centigrade u

$$L = 605{\cdot}6 - 0{\cdot}695\,\theta - 3{\cdot}3 \times 10^{-7}\,(\theta - 4)^3$$

change this to Fahrenheit. *Answer.* $L = 1091{\cdot}7 - 0{\cdot}695\,(\theta -$ $- 1{\cdot}03 \times 10^{-7}\,(\theta - 39)^3$.

Commonly we use in Fahrenheit units

$$L = 966 - 0{\cdot}7\,(\theta - 212) \text{ or } 1092 - 0{\cdot}7\,(\theta - 32) \text{ or}$$
$$1114{\cdot}4 - 0{\cdot}7\,\theta \text{ or } 1436{\cdot}8 - 0{\cdot}7\,t$$

change these to Centigrade units and also to foot-pounds.

Rankine's formula for saturated steam, $p$ being in lbs. per squ foot, and $t$ the absolute temperature Fahrenheit ($t = \theta°\text{ F.} + 46($ is

$$\log_{10} p = 8{\cdot}28203 - \frac{B}{t} - \frac{C}{t^2}$$

where $\log_{10} B = 3{\cdot}441474$, $\log_{10} C = 5{\cdot}583973$

alter these to Centigrade and pounds per square inch.

Many temperatures are stated in this book, sometimes on the Fahrenheit, sometimes on the Centigrade scale; convert them into the other scale. For example test the numbers given for temperatures in the Tables, Art. 180. It will be observed that I use $\theta$ for the ordinary and $t$ for the absolute temperature on either scale.

**171. Expansion of Solids and Liquids.** The linear expansion of bodies by heat is practically proportional to the rise of temperature. The values of $a$, the co-efficient for linear expansion (the fractional increase in length for a rise in temperature of 1° Centigrade), are supposed, I have no doubt quite incorrectly, to be the following numbers divided by $10^5$:—Aluminium, 2·34; copper 1·79; gold, 1·45; iron, 1·2; lead, 2·95; platinum, 0·9; silver, 1·94; tin, 2·27; zinc, 2·9; brass (71 copper to 29 zinc), 1·87; bronze (86 copper to 10 tin to 4 zinc), 1·8; German silver, 1·8; steel, 1·11; brick, 0·5; glass, 0·9; granite, 0·9; sandstone, 1·2; slate, 1·04; boxwood (across the fibre), 6·1; boxwood (along the fibre), 0·3; oak (across), 5·4; oak (along), 0·5; pine (across), 3·4; pine (along), 0·5.

The co-efficient, $k$, of cubical expansion is three times the co-efficient of linear expansion, because $(1+a)^3 = 1+3a$, is practically correct for these small values of $a$. The average values of $k$ between 0° and 100° C. are the following numbers divided by $10^3$:—Alcohol, 1·26; mercury, 0·18; olive oil, 0·8; petroleum, 1·04; pure water, 0·43; sea water, 0·5. Column 7 of the table, Art. 180, gives more exactly the volume of water when subjected to the pressure corresponding to its temperature.

The student is supposed to have worked many exercises like the following ones:—

1. Steel rails at 0° C. have an aggregate length of 1 mile. What is the length at 33° C.? *Answer.* 1 mile 23·2 inches.

2. A vertical column of water 12 feet high is heated from 4° C. to 210° C. under steam pressure. If its section remains constant, what is its increase in length? *Answer.* 2·06 feet.

3. A cylindric plug of copper just fits into a hole 4″ diameter in a piece of cast iron. After heating the mass to 1240° C. by how much is the diameter of the hole too small for the plug? *Answer.* ·0293 inches.

4. A bar of iron is 70 centimètres long at 0° C. What is its length in boiling water (100° C.)? What is its length at 50° C.? *Answer.* 70·084, 70·042.

5. Two rods, one of copper, the other of iron, measure 98 centimètres each at 0° C.; what is the difference in their lengths at 57° C.? *Answer.* ·033 *cm.*

6. Rails of wrought iron each 30 feet long are laid dow the temperature of 10° C. What space is left between every if they are intended to close up completely at 40° C.? *An* 0·13 inch.

7. A wrought iron connecting rod is 12 feet long at 10° C. W is its increase in length at 80° C.? *Answer.* 0·121 inch.

8. An iron Cornish boiler 33 feet long, the shell at 0° C., the at 100° C.; what would the difference of length be if the flue not prevented from expansion? *Answer.* 0·475 inch.

9. A steel pump rod 1,000 feet long, what is its change of le for a change of 10 Centigrade degrees? *Answer.* 1·33 inch.

10. In a thermometer ·01 cubic inch of mercury at 10° ( raised to 15° C., and rises 1 inch in the tube. What is the c section of the tube? *Answer.* $9 \times 10^{-6}$ square inch.

11. The volume of a mass of iron being 5 cubic feet at 10° C., its volume at 80° C. *Answer.* 5·0126 cubic feet.

**172. Expansion of Gases.** Many gases follow closely a which is said to be the law for a perfect gas, namely, that quantity of gas at the volume $v_0$, pressure (absolute, that i vacuum is the zero), $p_0$, and absolute temperature, $t_0$, changes to and $t$, then

$$\frac{vp}{t} = \frac{v_0 p_0}{t_0}$$

When we deal with 1 lb. of gas, the constant quantity, $vp/t$, is c $R$, and it has the values given in Art. 187 for various gases, $v$ b in cubic feet, $p$ being in lbs. per sq. foot, and $t$ being abs Centigrade temperature. When we deal with any other quan than 1 lb. of stuff, or any other units of pressure and volume, remains constant, but this constant is no longer the $R$ of the tabl

## Exercises.

1. A cubic foot of gas at 27° C. is heated to 137° C., and an variable pressure is maintained by using a movable piston in a t from the containing vessel; what is the new volume? *Answer.* 1 cubic feet.

2. If in the last question the pressure becomes half what it before (shown by using the proper weights in loading the pisto it becomes twice as great, or if its new pressure is to its old as what are the corresponding volumes? *Answer.* 2·734, 0·683,

3. Air goes into a furnace at 16° C., and reaches the chi ·03° C. The chimney contains 2,200 cubic feet of this hot a

ıe difference between the weight of this hot air and of an equal of cold air? A cubic foot of air at 0° C., and at the ordinary ɜure, weighs ·0807 lb. *Answer.* 126·5 lbs.

Ⓒ 500 litres of hydrogen at 60 C., and a pressure of 750 milli-ᵉes, being cooled to 20° C. under 840· mm., what is the new me? *Answer.* 392 litres.

ɨ. 100 cubic feet of steam at 100° C., and 15 lbs. pressure, is ed to 160° C. at 17 lbs. pressure: what is its volume? *Answer* 4 cubic feet. The student will notice that the steam is super-ed.

**173.** The following measurements of pressure and volume were e upon a **gas engine diagram** in 1883 from the beginning of compression until the exhaust opened. Assuming that the unt of stuff remained constant, and that it behaved like a perfect throughout, find the temperatures.

The actual scales of $v$ and $p$ are unimportant. The temperature, ' C., where $v$ = 25 before compression began, is the only tempera- known beforehand; calculate the temperature at every other ıt. One of my students found the following answers:—

| | Compression. | | Ignition and expansion. | |
|---|---|---|---|---|
| $v$. | $p$. | $\theta$° C. | $p$. | $\theta$° C. |
| 10 | 45·2 | 210 | 45·2 | 210 |
| 10·4 | — | — | 123·2 | 1096 |
| 10·6 | — | — | 157·7 | 1515 |
| 10·8 | — | — | 181·7 | 1825 |
| 11 | 39·7 | 194 | 188·2 | 1943 |
| 12 | 35·7 | 185 | 166·2 | 1860 |
| 13 | 32·2 | 175 | 146·2 | 1759 |
| 14 | 29·7 | 171 | 129·7 | 1669 |
| 16 | 24·7 | 150 | 105·7 | 1536 |
| 18 | 21 | 131 | 87·2 | 1406 |
| 20 | 19·5 | 144 | 74·2 | 1308 |
| 23 | — | — | 58·7 | 1171 |
| 25 | 14·7 | 120 | — | — |

Plot the values of $p$ and $v$ to scale on squared paper.

Plot also the values of the temperature and of $v$.

Plot also log. $p$ and log. $v$ on squared paper to see if the expansion rve follows any such law as $pv^s$ = constant. Also see if the com-ssion curve follows some such law. Some of my students plot temperature and time, assuming simple harmonic motion and revolutions per minute.

**174.** Exercise. It is sometimes said that the weight of a cu
foot of steam is about ⅝th of the weight of a cubic foot of air at t
same temperature and pressure. This is fairly true except with hi
pressure saturated steam. But for high pressure steam, if we wa
an easy rule of this kind we had better use ·6546 instead of
Calculate the volume of 1 lb. of air at each of the following pressu
and temperatures; divide by ·6546, or multiply by 1·528 and co
pare with the values for steam taken from the table, Art. 180.
we use $p$ in lbs. per square inch we must divide 95·67 (the $R$ giv
for air in Art. 187) by 144; multiplying by 1·528 we get the volu
of ·6546 lb. of air as $v = 1{\cdot}015\ t/p$. Use $t = \theta + 273{\cdot}7$.

| Temp. $\theta$° C. | Pressure in lbs. per sq. inch. | Volume of 1 lb. of steam from Table 180. | Volume of 1·528 lb. of air. |
|---|---|---|---|
| 100° C. | 14·70 | 26·43 | 25·8 |
| 120° C. | 28·83 | 14·04 | 13·9 |
| 140° C. | 52·52 | 7·993 | 7·99 |
| 160° C. | 89·86 | 4·828 | 4·90 |
| 180° C. | 145·8 | 3·065 | 3·16 |
| 200° C. | 225·9 | 2·030 | 2·12 |

**175. The Measurement of Heat.** This subject must rema
quite unknown to all students who get their information by me
reading. What I write is merely to remind students of some of t
facts learnt by them in their study of heat.

Heat which is measured by $C$ units on the Centigrade scale is
units on the Fahrenheit scale if $\frac{C}{100} = \frac{F}{180}$. To convert heat in
foot-pounds we multiply by Joule's equivalent, which is 774 or 1,39

Exercise 1. A unit of heat is the heat given to 1 lb. of water
raise its temperature 1° Centigrade; what is the heat required
raise 3 lbs. of water through 30 of the F. degrees?

*Answer.* 50 centigrade heat units.

Exercise 2. The latent heats of 1 lb. of water and 1 lb. of stea
(at atmospheric pressure) are respectively 79 and 537 Centigrad
units; convert these into Fahrenheit heat units.

*Answer.* 142 and 967 units.

Exercise 3. How many Fahrenheit and Centigrade heat uni
(as used by Regnault) per second and per minute correspond t
1 horse-power?

*Answer.* 0·712, 42·75 Fahr.; 0·396, 23·75 Cent.

For academic exercise work the student may use the followin

numbers. The heat energy required to raise $m$ lb. of any solid or liquid substance $n$ degrees in temperature is $mns$ units of heat if $s$ is **the specific heat** of the substance as given in this table.

| Substance. | Specific heat. | Substance. | Specific heat. |
|---|---|---|---|
| Brass or bronze | 0·088 | Aluminium | 0·214 |
| 89 copper + 11 aluminium | 0·104 | Copper | 0·092 |
| German silver | 0·095 | Gold | 0·030 |
| Rose's and Wood's alloys | 0·036 | Cast steel, hard | 0·119 |
| Glass (crown) | 0·161 | ,, soft | 0·117 |
| ,, (flint) | 0·117 | Rolled steel | 0·116 |
| Wood | ·5 to ·7 | Iron (wrought) | 0·11 |
| Ice | 0·5 | ,, (white cast) | 0·13 |
| Carbon | ·25 | ,, (grey cast) | 0·122 |
| Coal | ·2 to ·25 | Lead | 0·03 |
| Olive oil | 0·471 | Mercury | 0·033 |
| Petroleum | 0·511 | Platinum, 0° to 1000° C. | 0·032 |
| Sea water | 0·938 | | |

Gases at Constant Pressure.

| | |
|---|---|
| Air | ·238 |
| Oxygen | ·218 |
| Hydrogen | 3·406 |
| Superheated steam (doubtful) | ·37 to ·48 |
| Carbonic oxide | 0·245 |
| Carbonic acid | 0·216 |

The specific heats of some other gases are given in Art. 187. In the case of very expansible bodies like gases it is very important to note that heat given during a change of state depends on something more than on the change of temperature. $C_p$ of Art. 187 means the specific heat if the pressure is constant, $C_v$ if the volume is constant during the rise of temperature.

*Example.* 3 lbs. of mercury at 96° C. is thrown into 2 lbs. of water at 5° C.; what is the temperature of the mixture?

Let $x$° C. be the common temperature. The water rises through $x-5°$ and therefore receives $2(x-5)$ units of heat. The mercury falls $96-x°$ and therefore gives out $3(96-x)\times{}·033$ units of heat. Putting these quantities equal we have $2(x-5)=3(96-x)\times{}·033$, and we find $x=9°·33$ C.

*Example.* 1 lb. of iron at its welding-point, 1,500° C., is thrown into 100 lbs. of water at 0° C.; find the temperature of the mixture. Let $x°$ be the temperature of the mixture, and since about ·122 is the mean specific heat of iron $(1{,}500-x)\times{}·122=x\times 100$, from which $x=1°·83$ the answer.

Exercises.

1. A ton of air at 630° C. at the ordinary pressure is passed through oil originally at 7° C. The air is allowed to sink to 58° C. How much oil will it raise to the temperature of 28° C.?

*Answer.* 30,800 lbs.

2. How much wrought iron will be raised from 18° C. to 30° C. with the heat given out by 3 tons of water sinking from 60° C. to 30° C.?

*Answer.* 68·3 tons.

3. While 1 lb. of air at 700° C. is passing round a superheater, it sinks to 430° C. What weight of dry steam will this raise from 100° C. to 140° C., at the pressure of one atmosphere? And what will be the new volume of the steam, supposing steam to have $\frac{5}{8}$th of the density of air at the same temperature and pressure?

*Answer.* 3·346 lbs.; 100·4 cubic feet.

4. Twenty grammes of carbonic oxide at 680 °C., and at the ordinary pressure, is passed through a kilogramme of water at 0° C., and escapes at the temperature of 30° C.; what will be the temperature of the water?

*Answer.* 3·185°.

5. How many units of heat are required to raise the temperature of 1 lb. of air from 20° C. to 600° C.? What will be the volume of the heated air?

*Answer.* 138·04; 39·6 cubic feet.

6. What will be the relative capacities for heat of the same volumes of air, carbonic oxide, steam, and hydrogen, at the same pressures, if their densities are 14·4, 14, 9, and 1 respectively?

*Answer.* All equal.

7. What is the capacity for heat of a cubic foot of air, and hence (Exercise 6) of a cubic foot of any other gas at the ordinary temperature and pressure?

*Answer.* ·0192 heat units.

From the answers to Exercises 6 and 7 just preceding, it is seen *that a cubic foot of any gas requires the same amount of heat to raise its temperature one degree as a cubic foot of air requires, provided we have the same pressure at all times in both cases.* This amount of heat is expressed by the decimal ·0192, when the air is at the ordinary pressure and temperature.

**176. Latent Heat.** The work done by heat in the molecules of a body is not always measurable as a rise of temperature, for heat may enter into a body doing work among the molecules without raising the temperature. A mass of ice may absorb much heat, its temperature never rising above 0°. In fact, heat may enter into ice, doing work among its molecules, converting it into water, the melting being the only indication of the entrance of heat.

*Latent heat is the heat which enters into a body without increasing its temperature, being necessary for its condition*, or in producing a change in the state of aggregation of its molecules.

ı we say that the latent heat of water is 79° C. we mean
ıelt a quantity of ice at 0° C. without raising it in temper-
ıuires as much heat as would raise the temperature of an
ight of water 79 degrees.
water at 0° and 1 lb. of water at 79° C., when mixed, form
water at 39°·5 C.; but 1 lb. of ice at 0° and 1 lb. of water at
m 2 lbs. of water at 0° C., the water having fallen in temper-
degrees to melt the ice.
e same way we say that the latent heat of 1 lb. of steam is
we measure the amount of heat necessary to raise 1 lb. of
m 0° to 100°, it will take about 5·36 times this measured
onvert the whole of the water into steam under atmospheric

condense all the steam from 1 lb. of water boiling at the
pressure of the atmosphere, by passing it into a large vessel
ater, it will be found that the steam has given up 536 units
ı condensation, besides a certain amount of heat in passing
rom 100° to the new temperature of the water in the cistern.
ult found that the total quantity of heat in 1 lb. of steam—
e number of units of heat which it is capable of giving out
d at constant temperature, and then cooled to 0°: — was
5 + ·305 $\theta$, where $\theta$° C. is the temperature of the steam.
t heat and other properties of steam not at atmospheric
re more fully considered in Arts. 180—1.
*ple.* How many pounds of ice at 0° C. will be melted and raised
ature to 9° C. by 90 lbs. of water at 87° C. falling in tempera-
° C.? Let there be $x$ lb. of ice, then the heat received by

$$\underbrace{79x}_{\text{latent heat}} + \underbrace{9x}_{\text{to raise to 9° C.}}$$

given out is 90 × 78, hence 90 × 78 = $79x + 9x$, from
= 79·77 lbs.

EXERCISES.

w much ice will be converted into water at 4° C. by 6 lbs.
t 70° C.?
*r.* 4·77 lbs.
en 10 lbs. of water is converted into steam at atmospheric
how many units of heat does it take from the source of heat
unding bodies? use C + F
*r.* 5,360 units.

3. 6 lbs. of superheated steam at 122° and atmospheric pressure is passed into 1,250 lbs. of water at 4° and 20 lbs. of floating ice at 0°; to what height will the water be raised in temperature?

*Answer.* 5°·7 C.

4. 20 lbs. of saturated steam at 144° is converted into water at 30°; how many heat units has it given out? (3 r d all)

*Answer.* 1,2410.

5. 20 lbs. of steam from a boiler at the pressure of $1\frac{1}{2}$ atmospheres condenses in passing into 1,735 lbs. of water originally at the temperature of 16°; what is the new temperature of the water?

*Answer.* 23°·2 C.

6. 600 lbs. of mercury at 130° C., and 723 lbs. of olive oil at 110° C., are poured into a vessel containing 165 lbs. of water at 0° C., and 20 lbs. of floating ice; what will be the temperature of the mixture?

*Answer.* 70°·5 C.

**177.** The **specific heat** of a substance is usually not at all constant. Thus ice from − 78° C. to 0° C. has an average specific heat ·463, but from − 21° to 0° C. it is 0·502. Of aluminium at about 20° C. it is 0·2135, whereas about 300° C. it is ·2401. Copper about 0° C. is ·090, whereas about 300° C. it is ·0985. The best wrought iron about 15° C. is ·1091, whereas about 200° C. it is ·1249; about 850° C. it is ·218, about 1,100° C. it is ·200, and it has the extraordinarily high value of ·3243 about 700° C.

All the values of the specific heats of substances quoted by me are vitiated by uncertainty as to their chemical purity and the specific heat of the water with which they were compared. I give the received values, knowing their untrustworthiness, which, however, is not very important in ordinary steam engine calculations.

The heat required to raise a pound of water one degree may be taken to be $1 + 10^{-6}\,\theta^2$ in Centigrade units and on the Centigrade scale: $1 + 3{\cdot}09 \times 10^{-7}\,(\theta - 39)^2$ in Fahrenheit units and on the Fahrenheit scale. These empirical formulæ may be taken as according with Regnault's measurements. It is difficult to say exactly what these units of heat mean, because Regnault did not pay much attention to the variation in the specific heat of water below 100° C.

The latest determination of the average heat energy required to raise one pound of water one degree (called **Joule's Equivalent**) from 0° C. to 100° C., is by Professor O. Reynolds, and is 1,399 foot-pounds; or for 1 gramme it is 0·995 calorie. One calorie, the heat required to raise 1 gramme from 10° C. to 11° C., is 4·2 Joules or $4{\cdot}2 \times 10^7$ Ergs. The heat from 20° C. to 21° C. is $\frac{1}{10}$th of one per cent. less. Regnault's value of $h$ in the table, Art. 180, shows the heat given to one pound of water to raise it to $\theta°$ C. under the constant pressure corresponding to that temperature. The best thing in my power is to notice that Regnault's heat given to water from 0° to 100° C. is 100·5 units. According to Reynolds, this is 139,900 foot-pounds, and so I shall take one of what I call Regnault's units to be 1,393 (or 774 on the Fahr. scale) foot-pounds. There is no present possibility of comparing the Reynolds's measurement with those so carefully made between 10° C. and 25° C. by Rowland and Griffiths.

**178.** I must warn students that the tables of numbers given in engineering books as the heat properties of water and steam and other substances are generally wrong; often very greatly wrong. The tables, pages 320—3, have given me and one of my assistants (Mr. D. Baxandall) an enormous amount of trouble, because we ventured once or twice to assume numbers to be correct which we found published in treatises and scientific papers by the most noted of English and American and other writers. It is particularly annoying when a result of this dependence on others is the necessity of altering some hundreds of scattered calculations. I am sorry to say that one writer on whom I usually place great reliance has increased Rankine's $u$ of the table, Art. 180, in the ratio 778/772. I have just shown that the ratio ought to be 774/772, and this is what I have used.

It may be remarked that almost all calorimetric measurements made until quite recently are open to suspicion, if for nothing else than the **errors of the thermometers.** Now that the German Reichsanstallt is improving the glass manufacture, it may be hoped that in time thermometers of mercury in glass may be depended on to give always the same reading at the same temperature. For all temperatures and for the most exact readings, practical men will find the "Electrical Resistance of Platinum" thermometer better than any other. A handy "Thermal Junction" thermometer reading in degrees of the hydrogen thermometer is greatly wanted. There is a German glass mercury thermometer of the ordinary kind which reads to 1022° F., or 550° C., with considerable accuracy, using a zero correction.

In time our measurement of temperature will probably be, not by marks on an instrument showing expansion due to heat, but by the pressure of some vapour or vapours. The melting points of various substances in degrees on the air thermometer, furnish the best standards for practical men at the present time.

Rowland has shown that when 0° C. and 100° C. are the same on the air and mercury (in glass) thermometers, the readings $t$° C. (air) and $T$° C. (mercury) are connected by

$$t = T - a\,t\,(100 - t)(b - t)$$

where $a$ and $b$ are constants. Thus, with some kinds of glass,

$$a = 44 \times 10^{-8},\ b = 260.$$

It is to be remembered that a mercury thermometer has its stem between the 0° C. and 100° C. marks divided into parts of equal volume.

A mercury thermometer kept for five hours at a high temperature will often have its freezing point of water depressed one to two degrees, but it will recover. After forty years of use a mercury thermometer may read 1° C., instead of 0° C. In fairly exact work the stem is supposed immersed to the level of the top of the mercury; in the most exact work the whole thermometer is supposed to be at the same temperature.

But, indeed, remarks like this are misleading. The errors of English thermometers are very great, and must remain great until we have a physical laboratory which will do for our heat measurements what Whitworth did for our measurements of length. The glasses used differ greatly, and even the best thermometers at Paris and Berlin show remarkable secular changes of behaviour. Rises of 7°, or more, occur after exposure to high temperatures, so that all readings have to be checked by taking an ice reading afterwards. That pressure coefficients must be used is obvious, if one remembers that even the effect of the pressure of the mercury column itself is very evident when we change a thermometer from its vertical to a horizontal position; and barometric pressure

alters sometimes in these islands by 3½ inches. It is true that at the I International they are able, with a *verre dur* glass mercury thermomete possibly at Berlin, to get readings which approach those of the hy thermometer with errors ± ·002° C.; but it is absurd for any experim using English glass mercury thermometers to pretend to a greater accurac a tenth of a degree.

Dr. Harker, of Kew Observatory, has given me the latest comparis gas and mercury *verre-dur* thermometers. From his curves I give the foll readings for the same temperatures. The initial pressure in the gas tl meter is 1 metre of mercury, and changes occur at constant volume. Th reason to believe that the difference between the hydrogen and nitroger (const. vol.) in no case exceeds 0·1° C. below 600° C. The numbers for t scale are somewhat doubtful.

| Mercury Verre-dur. | Nitrogen. | Hydrogen. | Air. |
|---|---|---|---|
| - 20° C. | - 19·841 | - 19·828 | ? |
| - 10° | - 9·934 | - 9·927 | ? |
| 0° | 0 | 0 | 0 |
| 10° | 9·954 | 9·948 | 9·949 |
| 20° | 19·925 | 19·915 | 19·923 |
| 30° | 29·909 | 29·898 | 29·904 |
| 40° | 39·903 | 39·893 | 39·898 |
| 50° | 49·906 | 49·897 | ? |
| 60° | 59·915 | 59·910 | ? |
| 70° | 69·929 | 69·928 | ? |
| 74° | 73·935 | 73·935 | ? |
| 80° | 79·948 | 79·950 | ? |
| 90° | 89·971 | 89·975 | ? |
| 100° | 100· | 100· | 100° |

## CHAPTER XIX.

### PROPERTIES OF STEAM.

**179. Squared paper** is now in very common use to show how one thing depends upon another. I assume that students have already used it to express the results of experiments in a mechanics or heat laboratory.

The following **experiment** must be made by every student who hopes to understand the steam engine.

Get a little boiler with some water in it and a gas flame to heat it with. Most of the air must have been driven out by escaping steam before the following observations are made. There must be a safety valve. A thermometer (its bulb protected so that pressure shall not alter its readings) measures the temperature of the steam. It is worth while to have two thermometers, one reading Centigrade, the other Fahrenheit. A Bourdon's pressure gauge, whose construction is described in Art. 110, may be used for the roughly correct measurement of pressure. I use also a mercury gauge which may be graduated so as to measure the absolute pressure; the outer end of the tube being closed and containing air kept at constant temperature by a bath. It is an ordinary part of the work in a mechanical laboratory to test the readings of pressure gauges by the use of a column of mercury and cistern gauge, and in accurate work a barometer must be observed for the atmospheric pressure at the time. I shall quote only the absolute pressure. The first thing which the student ought to note is that when the temperature is 120° C. the pressure is 28·8 lbs. per square inch, and if the temperature alters and comes back again to 120° C. the pressure returns to 28·8. He will make out a table showing what the pressures are for many temperatures, so that by means of it he knows the pressure of steam for any temperature if the steam is in the presence of water or what we call saturated steam. This is a very important fact, and

shows that a thermometer may be used on a boiler instead of a pressure gauge to tell what the pressure is, but of course a table of the corresponding numbers is needed. The student ought to verify columns 1 and 2 of Table I. or of Table II. These tables are due to the very careful measurements of Regnault.

Now, having his table of numbers, let a student get a sheet of squared paper. Such sheets in which the length is 17 inches and the breadth is 10 inches, divided into tenths of an inch so that there are horizontal and vertical lines forming little squares each one-tenth of an inch in side, may be bought for sixpence a quire. Mark off a scale of temperatures horizontally and another of pressures vertically. Any scales will do, depending upon the ranges of values in our experimental numbers. Now plot a point for the temperature 120° C. and the pressure 28·8 lbs. per square inch; and each point in the curve marked $P$ and $\theta$, Fig. 216, represents one pair of observed numbers. Having plotted all such points use a thin batten of wood to help in drawing the curve which passes through all the points. If there are errors of observation, their probable values will be seen when we have drawn the curve which lies *most evenly* among the points. The curve not only represents the observed numbers, but corresponding values of temperatures and pressures lying between the observed ones.

There are some men who keep curves of this kind hanging up on the walls of their rooms for ready reference, not merely to show how pressure and temperature of steam depend upon one another, but many other corresponding quantities. Every morning you will find in the newspaper, curves showing the heights of the barometer and of atmospheric temperature, &c., at various times. A merchant plots the price of silk or cotton yarn or copper, showing by curves how it varies from day to day.

Squared paper is peculiarly valuable to the engineer. A curve shows at a glance the general nature of the relationship of one thing to another, and if it is drawn to a large enough scale, there is often as much accuracy as with a table of numbers. Besides, a table does not give one the intermediate values, and it is troublesome sometimes to interpolate. The curve shows to the eye the rate of increase also, of one quantity relatively to the other.

**180.** In a laboratory for the use of students of the steam engine there is supposed to be simple apparatus for measuring, not merely pressures above that of the atmosphere, but less pressures, and also some of the other columns of numbers shown in Tables I. and II., Art. 80, and it is only by such measurement that a student gets a ge-

working knowledge of the **properties of steam.** Of course there are men of genius, one in a century, who may know of things through mere reading; but the engineering world is getting too much filled up by men whose knowledge, or rather ignorance, of practical physics is due merely to reading. One cannot in a few words describe why actual observation should be so necessary for true knowledge; pro-

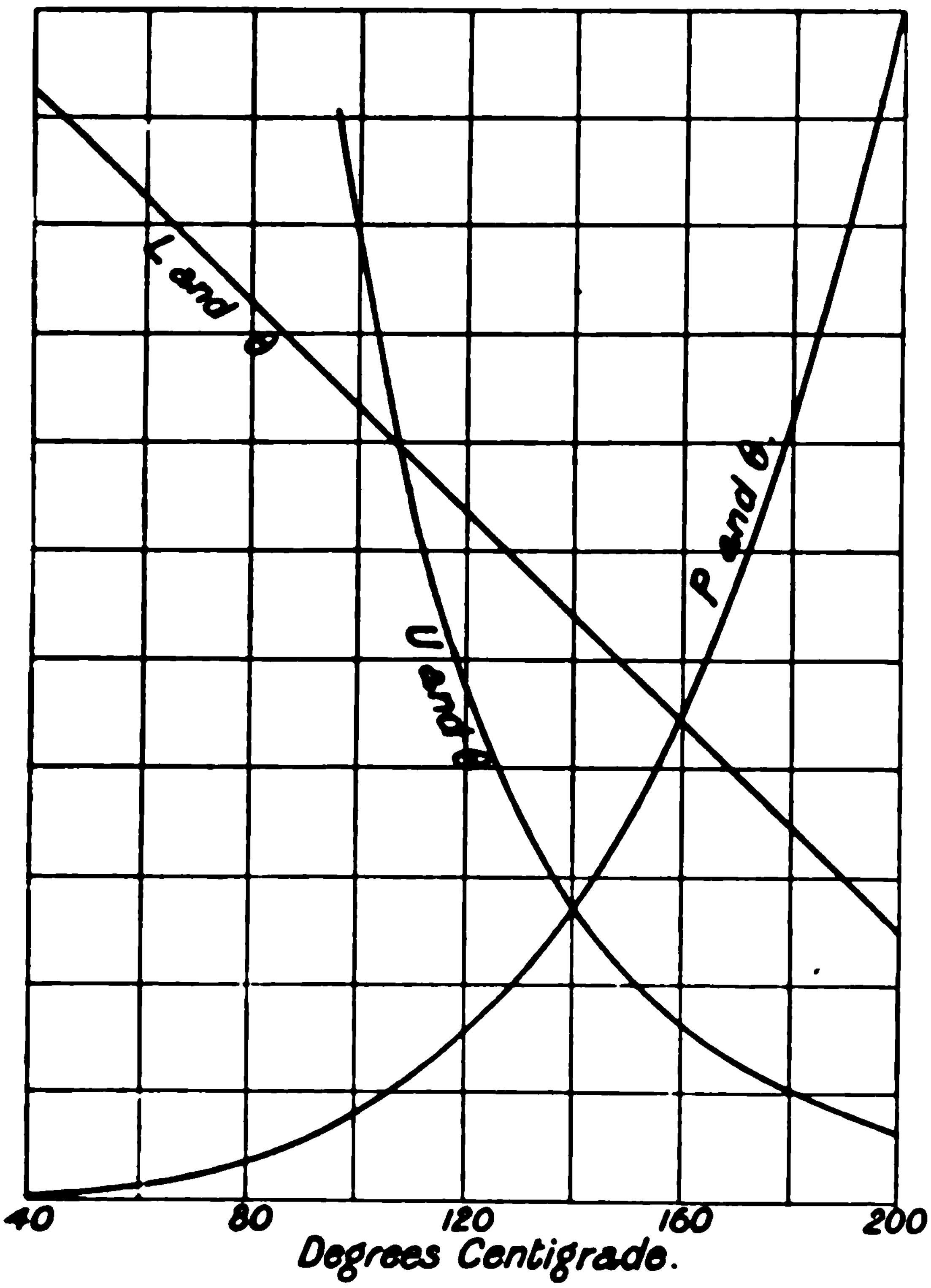


FIG. 216.

bably a lying witness after a simple cross-examination by a skilful barrister would be able to describe it easily enough.

There is no better practice for the student than to plot the various columns of numbers given in the following table, so that one may find the value of $p$, $u$, $H$, $l$, $\phi$ for water or $\phi$ for steam, rapidly, for any value of the temperature or for any value of the pressure. Some of these are shown in Figs. 216 and 217. Notice how $p$ increases more and more rapidly as $\theta$ increases, and how $l$ regularly diminishes as $\theta$ increases. Table II. is obtained from Table I. by

interpolation. Very often it is a definite pressure that is give we wish to find the other properties, and then such a table found valuable.

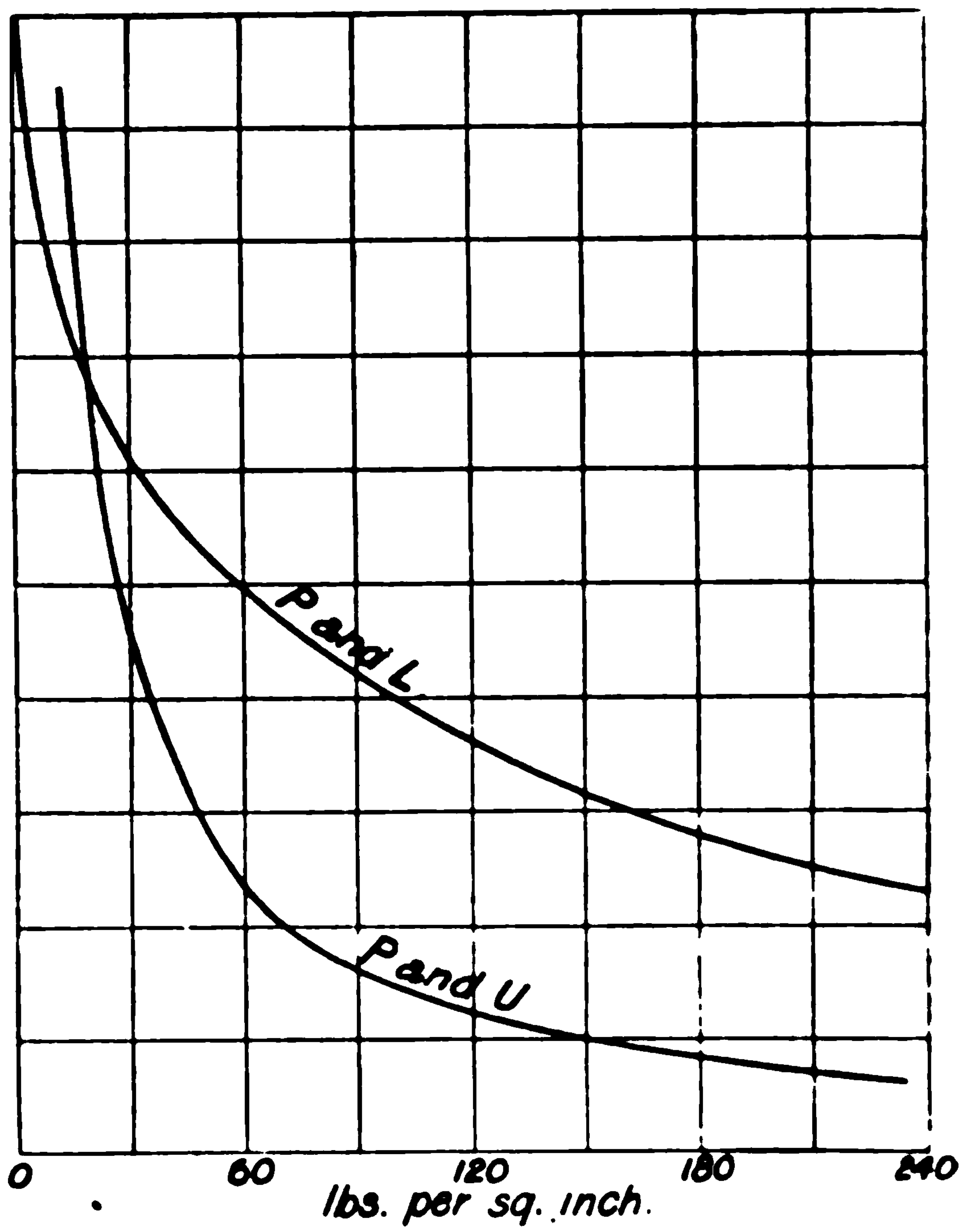


FIG. 217.

*Water-Steam.*

Although the properties of water-steam used in the steam e are tabulated, it sometimes saves trouble to use Regnault's fo in many calculations. These are :—

Total heat of a pound of steam $H$ at $\theta$° C. or $\theta$° F., ab temperature being $t$

$$(1) \quad \ldots \quad \left.\begin{aligned} H &= 606{\cdot}5 + {\cdot}305\,\theta \\ H &= 522{\cdot}5 + {\cdot}305\,t \end{aligned}\right\}\text{ in Centigrade un}$$

$$\left.\begin{aligned} H &= 1082 + .305\,\theta \\ H &= 941{\cdot}4 + {\cdot}305\,t \end{aligned}\right\}\text{ in Fahrenheit un}$$

The heat for water from 0° C. to $\theta$° C. may be taken to be

$$h = \theta + {\cdot}00002\,\theta^2 + {\cdot}0000003\,\theta^3 \quad \ldots \quad (2)$$

The latent heat $l$ is

$$l = H - h \quad \ldots \quad (3)$$

In almost all calculations on the steam engine we may take $h$ as equal to $\theta$ on the Centigrade scale or to $\theta - 32$ on the Fahrenheit scale, so that

$$(4) \quad \ldots \quad \left.\begin{array}{l} l = 606{\cdot}5 - {\cdot}695\,\theta \\ l = 796{\cdot}2 - {\cdot}695\,t \end{array}\right\} \text{Centigrade.} \qquad \left.\begin{array}{l} l = 1114 - {\cdot}695\,\theta \\ l = 1433 - {\cdot}695\,t \end{array}\right\} \text{Fahrenheit.}$$

It is convenient in this place to give other formulæ which we are likely to employ in our heat calculations.

Rankine's formula, probably the most accurate, is

$$log._{10}\, p = A - \frac{B}{t} - \frac{C}{t^2} \quad \ldots \quad (5)$$

Where, if $p$ is in pounds per square inch, and $t$ is absolute temperature Fahrenheit, we have

| | A | Log. $_{10}$ B. | Log. $_{10}$ C. |
|---|---|---|---|
| Water and steam | 6·1007 | 3·43642 | 5·59873 |
| Alcohol | 5·8123 | 3·31233 | 5·75323 |
| Ether | 5·4148 | 3·31492 | 5·21706 |
| Bisulphide of carbon | 5·1854 | 3·30728 | 5·21839 |

With Centigrade temperature subtract ·25527 from $log.\,B$, and ·51054 from $log.\,C$.

A fairly accurate formula, of easy application, is that of Professor Unwin $p$ is in pounds per square inch,

$$\log._{10}\, p = 5{\cdot}8031 - \frac{15900}{t^{1{\cdot}25}} \quad \ldots\ldots \quad (6)$$

if $t$ is absolute temperature Fahrenheit :

$$\text{or } \log._{10}\, p = 5{\cdot}8031 - \frac{7625}{t^{1{\cdot}25}} \quad \ldots\ldots \quad (6)$$

if $t$ is absolute temperature Centigrade.

A formula sometimes used is this, $p$ being in atmospheres,

$$\log._{10}\, p = 5\frac{\theta\ \text{F.} - 212}{\theta\ \text{F.} + 365} = 5\frac{\theta°\ \text{C} - 100}{\theta\ \text{C} + 221{\cdot}7} \quad \ldots \quad (7)$$

There are formulæ often used which are of the type

$$p = a\,(\theta + b)^c \quad \ldots\ldots\ldots \quad (8)$$

where $a$, $b$ and $c$ may be found from the table by a class of students. In many cases $c$ is taken as 5. But any value of $c$ between 5·7 and 4·7 may be taken, and values of $b$ and $a$ are easily found which will give a useful formula. It is,

# I. PROPERTIES OF WATER-STEAM.

| 0° C. | $p$ lbs. per sq. inch. | $p$ lbs. per sq. foot. | $\frac{dp}{dt}$ lbs. per sq. foot per degree Centigrade. | $u$ volume of 1 lb. in cubic feet. | $v$ volume of 1 lb. of gaseous $H_2O$ in cubic feet. | Percentage increase in volume of water from volume at 4° C. | Centigrade Heat Units. $H$ | $h$ | $H - h$ or $l$ | External work during evaporation. | Intrinsic energy 1 lb. of steam. $H - pu/1393$. | Entropy (in Ranks) per pound. Water. $\phi_w$ | Steam. $\phi_s$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0·085 | 12·27 | ·9 | 3398 | 3436 | 0 | 606·5 | 0·00 | 606·5 | 29·92 | 576·6 | 0 | 2·216 |
| 5 | 0·122 | 17·62 | 1·3 | 2412 | 2435 | — | 608·0 | 5·00 | 603·0 | 30·51 | 577·5 | ·018 | 2·183 |
| 10 | 0·173 | 24·92 | 1·6 | 1736 | 1752 | ·03 | 609·5 | 10·00 | 599·5 | 31·06 | 578·4 | ·035 | 2·150 |
| 15 | 0·241 | 34·77 | 2·27 | 1268 | 1278 | ·08 | 611·1 | 15·00 | 596·0 | 31·65 | 579·4 | ·053 | 2·119 |
| 20 | 0·333 | 47·87 | 3·00 | 936·9 | 944·8 | ·17 | 612·6 | 20·01 | 592·6 | 32·19 | 580·4 | ·071 | 2·088 |
| 25 | 0·452 | 65·06 | 3·98 | 700·8 | 706·8 | ·29 | 614·1 | 25·02 | 589·1 | 32·73 | 581·4 | ·087 | 2·060 |
| 30 | 0·607 | 87·40 | 5·07 | 530·7 | 535·0 | ·42 | 615·6 | 30·03 | 585·6 | 33·30 | 582·3 | ·104 | 2·032 |
| 35 | 0·806 | 116·1 | 6·50 | 405·9 | 409·3 | — | 617·2 | 35·04 | 582·1 | 33·84 | 583·4 | ·120 | 2·006 |
| 40 | 1·06 | 152·6 | 8·18 | 313·6 | 316·4 | ·77 | 618·7 | 40·05 | 578·6 | 34·34 | 584·4 | ·137 | 1·981 |
| 45 | 1·38 | 198·6 | 10·2 | 244·6 | 247·0 | — | 620·2 | 45·07 | 575·1 | 34·87 | 585·3 | ·152 | 1·958 |
| 50 | 1·78 | 256·0 | 12·7 | 192·5 | 194·7 | 1·20 | 621·7 | 50·09 | 571·7 | 35·37 | 586·3 | ·168 | 1·935 |
| 55 | 2·27 | 327·0 | 15·7 | 152·8 | 154·7 | — | 623·3 | 55·11 | 568·2 | 35·85 | 587·4 | ·183 | 1·913 |
| 60 | 2·88 | 414·3 | 19·2 | 122·3 | 124·0 | 1·69 | 624·8 | 60·13 | 564·7 | 36·37 | 588·4 | ·199 | 1·891 |
| 65 | 3·62 | 520·6 | 23·3 | 98·7 | 100·1 | — | 626·3 | 65·17 | 561·1 | 36·88 | 589·4 | ·213 | 1·871 |
| 70 | 4·51 | 649·1 | 28·0 | 80·23 | 81·51 | 2·26 | 627·8 | 70·20 | 557·6 | 37·39 | 590·4 | ·228 | 1·850 |
| 75 | 5·58 | 803·3 | 33·6 | 65·64 | 66·82 | — | 629·4 | 75·24 | 554·1 | 37·85 | 591·5 | ·243 | 1·832 |
| 80 | 6·86 | 987·6 | 39·9 | 54·06 | 55·12 | 2·89 | 630·9 | 80·28 | 550·6 | 38·32 | 592·6 | ·257 | 1·814 |
| 85 | 8·38 | 1206 | 47·3 | 44·81 | 45·78 | — | 632·4 | 85·33 | 547·1 | 38·79 | 593·6 | ·271 | 1·798 |
| 90 | 10·16 | 1463 | 55·5 | 37·36 | 38·25 | 3·57 | 633·9 | 90·38 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| [illegible] | 14·70 | 2116·4 | 75·4 | 26·43 | 27·17 | 4·3 | 637·0 | 100·5 | 536·5 | 40·15 | 596·8 | ·313 | 1·749 |
| 105 | 17·53 | 2524 | 87·6 | 22·40 | 23·10 | | 638·5 | 105·6 | 533·0 | 40·58 | 597·9 | ·326 | 1·734 |
| 110 | 20·80 | 2994 | 100·4 | 19·08 | 19·72 | 5·1 | 640·9 | 110·6 | 529·4 | 41·02 | 599·0 | ·339 | 1·719 |
| 115 | 24·54 | 3534 | 115·0 | 16·32 | 16·92 | — | 641·6 | 115·7 | 525·8 | 41·40 | 600·2 | ·353 | 1·705 |
| 120 | 28·83 | 4152 | 131·5 | 14·04 | 14·60 | 6·0 | 643·1 | 120·8 | 522·3 | 41·83 | 601·3 | ·366 | 1·692 |
| 125 | 33·71 | 4854 | 149·4 | 12·12 | 12·64 | — | 644·6 | 125·9 | 518·7 | 42·23 | 602·4 | ·378 | 1·679 |
| 130 | 39·25 | 5652 | 169·0 | 10·51 | 10·99 | 7·0 | 646·1 | 131·0 | 515·1 | 42·64 | 603·5 | ·391 | 1·668 |
| 135 | 45·49 | 6551 | 190·0 | 9·147 | 9·603 | — | 647·7 | 136·1 | 511·6 | 43·00 | 604·7 | ·404 | 1·656 |
| 140 | 52·52 | 7563 | 214·0 | 7·995 | 8·420 | 8·1 | 649·2 | 141·2 | 508·0 | 43·40 | 605·8 | ·416 | 1·644 |
| 145 | 60·40 | 8698 | 240·0 | 7·009 | 7·409 | — | 650·7 | 146·3 | 504·4 | 43·76 | 606·9 | ·429 | 1·633 |
| 150 | 69·21 | 9966 | 268 | 6·168 | 6·548 | 9·2 | 652·2 | 151·5 | 500·8 | 44·14 | 608·1 | ·441 | 1·623 |
| 155 | 79·03 | 11380 | 297 | 5·446 | 5·798 | — | 653·8 | 156·5 | 497·2 | 44·48 | 609·3 | ·453 | 1·613 |
| 160 | 89·86 | 12940 | 329 | 4·827 | 5·158 | 10·3 | 655·3 | 161·7 | 493·5 | 44·83 | 610·5 | ·465 | 1·603 |
| 165 | 101·9 | 14680 | 363 | 4·290 | 4·598 | — | 656·8 | 166·9 | 489·9 | 45·20 | 611·6 | ·476 | 1·593 |
| 170 | 115·1 | 16580 | 400 | 3·823 | 4·120 | 11·6 | 658·3 | 172·0 | 486·3 | 45·50 | 612·8 | ·488 | 1·584 |
| 175 | 129·8 | 18690 | 439 | 3·419 | 3·695 | — | 659·9 | 177·2 | 482·7 | 45·86 | 614·0 | ·500 | 1·575 |
| 180 | 145·8 | 20990 | 482 | 3·065 | 3·327 | 12·9 | 661·4 | 182·4 | 479·0 | 46·17 | 615·2 | ·511 | 1·567 |
| 185 | 163·3 | 23520 | 528 | 2·756 | 3·001 | — | 662·9 | 187·6 | 475·3 | 46·52 | 616·4 | ·523 | 1·559 |
| 190 | 182·4 | 26270 | 575 | 2·482 | 2·717 | 14·3 | 664·4 | 192·8 | 471·7 | 46·80 | 617·6 | ·534 | 1·551 |
| 195 | 203·3 | 29270 | 625 | 2·242 | 2·464 | — | 666·0 | 198·0 | 468·0 | 47·10 | 618·9 | ·545 | 1·543 |
| 200 | 225·9 | 32520 | 677 | 2·031 | 2·242 | 15·7 | 667·5 | 203·2 | 464·3 | 47·42 | 620·1 | ·556 | 1·536 |
| 205 | 250·3 | 36050 | — | 1·843 | 2·046 | — | 669·0 | 208·4 | 460·6 | 47·68 | 621·3 | ·567 | 1·529 |
| 210 | 276·9 | 39870 | — | 1·676 | 1·866 | 17·2 | 670·6 | 213·7 | 456·9 | 47·97 | 622·6 | ·578 | 1·522 |
| 215 | 305·5 | 43990 | — | 1.529 | 1·670 | — | 672·1 | 218·9 | 453·2 | 48·27 | 623·8 | ·589 | 1·515 |

Y

## II. PROPERTIES OF WATER-STEAM.

| Pressure in pounds per square inch $p$ | Temperature. $\theta$° F. | Pounds per square foot per degree Fahrenheit. $dp/dt$. | $u$ | Fahrenheit Heat Units. $H$ | $h$ | $l$ | Entropy (in Ranks) per pound. Water. $\phi_w$ | Steam. $\phi_s$ | Pounds of steam per hour per h.-p. (Rankine Cycle). Con-densing. $w$ | Non-con-densing. $w$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 102 | 4·28 | 334·2 | 1113·0 | 70·1 | 1043 | ·134 | 1·987 | | |
| 2 | 126·3 | 7·8 | 173·2 | 1120·4 | 94·4 | 1026 | ·175 | 1·924 | | |
| 3 | 141·6 | 10·8 | 118·0 | 1127·0 | 109·9 | 1011 | ·201 | 1·887 | | |
| 4 | 153·1 | 14·1 | 89·80 | 1128·6 | 121·4 | 1007 | ·220 | 1·861 | | |
| 5 | 162·3 | 17·0 | 72·50 | 1131·4 | 130·7 | 1001 | ·235 | 1·841 | | |
| 6 | 170·1 | 19·8 | 61·10 | 1133·8 | 138·6 | 995·2 | ·247 | 1·825 | | |
| 7 | 176·9 | 22·1 | 53·00 | 1135·9 | 145·4 | 990·5 | ·257 | 1·814 | | |
| 8 | 182·9 | 25·3 | 46·60 | 1137·7 | 151·5 | 986·2 | ·268 | 1·800 | | |
| 9 | 188·3 | 27·9 | 41·82 | 1139·4 | 156·9 | 982·4 | ·277 | 1·790 | | |
| 10 | 193·2 | 30·5 | 37·80 | 1140·9 | 161·9 | 979·0 | ·285 | 1·781 | | |
| 15 | 213·0 | 42·5 | 25·87 | 1146·9 | 181·9 | 965·0 | ·315 | 1·747 | | |
| 20 | 227·9 | 54·2 | 19·72 | 1151·4 | 197·0 | 954·4 | ·338 | 1·722 | | |
| 25 | 240·0 | 64·8 | 15·99 | 1155·1 | 209·3 | 945·8 | ·356 | 1·704 | | |
| 30 | 250·2 | 75·5 | 13·48 | 1158·3 | 219·7 | 938·5 | ·370 | 1·689 | | |
| 35 | 259·2 | 85·5 | 11·66 | 1161·0 | 228·8 | 932·1 | ·384 | 1·677 | | |
| 40 | 267·1 | 95·0 | 10·29 | 1163·4 | 236·9 | 926·5 | ·395 | 1·666 | | |
| 45 | 274·3 | 105·0 | 9·21 | 1165·6 | 244·3 | 921·3 | ·405 | 1·657 | | |
| 50 | 280·8 | 114·4 | 8·34 | 1167·6 | 251·0 | 916·6 | ·415 | 1·649 | 10·33 | 28·4 |
| 55 | 286·9 | 124 | 7·63 | 1169·4 | 257·1 | 912·3 | ·423 | 1·641 | 10·09 | 26·8 |
| 60 | 292·5 | 133 | 7·03 | 1171·2 | 262·9 | 908·2 | ·431 | 1·634 | 9·87 | 25·2 |
| 65 | 297·8 | 142 | 6·52 | 1172·8 | 268·3 | 904·5 | ·438 | 1·628 | 9·68 | 23·8 |
| 70 | 302·7 | 150 | 6·09 | 1174·3 | 273·4 | 900·9 | ·444 | 1·623 | 9·52 | 22·7 |
| 75 | 307·4 | 158 | 5·70 | 1175·7 | 278·2 | 897·5 | ·450 | 1·617 | 9·36 | 21·8 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 80 | 311·8 | 167 | 5·37 | 1177·0 | 282·7 | 894·3 | ·455 | 1·612 | 9·23 | 20·65 |
| 85 | 316·0 | 175 | 5·07 | 1178·3 | 287·0 | 891·3 | ·461 | 1·608 | 9·10 | 20·25 |
| 90 | 320·0 | 183 | 4·81 | 1179·5 | 291·2 | 888·4 | ·466 | 1·604 | 8·99 | 19·62 |
| 95 | 323·9 | 191 | 4·57 | 1180·7 | 295·1 | 885·6 | ·471 | 1·600 | 8·88 | 19·05 |
| 100 | 327·6 | 199 | 4·356 | 1181·8 | 298·9 | 882·9 | ·476 | 1·596 | 8·77 | 18·54 |
| 105 | 331·1 | 206 | 4·161 | 1182·9 | 302·6 | 880·3 | ·480 | 1·593 | 8·68 | 18·06 |
| 110 | 334·5 | 214 | 3·984 | 1184·0 | 306·1 | 877·9 | ·485 | 1·590 | 8·60 | 17·65 |
| 115 | 337·8 | 222 | 3·821 | 1185·0 | 309·5 | 875·5 | ·490 | 1·586 | 8·51 | 17·25 |
| 120 | 341·0 | 229 | 3·671 | 1185·9 | 312·8 | 873·2 | ·494 | 1·583 | 8·44 | 16·90 |
| 125 | 344·1 | 237 | 3·534 | 1186·9 | 316·0 | 870·9 | ·498 | 1·580 | 8·37 | 16·58 |
| 130 | 347·1 | 244 | 3·406 | 1187·8 | 319·0 | 868·7 | ·501 | 1·577 | 8·30 | 16·29 |
| 135 | 350·0 | 252 | 3·287 | 1188·7 | 322·1 | 866·6 | ·505 | 1·574 | 8·24 | 16·02 |
| 140 | 352·8 | 259 | 3·177 | 1189·5 | 325·0 | 864·6 | ·508 | 1·571 | 8·18 | 15·75 |
| 145 | 355·5 | 266 | 3·074 | 1190·4 | 327·8 | 862·6 | ·512 | 1·569 | 8·12 | 15·51 |
| 150 | 358·2 | 274 | 2·978 | 1191·2 | 330·6 | 860·6 | ·515 | 1·566 | 8·06 | 15·28 |
| 155 | 360·7 | 281 | 2·888 | 1192·0 | 333·2 | 858·7 | ·518 | 1·563 | 8·01 | 15·07 |
| 160 | 363·3 | 288 | 2·803 | 1192·7 | 335·9 | 856·9 | ·521 | 1·561 | 7·96 | 14·88 |
| 165 | 365·7 | 295 | 2·724 | 1193·5 | 338·4 | 855·1 | ·524 | 1·559 | 7·91 | 14·69 |
| 170 | 368·2 | 302 | 2·649 | 1194·2 | 340·9 | 853·3 | ·527 | 1·557 | 7·86 | 14·50 |
| 175 | 370·5 | 309 | 2·578 | 1194·9 | 343·4 | 851·6 | ·530 | 1·555 | 7·81 | 14·33 |
| 180 | 372·8 | 316 | 2·510 | 1195·6 | 345·8 | 849·9 | ·533 | 1·552 | 7·77 | 14·17 |
| 185 | 375·1 | 323 | 2·447 | 1196·3 | 348·1 | 848·2 | ·536 | 1·550 | 7·73 | 14·02 |
| 190 | 377·3 | 330 | 2·386 | 1197·0 | 350·4 | 846·6 | ·539 | 1·548 | 7·69 | 13·88 |
| 195 | 379·4 | 336 | 2·328 | 1197·7 | 352·7 | 845·0 | ·542 | 1·546 | 7·66 | 13·74 |
| 200 | 381·6 | 343 | 2·273 | 1198·3 | 354·9 | 843·4 | ·544 | 1·545 | 7·63 | 13·61 |
| 205 | 383·7 | 349 | 2·221 | 1199·0 | 357·1 | 841·9 | ·547 | 1·543 | 7·59 | 13·48 |
| 210 | 385·7 | 356 | 2·171 | 1199·6 | 359·2 | 840·4 | ·549 | 1·541 | 7·56 | 13·35 |
| 215 | 387·7 | 362 | 2·124 | 1200·2 | 361·3 | 838·9 | ·551 | 1·540 | 7·53 | 13·22 |
| 220 | [illegible] | 369 | 2·07 | 1200·8 | | | ·554 | 1·538 | 7·49 | 13·10 |
| 230 | 394 | 382 | 1·99 | 1201·9 | | | ·559 | 1·535 | 7·43 | 12·90 |
| 240 | 397·5 | | 1·91 | 1203·0 | | | ·563 | 1·532 | 7·38 | 12·70 |
| 250 | 401 | | 1·84 | 1204·0 | | | ·567 | 1·529 | 7·32 | 12·51 |
| 260 | 404·5 | | 1·77 | 1205·0 | | | ·571 | 1·526 | 7·27 | 12·35 |
| 270 | 408 | | 1·71 | 1206·0 | | | ·575 | 1·523 | 7·21 | 12·20 |
| 280 | 411 | | 1·65 | 1207·0 | | | ·579 | 1·520 | 7·16 | 12·05 |
| 290 | 414·4 | | 1·60 | 1207·9 | | | ·583 | 1·518 | 7·10 | 11·91 |
| 300 | 417·5 | | 1·55 | 1208·9 | | | ·587 | 1·515 | 7·04 | 11·77 |

however, easy to show that no such formula can be satisfactory; for, if true, then $p\frac{d\theta}{dp}$ ought to be a linear function of $\theta$. Now, my students h culated $\frac{dp}{d\theta}$ very carefully from Regnault's values, using the method of Ex 128, and they have plotted the values of $p\frac{d\theta}{dp}$ and $\theta$ on squared pa they do not get a straight line: the departure from a linear law is very I therefore use (8) only when I wish to interpolate, but when I wish to calculate $p$ from $\theta$, or $\theta$ from $p$, I use the Rankine formula, or in less work I use (6) or (7).

In Art. 366 we see how to calculate $u$ the volume in cub of a pound of steam at the pressure $p$ lbs. per square inch. values so calculated are given in our table. We find that numbers satisfy the rule

$$pu^{1\cdot0646}=479 \quad . \quad . \quad . \quad . \quad (9)$$

**181.** Imagine $A$ (Fig. 218) to be a cylinder of one squa in cross section with a piston, containing **one pound of** w

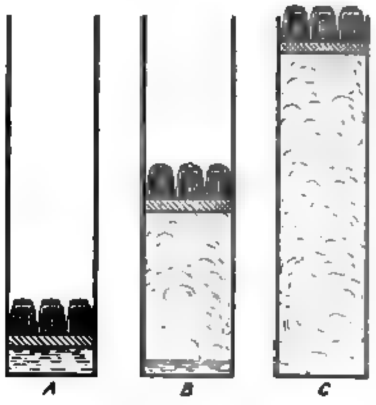


FIG. 218.

**stuff,** and let us suppose it to be surrounded by a bath which it at any known temperature.

The total load on the piston, including its own weight, 2737·6 lbs., I know that the normal atmospheric pressure 2116·4 lbs. per square foot there is a total downward pressure o water of 4,854 lbs. per square foot, or 33·71 lbs. per square inch. if the bath and water were originally at 0° C., the water stuff liquid, if the water is raised gradually in temperature to 125°

ets a little larger in volume, but this small change of volume, and ndeed, the whole volume of the water I shall neglect. The heat iven to the water is called $h$ in the table, and is 125·6 units, or 25·6 × 1,393 foot-pounds. The bath is supposed to keep the mperature of the stuff exactly at 125° C. in all that follows, and erefore our changes must proceed very slowly. The slightest ssening of the load will cause the piston to rise and part of the ater becomes steam, and, although the temperature remains nstant, the bath must give heat, called **latent heat** to the stuff. hen as shown in C. the stuff is all steam, it has received from 0° C. e total heat 644·5 units, called $H$ in the table. That is, it has ceived the additional heat called latent heat, 519 units, called $l$ in e table, being $H - h$. The smallest increase of pressure will cause e piston to fall, and as the bath keeps the temperature constant at 25° C. the steam becomes water again, giving up its latent heat. n the state $B$, suppose that there is 0·4 lb. of water and 0·6 lb. of eam at 125° C., the water has received the total heat $h \times \cdot 4$ nd the steam $H \times \cdot 6$, so that the total heat of the pound of uff is easily calculated, and if we start in the condition $A$, all water t 0° C., and get to the condition $B$ at 125° C., it is this total mount $\cdot 4\, h + \cdot 6\, H$, which has been given to the stuff from the ath.

It is well to remember that the steam has not this total amount f energy in it, for although it has received this or ($\cdot 4\, h + \cdot 6\, H$ ,393 foot-pounds, it has done work on the piston, whose amount is ,854 × the change of volume. Now, I shall neglect the volume of e water, and the volume of one pound of this kind of steam is 2·12 cubic feet, so that the increase of volume has been 0·6 × 12·12. ence to get the actual energy in our pound of stuff we must ubtract 4,854 × 0·6 × 12·12, or 35,300 foot-pounds.

**182.** I have sometimes had tables printed giving the values of and $h$ and $l$ in foot-pounds; every $H$ and $h$ and $l$ of Table I., t. 180, being multiplied by 1393, Joule's Equivalent; but in actice I find that everybody prefers to use heat units. The value u was calculated by Rankine from the thermodynamic formula—t. 366. The values of $\frac{dp}{dt}$ have been worked out by my students in Ex. 7, Art. 128, and tabulated after correction by a curve. The ernal work done by the steam in its formation is $pu$ foot-pounds, if s in pounds per square foot; I have converted it into heat units by iding by Joule's Equivalent. I have subtracted this from $H$ find the intrinsic energy $E$, or energy actually possessed by a

pound of steam in excess of the energy possessed by a pound of water at 0° C.

**183.** In Chap. XXIII. I endeavour to describe the use of $\phi_w$ and $\phi_s$. $\phi_w$ is the **entropy** of a pound of water calculated as in Art. 208; $\phi_s$ is the entropy of a pound of steam. It will be seen that

$$\phi_s = \phi_w + \frac{\text{latent heat}}{\text{absolute temperature}}$$

The student must understand that $\phi_w$ and $\phi_s$ are properties possessed by a pound of water and by a pound of steam, respectively.

*Example.* What is the entropy of 1 lb. of water-steam containing 0·4 lb. of water, and 0·6 lb. of steam at 125° C.? *Answer* (see Table I.). One way of working is to say $0·4 \times ·380 + 0·6 \times 1·680 = 1·160$

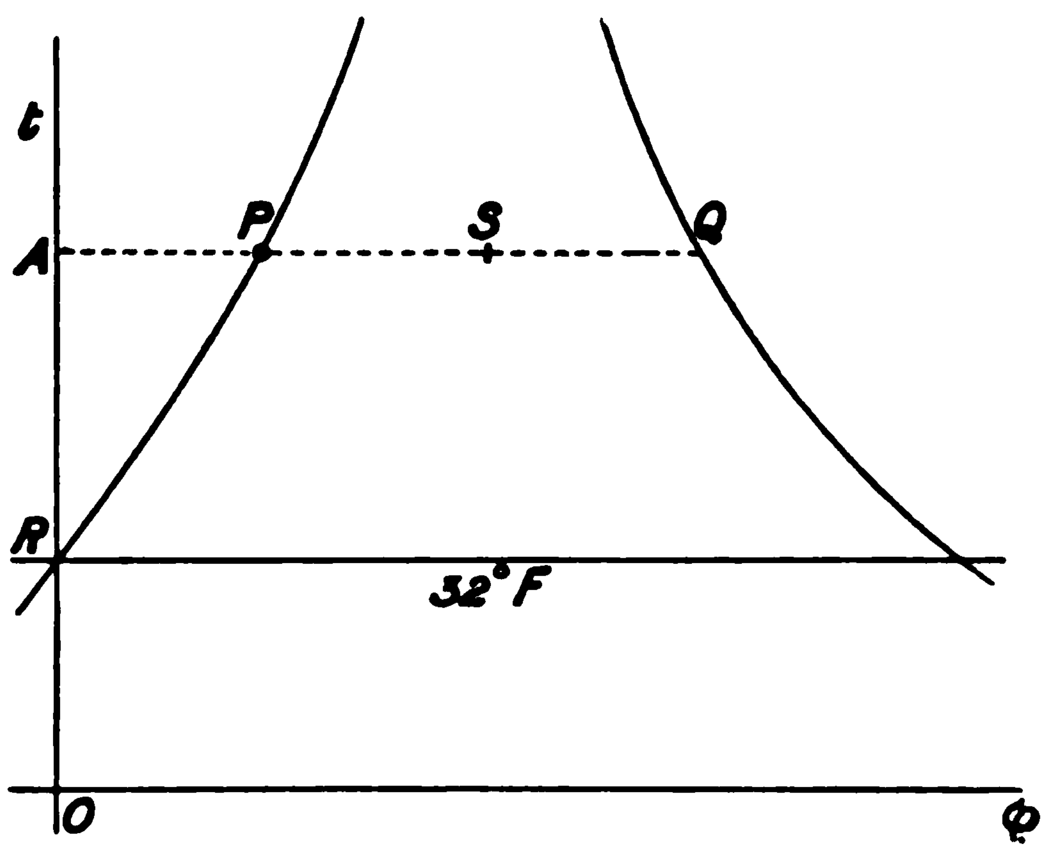


FIG. 219.

ranks. Another equally good is to say:—There is the entropy of 1 lb. of water at 125° C., or 0·380, and addition of entropy due to the formation of 0·6 lb. of steam, or 0·6 × the latent heat 519 divided by the absolute temperature 399.

Curves $P$ and $Q$, showing the values of $\phi_w$ and $\phi_s$ for every temperature in the table, are usually drawn by my students (see Fig. 219) upon somewhat larger and more expensive squared paper than what they use for ordinary calculation.

Intermediate curves are also drawn, dividing every horizontal distance between $P$ and $Q$ into ten equal parts. Indeed, some of my students who use the $\theta\ \phi$ diagram for many practical purposes, divide the spaces into many more parts than ten, so that for example

n the above case they have only to look for the line $APQ$ which corresponds to 125° C.; they take $SQ$ as 0·4 of $QP$, so that the point $S$ represents the state of the pound of water stuff given bove.

A carefully prepared $\theta\phi$ diagram will also have rawn upon it urves of constant volume for a pound of mixed steam and water. t will also have curves of constant volume and pressure of superated steam as described in Art. 205. I find that a blackboard ith all these lines upon it is very useful. The lines of the squared aper are especially useful as the area of each square represents nergy (see Art. 203).

**184.** The numbers in the last two columns of Table II. are escribed in Art. 214, 1st case. For example take 100 lbs. pressure. **perfect** non-condensing engine using the Rankine cycle would se 18·54 lbs. of this steam per hour to produce one horse-power, nd this number serves as a standard. Thus, suppose some nonondensing engine to give one horse-power for 25 lbs. of such steam, re should say that its efficiency as compared with the most perfect on-condensing engine using such steam is 18·54 ÷ 25, or 0·7416, r 74·16 per cent.

It is the fashion just now to use this kind of standard. A etter one is illustrated in the following exercise:—

EXERCISE. Feed water is supplied to the boiler at 60° F.: 0 lbs. of steam at 100 lbs. pressure are used per hour per brake orse-power. What is the efficiency?

The total heat of 1 lb. of such steam is 1,182 in Fah. units. ubtracting 60 − 32, or 28, we get 1,154 units as the heat given to rm each pound of steam, or 1,154 × 30, or 34,620 units per hour. ow one horse-power is 33,000 × 60 ÷ 774, or 2,558 heat units per our, so that the efficiency is 2,558 ÷ 34,620, or 0739, or 7·39 per nt., a very different sort of answer from the last.

The student will find it well at this point to work the exercises ven in Arts. 248 and 249.

In my steam-water calculations, I almost always neglect the **lume of water** present. In other calculations we need to know e volume of a pound of water (see Table I.). At ordinary temratures, about 60° F., we take ·016 cubic feet. At the following mperatures we multiply ·016 by the following numbers:—

| | | |
|---|---|---|
| 212° F. or 100° C. | multiply by | 1·04 |
| 284° F. or 140° C. | ,, | 1·08 |
| 356° F. or 180° C. | ,, | 1·13 |
| 392° F. or 200° C. | ,, | 1·16 |

Rankine gives the following formula for the volume
feet of one pound of liquid water at any absolute Fa
temperature $t$:—

$$V_w = 0{\cdot}0080 \left(\frac{t}{500} + \frac{500}{t}\right)$$

EXERCISE. A cylinder is 12 inches diameter. The bounding surface of the clearance space, including the are piston, is 350 square inches. What is the total area expos cut off takes place, if the crank is 1 foot and cut off takes one-third of the stroke? If the initial steam is 120 lbs. what is the weight of indicated steam? If 35 per cent. of t admitted is condensed, what is the weight of condensed stea

Take twice this quantity of water, and imagine it spreac the surface exposed at cut off, what would be the thicknes water? What thickness of cast iron would have the same for heat as this thickness of water? If the exhaust pres 4 lbs. per square inch, what thickness of iron would be chan the exhaust to the admission temperature by the same ar heat as the difference in total heat of the condensed steam?

EXERCISE. In one stroke 0·7 cubic feet of steam at 1 supplied to a cylinder during $\frac{1}{15}$th of a second. Half condensed. The exposed area of metal is 450 square inches temperature is nearly constant 110° C. How much heat en metal per second, per square $c\ m$ of surface, per degree diffe temperature?

This steam is 6·17 cubic feet to the pound, so that ·0 condensed. In condensing, each pound gives out the heat

$$606{\cdot}5 + {\cdot}305\ (150) - 110, \text{ or } 542 \text{ units},$$

so that the heat given to the metal is 30·9 units. T exposed is 450 × 6·45 square $c\ m$, and the answer is e

$\frac{30{\cdot}9 \times 15}{450 \times 6{\cdot}45 \times 40}$ or ·004 units of heat per second per squa metre per degree difference of temperature. This is twen the greatest emissivity observed between a small polishe ball and the atmosphere, and such a ball owes half its emis having a great curvature of surface, so that the above nu about forty times what we might have expected the emissiv between air and polished metal.

**185.** If a point $P$ is given, to draw through it a curve representing the pressure and volume of a quantity of s

steam where $PB$ represents pressure and $PA$ represents volume, to any scale.

Find the pressure represented by $PB$: let it be, say 89·86 lbs. per quare inch; $PA$ is given in inches as the linear representation of he volume. Let $F^1F$ represent the pressure 69·21 to find $OF$. Ve note that

$$\frac{OF}{OB} = \frac{\text{vol. of 1 lb. of steam at 69·21}}{\text{vol. of 1 lb. of steam at 89·86}} = \frac{6·153}{4·816}$$ by the table, so that as $OB$ is known, $OF$ can be found.

In this way, using the tables, we can find $OF$ for any pressure nd plot the point $F^1$ and so get the saturation curve $PF^1N^1$. I refer this method of drawing the curve. It may be more tedious han some other, but it keeps one in touch with necessary ideas conerning the properties of steam.

I do not know to what extent the following exercises are worth loing by students.

*Expansion curves in Steam and Gas Engine Cylinders.*

It often happens that we are asked to draw a $p$, $v$ curve through the point $P$ such that $pv^k$ is constant, where, for any point $F^1$ on the curve, the distance $OF$ represents $v$ to some scale, and the distance $FF^1$ represents $p$ to some scale. It is not necessary to pay any attention to these scales.

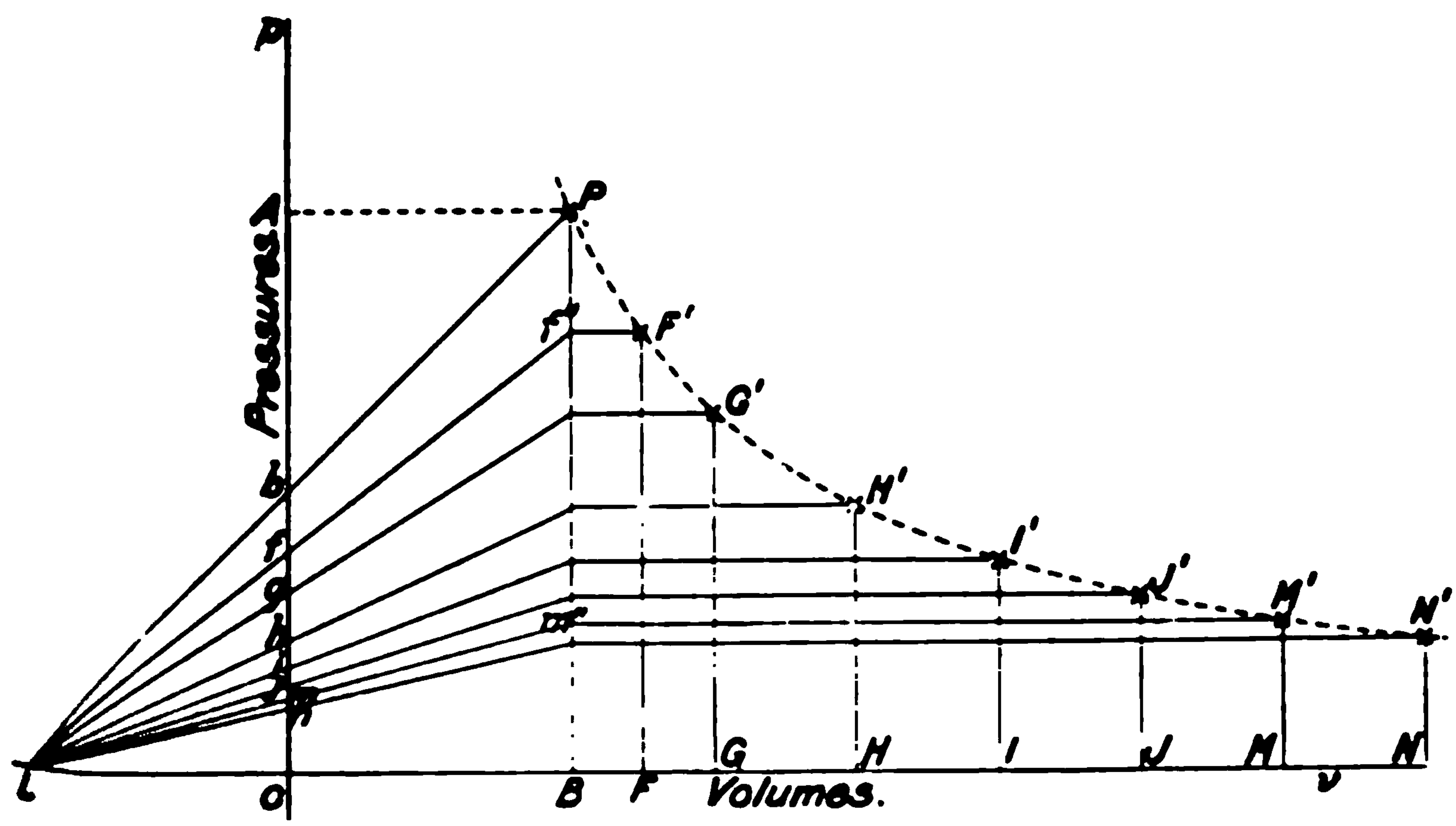


Fig. 220.

Given $P$. Draw $PB$ at right angles to $ov$.

Take points $F$, $G$, $H$, $I$, $J$, $M$, $N$, &c., at distances from $O$, $1\frac{1}{4}$, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3 $3\frac{1}{2}$, 4, or &c. times $OB$.

Thus, suppose $OM$ is $3\frac{1}{2}$ times $OB$,

To find $M^1$ the corresponding point in the curve,

Set off $ob = 1$ along $Op$, to any convenient scale, say $ob = 1$ inch.

Join $Pb$ and produce to $L$.

Choose the column of numbers on the following table under any particu value of $k$ that may be given. Thus, if $k$ is 0·9, then as $\frac{OM}{OB}$ is $3\frac{1}{2}$, find $\frac{om}{ob} = 0{\cdot}324$, and hence we set up $om$ as 0·324 inch if $ob$ is 1 inch.

Join $Lm$ and produce to $m''$. Project horizontally and vertically from and $M$ to find the point $M'$ which is on the curve.

A number of points like $M$ ought to be set off at starting, so that the poi like $m$ may be set off rapidly.

In Fig. 220 we wanted to draw the curve $pv^{0\cdot9}$ constant through the gi point $P$.

We made $ob = 1$, $of =$ ·818, $og =$ ·694, $oh =$ ·536, $oi =$ ·438, and so on, numbers in the column headed ·9 in our table, and so found all the poi quickly on any scale whatever.

We give, among other values of $k$ in our table, $k = 1{\cdot}0646$, that ste saturation curves may be easily drawn : $k = 1{\cdot}130$, because this gives a fair proximation to many adiabatic curves (see Art. 211), when little water is pres at the beginning of the expansion. We give $k = 1{\cdot}3$ and $k = 1{\cdot}414$, because tl are the adiabatics for superheated steam (?) and for air ; also we give $k = 1{\cdot}37$, cause it is the adiabatic for the usual mixture found in gas and oil engine cylinde

$k = 1$ gives the rectangular hyperbola ; the curve of expansion of perf gases at constant temperature, easily drawn in other ways.

It is a good exercise for the student to draw all these curves to a la scale from the same point $P$, so that he may have a working notion of differences between them.

| | ·7 | ·8 | ·9 | 1·0 | 1·0646 | 1·1 | 1·130 | 1·2 | 1·3 | 1·37 | 1·4 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $1\frac{1}{4}$ | ·855 | ·837 | ·818 | ·800 | ·789 | ·782 | ·777 | ·765 | ·748 | ·737 | ·732 | [illegible] |
| $1\frac{1}{2}$ | ·753 | ·723 | ·694 | ·667 | ·649 | ·640 | ·632 | ·615 | ·590 | ·574 | ·567 | [illegible] |
| 2 | ·616 | ·574 | ·536 | ·500 | ·478 | ·467 | ·457 | ·435 | ·407 | ·387 | ·379 | [illegible] |
| $2\frac{1}{2}$ | ·527 | ·481 | ·438 | ·400 | ·377 | ·365 | ·355 | ·333 | ·304 | ·285 | ·277 | [illegible] |
| 3 | ·463 | ·415 | ·372 | ·333 | ·311 | ·299 | ·289 | ·268 | ·240 | ·222 | ·215 | [illegible] |
| $3\frac{1}{2}$ | ·416 | ·367 | ·324 | ·286 | ·263 | ·252 | ·243 | ·222 | ·196 | ·180 | ·173 | [illegible] |
| 4 | ·379 | ·330 | ·287 | ·250 | ·229 | ·218 | ·209 | ·189 | ·166 | ·149 | ·144 | [illegible] |
| 5 | ·324 | ·276 | ·235 | ·200 | ·180 | ·170 | ·162 | ·145 | ·124 | ·113 | ·105 | [illegible] |
| 6 | ·285 | ·238 | ·199 | ·167 | ·148 | ·139 | ·132 | ·116 | ·0974 | ·0859 | ·0814 | [illegible] |
| 8 | ·233 | ·189 | ·154 | ·125 | ·109 | ·102 | ·0954 | ·0825 | ·0670 | ·0579 | ·0544 | [illegible] |
| 10 | ·200 | ·158 | ·126 | ·100 | ·0862 | ·0794 | ·0741 | ·0631 | ·0501 | ·0427 | ·0398 | [illegible] |

Let the student notice the sort of difference that exists between a cu $pv$ = constant, and $pv^{1\cdot0646}$ = constant, and remember that there are some practi men who treat $pv$ = constant, as if it were the saturation curve ; some peo treat it as if it were an adiabatic curve for steam, and some others call vaguely "the theoretical curve for expansion."

# CHAPTER XX.

## PROPERTIES OF GASEOUS FLUIDS.

**186. A pound of fluid stuff** has three qualities, its pressure ssumed to be the same everywhere in it, its temperature assumed o be the same everywhere in it, and its volume. Thus a pound of ir at 0° C. (or $t=274$) and at atmospheric pressure (or $p=2{,}116$ lbs. er square foot) has a volume $v$ of 12·39 cubic feet, and it is very early true that for all values of $p$, $v$ and $t$

$$\frac{pv}{t}=95{\cdot}7 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Hence if we know any two of $p$, $v$ and $t$ we can calculate the ther. And so we say that if any two are known, the state of the tuff is known.

Again, a pound of any of the following gases has a law like

$$\frac{pv}{t}=R \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Where $R$ is given in the following table. The law is not strictly ue for any gas, but it is so nearly true that (2) may be used in all gineering calculations. The law connecting $p$, $v$ and $t$ for any ibstance is called its **characteristic.**

In Art. 172 I give some exercises on the calculation of $p$ or $v$ or when the other two are given. I give here a table of such operties and laws of the gases with which engineers concern themlves, as are necessary in engineering calculations. The reasoning hich has led us from experimental facts to these laws or rules will found in Chap. XXXI. The student will find his knowledge of e subject and security in thinking about it greatly increased by ading Chap. XXX. on the Kinetic theory.

**187. Properties of Gases.** The unit of heat is what is equivalent to 774 or 1,393 foot-pounds. $C_v$ and $k$ are the specific heats at constant volume in heat units and foot-pounds. $C_p$ and $K$ are the specific heats at constant pressure in heat units and foot-pounds.

$p$ is pressure in pounds per square foot at London.

$v$ volume in cubic feet of 1 lb. of stuff.

$t$ absolute temperature centigrade.

$$\frac{pv}{t}=R=K-k,\ \frac{K}{k}=\gamma$$

| Substance. | $C_v$ | $C_p$ | $k$ | $K$ | $R$ | $\gamma$ |
|---|---|---|---|---|---|---|
| Air | ·169 | ·238 | 234·5 | 330·1 | 95·67 | 1·407 |
| Oxygen | ·156 | ·218 | 216·3 | 302·9 | 86·60 | 1·4[illegible] |
| Hydrogen | 2·416 | 3·406 | 3354 | 4729 | 1375·0 | 1·410 |
| A | ·1802 | ·250 | 252·3 | 349·4 | 97·16 | 1·3[illegible] |
| B | ·1889 | ·258 | 264·5 | 361·3 | 96·88 | 1·3[illegible] |
| D | ·1803 | ·250 | 252·4 | 349·4 | 97·01 | 1·3[illegible] |
| E | ·1902 | ·260 | 266·3 | 363·2 | 96·88 | 1·3[illegible] |
| F | — | ·25 | — | — | — | — |
| Carbonic acid[1] | — | ·216 | — | — | 62·58 | — |
| Carbonic oxide | ·173 | ·243 | 241 | 338 | 98·9 | 1·4[illegible] |

Superheated steam. $C_p$ is usually supposed to be 0·475, but this is more than doubtful. Mr. McFarlane Gray thinks $C_p$ to be $0{\cdot}3864+9\times 10^6 pt^{-3\cdot5}$. In Chapter XXXI. I show that if Regnault's results are to be relied upon, then $C_p$ is ·305 at 0° C., ·36 at 100° C., ·43 at 150° C.

I give a characteristic for steam in (2) Art. 371, which is however probably untrue except near saturation. This seems the best result at present available, and yet there is a consensus of opinion among physicists that vapours tend to become more and more nearly constant in their specific heat at constant pressure as the temperature increases.

$A$ is the usual mixture in gas or oil engine cylinders using coal gas, before ignition; and $B$ is the mixture after ignition.

$D$ is the usual mixture in gas engine cylinders using Dowson gas before ignition; and $E$ is the mixture after ignition.

$F$ is the usual mixture of furnace gases from boilers when 24 lb. of air is admitted per pound of coal.

[1] Carbonic acid is so far from being a perfect gas that we can only say that from 15° C. to 100° C. is ·2025, and from 11° C. to 214° C. it is ·2169, the mean ratio of its specific heats being 1·30. It is my opinion that there is no possible explanation of the increasing values of $C_p$ both for carbonic acid and steam except that of dissociation, although chemists ridicule the idea of possible dissociation at these low temperatures.

**188. Formulæ for Gases.** All energy in foot-pounds. One pound of gas. Only true for gases which satisfy (2).

$$dH = k\,.\,dt + p\,.\,dv$$
$$= K\,.\,dt - v\,.\,dp$$
$$= \frac{1}{\gamma - 1}\,d(pv) + p\,.\,dv$$
$$H_{12} = \underbrace{\frac{1}{\gamma-1}(p_2v_2 - p_1v_1)}_{\text{goes to temp change}} + \text{work done}$$

$H_{12}$ is the total heat given in any kind of change from the state $p_1, v_1, t_1$ to $p_2, v_2, t_2$.

Expansion according to the law $pv^s = c$, a constant, the work done is $\frac{c}{1-s}\left(v_2^{1-s} - v_1^{1-s}\right)$ and the heat given to the gas during expansion $= \frac{\gamma - s}{\gamma - 1} \times$ work done.

Expansion according to the law $pv = c$, a constant, the work done is $c \log_e \frac{v_2}{v_1}$ and the heat given to the gas during expansion is equal to the work done.

In gases the entropy $\phi = k \log_e t + R \log_e v +$ constant.

The intrinsic energy $E = kt +$ constant.

EXERCISE. If $H_2O$ can be in the state of a perfect gas, its density relatively to hydrogen is in the proportion of 2 + 15·88, or 17·88 to 2, or 8·94. Hence if the $R$ of hydrogen is 1375·0, the $R$ of gaseous $H_2O$ is 154.

For the various values of the pressure and temperature of Table I., Art. 180, calculate $v$ if $pv/t = 154$. The answers are headed $v$ in the table.

EXERCISE. The fractional difference between the volume of a pound of saturated steam $u$, and of gaseous $H_2O$, or $\frac{v-u}{v}$ being called $x$; plot log. $x$ and log. $p$ on squared paper and see if there is such a law connecting them as

$$x = \cdot000101\, p^{0.443}$$

which has been found by one of my students.

**189.** To find the specific heats, $K$ and $k$, of a mixture of gases. If we have $w_1$, $w_2$, $w_3$, &c., lb. of gases whose specific heats are $K_1$, $K_2$, &c. ; $k_1$, $k_2$, &c. Then

$K = (w_1K_1 + w_2K_2 + \&c.) \div (w_1 + w_2 + \&c.)$ and $k = (w_1k_1 + w_2k_2 + \&c.) \div (w_1 + w_2 + \&c.)$

*Important Results to be Checked by Students.*

Table I.—One cubic foot of coal gas with the following composition (by volume), and 5·76 cubic feet of air, and 4·5 cubic feet of the products of a previous

combustion. What I call $c_p$ and $c_v$ for each kind of gas, are capaciti heat per cubic foot. $q$ is the amount of each constituent in cubic feet t cubic foot of coal gas.

COAL GAS ENGINE MIXTURE BEFORE COMBUSTION.

| | cubic ft. $q$ | $c_p$ | $c_v$ | $qc_p$ | $qc_v$ |
|---|---|---|---|---|---|
| Hydrogen . . . . | 0·46 | ·2359 | ·99 × ·168 | ·1085 | ·4554 × |
| Carbon monoxide. | 0·075 | ·237 | 1 ,, | ·0178 | ·0750 |
| Marsh gas . . . | 0·3950 | ·3277 | 1·54 ,, | ·1294 | ·6082 |
| Olefiant gas . . . | 0·0380 | ·4106 | 2·03 ,, | ·0156 | ·0771 |
| Nitrogen . . . . | 0·0050 | ·237 | 1 ,, | ·0012 | ·0050 |
| $H_2O$ vapour . . . | 0·0200 | ·2984 | 1·36 ,, | ·0060 | ·0272 |
| Air. . . . . . . . | 5·76 | ·2374 | 1 ,, | 1·3680 | 5·760 |
| Products . . . . | 4·5 | ·2581 | 1·124 ,, | 1·1614 | 5·058 |
| Total . . . . . . | 11·253 | — | — | 2·8079 | 12·066 × |

Hence for the mixture $c_p = 0·2496$, $c_v = 0·1802$; ratio 1·385 ; difference ·06

COAL GAS ENGINE MIXTURE AFTER COMBUSTION.

| | cubic ft. $q$ | $c_p$ | $c_v$ | $qc_p$ | $qc_v$ |
|---|---|---|---|---|---|
| $H_2O$ vapour . | 1·3714 | ·2984 | 1·36 × ·168 | ·4092 | 1·865 × |
| Carbon dioxide . | 0·5714 | ·3307 | 1·55 ,, | ·1889 | ·8855 |
| Nitrogen . . . . | 4·5554 | ·2370 | 1 ,, | 1·0790 | 4·5554 |
| Total . . . . . . | 6·4982 | — | — | 1·6771 | 7·3059 × |

Or for the mixture $c_p = 0·2581$, $c_v = 0·1889$; ratio 1·367 ; difference ·0692

DOWSON GAS ENGINE MIXTURE BEFORE COMBUSTION.

| | cubic ft. $q$ | $c_p$ | $c_v$ | $qc_p$ | $qc_v$ |
|---|---|---|---|---|---|
| Hydrogen . . . . | ·1873 | ·2359 | ·99 × ·168 | ·0442 | ·1854 × |
| Carbon monoxide | ·2507 | ·237 | 1 ,, | ·0594 | ·2507 |
| Marsh gas . . . . | ·0031 | ·3277 | 1·54 ,, | ·0010 | ·0048 |
| Olefiant gas . . . | ·0031 | ·4106 | 2·03 ,, | ·0013 | ·0063 |
| Nitrogen . . . . | ·4898 | ·237 | 1 ,, | ·1161 | ·4898 |
| Carbon dioxide | ·0657 | ·3307 | 1·55 ,, | ·0217 | ·1018 |
| Air . . . . . . . | 1·1325 | ·2374 | 1 ,, | ·2689 | 1·1325 |
| Products . . . | 2 | ·2594 | 1·1323 ,, | ·5188 | 2·2646 |
| Total . . . . . | 4·1322 | — | — | 1·0314 | 4·4359 × |

$c_p = ·2496$, $c_v = ·1803$; ratio 1·385 ; difference ·0693.

DOWSON GAS ENGINE MIXTURE AFTER COMBUSTION.

| | cubic ft. $q$ | $c_p$ | $c_v$ | $qc_p$ | $qc_v$ |
|---|---|---|---|---|---|
| Water vapour . . | 0·2019 | ·2984 | 1·36 × ·168 | ·0602 | ·2746 × ·168 |
| Carbon dioxide . | 0·3279 | ·3307 | 1·55 ,, | ·1084 | ·5083 ,, |
| Nitrogen . . . . | 1·3845 | ·2370 | 1 ,, | ·3281 | 1·3845 ,, |
| Total . . . . . | 1·9143 | — | — | ·4967 | 2·1674 × ·168 |

$c_p$ = ·2594, $c_v$ = ·1902 ; ratio 1·3637 ; difference ·0692.

**190.** When we develop thermodynamic rules (Chap. XXX.) for all kinds of stuff, it is an excellent exercise to apply them to the case of a gas which approximately satisfies (2), Art. 186. But the student must remember that (2) is only approximately true for any substance. It is very nearly true in air, nitrogen, oxygen, and hydrogen. In nitrogen and air, $\frac{pv}{t}$ decreases slightly as $p$ increases. For hydrogen, $\frac{pv}{t}$ increases as $p$ increases. An examination of the more correct characteristic for carbonic acid will show that (2) is nearly true at the temperatures and pressures which exist in ordinary chimneys and flues. It is not very wrong to assume, as I shall do in exercise work, that (2) is true for superheated steam ; our knowledge of water in this state is described in Chap. XXXII.

The important fact to remember is this, that there is some law (called its *characteristic*) connecting the $p$, $v$ and $t$ of a pound of any kind of stuff, although our knowledge of it may be quite defective.

Again, the state of a pound of water stuff consisting of $x$ lb. of steam, and $1 - x$ lb. of water is supposed to be completely known to us (it is well to recollect that we suppose the temperature the same everywhere in the stuff) if we know its $v$ and its $p$, or its $v$ and its $t$. It is a peculiar case this of change of state, because there is a discontinuity, a sudden change from water to steam, and if the pressure is known the temperature is already known, so that there must be a second independent thing given, such as $v$ or $x$. If $v$ is known, $x$ can be found (or indeed if $x$ is known, $v$ may be found). For in the tables of Art. 180 if we know $p$ or $t$, we know $u$, the volume of a pound of steam : here we have $x$ lb. of steam, so that its volume is $xu$, and as the volume of the water is very small, we neglect it in our steam engine calculations, so that $v = xu$.

## CHAPTER XXI.

### WORK AND HEAT.

**191.** THERE are many forms of energy which may be giv or given out by bodies in Nature, but in our study of the dynamics we recognise only two:—

1. **Mechanical work** done by a fluid. If the volume incr from $v$ to $v + \delta v$, we say that the work done by the fluid is and more nearly $p . \delta v$ foot-pounds, as the change of volume considered to be smaller and smaller. Indeed, I am not sure the best definition of pressure is not this: If fluid has already work $W$, and if in the increase of volume $\delta v$ the extra work $\delta$ done, then $p . \delta v = \delta W$, or rather

$$p = \frac{dW}{dv}$$

That is, pressure is the rate at which work is done per cubic of expansion.

Of course if $\delta v$ is negative; if the volume gets less, the done *by* the fluid is negative; that is, work is done *upon* it.

Observe that the most immediate way of finding $\delta W$ is thr the infinitely small change of volume $\delta v$. We could calculate $\delta$ more laborious ways from knowing infinitely small changes in pre $p$ and the temperature $\delta t$.

2. **Heat** $\delta H$ given to the fluid when it changes its state in any If the change of state is an infinitely small one we can calc $\delta H$ from our knowing any two of the changes $\delta t$ or $\delta v$ or $\delta p$. changes being infinitely small we can say that

$$\delta H = k . \delta t + l . \delta v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$= K . \delta t + L . \delta p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$= P . \delta p + V . \delta v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $k$, $l$, $K$, $L$, $P$ and $V$ are numbers which we know if we kno the properties of the stuff. These numbers are called specific l or latent heats or capacities, and they may be quite different in

f the stuff from what they are in another. We might have ed $\delta W$ in some similar way, but how cumbrous and un- ry it would have been! Now, just as $\delta W = p.\delta v$, so there ich quicker way of calculating $\delta H$ than by either (1), (2)

There is a property of the stuff called its **entropy** $\phi$, which that any change in it, $\delta\phi$, if multiplied by $t$ the absolute iture, gives $\delta H$ or

$$\delta H = t.\delta\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

en stuff changes in state we can use either (1) or (2) or (3) to e the amount of heat given to it, but if we only know the in $\phi$, the rule (4) is of all ways the easiest for calculation.

most general statement of the laws of thermodynamics is this: en a body changes its state and has heat energy $\delta H$ given to it gives out mechanical energy $\delta W$, the intrinsic gain of is $\delta H - \delta W$; call this $\delta E$ and use the name "intrinsic ' for $E$. This is the total energy actually in the stuff.

Law. The $E$ in the stuff is always the same when the stuff to the same state; in fact, $E$ can be calculated if we know , or $p$ and $t$, or $v$ and $t$.

Law. The $\phi$ of the stuff is always the same when the stuff to the same state; in fact, $\phi$ can be calculated if we know e.

**3. First Law.** A great number of practical problems are it once if we remember the first law, and if we know how to e the intrinsic energy.

r we do not know the real intrinsic energy of any stuff, but now in many cases how much greater it is in one state than her. For example: In air, oxygen, nitrogen, hydrogen, and ses we find it nearly true that the intrinsic energy depends only temperature. Thus, when the temperature keeps constant, if f expands doing work, the amount of heat given is exactly equal vork done, that is, there is no gain or loss of intrinsic energy. io work is done (volume constant) the heat given to a gas is d as intrinsic energy. Now it is found that the heat given is at constant volume to raise it from $t_0$ to $t$ is $k\,(t - t_0)$ : is a constant quantity called the specific heat at constant As we are only concerned with differences we may say that insic energy in a pound of stuff is $kt$, although we can attach ing to such a statement at such low temperatures that the longer behaves like the mathematical substance called a gas.

ERCISE 1. What heat must be given to a pound of gas when es in volume from $v_1$ to $v_2$, its pressure $p$ remaining constant?

*Answer*. Heat = gain of intrinsic energy + work done. Th work done $= p(v_2 - v_1)$. To find the gain of intrinsic energy we mu find the change of temperature. The stuff follows the law $pv = R$

Hence $t_1 = \frac{pv_1}{R}$, $t_2 = \frac{pv_2}{R}$; gain of intrinsic energy $= k(t_2 - t_1) =$ $\frac{k}{R} p(v_2 - v_1)$.

$$\text{Hence heat} = \frac{k}{R} p(v_2 - v_1) + p(v_2 - v_1)$$
$$= p\left(\frac{k}{R} + 1\right)(v_2 - v_1).$$

This may be put in many shapes. Thus $pv_2 = Rt_2$, $pv_1 = Rt_1$ an the above becomes $(k + R)(t_2 - t_1)$. Now, if we say "Heat given = specific heat $K$ at constant pressure, multiplied by change of tem perature" we see that $k + R = K$.

Exercise 2. When a pound of gas changes in any way, what i the heat given to it? *Answer*. Heat $= k(t_2 - t_1)$ + work done.

We see therefore that the question cannot be answered in numbe unless we know the work done. Calling the work done $W$, an seeing that $t_2 = \frac{p_2 v_2}{R}$, $t_1 = \frac{p_1 v_1}{R}$.

$$\text{Heat} = \frac{k}{R}(p_2 v_2 - p_1 v_1) + W.$$

This formula is of great value in air engine, gas engine and oi engine work.

Exercise 3. A pound of gas at 400° C., $p = 10{,}000$ lbs. per squar foot whose $R$ is 95·67, what is its volume?

*Answer*. As $\frac{pv}{T} = 95{\cdot}67$. $v = 5{\cdot}729$.

It receives $7 \times 10^5$ foot-pounds of energy as heat at consta volume, find its new pressure and temperature.

*Answer*. The heat is all stored as intrinsic energy, and a $k = 252{\cdot}3$ (see Art. 187), the change of temperature is $7 \times 10^5 \div 252$ or 2,775° C. It is easily seen that the new pressure is 3,300 lb per square inch.

Exercise 4. Given a $p, v$ diagram for a pound of one of the gas of the table Art. 187, find the rate of reception of heat.

$$\delta H = k \,.\, \delta t + p \,.\, \delta v$$

or
$$\frac{dH}{dv} = k\frac{dt}{dv} + p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

We shall call this $h$.

It is evident then that our answer is just of the **same dimen sions as a pressure**: the one being "mechanical energy given o

by the stuff per unit increase of volume," the other being "heat energy taken in by the stuff per unit increase of volume." Suppose we get an indicator diagram and we do not know the temperature anywhere, we only see $p$ drawn to scale, we know not what scale, except that $\frac{dH}{dv}$ or $h$ must be shown to the same scale

It is convenient to change (1).

As $t = \frac{pv}{R}$, $\frac{dt}{dv} = \frac{1}{R}\left(p + v\frac{dp}{dv}\right)$. Hence (1) may be written, nce $R = K - k$ and $K/k$ is called $\gamma$

$$h = \frac{k}{R}\left(p + v\frac{dp}{dv}\right) + p$$

r

$$h = \frac{1}{\gamma - 1}\left(v\frac{dp}{dv} + \gamma p\right) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This is really the same as (1), but it will be observed that we can get $h$ in terms of $p$ without knowing how much stuff is present, and we need not care what are the scales of $p$ or $v$.

**193.** The following numbers were measured on a **gas engine indicator** card of the stuff $A$ of the table Art. 187, whose $\gamma$ is 1·385. The stuff was in a cylinder whose clearance volume was known, and of course this is included. The student will do well to draw the diagram from the dimensions given. If he does so he will get much more accurate answers.

| $v$. | $p$. | $\delta p/\delta v$. | average $v$. | average $p$. | $h$. |
|---|---|---|---|---|---|
| **Compression** | | | | | |
| 25 | 14·7 | −0·96 | 22·5 | 17·1 | 5·92 |
| 20 | 19·5 | −1·70 | 17 | 24·6 | 13·8 |
| 14 | 29·7 | −3·88 | 12 | 37·5 | 14·6 |
| 10 | 45·2 | | | | |
| **Expansion** | | | | | |
| 10 | 45·2 | 173 | 10·1 | 62·4 | 4760 |
| 10·2 | 79·7 | 218 | 10·3 | 101·5 | 6210 |
| 10·4 | 123·2 | 173 | 10·5 | 140·4 | 5230 |
| 10·6 | 157·7 | 120 | 10·7 | 169·7 | 3930 |
| 10·8 | 181·7 | 33 | 10·9 | 184·9 | 1590 |
| 11·0 | 188·2 | −22 | 11·5 | 177·2 | −20·8 |
| 12·0 | 166·2 | −20 | 12·5 | 156·2 | −85·8 |
| 13 | 146·2 | −14·8 | 14 | 131·5 | −64·9 |
| 15 | 116·7 | −10·5 | 16 | 106·2 | −54·5 |
| 17 | 95·7 | −7·5 | 18 | 88·2 | −33·8 |
| 19 | 80·7 | −6·0 | 20 | 74·7 | −41·5 |
| 21 | 68·7 | −5·0 | 22 | 63·7 | −57·1 |
| 23 | 58·7 | | | | |

It is to be noticed that during compression as $v$ is diminishing $\delta v$ is negative, since $\frac{dH}{dv}$ is positive, it means that heat is being l by the stuff. Until $v = 10{\cdot}9$ in the expansion, notice that the st first receives heat and thereafter loses heat. The student ought draw $h$ to the same scale as that to which pressure is drawn.

If it is required to know rate of reception of heat *per seco* $h \times$ *velocity of piston*, evidently represents what is wanted. For t purpose we may without much inaccuracy imagine the connect rod to be infinitely long; therefore we describe a semicircle on t distance which represents to scale the length of the stroke, and multiply the ordinate of our $h$ diagram by the ordinate of the sem circle for any position in the stroke.

It is obvious that an exercise like this well carried out will tea students a great deal more than may be described here.

If an expansion curve follows the law $pv^s = c$ a constant, $p = cv^{-s}$.

$$v\frac{dp}{dv} = -sp$$

Hence (2) becomes

$$h = \frac{1}{\gamma - 1}(-sp + \gamma p) = \frac{\gamma - s}{\gamma - 1}p$$

Thus in the latter part of the above *expansion*, if we plot log. and log. $v$ on squared paper, we shall find $pv^{1{\cdot}575}$ = constant, and henc $h = -0{\cdot}50\,p$.

Again, in the above *compression* it will be found that $pv^{1{\cdot}206}$ i constant, and therefore $\frac{dH}{dv} = 0{\cdot}47\,p$.

**194.** I give a little **thermodynamics** in Chap. XXXI., but write for students who are supposed to know something of thermo dynamics already, and especially the proof of the second law Elementary students of heat and advanced students who wish t study the philosophy of this subject will find no great help here I think that the mathematical basis of the second law as given i my book on the Calculus is well worth study. **The course o one's elementary study** is usually this:—1. The equivalence o mechanical and heat forms of energy. 2. When change of state of a body occurs; what is the heat given? what is the work done? how are these usually calculated? 3. The mathematical conception, a Carnot cycle, stuff taking in heat $H$ when expanding at constant temperature $T$; giving out heat $h$ when being compressed at constant temperature $t$; when change of temperature occurs it is due to

iabatic expansion or compression. 4. An engine which could llow a Carnot cycle is reversible. The study of this section of the bject may break over the barriers of our mathematical assumptions regard to the nature of matter and energy and become a study of e universe. Keeping to mathematics we are led to:—5. All reverble engines working between the same higher and lower temperares are equally efficient, and therefore this efficiency depends pon the temperatures alone. 6. Define temperature to be such at in a reversible engine

$$\frac{\textit{Nett work}}{H} = \frac{T-t}{T}$$

7. Calculate what this scale of temperature must be by calculating ett work and $H$ when some particular substance is used whose roperties are known. 8. The scale of temperature so found is such hat if we can imagine the substance to be one whose intrinsic nergy depends only upon its temperature, and if it is also such a ubstance that its $p\left(\frac{dt}{dp}\right)_{v\ const}$ is a linear function of the scale of emperature employed, then the value of $p\left(\frac{dt}{dp}\right)_{v\ const}$ being called he absolute temperature, the above condition is satisfied. 9. The roperties of air, nitrogen and hydrogen are such that we can approximate very closely to the scale of temperature required, and as a help o our recollection of our results we have invented an ideal substance, alled a perfect gas, which is such that if $t$ is our absolute temperaure, and if $v$ and $p$ are its volume and pressure

$$vp/t = R$$

here $R$ is constant and where $t$ may be taken as $\theta^\circ$ C.+273·7. ° C. being the reading on what we call sometimes an air thermometer nd sometimes a nitrogen or hydrogen thermometer with delightful agueness.

**195.** The fact most impressed upon the young engineer is this, hat in trying to convert as much of the heat energy $H$ as possible nto the mechanical form, **the temperatures limit our power,** and e can only in the most perfect heat engine convert the fraction $\frac{T-t}{T}$ of the whole.

Carnot thought that when heat fell in temperature and work as done, it was like water falling down a height in a water-wheel. Ie was wrong. $H$ at the higher temperature $T$ becomes only $h$ at he lower temperature $t$, the difference $H-h$ being converted into

work $w$ in a perfect engine. Taking it that all energy is in the s units, we have $\frac{w}{H} = \frac{T-t}{T}$.

Instead of thinking of $H$ as analogous with weight of wate: us take $\frac{H}{T}$ as analogous with weight of water.

A weight $\frac{H}{T}$ falling through the height $T-t$ would do the

$\frac{H}{T}(T-t)$, so that the analogy is complete.

**196. Lord Kelvin** put forward a **suggestion** once that not probably be acted upon much, until coal is more expensive. this. Just as in a heat engine we take in heat $H$ at $T$, give heat $h$ at $t$, converting only the small quantity $w = H - H\frac{T-t}{T}$ into work; so in a reversed heat engine, we might tal $h$ at the lower temperature $t$, do work $w$ and deliver the l amount of heat $h + w$, or $w \frac{T}{T-t}$ at the higher tempera Many refrigerating machines already work on the principle. L take a concrete example.

EXERCISE. Suppose that for 1 lb. of coal whose calorific en is 8,300 centigrade units of heat, we get 1 brake power hour, u Dowson gas and a gas engine; that is, we get work equivaler $\frac{1,980,000}{1,393}$ or 1,422 heat units. Suppose that this work is given reversed heat engine taking in heat $h$ in air on a cold day at 1 the atmospheric temperature, and by compression giving it ou 20° C. Let us imagine this to be done with an efficiency of 90 cent., which is quite practical. Then the work 1,422 will allow heat $1,422 \frac{274+20}{10} \times \cdot 9$ or 37,620 to be given to the air.

Here then is a comparison:

By direct heating, the usual way, all the heat of the coal b given to the air (it is unusual to give nearly so much), the air 8,300 units of heat.

By using a gas engine and reversed heat engine, the 37,620 is given to the air.

It looks at first sight like a creation of energy, but the stu will see that the heat energy is not created; we have the 1,422, this is changed into heat, and the extra heat 36,198 is n in temperature. All that is disadvantageous in the heat en

**becomes advantageous** in the reversed heat engine, whether it is used for heating or for refrigerating.

The comparison would be more striking if we assumed that by some electric battery method we could get more useful work from 1 lb. of coal than we can get by using Dowson gas in a gas engine.

**197.** The reversibility of a heat engine depends upon this, that when the stuff gains or loses heat it shall do so to a body of infinite capacity for heat at the same temperature. In the **Carnot cycle** heat $H$ is taken in at the higher temperature $T$, heat $h$ is given out at the lower temperature $t$; change of temperature occurs adiabatically.

**Stirling's regenerator** produces a reversible heat engine in the same way. Imagine air to be the stuff used. A pound of air expands from $v_1$ to $v_2$ at $T$, taking in the heat $H = RT \log. \frac{v_2}{v_1}$, and doing work equal to $H$. The air then goes through a passage whose walls have infinite capacity and gradually alter in temperature from $T$ to $t$, so that the air gets lowered to $t$ in passing through, its volume keeping constant.[1] The heat given up by the air and stored in the regenerator is $k\ (T-t)$ the pressure falling. The air is now compressed at the constant temperature $t$ from $v_2$ to $v_1$, giving out the heat $h = Rt \log. \frac{v_2}{v_1}$, the work done upon it being equal to $h$. It is now passed in the reversed way through the regenerator, taking in the heat $k\ (T-t)$ in reaching its initial condition, volume $v_1$, temperature $T$.

The regenerator gives out the same heat that it took in. At every point in the passage through it, the air gives up or takes heat from a part of the regenerator which is at the same temperature as itself. The heat taken in was $H$; the heat given out was $h$; the net work done was $H-h$, and we see that $\frac{H}{h} = \frac{T}{t}$, so we have the efficiency $\frac{T-t}{T}$ as before. The student can work out the Ericsson form for himself.

**The Joule air engine** is not reversible. The stuff takes in heat at constant pressure and gives it out at lower constant pressure, the other two parts of the cycle being adiabatics. It is specially interesting because in its reversed form it is a well-known form of refrigerating machine.

[1] Ericsson let its pressure keep constant.

# CHAPTER XXII.

## WORK AND HEAT. ENTROPY.

**198.** I TAKE it that my readers know something of thermody namics already. The application of the above notions to chemical an physical questions generally will lead to the study of the *availabilit* of the heat in a system of bodies whose temperatures are not th same. With this matter, so all-important in physical chemistry, th engineer need not concern himself; he is more concerned to stud thermodynamics from the entropy point of view, because he ha one stuff at the same temperature and pressure throughout. I hav given the mathematics of the subject in Chap. XXXI.

**199.** If stuff is at the absolute temperature $t$ and we give th small amount of heat $\delta H$ to it, we say that we give it the entrop $\frac{\delta H}{t}$. Engineers seem to have great difficulty in understanding wh we introduce the notion of this ghostly quantity, but they must g accustomed to it. The entropy of a body is said to be its $\phi$. a body has the entropy $\phi$, the pressure $p$, the temperature $t$, th volume $v$, and the intrinsic energy $E$, and receives heat, does wor goes through all sorts of changes, and is brought back to the sam $p$ and $v$ again, it will be found that it is also at its old $t$, that its is the same, and also its $\phi$ is the same. The heat given to an taken from the body are by no means the same; the work done b and upon the body are by no means the same; but the entrop given to and taken from the body are exactly the same.

It is a mathematical idea which must be taken in, and it i a most impossible to get the idea without working exercises on hea engines. There is **no good analogy** to help the beginner, but may try this one.

When a body changes its state by a small amount and we hav

given to it the heat energy $\delta H$, and let it give out the mechanical energy $\delta W$, and if all sorts of such changes take place and the body comes back to its old state again, how do we take account of what has happened :—

1. If we reckon up all the work done *by*, and done *on* the stuff we do not find that the accounts balance.

2. If we reckon up all the heat given to, and given out by the stuff we do not find that the accounts balance.

3. If we look upon all the work and heat as energy and calculate it all in foot-pounds, we find that the account does balance.

Now, is there any way in which we can make the work account balance by itself? Yes; when the work $\delta W$ is done, do not reckon it up directly, but divide by the $p$ at the time, and *then* reckon up: what we really reckon up is $\delta W \div p$ or $\delta v$, the mere change of volume, and this *must* come back to the same value again.

Similarly, if we divide every $\delta H$ by $t$, so that when 1,000 units of heat are taken in at the constant temperature 500, we say "the entropy added is $\frac{1000}{500}$ or 2," and again when we take out the heat 800 at the constant temperature 400 we say, "the entropy taken away is $\frac{800}{400}$ or 2"; if we take care to reckon in this fashion, every amount $\delta H$ being divided by the $t$ at the time, and if we call the $\delta H$ divided by the $t$ by the name, entropy, we shall find that when the stuff is brought back to its old state again, we have just given out as much entropy as we have taken in. The account balances exactly.

Is there any other good analogy? Many a time have I worried over this pedagogic difficulty. How to give this powerful idea in a simple way. What is the use of trying to prove this second law of thermodynamics unless one knows that one can comprehend it when one has proved it? And so many men prove it in books and talk glibly about it, to whom it is a mere bit of mathematics! Is it a name for its unit that is wanted—then here I give it a name for the first time. When 1,800 units of heat are given at the absolute temperature 600, I shall say that entropy of the amount 1,800 ÷ 600 or 3 *Ranks* is given to the body. This will be 3 Ranks whether the heat is in Fahrenheit units at absolute Fahrenheit temperature, or Centigrade units at absolute Centigrade temperature. The name **Rank** I take from the name of **Rankine** who first used $\phi$ and gave it a name which I need not now

mention, as everybody uses another name '**entropy**.' In general equations entropy is measured as

$$\frac{\text{heat received in work units}}{\text{absolute temperature of reception}}$$

so that Ranks must be multiplied by Joule's equivalent.

**200.** Latent heat is usually given to water kept at constant temperature, to convert it into steam; in this case the gain of entropy is easily calculated. It is the latent heat divided by the absolute temperature.

When the temperature of a body changes as it receives heat, we have to calculate the gain of entropy by small amounts and add up. The gain $\delta\phi$ is the gain of heat $\delta H$, divided by the absolute temperature $t$. Thus a pound of water receives heat $\delta H$, which in heat units is very nearly $\delta t$ when being heated from $t$ to $t+\delta t$ (see Art. 208). We say that it has gained the entropy $d\phi = \frac{dt}{t}$ and we must integrate this to get the total gain from the temperature $t_0$ or

$$\phi - \phi_0 = \log. t - \log. t_0$$

If $t_0$ is 461 + 32 Fahrenheit or 273·7 Centigrade, the freezing point of water, and $\phi$ is counted from this, so that $\phi_0$ is 0, as I usually employ $\phi_w$ to denote **the entropy of a pound of water**,

$$\phi_w = \log_e \frac{t}{493} \text{ or } \log_e \frac{t}{273 \cdot 7}$$

Of course $\phi_s$ for **a pound of steam** is $\phi_w + \frac{l}{t}$. The $\phi$ of a pound of stuff made up of $x$ lb. of steam and $1 - x$ lb. of water is evidently

$$\phi_w + x\frac{l}{t}$$

**201.** We shall see in Art. 362 that in **a pound of perfect gas** whose law is $\frac{pv}{t} = R$ where $p$ is in pounds per square foot, and $v$ in cubic feet, if the entropy was $\phi_0$ when the stuff was in the state $p_0$, $v_0$ and $t_0$

$$\phi - \phi_0 = \gamma \log. \frac{p}{p_0} + K \log \frac{v}{v_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{or} = K \log. \frac{t}{t_0} - R \log \frac{p}{p_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{or} = \gamma \log. \frac{t}{t_0} + R \log \frac{v}{v_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Here $k$, $K$ and $R$ are in foot-pound units. Or if we divide all across by Joule's equivalent, we get $\phi$ in Ranks, and we may still use $k$ and $K$ for the specific heat at constant volume, and constant pressure, respectively, in units equivalent to *Ranks*, and if $R$ is known, it is easy to divide it by Joule's equivalent; thus for air, $R/J$ becomes ·0687 ranks. Our numbers are now the same for either scale of temperature.

EXERCISE. One pound of air, $p_0 = 2116$ (one atmosphere), $v_0 = 12{\cdot}39$, $t_0 = 493$ (Fah.) and $R = 53{\cdot}15$, let $\phi_0$ be called 0; find $\phi$ in ranks when $v = 3$, $t = 900$. We find that $p$ would then be $\frac{Rt}{v}$ or 15,950 lbs. per square foot.

$K$ for air in heat units is ·2375, $\frac{R}{774} = {\cdot}0687$, and as $K - k = R$ for any perfect gas when in foot-pound units

$$k = K - R = {\cdot}2375 - {\cdot}0687 = {\cdot}1688 \text{ in heat units.}$$

Hence (1), (2) and (3) become

$$\phi \text{ ranks} = {\cdot}1688 \log. \frac{15{,}950}{2116} + {\cdot}2375 \log. \frac{3}{12{\cdot}39} = 0{\cdot}0041$$

$$= {\cdot}2375 \log. \frac{900}{493} - {\cdot}0687 \log. \frac{15{,}950}{2116} = 0{\cdot}0041$$

$$= {\cdot}1688 \log. \frac{900}{493} + {\cdot}0687 \log. \frac{3}{12{\cdot}39} = 0{\cdot}0041$$

so that we see we get the same answer by all the ways of working.

**202.** The statement that $\phi$ depends on the state of the stuff, is often put in other ways.

Thus in classes in physics we are taught how Carnot conceived of stuff working in an engine under these conditions;—

1. Receiving heat $H$ at constant temperature $T$, from a source of heat at the same temperature, expanding and doing work.

2. Expanding adiabatically in a non-conducting vessel, and doing further work, till it reaches the temperature $t$.

3. Being compressed (having work done upon it) at constant temperature $t$, and giving up heat $h$ to a refrigerator at this lower temperature.

4. Being further compressed adiabatically so that it shall return to its first condition again.

Carnot showed that this engine is reversible, and that it is not possible to conceive of an engine taking the heat $H$ at $T$, and giving up heat at $t$, which would do more work. We know that the nett

work done by this perfect engine is $H - h$ (all our measure energy being mechanical).

The gain of $\phi$ in the first operation is $\frac{H}{T}$ and its los third operation is $\frac{h}{t}$ and there is no gain or loss of $\phi$ in the fourth operations.

Our statement as to $\phi$ is that $\frac{H}{T} = \frac{h}{t}$

$$\text{or } \frac{H-h}{H} = \frac{T-t}{T}$$

That is, the efficiency of our perfect heat engine or of a sible engine working between the absolute temperatures T $(T - t)/T$. Perhaps th law of thermodynamics better known to stu this form than in the Art. 191.

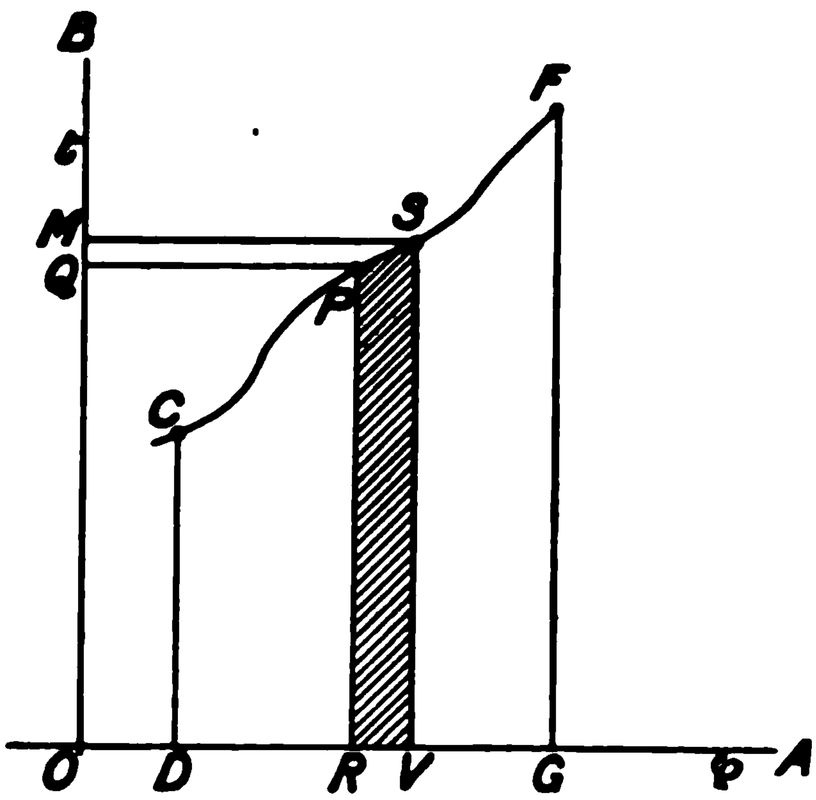


Fig. 221.

**203.** If we are g values of any two of t ties $v$, $p$, $t$, $E$, and $\phi$ for of stuff, we are suppos able to find all the oth

This statement may to be the most genera presenting the two thermodynamics. Indi grams show the stat values of $p$ and $v$, and areas represent work done. Many gators have in a general way used other diagrams, and diagram connecting any two of the above properties may in studying the behaviour of a pound of stuff.

The $t\phi$ diagram is not particularly valuable in calculatic or other gases, but for stuff which is in two forms, water a for example, the changes which Rankine and Clausius had difficulty in calculating, go on visibly on the diagram.

It is to Mr. MacFarlane Gray's persistence that we owe common use of the $t\phi$ diagram, so directly applicable engine problems. Even when steam is superheated a good probably still have both steam and water always presen cylinder of an engine.

[1] During change of state $p$ and $t$ are not independent.

If we make a $t\phi$ diagram for a pound of any kind of stuff. If the stuff changes in state (Fig. 221) from

$$\phi \text{ or } PQ \text{ and } t \text{ or } PR, \text{ to } \phi + \delta\phi \text{ or } MS \text{ and } t + \delta t \text{ or } SV$$

if the heat taken in is $\delta H$, the definition of entropy is that $\delta\phi = \frac{dH}{t}$ so that $t \cdot \delta\phi = \delta H$, and therefore the area $PSVR$ represents the heat taken in during the change. Hence in any great change, say from $C$ to $F$, the total heat taken in is represented by the area $CPSFGDC$.

**204.** Thus the rectangle $DEFC$, Fig. 222, shows a **Carnot cycle**; heat $H$ is taken in during the isothermal operation $DE$, at the absolute temperature $t_1$; heat $H_3$ is given out during the isothermal operation $FC$, at the absolute temperature $t_3$. Now, the distances $DG$ and $CG$ represent these absolute temperatures, and it is evident that as $H_1$ is represented to scale by the area $DEJG$, and $H_3$ by $CFJG$ then $H_1 - H_3$ or the area $DEFC$ is the work done.

We have
$$\frac{\text{work done}}{H_1} = \frac{DEFC}{DEJG} \text{ or } \frac{DC}{DG}$$
or
$$\frac{t_1 - t_3}{t_1}$$

It is worth while for the student to study the figure more carefully, writing 1, 2, 3, 4 for the operations, writing the value of the entropy at each corner and noting that

$$H_1 = t_1(\phi_2 - \phi_4), \ H_3 = t_3(\phi_2 - \phi_4)$$

It makes an excellent set of exercises—I hope that they will not be thought too tedious—to work out very carefully all that occurs in a Carnot cycle performed upon a pound of air; calculating both from the $pv$ diagram point of view and the $\theta\phi$ point of view. The student had better illustrate the work with two figures, one like Fig. 222, the other a $pv$ diagram, both drawn to scale.

We can find the values given in the following table in various ways. In these four exercises I take the most easy way for each operation, but the student ought to accustom himself to all the ways suggested in Art. 192.

**Exercise 1.** A pound of air $v = 3$ cubic feet, $p = 15{,}950$ lbs. per square foot, $t$ (absolute Fahrenheit) $= 900$ (these agree with $R = 53{\cdot}15$ Art. 186) expands at constant temperature to $v = 12$, find the new

$p$, the heat taken in, the work done, the gain in $E$ the energy, and the gain in entropy.

*Answer.*

$$\frac{p \times 12}{900} = \frac{15,950 \times 3}{900} \text{ or } p = 3,988.$$

Work done $= pv$ log. 4 (see Art. 188), or 66,310 foot-po
85·68 in heat units). Gain in entropy = heat 85·68 ÷ ten

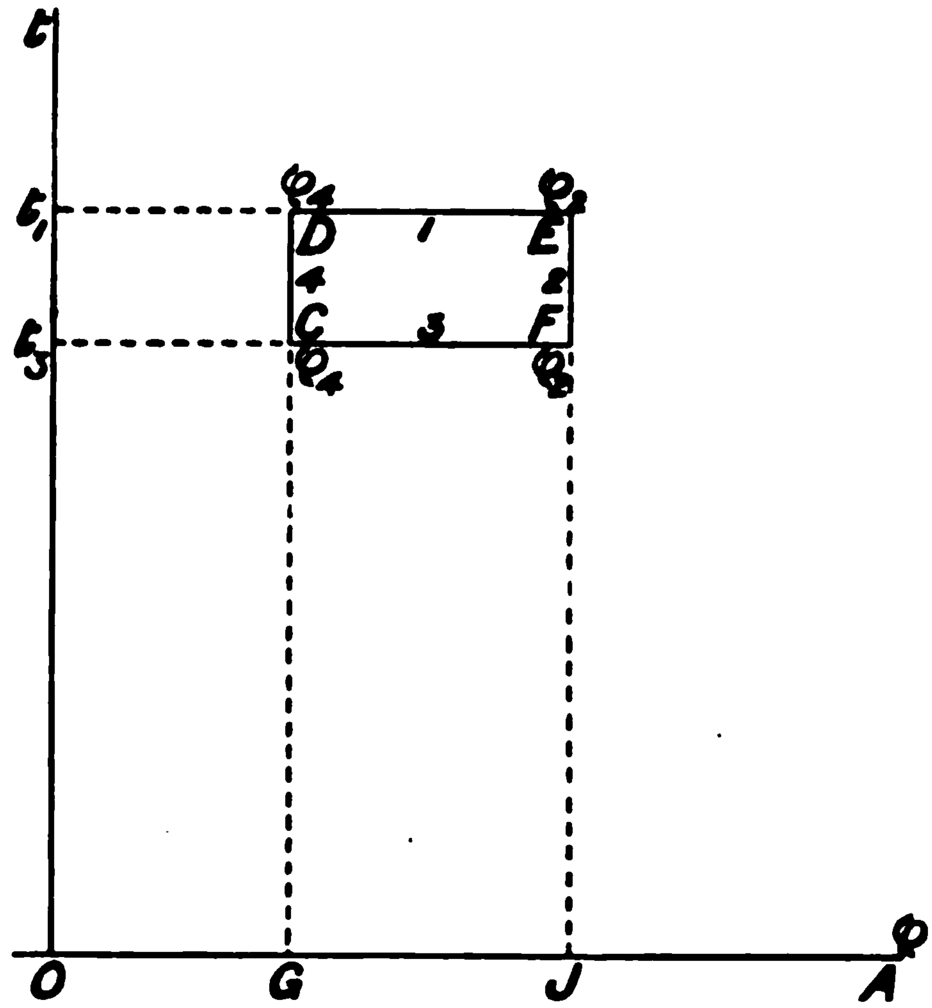


FIG. 222.

900 = 0·0952. Gain in $E = 0$. Let these results be w
the table.

We had better count entropy $\phi$ as 0 at atmospheric pres
0° C. It is easy to show as in Art. 362 that $\phi = K \log. \frac{t}{493}$ —
so that at $D$, $\phi$ is 0·00415.

EXERCISE 2. A pound of air $v = 12$, $p = 3,988$, $t = 900$,
adiabatically to $v = 42·46$, find the new $p$ and $t$, &c.

*Answer.* Expansion being according to the law $pv^{1·405}$
$p\,(42·46)^{1·405} = 3,988\,(12)^{1·405}$, so that $p = 676$. Hence $t =$

EXERCISE 3. A pound of air at $v = 42·46$, $p = 676$, $t$
compressed at constant temperature to the volume 10·62: w
pressure, the work done upon it, the heat taken from it and t
entropy? *Answer.* Its pressure is 2,704. The work done u
$Rt \log. \frac{42·46}{10·62}$ or 53·15 × 539 log. 4 or 39,720. This is also

taken from it, or dividing by 774 we have 51·32 units of heat. Dividing this by 539 we find ·0952 the loss of entropy.

Exercise 4. A pound of air at $v = 10{\cdot}62$, $p = 2{,}704$, $t = 539$, is compressed adiabatically to $t = 900$: find its $v$ and $p$ and the work done upon it. There is no loss or gain of heat or entropy. *Answer.* $v = 3$, $p = 15{,}950$, $w = -47{,}350$.

| Points. | $v$. | $p$. | $t$ | $h$. | $\phi$ | Heat taken in (heat and work units). | Work done by stuff. |
|---|---|---|---|---|---|---|---|
| $D$ | 3 | 15950 | 900 | 38·21 | ·00415 | | |
| | | | | | | 47·40<br>66310 | 66310 |
| $E$ | 12 | 3988 | 900 | 38·21 | ·09935 | | |
| | | | | | | | 47350 |
| $F$ | 42·46 | 676 | 540 | 5·099 | ·09935 | | |
| | | | | | | - 28·44<br>- 39820 | - 39720 |
| $C$ | 10·62 | 2704 | 540 | 5·099 | ·00415 | | |
| | | | | | | | - 47350 |
| $D$ | — | — | — | — | — | | |

The student will notice that in the Carnot cycle of a perfect gas the works of the two adiabatic operations are equal. This becomes clearer when we recollect that work done in an adiabatic operation is at the expense of intrinsic energy, and intrinsic energy of a perfect gas depends only upon temperature.

He is not likely to spend too much time in all kinds of study of the Carnot cycle of a perfect gas. All the calculations are of a nature likely to teach useful lessons, both when they are being carried out and in the study of their results.

### 205. Gases. $t\phi$ *diagrams.*

1. Take $R = 53{\cdot}15$, $t_0 = 493$, $p_0 = 2116$, $K = {\cdot}238$. Plot $\phi$ for values of $t = 493, 550, 600, 650, 700, 750, 800$, &c., if $\phi = K \log. \frac{t}{493}$; cut this curve out of a sheet of zinc as a template.

For the values of $p$, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, &c., atmospheres, calculate the value of $R \log. \frac{p}{p_0}$.

Now draw $t\phi$ curves of equal pressure, sliding the template horizontally so that each shall represent

$$\phi = K \log. \frac{t}{t_0} - R \log. \frac{p}{p_0}.$$

2. In the same way make a template for $\phi = k \log. \frac{t}{t_0}$ and for $v = 1\frac{1}{2}\, v_0,\ 2\, v_0,\ 2\frac{1}{2}\, v_0$, &c., $\frac{3}{4}\, v_0,\ \frac{1}{2}\, v_0,\ \frac{1}{4}\, v_0,\ \frac{1}{8}\, v_0$, &c., calculate $R \log. \frac{v}{v_0}$ the distance through which the template must slide horizontally.

In this way my students have obtained sheets of curves which they use for rapid calculation of difficult looking problems. Of course isothermal and adiabatic lines are straight horizontal and vertical lines. On such a diagram it is easy to lay out the $t\phi$ expansion curve of a given gas engine indicator diagram.

### *Superheated Steam.*

If from the point of saturation we may imagine the stuff to behave as a perfect gas, the intrinsic energy of 1 lb. of superheated steam is the same as that of 1 lb. of saturated steam at the same temperature, because intrinsic energy of a gas depends upon temperature only. This assumption is good enough for many steam engine calculations. Hence then a $t\phi$ diagram for a perfect gas is also an $E\phi$ diagram. I think that to assume $K$ to be ·475 in any important calculation is very wrong (see Chap. XXXI.), but until a proper measurement is made we may adopt it for academic purposes. The density of steam being taken as $\frac{5}{8}$ that of air, the $R$ of a pound of superheated steam may be taken to be 153. Also $(K - k)\, 1393 = R$ so that $k = 0{\cdot}359$.

My students have added to the ordinary $t\phi$ diagram for water and steam, the $t\phi$ diagrams for constant pressure and volume of superheated steam to facilitate some exercise work that is really somewhat misleading. For example: If there is only a pound of dry steam in a cylinder, how does it receive heat if it expands according to the law $pv$ constant, or $pv^s$ constant, if $s$ is less than 1·13 so that we know there is heat received during expansion. As I believe that there is always some water present in cylinders I look upon this as an academic exercise. If it must be worked, I say that we may take it as the case of a perfect gas and the rate of reception of heat per unit change of volume is $\frac{\gamma - s}{\gamma - 1}\, p$ where $\gamma$ is 1·3.

In calculating the total heat required for the production of 1 lb. of superheated steam of pressure $p$ and temperature $\theta_1$, I usually assume that water at 0° C. is first converted into saturated steam at the pressure $p$ and the temperature $\theta$° C. receiving Regnault's $H$, and

that it then receives the further heat $(\theta_1 - \theta) \times 0{\cdot}48$, assuming $0{\cdot}48$ as the constant specific heat of superheated steam. This gives us $606{\cdot}5 + 0{\cdot}305\theta + {\cdot}48\,(\theta_1 - \theta)$. Rankine on the assumption of $0{\cdot}48$ being the constant specific heat of superheated steam from 0° C. gives another formula. But we know that it is wrong to assume $0{\cdot}48$ as the constant specific heat from $\theta$ to $\theta_1$; Rankine assumed it correct from 0° C. to $\theta_1$° C., and he is much more incorrect than we.

## 206. Intrinsic Energy *E of Water-Steam.*

The intrinsic energy of a pound of water at $t$° F. is the heat, $h$, of the table, Art. 180. We ought to subtract the work done in expansion, but this is evidently very small.

The intrinsic energy of a pound of steam at $t$° F. is the heat, $H$, of the table, (in foot pounds) minus the work done by it in its formation which is $pu$ foot-pounds, or $H - pu$.

The intrinsic energy therefore of 1 lb. of stuff consisting of $x$ lb. of steam, $1 - x$ lb. of water is

$$E = x\,(H - pu) + (1 - x)\,h, \text{ or}$$
$$E = h + x(l - pu) \quad . \quad . \quad . \quad . \quad (1)$$

$h$ and $l$ are in work units or $pu$ is divided by Joule's equivalent 774 if $E$ is to be in heat units. Notice that values of $l - pu$ (called $E$ in the table) are given in heat units.

EXERCISE. A pound of stuff ·7 of steam, ·3 of water, at 95 lbs. per sq. in. (or 323°·9 F.) expands, becoming ·8 of steam, ·2 of water, at 50 lbs. per square in. (or 280°·8 F.); what heat has been given? Consulting the table we see that the gain of intrinsic energy is $251 + {\cdot}8\,(839) - (295{\cdot}1 + {\cdot}7 \times 804{\cdot}9)$ or $63{\cdot}7$ heat units: this is to be added to the work done and the work cannot be calculated without more data.

## 207. Exercises Illustrating Tests of Wetness of Steam.

1. *Condensing Method.* A **well-lagged tank** containing 200 lbs. of water at 60° F. increases 5 lbs. in weight by the reception of wet steam at 101·9 lbs. per sq. in. pressure, brought by a small connection from the steam pipe: the temperature at the end being 83° F. If no heat has been lost find the wetness of the steam.

*Answer.* $x$ lb. of steam and $1 - x$ lb. of water cooling to 32° F. from 329° F. would give out the heat $1182{\cdot}2\,x + 299{\cdot}5\,(1 - x)$ heat units, and subtracting $83 - 32$ or 51, because each pound of stuff is only reduced to 83° F., we have $5\,(882{\cdot}7\,x + 248{\cdot}5)$ as the total heat

given to the 200 lbs. of water, which being raised from 60° F. to 83°, receives 200 (83 − 60), or 200 × 23, or 4,600 units. Hence 5 (882·7 $x$ + 248·5) = 4,600. Hence $x$ = 0·761, or 76·1 per cent. of the stuff is steam, and 23·9 per cent. is water. The student will notice that the most important defect of this method lies in the difficulty of measuring accurately the increased weight of the tank.

2. *Condensing Method.* Some steam is continually being drawn off from the steam pipe into a **small surface condenser.** Suppose the pressure in the steam pipe to be 101·9 lbs. per sq. in. The water in one hour is weighed and found to be 5 lbs., its temperature being 110° F. The condensing water which passes during the hour is measured and found to be 300 lbs., its temperature upon entering being 60° F., and on leaving being 75° F. What is the wetness of the steam in the pipe?

*Answer.* Calculating as in the last exercise, if in each pound of stuff we have $x$ lb. of steam, and 1 − $x$ lb. of water: this at 329° F. cooling all to water at 110° F. gives out, per pound,

$$1{,}182\,x + 299{\cdot}5\,(1 - x) - (110 - 32)$$

units of heat. Five times this is equal to the heat given to 300 lbs. of water to raise it 15 Fahrenheit degrees, or 4,500 heat units. Solving the equation, $x$ = 0·769, or 76·9 per cent. of the stuff is steam.

3. *Throttling Method.* A small supply of steam is drawn off from the steam pipe and **throttled** in passing through a well-lagged tap into a well-lagged chamber from which it can escape freely into the atmosphere. If the original steam does not contain much moisture it will be superheated after the throttling, and the temperature of it enables us to calculate the previous wetness.

Suppose the steam at 101·9 lbs. per square inch and 329° F., and that in the chamber at atmospheric pressure the temperature is found on a very accurate thermometer to be 218°·5 F. Very careful measurement of the actual pressure in the chamber must be made by a barometer; suppose that this is found to be 14·35 lbs. per square inch [prove that a barometric height of 29·14 inches corresponds to 14·35 lbs. per square inch.] Now find by the table, Art. 180, the energy in 1 lb. of superheated steam of the pressure 14·35 and temperature 218°·5 F. Saturated steam at this pressure would be at the temperature 210·8. For a pound of such saturated steam $H$ of table would be 1145·4; add to this the heat required to superheat it from 210°·8 F. to 218°·5 or 0·48 × 7·7, or 3·7 units, so that the heat of formation of such superheated steam from 32° F. is 1149·1. Now $x$ lb. of steam, and

$-x$ lbs. of water had the total heat $1182{\cdot}2\,x + 299{\cdot}5\,(1-x)$ itting this equal to $1149{\cdot}1$ we find $x = {\cdot}9626$ or $96{\cdot}26$ per cent. the stuff is steam.

The thoughtful student must have met with some difficulty in rking the above exercise, which will be cleared by the following.

**Exercise.** Steam at $p_1$, $\theta_1°$ F. and dryness $x_1$, is throttled, bemning steam at $p_2$, $\theta_2°$ F. and dryness $x_2$. If the other numbers are ven, calculate $x_2$ on the assumption of a perfectly non-conducting pe and valve.

Let us study what occurs at a cross-section where the steam is at . Every pound that crosses this section carries with it its intrinsic ergy which is

$$J(\theta_1 - 32) + x_1(l_1 J - u_1 p_1),$$

$J$ is Joule's equivalent, $l$ the latent heat, $u$ the volume of a pound f steam. But it also has the work done upon it, the pressure nultiplied by the volume, which is $x_1\,u_1\,p_1$. Hence the energy ntering at the section is $J(\theta_1 - 32 + x_1 l_1)$, or its total heat. Similarly oming out at a section where the pressure is $p_2$ we have per pound of stuff the energy

$$J(\theta_2 - 32 + x l_2).$$

And as we assume just as much energy to leave as to enter,

$$\theta_1 + x_1\,l_1 = \theta_2 + x_2\,l_2$$

and so $x_2$ may be calculated.

If at the lower pressure, it is at $\theta_2°$ F. but is superheated to $\theta_3°$ F., its intrinsic energy is

$$J(\theta_2 - 32 + l_2) + JK(\theta_3 - \theta_2) - p_2 v$$

if $v$ is the volume of 1 lb. of it; but it does work $p_2 v$ in leaving the space, hence we take

$$\theta_1 + x_1\,l_1 = \theta_2 + l_2 + K(\theta_3 - \theta_2).$$

Of course our want of exact knowledge of the value of $K$ causes s in such measurements to reduce the amount of super-heating as nuch as we possibly can.

4. *Melting of ice method.* A well-lagged case contains 30 lbs. f broken ice separated by wire gauze partitions so that the ice xposes a very great surface. The case is exhausted of air, and team is admitted in such a way as to melt the ice quickly. The otal amount of water coming from the box is $36{\cdot}8$ lbs. at $100°$ F.

Each pound of ice received latent heat 142 units $+ (100 - 32)$ or

A A 2

210 units. Hence the heat received by the ice is $30 \times 210$ or 6,300 units.

In each pound of fresh water stuff, if we have $x$ lb. of steam and $1-x$ lb. of water, the total heat given out in cooling to 100° F. is

$$1{,}182{\cdot}2x + 299{\cdot}5\,(1-x) - (100-32)$$

$$\text{or } 882{\cdot}7x + 231{\cdot}5$$

and as we have 6·8 lbs. of this fresh water stuff

$$6{\cdot}8\,(882{\cdot}7\,x + 231{\cdot}5) = 6{,}300,$$

so that $x = 0{\cdot}787$, or 78·7 per cent. of the stuff entering the box was steam.

5. Let $W_1$ lbs. of water stuff (each pound of which has $x_1$ lbs. of steam) enter a well-lagged vessel in which there is already $W_2$ lbs. of water stuff (each pound of which has $x_2$ lb. of steam) forming a mixture. What is the dryness of the mixture? Here we say:— The heat of formation of $W_1$ + the intrinsic energy of the $W_2$ = the intrinsic energy of the resulting mixture.

*Example.* The vessel, well-lagged, contains at first 11 lbs. of water (as noted on a gauge glass tube) and 6·64 cubic feet of steam (or 0·25 lb.) at 212° F. A part of the metal of the vessel is exposed to a flame which may be so regulated that for ten minutes there is no alteration in the visible height of the water, the pressure remaining constant. I assume that this flame just compensates for loss of heat by the vessel. Connection is now made with the steam pipe where the pressure is [illegible] lbs. per square inch so that the steam to be tested passes through a thin coil of pipes in the water without much disturbance. At the end of a convenient time the connection is shut off. The gauge glass now indicates that there are 11·6 lbs. of water [illegible] the pressure being [illegible] lbs. per square inch. What was the dryness of the incoming steam?

[illegible] of water, there is now 6·64 [illegible] lbs. per square inch, or 6·64 ÷ 14·04 or [illegible] 12·07 lbs. [illegible] 11·25 lbs. so that 0·82 lb. [illegible]

[illegible]

[illegible]

The intrinsic energy now is

$$0{\cdot}47\left(1157{\cdot}6 - \frac{28{\cdot}83 \times 144 \times 14{\cdot}04}{780}\right) + 11{\cdot}6 \times 216{\cdot}9.$$

The incoming energy was

$$0{\cdot}82\left\{x(1182{\cdot}2) + (1 - x)\ 299{\cdot}5\right\}$$

Putting the sum of the first and third equal to the second we have $x = 0{\cdot}725$, or 72·5 per cent. of the entering stuff was steam.

I do not describe here the so-called chemical tests, as they are quite valueless.

## CHAPTER XXIII.

### WATER—STEAM, $\theta\phi$ DIAGRAM. EXERCISES.

**208.** WE find that the law

$$h = \theta + \cdot 00002\,\theta^2 + \cdot 0000003\,\theta^3 \ldots\ldots \quad (1),$$

the temperature being $\theta°$ C., $h$ being the heat given to a **pound of water** at 0° C. to raise it to $\theta°$ C. under gradually increasing pressure, is fairly well satisfied. If $t$ is the absolute temperature, $t = \theta + 273{\cdot}7$.

Now $d\phi = \frac{dh}{t}$. Hence if we were to take it that in water $dh = dt$, $\phi = \log. t +$ constant. If $\phi$ is taken to be 0 at 0° C. then $\phi = \log. \frac{t}{274} \ldots\ldots$ (2). It will be found that above 120° C. there is a noticeable error in using this simple formula.

If however we take the more exact rule given above

$$\phi = 1{\cdot}0565 \log. \frac{t}{273{\cdot}7} + 9 \times 10^{-7}\left(\frac{t^2}{2} - 502{\cdot}96\,t\right) + 0{\cdot}0902 \;.\;.\; (3).$$

The student will find it an excellent exercise to use this formula in calculating the numbers in the table, pages 320–321.

I shall often take the simpler formula (2) as stated in Art. 200.

**209.** In all the following academic exercises it is to be understood that the stuff—water and steam—is all at the same temperature. We must be cautious in using the results of such calculations in the consideration of actual steam engine problems.

All the calculations made by Rankine and others proceed on certain assumptions. One assumption made by everybody is that the stuff, water and steam, is **all at the same temperature** at the same instant. Now if it is also assumed that we know exactly **how much water is with the steam,** we have seen that MacFarlane Gray's $\theta, \phi$ diagram (a method which supersedes other more cumbrous methods) enables us to say exactly how much heat is being given to or given

e stuff to the metal of the cylinder at every instant during ısion, and indeed during all the cycle if we still assume that exactly how much water and steam we are dealing with. n exact account in this way from experimental results of an ıgine was first done, I think, by Hirn, and the method has ɔorately developed by his pupils. The method is called ethod although it is what any student of Rankine would do ɔeing told. I feel quite sure that a great deal too much has de of it, and that the results of the elaborate analyses of Hirn's followers are of no practical use and indeed give a ;rue account of what occurs inside the cylinder of a steam I would beg of the student to use these assumptions only

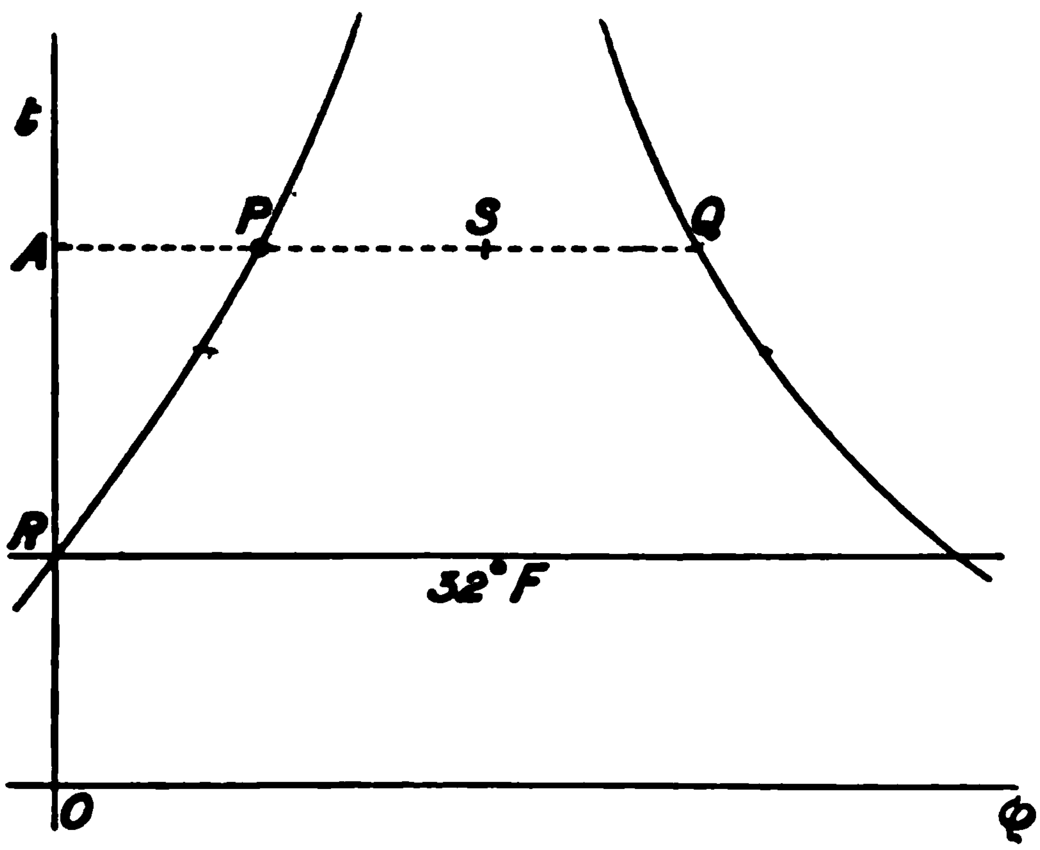


FIG. 223.

ɔrking of suggestive exercises like those that I have given V., and in what follows.

. A pound of water stuff containing $x$ lb. of steam and ɔf water, at the temperature $t$ has entropy $xl/t$ in addition 1 lb. of water has; if $l$ is the latent heat of 1 lb. of steam. ı Fig. 223 if the point $P$ represents the $\phi$ and $t$ of 1 lb. and $Q$ represents that of 1 lb. of steam, $S$ will represent ater stuff of which the fraction $\frac{PS}{PQ}$ is steam and the fraction

ter.

CISE 1. A pound of water stuff at $\theta^\circ$ F. contains $x$ lb. of d $1 - x$ of water, find $\phi$, if $\phi$ is 0 for 1 lb. of water at

*Graphical Method.* In Fig. 223A, where 0 represents 32° F., and any line like $ABC$ is at any particular temperature such as $\theta$° F.; $AB$ represents $\phi_w$, $AC$ represents $\phi_s$; make $BP/BC = x$. Then $AP$ shows the value of $\phi$.

*Algebraic Method.*—We saw in Art. 200 that the answer is

$$\phi = (1 - x)\phi_w + x\phi_s$$

In case the table is not at hand we may use

$$\phi = \log. \frac{t}{493} + x\frac{l}{t}$$

EXERCISE. One pound of water stuff at 392° F., or $t = 753$ contains 0·9 lb. steam, 0·1 lb. water; it expands adiabatically (that

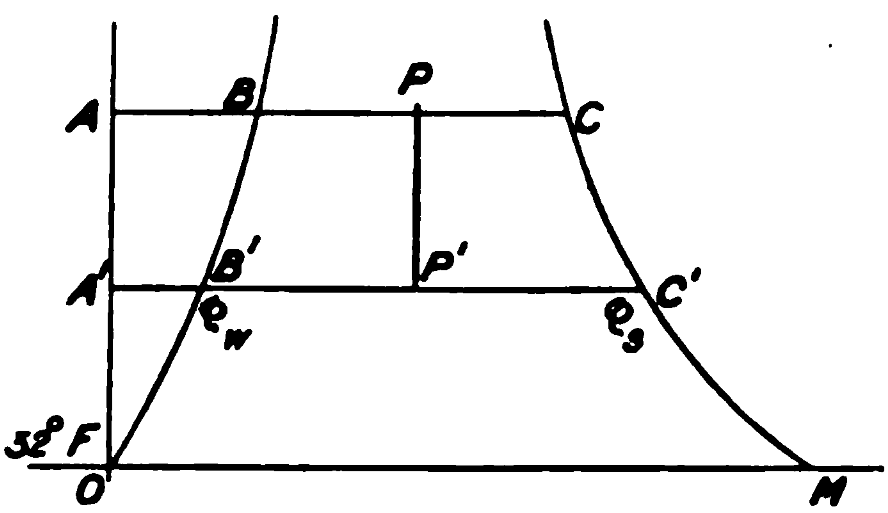


FIG. 223A.

is keeping its $\phi$ constant) to 216° F., and then contains $x$ lb. of steam, find $x$.

1st, Graphically. Draw $AC$, Fig. 223A, for 392° F., and $A^1C^1$ for 216° F. Let $PB = \cdot 9 \times BC$; draw $PP^1$ vertically; $x$ is the value of $P^1B^1 \div B^1C^1$, and in this case I find it to be 0·78.

2nd, Algebraically. $l$ for 392° F. (or $t = 853$ abs. Fah.) is 836 by the table, and $l$ for 216° F. (or $t = 677$) is 961, and hence

$$\log. \frac{853}{493} + 0{\cdot}9\,\frac{836}{853} = \log. \frac{677}{493} + x\,\frac{961}{677}$$

Hence $x = 0{\cdot}78$.

## 211. Exercise for a Class of Students.

Let 1 lb. of water stuff, consisting of $s$ lb. of steam and $1-s$ of water at $\theta$° C., expand adiabatically to $\theta_0$° C. Find the $p$, $v$ diagram, and assuming that the expansion curve follows a law like $pv^k =$ a constant, find $k$. How does condensation or evaporation go on during the adiabatic expansion ?

*tod.* At the temperature $\theta°$ C. on the $t\,\phi$ diagram, Fig. 224, e horizontal $ABD$. Find $C$ so that $BC/BD = s$; draw other tals $A_1B_1D_1$ at various temperatures and the adiabatic vertical $CC_1C_0$. The ratio of any $B_1C_1$ to its $B_1D_1$ is the fractional quantity of steam present and if we neglect the volume of water present, the volume of this steam is the whole volume.

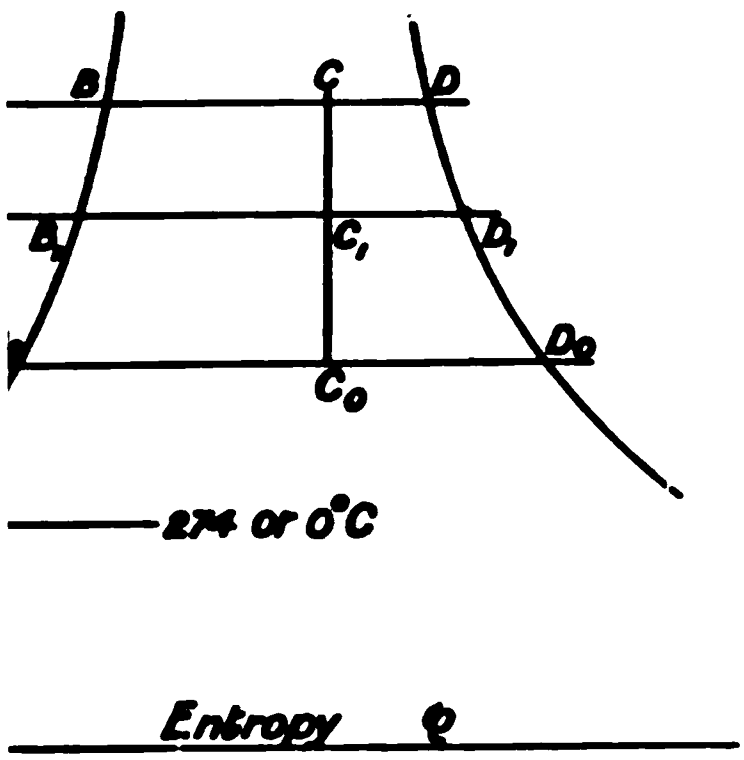


Fig. 224.

*Example I.* (1). Let $\theta°$ C. be 195° C. (pressure 203·3 lbs. per square inch). Let $s = 1$ so that there is no water present at the beginning of the expansion. Proceeding as directed, a student finds the following

$u$ is the volume of 1 lb. of dry steam at each of the tures at which a measurement is made, $v$ is the actual of steam present. Plotting log. $p$ and log. $v$ on squared nables us to find $k$. $x$ is the amount of stuff in the steam every point, and its value shows therefore whether there is tion or condensation going on. $s$ is the value of $x$ at the ng of the expansion.

| $p$. | $x$. | $u$. | $v$. | log. $p$. | log. $v$. |
|---|---|---|---|---|---|
| 203·3 | 1 | 2·242 | 2·242 | 2·3081 | ·3506 |
| 145·8 | ·977 | 3·065 | 2·994 | 2·1638 | ·4763 |
| 89·86 | ·944 | 4·827 | 4·556 | 1·9536 | ·6586 |
| 52·52 | ·918 | 7·995 | 7·338 | 1·7204 | ·8656 |
| 28·83 | ·886 | 14·04 | 12·44 | 1·4599 | 1·0947 |
| 14·7 | ·858 | 26·43 | 22·68 | 1·1673 | 1·3556 |

*Example I.* (2) Same as *I.* (1) but begin with $s = 0{\cdot}75$.

*Example I.* (3) Same as *I.* (1) but begin with $s = 0{\cdot}5$.

*Example I.* (4) Same as *I.* (1) but begin with $s = 0{\cdot}25$.

*Example I.* (5) Same as *I.* (1) but begin with $s = 0$.

*Example II.* Let $\theta°$ C. be 165° C. (101·9 lbs. per square inch). Let lowest temperature be 85° C. and use the above values for $s$.

*Example III.* Let $\theta°$ C. be 140° C. (52·52 lbs. per square inch). Let lowest temperature be 85° C.

In every case, plot log. $p$ and log. $v$ on squared paper and find if there is any such law as $pv^k$ = a constant.

Each of the answers in the following table is the mean of the results of four students. They were elementary students and the results are likely to be not quite so correct as those obtained by advanced students.[1]

BEST VALUE OF $k$ IF ADIABATIC EXPANSION IS SUPPOSED TO FOLLOW THE LAW $pv^k$ CONSTANT AS IT VERY NEARLY DOES.

| Range of Pressure. | Range of Temperature. | Best values of $k$ for the following values of dryness at beginning of expansion. | | | | |
|---|---|---|---|---|---|---|
| | | 1·0 | 0·75 | 0·50 | 0·25 | 0 |
| 203 to 15 | 195° C. to 100° C. | 1·129 | 1·113 | 1·054 | ·959 | Law untrue. |
| 102 to 8 | 165° C. to 85° C. | 1·129 | 1·108 | 1·110 | 1·022 | |
| 53 to 8 | 140° C. to 85° C. | 1·135 | 1·110 | 1·069 | 1·089 | |
| Average values of $k$ . . . | | 1·130 | 1·110 | 1·078 | 1·023 | |

Rankine gives the number $k = 10/9$ or 1·111 as correct for the adiabatic expansion of steam, but the details of his calculation are now lost. The formula, $k = 1{\cdot}035 + 0{\cdot}1s$ has been given. The MacFarlane Gray diagram enables elementary students to work easily for themselves what we used to be compelled to take on trust. The above values fit fairly well the rule $k = 1 + 0{\cdot}14s$ for any of the ranges of temperature.

**212. To quickly convert** a $pv$ diagram of steam into a $t\phi$

[1] Mr. F. W. Arnold assisted in the above work; in his holiday he has done the work very thoroughly, and obtained a most interesting set of relations between $k$ and $s$ and the range of temperature. I wish I could here find space to reproduce the beautiful curves he has drawn. I hope that they may be published elsewhere.

n the plan of Art. 55 is the one which I use myself. It may t shorter methods may be invented, but I like it because it to keep general principles well in the mind, and unless one g many exercises (a most unlikely thing unless one is engaged ecial kind of investigation) special rules are very easily for- . I have used the following method and it may give satisfaction e students.

t distances measured vertically from $O'G$, Fig. 225, represent ite temperature. Let distances measured from $OP$ represent ratures above 32° F.: the distance from $O$ to $O'$ represents 493·2

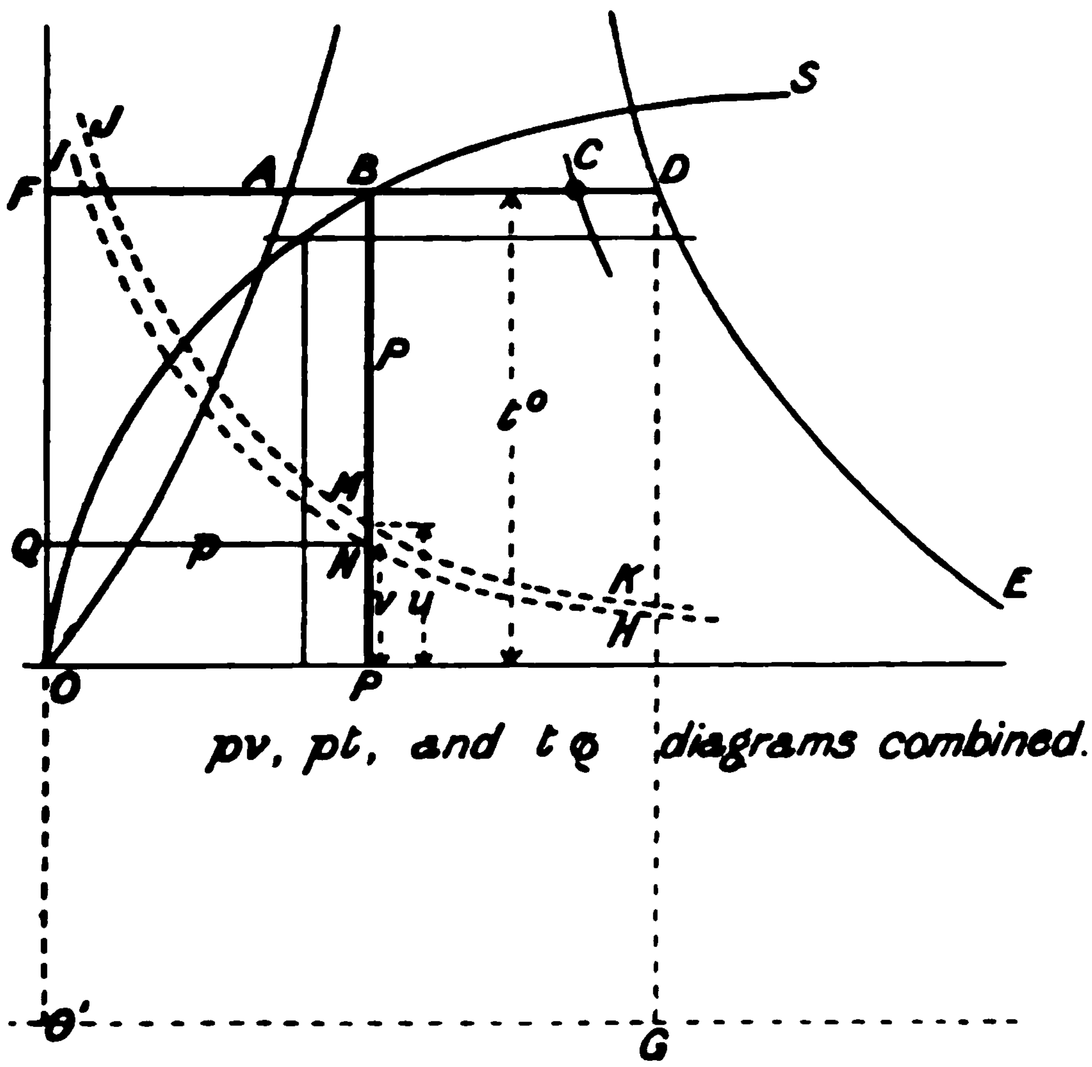


FIG. 225.

eed not be more than indicated. The abscissæ of the curves nd $DE$ show the values of $\phi_w$ and $\phi_s$. The scale for heat is hat the area of the rectangle $FDGO'$ represents $t \times \phi_s$ units. $HNI$ an actual expansion curve of an indicator diagram, $QN$ enting pressure and $PN$ volume, to any scale which is con- t. At some point $N$ let us know how much water is present cylinder and make $MN : NP$ in the ratio of water : steam. gh $M$ draw the curve $KMJ$ whose law is $pv^{1\cdot0646}$ constant. if any such line as $PNM$ is drawn, it will show the ratio of to steam.

t the curve $OBS$ whose ordinate $PB$ and abscissa $OP$ nt temperature and pressure of steam from the table, Art. 180. any point $P$ erect the perpendicular $PB$ meeting the curve

$OBS$ in $B$; draw the horizontal $FABD$ through $B$ and div
so that $AC : CD = PN : NM$. The point $C$ is a point in
diagram corresponding to $N$ on the indicator diagram.

All students accustomed to graphical methods are aware, (
to be aware, of quick methods of dividing lines proportionall
another; the best method requires a sheet of transparent
paper, or rather of tracing paper with a number of equ
parallel lines ruled upon it. It enables one to copy rapidly
whose ordinates and abscissæ are altered in any given pro
and is very valuable if one has much work to do of the san

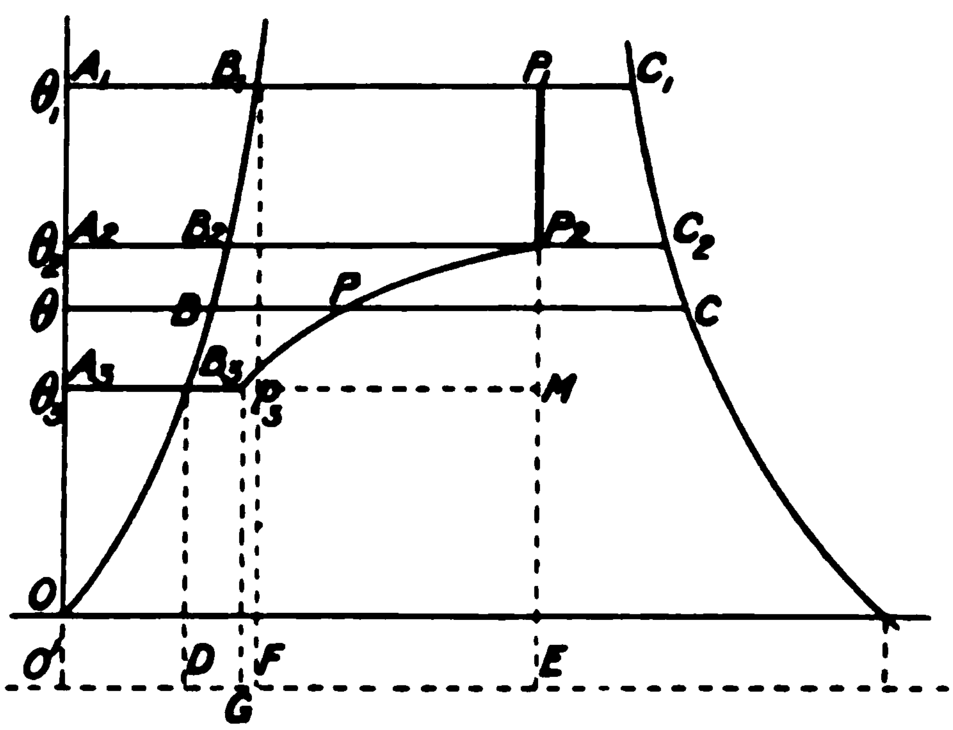


FIG. 226.

After all, however, a student may benefit more from the
clumsy method of his own as it keeps elementary princip
before his mind.

**213.** EXERCISES WITH THE $\theta\phi$ DIAGRAM I. $x_1$ lb. of
$1-x_1$ of water at $\theta_1$° C. expands adiabatically to $\theta_2$°C., is then
to a condenser at $\theta_3$° C. The $pv$ diagram for this is bou
straight lines and one curve.

Make $P_1B_1 \div B_1C_1 = x_1$ on the horizontal corresponding
The vertical line $P_1P_2$ shows adiabatic expansion to $P$
corresponds to $\theta_2$° C.

Let there now be the idea that we have a vessel with a
water stuff of which the fraction $B_2P_2/B_2C_2$ is steam, kept at
volume but lowered in temperature to $\theta_3$. To draw the curv
that is to find any point $P$ corresponding to any temperatu
$u_2$ and $u$ are the volumes of a pound of steam at $\theta_2$ and $\theta$ a
in the table, Art. 180; as we have the volume at release
constant $\frac{B_2P_2}{B_2C_2}u_2 = \frac{BP}{BC}u$, so that $BP$ may be calculated.

If $O^1DE$ is the horizontal corresponding to the absolute zero of temperature, the area $DB_3B_1F$ represents the heat given to raise the feed water from $\theta_3$ to $\theta_1$; $FB_1P_1E$, the heat to produce the $x_1$ lb. of steam, $EP_2P_3G$ is the heat taken *from* the stuff if it were kept in a vessel of constant volume and cooled to $\theta_3$; this we have taken to correspond with the real release; $GP_3B_3D$ is the heat taken from the stuff in the supposed compression of the remaining steam at $\theta_3$° C. till it is all condensed. Hence the work done per pound of steam in a perfect engine would be represented by the area $B_3B_1P_1P_2PP_3B_3$

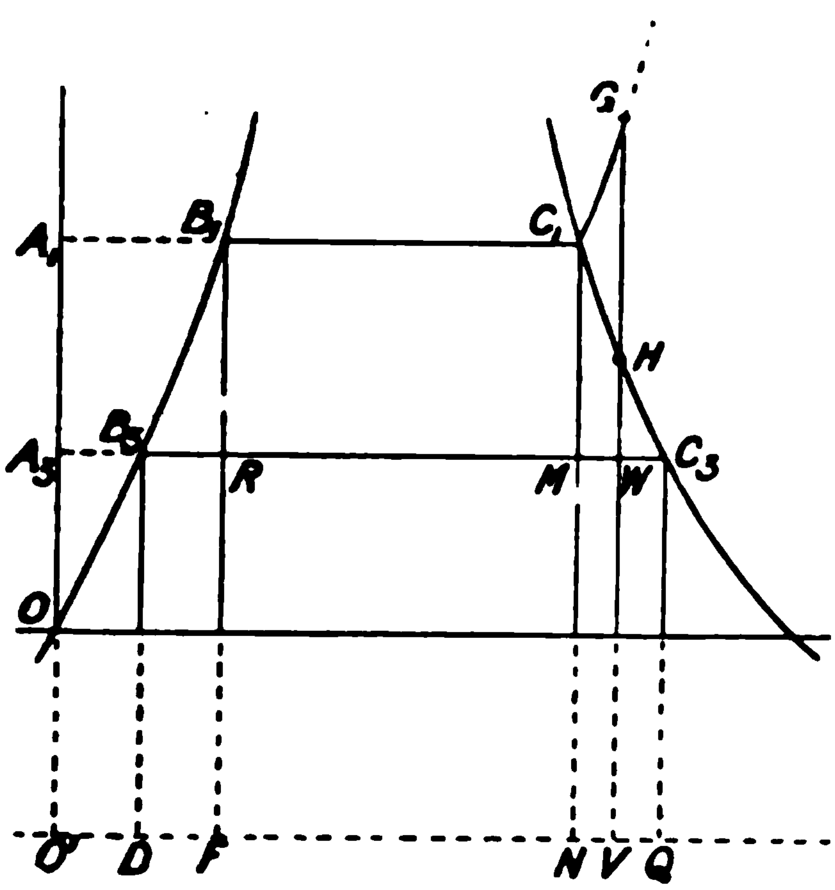


FIG. 227.

and the heat expended $DB_3B_1P_1ED$, the ratio between these being the efficiency.

The area of $P_3PP_2MP_3$ represents the loss of work because the adiabatic expansion has not continued to the temperature $\theta_3$.

If the student's prepared sheet of paper is provided with lines of constant volume, of course the drawing of the line $P_2PP_3$ gives no trouble.

**214.** EXERCISE II. A **perfect steam engine** uses steam under the following conditions, find in each case:—

$W$ the work (in heat units) done per pound of steam and $w$ the number of pounds of steam used per hour per horse-power.

$h$ the heat given per lb. of steam. $e = W/h$ the efficiency.

1. Feed water at 100° F. is heated to 329° F. and converted into steam; it is expanded adiabatically to 100° F. and released at 100° F. $W \equiv B_3B_1C_1MB_3$, Fig. 227; $h \equiv DB_3B_1C_1ND$|

*Answer.* $W = 293$, $h = 1114$, $e = 0{\cdot}262$.

2. During expansion the stuff receives just so much heat as keeps it in the condition of dry saturated steam. $W \equiv B_3B_1C_1C_3B_3$, $h \equiv DB_3B_1C_1C_3QD$.

*Answer.* $W = 331$, $h = 1379$, $e = 0{\cdot}240$.

3. The stuff is superheated to 410° F. and expands adiabatically to 100° F. Notice that the steam is wet towards the end of the expansion $W = B_3B_1C_1GHWB_3$, $h = DB_3B_1C_1GHWVD$.

*Answer.* $W = 312$, $h = 1172$, $e = {\cdot}266$.

4. The stuff is superheated to 410° F., expands adiabatically till it is just saturated at $H$; receives sufficient heat during the remainder of its expansion to 100° F. to keep it in the dry saturated condition. $W = B_3B_1C_1GHC_3B_3$, $h = DB_3B_1C_1GHC_3QD$.

*Answer.* $W = 336$, $h = 1382$, $e = {\cdot}242$.

5. To compare the above with a Carnot cycle. In the Carnot cycle all the heat is given at 329° F., and the heat is taken out at 100° F. $W = B_1C_1MR$, $h = B_1C_1NF$, $W/h = (t_1 - t_3)/t_1$ or $W = 256$, $h = 882$, $e = 0{\cdot}29$. The results are here tabulated :—

| | *W*. Work per pound of steam in Fah. heat units. | Energy expended in Fah. heat units. | *e*. efficiency. |
|---|---|---|---|
| 1st case, ordinary . . . . . . . . | 293 | 1114 | 0·262 |
| 2nd ,, with jacket . . . . . . | 331 | 1379 | 0·240 |
| 3rd ,, super-heating . . . . . | 312 | 1172 | 0·266 |
| 4th ,, super-heating and jacket | 336 | 1382 | 0·242 |
| 5th ,, Carnot cycle . . . . . | 256 | 882 | 0·290 |

Notice that although in all the other cases there is more work done per pound of steam, none of them is so efficient as the Carnot cycle. Cases (1) and (3) are said to be "standard or perfect steam engines following the **Rankine cycle.**"[1]

[1] Lord Rayleigh, in an article in *Nature* (February 18th, 1892), after pointing out that only a small amount of the heat received by the stuff in the formation of superheated steam, is received at the highest temperature [a fact known to every one who uses the $t\phi$ diagram], made the further very important statement :—

"If we wish effectively to raise the superior limit of temperature in a vapour-engine, we must make the boiler hotter. In a steam engine this means pressure that would soon become excessive. The only escape lies in the substitution for water of another and less volatile fluid. But, of liquids capable of distillation without change, it is not easy to find one suitable for the purpose. There is, however, another ... h we may look. The volatility of water may be restrained by the ... sters, such as chloride of calcium or acetate of soda. In ... rature may be raised without encountering excessive ... efficiency, according to Carnot, may be increased ... n of this method would involve the condensation of the

**215.** The following exercises are just like the above, but they e worked *algebraically*.

It is a good test of a student to find out to what extent he misapprehends the value of such calculations as these.

**T CASE.—RANKINE CYCLE. DRY STEAM. PERFECT STEAM ENGINE (ADIABATIC EXPANSION).**

Knowing the shape of the curve $B_2B_1$, Fig. 227, it is easy to calculate the area of the figure $B_2B_1C_1M$. But I prefer to take the matter up from first principles.

It is proved in thermodynamics that if in a heat engine the working stuff receives heat $H$ at the absolute temperature $t$ and if $t_2$ is the temperature of the refrigerator, then the work done by a perfect heat engine would be

$$H\frac{t-t_2}{t} \text{ or } H\left(1-\frac{t_2}{t}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If one pound of water at $t_2$ is heated to $t_1$, and we assume that the heat received per degree is constant, what is the work which a perfect heat engine would give out in equivalence for the total heat? Let all energy be expressed in heat units.

To raise the temperature from $t$ to $t+\delta t$ the heat given is $\delta t$, and this stands for $H$ in the above expression. Hence for this heat a perfect engine would give the work

$$\delta t\left(1-\frac{t_2}{t}\right)$$

and the integral of this from $t_2$ to $t_1$ is

$$t_1 - t_2 - t_2 \log.\frac{t_1}{t_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If now a pound of water at $t_1$ receives the heat $l_1$ (the latent heat), and is all converted into steam at the constant temperature $t_1$, the work that is thermodynamically equivalent to this is $l_1\left(1-\frac{t_2}{t_1}\right)$. We see then that the work which a perfect steam engine would give out as equivalent to the heat received per pound of steam is

$$t_1 - t_2 - t_2\log.\frac{t_1}{t_2} + l_1\left(1-\frac{t_2}{t_1}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

It will be found more correct to take $h_1 - h_2$ instead of $t_1 - t_2$ at the beginning.

This may be calculated either on the Centigrade or the Fahrenheit scale, and converted into foot-pounds. A horse-power hour is $33,000 \times 60$ foot-pounds; so dividing our work into this we find the number of pounds of steam or which a perfect steam engine would consume per horse-power hour when working between the temperatures $t_1$ and $t_2$.

---

am at a high temperature by reunion with the desiccating agent, and the communication of the heat evolved to pure water boiling at nearly the same temperature, t at a much higher pressure. But it is possible that, even without a duplication this kind, advantage might arise from the use of a restraining agent. The steam, erheated in a regular manner, would be less liable to premature condensation in cylinder, and the possibility of obtaining a good vacuum at a higher temperature n usual might be of service where the supply of water is short, or where it is ired to effect the condensation by air."

The numbers of columns 10 and 11 of Table II., Art. 180, have bee lated in this way. I assume that in a perfect non-condensing engine th temperature is 212° F., and in a condensing engine 100° F.

**Mr. Willans** used the above **as his standard of comparison** when lished his **non-condensing trials** in 1888; but when he came to pub **condensing trials** in 1893 he saw that as the perfect engine presu pansion to very large volumes indeed, no actual engine could approa efficiency. He therefore adopted arbitrarily as the standard condensin engine, one in which $\theta$ F., but adiabatic ex ceases and the stean leased at a tempera 170° F. The student done the Exercise, A will see that this the deduction of tl $MP_2P_3$, Fig. 228, fr whole work per po steam $MC_1B_1B_3$. He work graphically a fe cises making this assumption, so as to g idea of its effect in

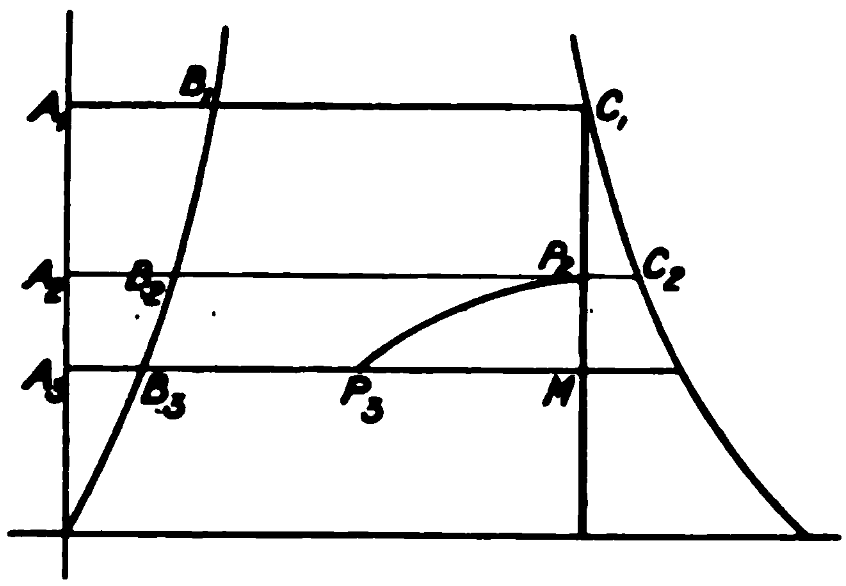


FIG. 228.

our standard. In any case, the standard or perfect condensing engine i an arbitrary standard. As I have already said, I prefer to take $\theta_3$ as and to imagine complete expansion down to that temperature. Any s of this kind is of a temporary character, and will be given up when it c be commercially profitable to use it. **The only scientific method of efficiency** is energy usefully given out by the engine ÷ total energy of t

In pages 257-8 I give results actually obtained from steam engines, each case I have written beside the actual $w$ the $w$ for a perfect steam er

EXERCISE 1. In condensing engines test the amount of error approximate formula

$$W = 155 + 13{\cdot}1\, p^{\frac{1}{4}}$$

$$W = 14 + {\cdot}872\, \theta.$$

Where $\theta$° F. is the temperature of the steam, $p$ the pressure in pounds pe inch, $W$ the work done (converted into Fahrenheit units of heat) per p steam in a perfect steam engine expanding adiabatically.

EXERCISE 2. Mr. Willans was of opinion that the standard or condensing engine ought only to be expected to expand its steam adiat to 170° F. and release at 110° F. By thus cutting off the "toe of the di show that, instead of getting the work $W$ we get the work $W_1$ per p steam.

| $p_1$ | $t_1$° F. | $W$. | $W_1$. |
|---|---|---|---|
| 250·6 | 401 | 335·6 | 300 |
| 203·5 | 383 | 322·4 | 286 |
| 146·0 | 356 | 301·4 | 264 |
| 102·0 | 329 | 278·4 | 240 |
| 52·6 | 284 | 236·0 | 197 |
| 33·7 | 257 | 207·6 | 168 |

2ND CASE.—EXPANSION AS DRY SATURATED STEAM. STEAM JACKETING.

**Our main reason for jacketing** is to prevent condensation and leakage, and these exercises are a good deal misleading; nevertheless it is well to do them, and they are not much more misleading than many other exercises.

A perfect steam engine has its limiting temperatures $t_1$ and $t_3$ (absolute). If there is just enough jacketing to keep the steam dry in its expansion, find the work done per pound of steam and the other numbers of the following table. The numbers in the second and third columns of the following table are in Fahrenheit heat units.

Evidently the heat required per pound of steam, in addition to what is wanted for Case 1, is represented by the area $NC_1C_3QN$ of Fig. 227, and the extra work is represented by the area of $C_1C_3M$.

The ordinate of the curve $C_1C_3$ is $t$ and its abscissa $\phi_s$ or, using Centigrade temperature,

$$\log. \frac{t}{273 \cdot 7} + \frac{l}{t}$$

where $l = 606 \cdot 5 - \cdot 695\theta$ or $797 - \cdot 695t$

$$\phi_s = \log. \frac{t}{273 \cdot 7} + \frac{797}{t} - \cdot 695.$$

The area is the integral of $t.d\phi$ or $t\frac{d\phi}{dt}.dt$

$$\frac{d\phi}{dt} = \frac{1}{t} - \frac{797}{t^2}$$

so that the area representing the extra heat given is

$$\int_{t_1}^{t_3}\left(1 - \frac{797}{t}\right)dt = 797 \log. \frac{t_1}{t_3} - (t_1 - t_3).$$

The extra work done is the integral of $(t - t_3)d\phi$ and is

$$(t_3 + 797) \log. \frac{t_1}{t_3} - \left(1 - \frac{t_3}{t_1}\right)(t_1 + 797)$$

When Fahrenheit absolute temperatures are taken, instead of 797 we have 1,434.

| | $p_1$. | Extra heat supplied to keep 1 lb. of steam dry. | Total work done per lb. of steam, including jacket steam. | Pounds of steam needed per horse-power hour. | Pounds of steam per horse-power hour if expansion is adiabatic. |
|---|---|---|---|---|---|
| Non-condensing, lowest temp. 212° F. | 250·3 | 164·9 | 189·4 | 13·4 | 12·3 |
| | 203·3 | 150·4 | 173·0 | 14·7 | 13·4 |
| | 145·8 | 131·4 | 153·8 | 16·5 | 15·3 |
| | 101·9 | 106·7 | 132·2 | 19·2 | 18·1 |
| | 52·5 | 71·5 | 88·48 | 28·7 | 27·6 |
| Condensing, lowest temp. 100° F. | 250·3 | 319·3 | 291·7 | 8·70 | 7·3 |
| | 203·3 | 305·8 | 283·8 | 9·01 | 7·6 |
| | 145·8 | 286·3 | 270·2 | 9·40 | 8·1 |
| | 101·9 | 264·1 | 254·9 | 9·96 | 8·7 |
| | 52·5 | 227·1 | 221·6 | 11·45 | 10·1 |

3RD CASE.—A LARGE AMOUNT OF SUPERHEATING.

In the following exercise the superheating is supposed to be so high the steam is just not wet at the end of the expansion; the student is exp to work out all the numbers. In these exercises, if the lower temperature and after the steam has been produced at $t_1$ if it is superheated at con pressure to such a temperature that it will be just saturated after adia expansion to $t_2$, then the perfect steam engine will, per pound of steam, do in heat units (Fahrenheit).

$$= t_1 - t_2 - t_2 \log_e \frac{t_1}{t_2} + \frac{L_1(t_1 - t_2)}{t_1} + Kt_1(e^{x/K} - 1) - t_2 x$$

$$= 0{\cdot}305\,(t_1 - t_2) + Kt_1(e^{x/K} - 1)$$

$$\text{where } x = 1434\left(\frac{1}{t_2} - \frac{1}{t_1}\right)\log.\frac{t_1}{t_2} \text{ and } K = 0{\cdot}48$$

the maximum superheating temperature being $t' = t_1 e^{x/K}$. The specific h superheated steam has been taken to be 0·48; the engine is non-conder that is, $t_2 = 673$ or 212° F.

| $p_1$. | Amount of superheating (Fahrenheit). | Work in Fahrenheit thermal units per pound. $W$ | Pounds of steam per horse-power hour. $w$ | Pounds of ste per horse-pow hour if not superheated. $w$ |
|---|---|---|---|---|
| 52·5 | 181° | 109 | 23·3 | 27·6 |
| 101·9 | 298° | 179 | 14·2 | 18·1 |
| 145·8 | 376° | 224 | 11·4 | 15·3 |
| 203·3 | 447° | 267 | 9·5 | 13·4 |
| 250·3 | 500° | 298 | 8·64 | 12·3 |

My students have found that the following formulæ give values of $W$ w differ less than one per cent. from the calculated values:

$$W = 1{\cdot}648\theta - 178 \text{ for non-condensing, and}$$
$$W = 2{\cdot}8\theta - 360 \text{ for condensing engines.}$$

**216.** EXERCISE 1. The following pressures and volumes be measured on the expansion curve of an indicator diagram, $p =$ : $v = 1$; $p = 40$, $v = 4$. If the expansion curve is adiabatic, l much water is present with the steam?

*Answer.* Water per pound of stuff at the beginning 0·896, an the end 0·838. Or water to steam 8·6 : 1 and 5·2 : 1.

EXERCISE 2. At $G$ (Fig. 82) presumably the end of admission, the vo is 1·4 cubic feet, pressure 51·9 lbs. per square inch (or 283°·2 F.), $u_1 = 8$ latent heat (Fahrenheit) = 914·5), and thus the indicated steam is $i = 0{\cdot}17$ At $F$ the volume is 4·323 cubic feet, pressure 21·69, and the steam pr weighs 0·235 lb.

Assume that the metal of the cylinder is non-conducting, that we $z$ lb. of *water* present before admission, that $i$ lb. is the indicated steam that the entering steam which has condensed is $yi$; neglect the steam pres

the end of the cushioning; assume that all the water stuff present is everywhere of the same temperature at any instant, and therefore that the expansion is adiabatic, it is evident that we have the means of calculating $y$ and $z$.

Note the nature of this assumption. As the metal always does take heat, this action will be represented by there being a little more water present. If

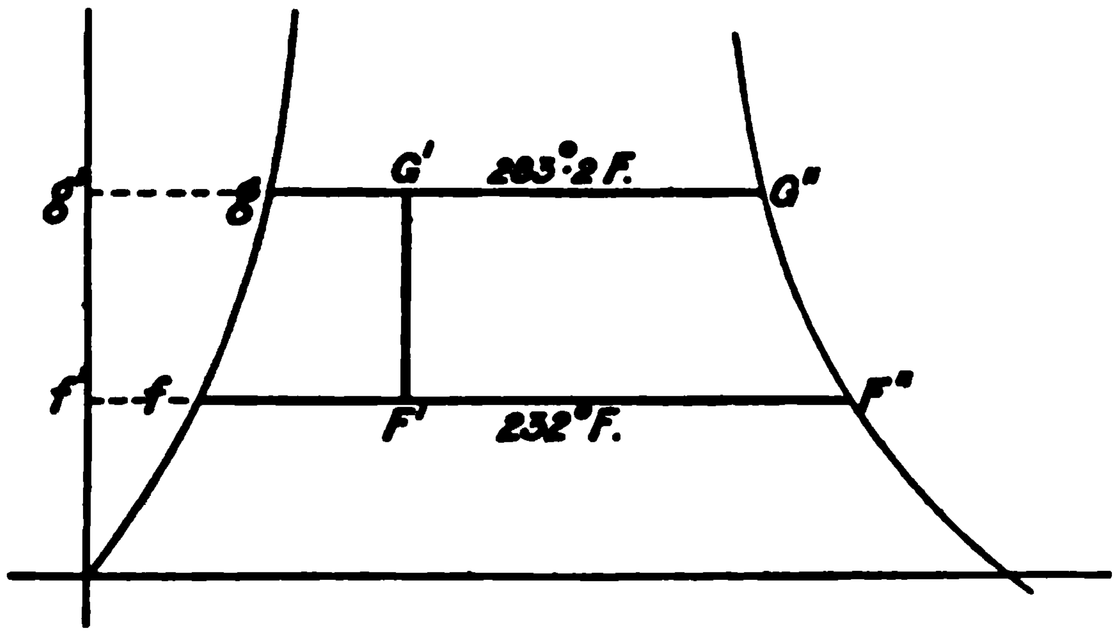


FIG. 220.

the metal really does give heat, because of a steam jacket, this action will be represented by there being a little less water present.

In Chap. XXIV we shall see that it is only for certain calculations that such an assumption is legitimate. The latent heat $lyi$ of $yi$ lb. of steam heats $z$ lb. of water from the back pressure temperature ($p_3 = 3{\cdot}4$ or $t_3 = 146^\circ{\cdot}3$ F. say) to 283°, so that

$$914{\cdot}5y \times 0{\cdot}173 = 137z, \text{ or } y = 0{\cdot}866z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let the whole water stuff present $\cdot 173 + z + \cdot 173y = w$.

Again, if the adiabatic expansion represented by $G\,E\,Q\,F$ on the indicator diagram is represented by $G'F'$ on the $t\phi$ diagram, $\dfrac{gG''}{gG'} = \dfrac{w}{\cdot 173}$

$$\frac{fF''}{fF'} = \frac{w}{\cdot 235}, \text{ so that } \frac{gG''}{gG'} \cdot \frac{fF'}{fF''} = \frac{\cdot 235}{\cdot 173}$$

Hence

$$\frac{gG'}{fF'} = \frac{gG''}{fF''} \cdot \frac{\cdot 173}{\cdot 235}.$$

Now

$$gG'' = \frac{\text{latent heat } 914{\cdot}5}{\text{absolute temp. } 744{\cdot}2} = 1{\cdot}229;\ fF'' = \frac{952}{693} = 1{\cdot}374$$

$$\frac{gG'}{fF'} = \frac{\cdot 173}{\cdot 235} \times \frac{1{\cdot}229}{1{\cdot}374} = \cdot 660.$$

We have now only to find the two points $G'$ and $F'$ so that they shall be in the same vertical and the distances in this ratio. I have found them by trial and get

$$\frac{gG''}{gG'} = 8{\cdot}72 = \frac{w}{\cdot 173}. \quad \text{Hence } w = 1{\cdot}508.$$

Or, without actual trial on a diagram;—By the table, Art. 180, $g'g$ for 283°·2 F. is ·415, $f'f$ for 232° F. is ·342.

Let $g'G' = f'F' = x$, then

$$\frac{x - \cdot 415}{x - \cdot 342} = \cdot 660, \text{ whence } x = \cdot 556$$

$$gG' = \cdot 556 - \cdot 415 = \cdot 141 \text{ or } gG'' = 1 \cdot 229.$$

So that there is 7·71 times as much water present as steam at the point o off. In fact

$$w \frac{1 \cdot 229}{\cdot 141} \times \cdot 173 = 1 \cdot 508 = \cdot 173 + z + \cdot 173y = \cdot 173 + z + \cdot 173 \times 866z,$$

Whence $z = 1 \cdot 16$, $y = 1 \cdot 005$ or $yi = \cdot 174$.

That is, the whole $w = 1 \cdot 508$ is made up of (at cut off) indicated 0·173, densed 0·174, water already there 1·16. **A very striking sort of result.**

**217.** Exercise. Given $v_1$, the volume in cubic feet of steam at the e admission, the indicated steam is $i_1 = \frac{v_1}{u_1}$ lb. Suppose $x$ lb. (we called it the last exercise) to have condensed during admission, its latent heat $l_1$ been given to $z$ lb. already in the cylinder to heat it from $t_2$ to $t_1$, so that

$$z(t_1 - t_2) = l_1 x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let $p_2$ and $v_2$ be the pressure and volume at any other part of the expa curve, $i_2$ being the weight of indicated steam there; if we assume that the of the cylinder is perfectly non-conducting, we can calculate $x$ and $z$.

Let the entropy of 1 lb. of water be $\phi$, and let $i_1 + x + z = w$.

Then

$$\frac{w}{i_1} = \frac{gG''}{gG'} \text{ and } \frac{w}{i_2} = \frac{fF''}{fF'}$$

and we find that letting $a$ be $l/t$

$$w = \frac{a_2 i_2 - a_1 i_1}{\phi_1 - \phi_2} = i_1 + x + z \quad . \quad . \quad . \quad . \quad . \quad (2)$$

(1) and (2) are equations connecting $x$ and $z$; we find the unknowns to be

$$z = \left(\frac{a_2 i_2 - a_1 i_1}{\phi_1 - \phi_2} - i_1\right) \div \left\{1 + \frac{1}{a}\left(1 - \frac{t_2}{t_1}\right)\right\}$$

and

$$x = z\left(1 - \frac{t_2}{t_1}\right) \div a_1.$$

Let us take the following examples.

In every case $p_1 = 101 \cdot 9$, $\theta_1 = 165°$ C., $p_2 = 52 \cdot 52$, $\theta_2 = 140°$ C.

Also let the indicated quantity of steam be such that $v_1 = 1$ cubic foot as $u_1 = 4 \cdot 302$, $i_1 = \cdot 232$). I find that if we let $v_2 = 2v_1(1 - \beta)$ the wo simplified. I get the following results when expansion is according to the $pv^k$ constant.

| $k$. | $z$. | $x$. | $w$. |
|---|---|---|---|
| 0·8 | 1·0760 | ·2195 | 1·5275 |
| 0·9 | 0·6391 | ·1303 | 1·0014 |
| 1·0 | 0·3225 | ·0658 | ·6203 |
| 1·1 | 0·0792 | ·0162 | ·3274 |
| 1·2 | −0·1124 | −·0229 | |
| 1·3 | −0·2696 | −·0550 | |

For values of $k$ greater than 1·130, as, indeed, we know by the table, 211, we see that there can be no adiabatic for steam of the shape $pv^k$ const

EXERCISE.

Mr. Willans took $pv^{7/6}$ = a constant, as the law of adiabatic expansion. If any such law as $pv^k$ constant, holds between two points in an adiabatic expansion curve, $p_1$ and $p_2$, find how much water must have been present at the beginning of the expansion, and how much at $p_2$.

*Answer.* Take $v$ to be the volume of the steam only, neglecting the volume of the water. Then $v_1 = u_1 x_1$ if there are $x_1$ lb. of steam in 1 lb. of the stuff and $v_2 = u_2 x_2$. If $\phi_1$ is the entropy of 1 lb. of water at $\theta_1$° F., the entropy of the stuff we deal with is

$$\phi_1 + \frac{l_1}{t_1}\frac{v_1}{u_1} = \phi_2 + \frac{l_2}{t_2}\frac{v_2}{u_2} \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$\text{also } p_1 v_1^k = p_2 v_2^k \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

And $u_1$ and $u_2$ are known as $p_1$ and $p_2$ are known, so that (1) and (2) enable the two unknowns $v_1$ and $v_2$ to be calculated. Thus from (2) we have

$$v_2 = v_1 \left(\frac{p_1}{p_2}\right)^{1/k}$$

and hence from (1), since $\frac{v_1}{u_1} = x_1$

$$x_1 = (\phi_1 - \phi_2) \div \left\{\frac{l_2}{t_2}\left(\frac{p_1}{p_2}\right)^{1/k}\frac{u_1}{u_2} - \frac{l_1}{t_1}\right\}$$

Similarly,

$$x_2 = (\phi_1 - \phi_2) \div \left\{\frac{l_2}{t_2} - \frac{l_1}{t_1}\left(\frac{p_2}{p_1}\right)^{1/k}\frac{u_2}{u_1}\right\}$$

Thus taking the Willans $pv^{7/6}$ constant as an adiabatic, $p_1 = 100$, $p_2 = 50$, then it is found that $x_1 = 1{\cdot}22$, and $x_2 = 1{\cdot}15$. That is, it is impossible for $pv^{7/6}$ to be an adiabatic for saturated steam, since $x_1$ and $x_2$ are greater than unity. Again no such law for the adiabatic can hold in superheated steam. Taking the ratio of the specific heats to be 1·3 (the usual assumption) $pv^{1{\cdot}3}$ constant, is the adiabatic. The table Art. 211, confirms this conclusion concerning the Willans' assumption.

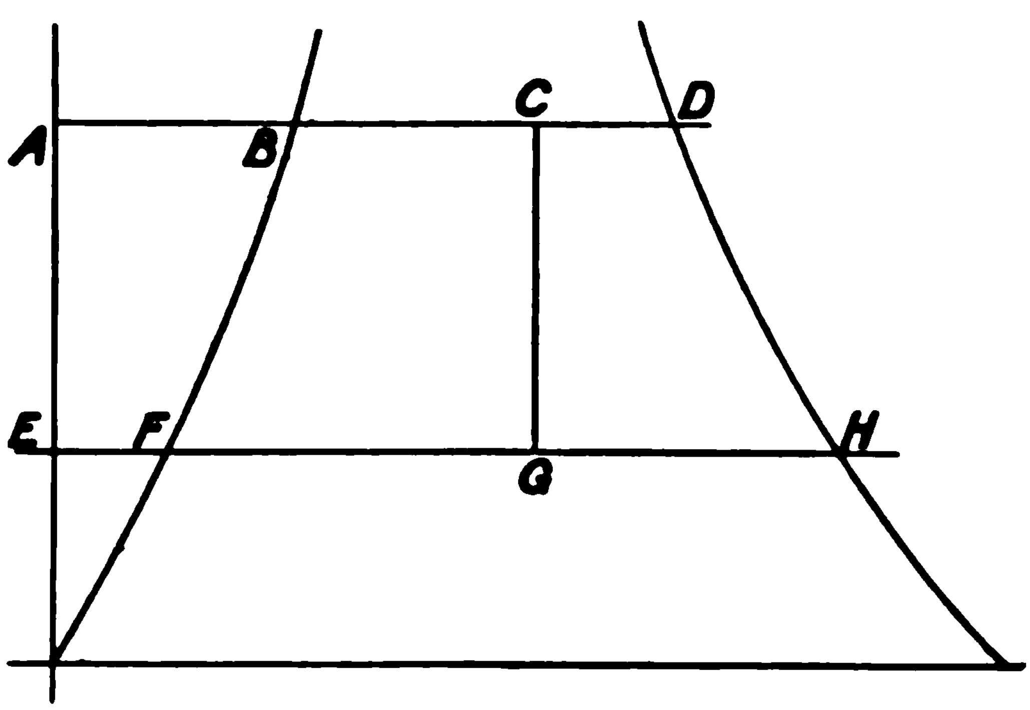


FIG. 230.

**218. Flow of Saturated Steam.** In Art. 387 it is shown that if steam at rest at $\theta_1$, whose state is $x_1$ lb. of steam to $1-x_1$ lb. of water, flows adiabatically to a place where the temperature is $\theta_2$ and the state is $x_2$ and the velocity $V$ feet per second, and if we neglect gravity, we can find $V$ and $x_2$ by the $\theta\phi$ diagram.

Let $AD$ and $EH$ correspond to the two temperatures.

Make $BC/BD = x_1$. Draw the adiabatic $CG$.

Then $FG/FH = x_2$. One of our answers.

Convert the area $BCGF$ into foot-pounds, multiply by 64 extract the square root and we find $V$.

**Exercise.** Steam with 10 per cent. of moisture, at 100 l square inch, escapes adiabatically to a place where the pres 16 lbs. per square inch, find the wetness and the velocity.

*Answer.* 21·5 per cent. wet; $V = 2{,}430$ feet per second. be seen in Art. 391 that this answer is misleading.

How much of this steam (pounds per second) will pass t

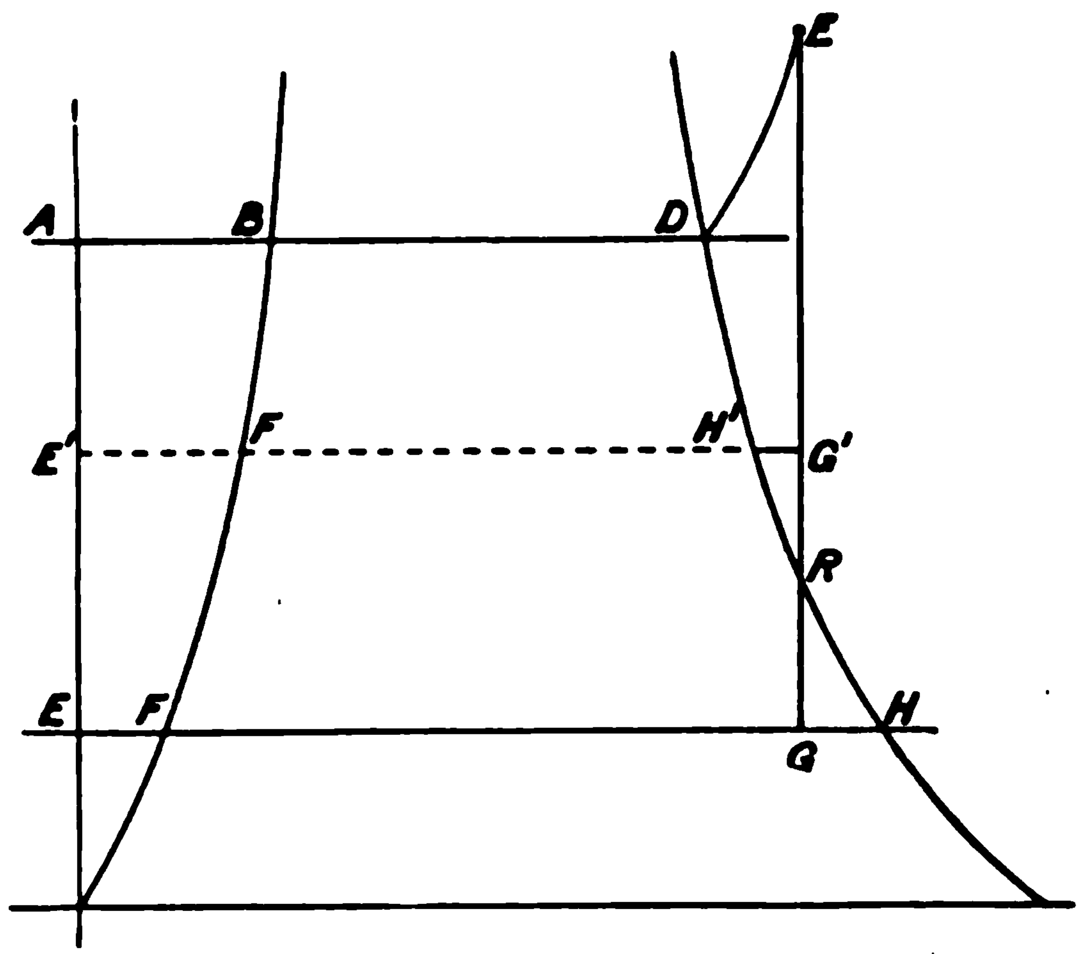


Fig. 231.

an orifice if the cross section of the jet where the stream line are nearly parallel just outside the orifice is 1 square inch?

**219. Flow of Superheated Steam.** In Art. 387 it is that if superheated steam at rest at $\theta_1$° C. and pressure $p$ adiabatically to a place where the temperature is $\theta_2$° C., and may neglect gravity, we can find the velocity $V$ and the s the steam. Let $E$, Fig. 231 show the state of 1 lb. superheated steam at $\theta_1$° C. and pressure $p$. Draw the ad $ERG$, the lower temperature being at $EH$. Then $FG/FH$ = dryness at the lower temperature.

Convert the area $BDERGFB$ into foot-pounds, multiply l and extract the square root, this gives $V$. If the stuff r superheated at the lower temperature as $E^1G^1$, we treat tl $BDEG^1H^1F^1B$ in the same way.

## CHAPTER XXIV.

### CYLINDER CONDENSATION.

**220.** WATT'S great improvement of the Newcomen engine consisted in **keeping the cylinder warm**; not condensing the steam in the cylinder itself, but using a separate condenser. Even now, however, a cylinder is heated up by the condensation of the entering steam, and the condensed water boils away during the exhaust. A cylinder is alternately a condenser and a boiler. If we could make its material absolutely non-conducting and keep it perfectly drained of water, we should get rid of this prejudicial action. Unfortunately, an amount of water, which forms only an exceedingly thin skin, may have sufficient capacity to produce great evil effects, and **non-conductivity of metal would then be an evil** (see Art. 399). It is my belief, based on a good deal of practical knowledge of conductivity of heat, that if the metal of a cylinder were quite dry, when fresh steam is admitted, the surface resistance to the passage of heat would be so great that almost no evil effects would be produced at the speeds usual in steam engines.

Probably, what would diminish it more than anything else would be the admixture with the steam of a small quantity of air (easily done on locomotives at the ordinary injector) or an injection of flaming gas, or some vapour less readily condensed than steam, or the use of the same cylinder as a steam and a gas engine in alternate strokes.

I am informed that some careful experiments made in America showed no great increase of economy due to the admission of air. I have been too busy to study the method of experimenting employed, and my attitude towards other people's experiments is that of Mrs. Bormalack on soup.

It was Mr. Clark who first drew attention to the missing water in cylinders, and the evil effects of too early a cut-off, but he states that the common engine-drivers were perfectly well aware of the phenomenon before he knew it. Mr. Isherwood showed that the missing water increased in proportion to the square root of $r$.

**221.** Every test yet made of the **effect of superheating** shows that it leads to greatly increased economy. From 12 to 20 per cent. increase is not uncommon when the superheating has only been about 40 to 100 degrees Fahrenheit.

A compound Corliss gear mill engine, with steam jacketed cylinder, gave on careful trials the following results:—

| | $I$. Indicated power. | $W$. lb. of steam per hour. | $\frac{W}{I}$ |
|---|---|---|---|
| Using saturated steam at 96 lbs. per sq. in. | 475 | 9380 | 19·75 |
| Using superheated steam at 99 lbs. per sq. in. superheated 118° Fahrenheit . . . . . . | 491 | 7675 | 15·63 |
| Using superheated steam at 94 lbs. per sq. in. superheated 127° Fahrenheit . . . . . . | 502 | 7835 | 15·61 |

Quite recently in the Schmidt compound condensing engine, of 75 indicated horse-power, only 10½ lbs. of steam was used per hour per indicated horse-power. The steam was of 170 lbs. pressure, and was superheated 300° Fahrenheit. An engine must be specially arranged for the use of such high temperature steam.

When condensation is exceptionally bad, the increase of economy due to the use of superheating is exceptionally marked. Mr. Ripper, in his tests of a small non-condensing one-expansion Schmidt engine (Proc. Inst. C.E., Vol. 128, of 1897), found a consumption of 38 lbs. of steam per horse-power hour, reduced to 17 by 300 degrees of superheating. The extra heat required is inconsiderable when we compare it with the advantage derived from superheating. It is to be remembered that on account of the small specific heat of steam, even a large amount of superheat only suffices to just prevent condensation in the cylinder. The evil action on the lubricant is not so great as it might seem to be.

**222. The effect of a steam jacket** is to cause a flow of heat into the cylinder which continually tends to diminish the amount of water present, not only in the cylinder but about the valves. In every case when an engine is tried without and with the jacket it is found that a small expenditure of steam in the jacket causes a great diminution of the missing water. In the Report of the Committee of the Institution of Mechanical Engineers, although the jacket feed was usually from 7 to 12 per cent. of the whole of the steam used by the engine, yet, on the whole, there was 9 to 25 per cent. diminutio

ːam per horse-power hour. The increased economy is most notice- in engines which are very uneconomical without the jacket. ʼrofessor O. Reynolds found in his engine, using three expan- ;, that without jackets, the missing water in his intermediate low pressure cylinders was ⅜ of the indicated water, whereas, ı they, as well as the high pressure cylinder, were jacketed with ull boiler pressure, the initial condensation in the intermediate ıder was only about 20 per cent. of the indicated steam, and e was, practically, no condensation in the low pressure cylinder.

ʿhe following experimental Results must also be studied :—

**Effects of Jackets in Condensing Engines.**

| ription. | Indicated horse-power. | Boiler pressure. | Piston speed. | Steam per indicated horse-power hour. | Coal per indicated horse-power hour. | ·r Steam per horse-power hour, perfect steam engine. | Simple, s. Compound, c. Triple, T. | |
|---|---|---|---|---|---|---|---|---|
| ızontal | 41·1 | 75 | 412 | 32·14 | 3·53 | 9·35 | S | No jacket. |
| | 38·0 | | 373 | 26·69 | 2·94 | | S | Jacket. |
| ıss . | 146 | 75 | 551 | 22·57 | 2·51[1] | 9·38 | S | No jacket. |
| | 159 | | 548 | 19·80 | 2·20[1] | 9·31 | S | Jacket. |
| ss . | 508 | 75 | 520 | 19·77 | 2·19[1] | 9·34 | S | No jacket. |
| | 488 | | 521 | 19·27 | 2·14[1] | | S | Jacket. |
| am | 65·8 | 77 | 176 | 23·84 | 2·65[1] | 9·65 | C | No jackets. |
| ping | 81·2 | | 221 | 19·41 | 2·16[1] | | C | Jackets. |
| am | 162 | 64 | 200 | 18·2 | 2·02[1] | 9·71 | C | No jackets. |
| ping | 168 | | 212 | 16·6 | 1·85[1] | | C | Jackets. |
| rted | 140 | 145 | 138 | 17·22 | 1·91[1] | 8·12 | T | No jackets. |
| ping | 138 | | 137 | 15·45 | 1·72[1] | | T | Jackets. |

**Effects of Superheating on Condensing Engines.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ı . . . | 136 | 66 | 335 | 21·5 | 2·39[1] | 9·66 | S | Saturated. |
| | 107 | 71 | 335 | 19·41 | 2·16[1] | 9·50 | S | Superheated. |
| | 99·5 | | 335 | 19·25 | 2·14[1] | | S | Saturated. |
| | 113 | | 335 | 16·16 | 1·79[1] | | S | Superheated. |
| ontal | 475 | 111 | 471 | 19·75 | 3·15 | 8·58 | C | Saturated. |
| | 496 | | 474 | 15·62 | 2·55 | | C | Superheated. |

ʿhese numbers for coal were not measured ; they are calculated at the rate of of steam per pound of coal.

The good effect produced by a jacket gives proof that—in spite of what is almost universally stated by men who have studied this subject—a cylinder, even when there is considerable superheating of the steam before it enters, is not free from water when admission occurs. It is my opinion that jacketing ought to be done by steam of a very much higher temperature than that of the steam which enters the cylinder.

**223. The good effects due to drainage,** or easy escape of water, are not sufficiently thought about. In my opinion it is to this easy drainage that the Willans' engine owes its superiority.

Rot

If a pound of steam entering at $\theta_1$° C. drains away at the exhaust temperature $\theta_3$° *C.*, it has given to the cylinder the heat

$$606{\cdot}5 + {\cdot}305\,\theta_1 - \theta_3$$

If a pound of steam entering at $\theta_1$° C. **condenses**, and if it evaporates and leaves the cylinder as steam at $\theta_2$° C., it has given to the cylinder the heat

$${\cdot}305\,(\theta_1 - \theta_3)$$

For example, let $\theta_1 = 165°$ C., $\theta_3 = 60°$; in the first case, the heat is 597 units; in the second case, it is 33 units.

We see by this crude calculation that in a condensing engine, water that drains away mechanically gives about 20 times as much heat to the cylinder as if it were condensed on admission and re-evaporated in exhaust.

I am even disposed to believe that steam used in a steam jacket is not much more efficient than, even if it is so efficient as, steam allowed to condense and drain away from a well-lagged cylinder.

**224.** In a steam engine cylinder there is a condition of things which **may almost be called instability.**

Joker.

It may almost be seen from the above figures how enormous condensation and evaporation may go on, doing great evil, for the purpose of supplying an amount of heat which a twentieth or a thirtieth of the amount of condensation would supply if there was drainage or a steam-jacket. I have heard of an agent who bought a hundred thousand pounds' worth of utterly unnecessary supplies for an army, which he knew would be wasted, because he had a perquisite of 5 per cent.; I have known of an admiral wasting eight days' coal of a fleet to prevent a two days' delay in the reception of a few private letters. Charles Lamb tells us how the first discoverer of the gastronomical value of roast pork burnt down a house every time he wanted a roast. These are not unfair illustrations of the economical conditions under which the cylinder of an ordinary *engine* is kept fairly dry.

**225. Benefit of Successive Expansion.** We find that the percentage of the total steam condensed increases if we cut off earlier in the stroke; possibly it is not that there is more steam actually condensed per stroke, but that it is in a greater ratio to what is indicated. Now it is evident from Art. 214 and elsewhere that we get more economy by using high pressure steam and great expansion, and as great expansion in one cylinder leads to great condensation, we **use two or three cylinders**, Fig. 65. To cut off at th of the stroke in a single cylinder is not very different from cutting off at half stroke in three successive cylinders. It makes a more complicated looking engine, but there are these great advantages:—

1. We are able to use a very simple kind of valve gear.
2. The loss by clearance is small.
3. There is a possibility of balancing the forces acting on the frame of the engine and ground; a possibility of obtaining more uniform turning moment on the crank shaft.
4. The range of temperature in each cylinder is only a third of what it is in a single cylinder. It is found that steam condensed in the high pressure cylinder is more or less completely evaporated before admission to the second.
5. The intermediate and low pressure cylinders may be [and always ought to be] jacketed with high pressure steam, so that in these there need be hardly any condensation.
6. There is less than one-third of the leakage past valves and pistons (see Art. 232).
7. Considerations such as (3) show that much higher speeds may be used.
8. In a great number of cases, the machines to be driven run at high speeds; the high speed of the engine allows of direct coupling and so there is much less loss of energy by friction and much greater convenience because of the smaller space occupied.
9. The cost of engines for the same power and economy is less.

There is a disadvantage due to drop of pressure after release in each cylinder, but in truth this is about counterbalanced by the drying of the steam which it produces. With more superheating, or better jacketing or drainage, these drops may be reduced with advantage.

As to the condensation being less when the expansion occurs in two or three cylinders instead of one, this has been proved by many careful tests. Thus Professor Unwin found that when a two-cylinder

engine was driven, and afterwards its larger cylinder alone was used with the same total expansion, he obtained the following results, $W$ being lb. of steam per hour and $I$ the indicated power:—

EFFECTS OF JACKETS AND SUCCESSIVE EXPANSION.

| | $W \div I$ Without steam in jackets. | $W \div I$ With steam in jackets. | Consumption of steam in jacket as a fraction of the whole. |
|---|---|---|---|
| | | | per cent. |
| Single . . . . . | 32·1 | 26·7 | 7 |
| Compound . . . | 22·1 | 19·5 | 12 |

Single cylinder engines are used when the initial pressure is not much more than 80 lbs. (condensing) or 90 lbs. (non-condensing). Two-expansion engines are used up to initial pressures of about 130 lbs per square inch. Three-expansion engines are used for higher pressures. There are no exact rules. The use of four-valve gear such as the Corliss, allows us to have economy with more expansion at considerably higher pressures than when the slide valve is used.

**226.** We find always that **increased speed** means increased economy, and this seems to be altogether due to the fact that at higher speeds there is less missing water per stroke.

The following figures from the non-condensing trials of Mr Willans illustrate the effect of speed, and also of compounding and tripling on the same engine. $y$ means the ratio of the missing steam at the cut-off in the cylinder of highest pressure to the indicated steam: $W$ is the total weight of steam used in pounds per hour, and $I$ is the indicated horse-power, $n$ being the revolutions per minute, $r$ the total ratio of expansion.

EFFECTS OF SUCCESSIVE EXPANSION AND SPEED.

| | $n$ | $r$ | $p_1$ | $y$ | $W/I$ |
|---|---|---|---|---|---|
| Simple . . . . . | 400 | 4·6 | 106 | ·420 | 26 |
| Compound . . | 400 | 4·9 | 109 | ·128 | 21·4 |
| Triple . . . . . | 400 | 6·0 | 152 | ·056 | 19·7 |
| Simple . . . . . | 138 | 4·32 | 109 | ·802 | 31·22 |
| Compound . . . | 124 | 4·36 | 110 | ·337 | 24·73 |

I find that as a rule in wet cylinders the condensation is halved when the speed is quadrupled, whereas in fairly dry cylinders, well jacketed and drained, the condensation is halved when the speed is doubled. I mean that there is a tendency to some such difference

of law, but the following results show that there is no very exact law.

WILLANS' CONDENSING COMPOUND. EFFECTS OF SPEED.

| $p_1$ | $n$ | $r$ | $y$ | $W/I$ |
|---|---|---|---|---|
| 90 | 401 | 4·8 | ·098 | 17·3 |
| | 301 | | ·139 | 17·6 |
| | 198 | | ·218 | 18·9 |
| | 116 | | ·264 | 20·0 |

**227.** The state of things inside a steam engine cylinder so nearly approaches instability that the student must be specially careful in adopting off-hand assumptions which may seem reasonable. For example, such a calculation as that of Art. 223, where I glibly speak of the heat given to the cylinder by steam condensing at the initial pressure, and evaporating at the exhaust pressure, is misleading, although it happens not to be utterly wrong, as so many reasonable looking assumptions are, which one finds in books and quasi-scientific papers. The neglected part of that calculation is what occurs at intermediate temperatures, and particularly in the expansion (see Art. 400).

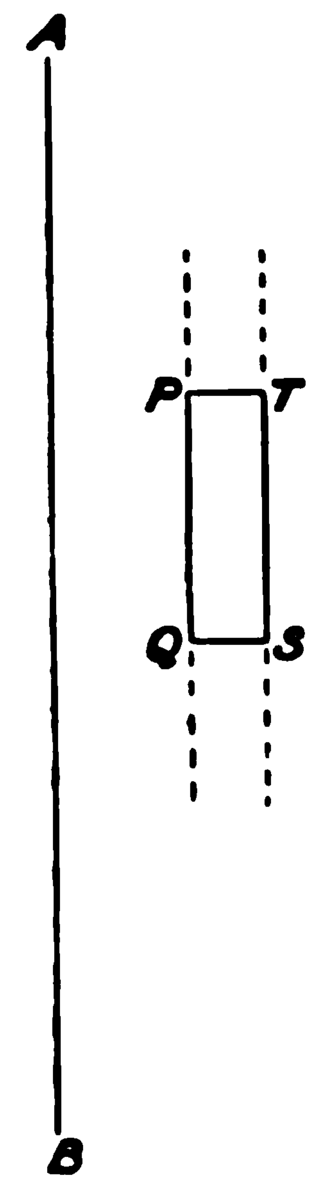


FIG. 232.

It is really necessary to take up **one or two problems which can be worked out accurately mathematically**, and use the answers merely as suggestions in our study of the cylinder.

If an infinite block of material, supposed to be homogeneous, has a plane face, $A B$. If at the point $P$, which is at the distance $x$ from $A B$, the temperature is $v$, and we imagine the temperature the same at all points in the same plane as $P$ parallel to $A B$ (that is, we are only considering flow of heat in a direction at right angles to the plane $A B$), and if $\frac{dv}{dx}$ is the temperature gradient at $P$, then $-k\frac{dv}{dx}$ is the amount flowing per second through unit area like $P Q$, in the direction of increasing $x$. This is really the definition of $k$, the conductivity of a material. I shall imagine $k$ to be constant. Let us imagine $P Q$ exactly a square centimetre in area. Now what is the flow across $T S$, or what is the value of $-k\frac{dv}{dx}$ at the new place, which is $x + \delta x$ from the plane

$AB$? Observe that $-k\frac{dv}{dx}$ is a function of $x$, call it $f(x)$ for a moment the space $PQTS$ receives heat $f(x)$ per second, and gives out $f(x+\delta x)$.

$$f(x+\delta x) - f(x) = dx \,.\, \frac{df(x)}{dx}.$$

This equation is, of course, true only when $\delta x$ is supposed to be small smaller without limit. We see then that $-\delta x\frac{d}{dx}f(x)$ is the heat being to the space $P\,Q\,T\,S$ every second; this is

$$-\delta x\frac{d}{dx}\left(-k\,.\,\frac{dv}{dx}\right),\text{ or } k\,.\,\delta x\,.\,\frac{d^2v}{dx^2}.$$

But the volume is $1 \times \delta x$, and if $\rho$ is the weight per cubic centimetre, and the specific heat, then if $t$ is time in seconds, $\rho\,.\,dx\,.\,s\,.\,\frac{dv}{dt}$ also measures tl per second at which the space receives heat. Hence

$$k\,.\,\delta x\,.\,\frac{d^2v}{dx^2} = \rho\,.\,\delta x\,.\,s\,.\,\frac{dv}{dt}$$

or

$$\frac{d^2v}{dx^2} = \frac{\rho s}{k}\,.\,\frac{dv}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

It will be found that there are innumerable solutions of this equatio there is only one which suits particular surface and other given cond The beginner ought to take up the following problem—

Imagine the average temperature everywhere to be 0, and that

$$v_0 = a \sin 2\pi nt, \text{ or } a \sin qt \quad \ldots\ldots\ldots\ldots \quad (2)$$

is the law according to which the temperature changes at the skin where : $n$ or $q/2\pi$ means the number of complete periodic changes per second. carefully examined the cycle of temperature change in the clearance spac steam cylinder, and it follows sufficiently closely a simple harmonic la Art. 229) for us to take this as a basis of calculation. Take any period one pleases, it consists of terms like this, and any complicated case is studied. Considering the great complexity of the phenomena occurrin steam cylinder, I think this idea of simple harmonic variation at the s of the metal to be a good enough hypothesis for our guidance. It is sho the note that the range ($2a$) of temperature of the actual skin is much les that of the steam, being the range in the steam multiplied by $e$, the emis at the surface, and divided by $\sqrt{2\pi ncsk}$. I am not now considering the in the cylinder, on the skin and in pockets, as requiring itself to be heate cooled; this heating and cooling occurs with enormous rapidity, and is pr nearly independent of the speed of the engine. Drainage will get rid of of this water, and drainage has another advantage so great that I am in to think drainage much more important than steam-jacketing. But besid evil function of the water, the layer on the skin acts as greatly increasing so causing the range ($2a$) to be greater.

The student ought to try if the equation (1) has a solution like

$$v = A\epsilon^{ax}\sin.\,(qt + \gamma x),$$

and if so, find $a$ and $\gamma$, and make it fit the case in which $v = 0$ when $x =$ ∞ $v = a\sin.\,qt$ where $x = 0$. By actual trial we find that

$$v = A\epsilon^{ax}\sin.\,(qt + ax) + B\epsilon^{-ax}\sin.\,(qt - ax) \quad \ldots \quad (3)$$

Where $A$ and $B$ are any constants, and $a = \sqrt{\frac{\pi n \rho s}{k}}$ if $q = 2\pi n$. Now if $v = 0$ when $x = \infty$, obviously $A$ is 0. If $v = a \sin. qt$ where $x = 0$, obviously $B$ is $a$, and hence at any place and at any time

$$v = a\epsilon^{-ax} \sin. (2\pi nt - ax) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)[1]$$

This is the answer for an infinite mass of material with one plane face. It is approximately true in the wall of a thick cylinder, if the outside is at the temperature 0. If the outside is, with very little fluctuation, at an average temperature $v'$, and the thickness of the metal is $b$, and if the inside skin has the average temperature $v''$ (in our case 0), we have only to add the terms $v'' + \frac{v' - v''}{b} x$ (in our case $0 + \frac{v'}{b} x$) to the expression (4). This shows how a steam-jacket affects $v$. If $v'$ is made negative, we have an approximate representation of what occurs in a well-lagged unjacketed cylinder.

The result ought to be very carefully studied. Take for example $a = 10°$ C. $v' = 50°$ C., $b = 3$ centimetres, $k = \cdot 16$, as it probably is in cast iron, although even in iron we do not know $k$ within 50 per cent. Take $n = 2$ which corresponds to 120 revolutions per minute; for any particular value of $t$ find $v$ for various values of $x$, and show your answers by a curve. Now take other values of $t$ and repeat, and show all the curves in different colours on one sheet of paper. I advise a curve for each of the following values of $t$—0, 0·1, 0·2, 0·3, 0·4, 0·5. I might waste ten pages of this book on an interesting study of these

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[1] The emissivity at the metal surface is $e$, which means that

$$(v_0 - \theta)\, e = k \frac{dv}{dx} \text{ where } x = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

if $\theta$ is the temperature of the steam at any instant, and $v_0$ that of the metal at the surface. The thickness of metal $b$ is supposed to be so great that there are no fluctuations of temperature where $x = b$. It is easy to show that the temperature at any point in the metal is

$$v = \theta_0 + \frac{kv'}{eb + k} + x \frac{v'e}{eb + k} + a\epsilon^{-ax} \sin. (2\pi nt - ax) \quad . \quad . \quad . \quad (2)$$

$$\theta = \theta_0 + \frac{aka}{e}\left\{\left(1 + \frac{e}{ka}\right) \sin. 2\pi nt + \cos. 2\pi nt\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Of course $\theta$ may also be written

$$\theta = \theta_0 + \frac{a}{e} \sqrt{\{(ka + e)^2 + k^2a^2\}} \sin. \left(2\pi nt + \tan.^{-1} \frac{ka}{ka + e}\right)$$

We see that the effect of the steam-jacket keeping the outer surface of the metal at a temperature which is higher than $\theta_0$ by the amount $v'$ is to raise the average temperature of the inner surface by $kv'/(eb + k)$ above that of the steam. As the surface resistance gets greater and greater ($e$ less and less) the mean inner surface temperature gets to be nearer and nearer that of the outer surface of the metal.

If the amplitude of the steam temperature is called $A$ (this is called $\frac{1}{2}(\theta_1 - \theta_3)$ elsewhere), the amplitude $a$ of the inner surface of the metal is, since $ka = \sqrt{\pi n \rho s k}$

$$a = \frac{Ae}{\sqrt{\pi n \rho s k + (e + \sqrt{\pi n \rho s k})^2}}$$

If $e$ is small $a \propto Ae/\sqrt{n}$

If $e$ is large $a = A$.

curves, but the student will get more good from his own study of them th
reading.

At any point at the depth $x$ there is a simple harmonic rise and fall i
time of one revolution of the engine; but the range gets less *rapidly* a
depth is greater; note also that the changes lag more as we go deeper. T
exactly the sort of thing observed in the buried thermometers at Crai
Quarry, Edinburgh. The changes of temperature were of twenty-four l
period, noticeable only at shallow depths, and also of one year period, notic
at considerable depths. I give the yearly periodic changes, the average resu
eighteen years' observations.

| Depth below surface. | Yearly range of temperature (Fahrenheit). | Time of highest temperature. |
|---|---|---|
| 3 feet | 16·14 | August 14 |
| 6 ,, | 12·30 | ,, 26 |
| 12 ,, | 8·43 | September 17 |
| 24 ,, | 3·67 | November 7 |

Observations at twenty-four feet below the surface at Calton Hill, Edinb
showed highest temperature on January 6th.

Now let us from (4) find the rate per second at which heat is flowing th
a square centimetre, that is, find $-k\dfrac{dv}{dx}$ at any instant, where $x = 0$, us
for $\sqrt{\pi n p s/k}$. I find it to be

$$k a a \sqrt{2} \sin.\left(2\pi n t + \frac{\pi}{4}\right).$$

The note gives the true form of the steam-jacket term, $e$ being the emi
at the surface. The steam-jacket sends in heat at the rate $kev'/(eb + k)$ per se
The amount flowing into the metal then during the half period $\frac{1}{2}T$ (or $\frac{1}{2n}$ i
the frequency or number of periods per second) is the integral of the rate, o

$$a\sqrt{2kps/n\pi} - kev'/2n(eb + k)$$

and the amount *out* of the metal is the same except that the jacket te
positive.

When $e$ is small, the note tells us that $a$ is

$$\frac{1}{2}(\theta_1 - \theta_3)e/\sqrt{2\pi n p s k}$$

so that the maximum amount of heat flowing into the metal in one cycle is

$$\frac{(\theta_1 - \theta_3)e}{2n\pi} - \frac{v'}{2nk}.$$

Again, when $e$ is very large, $a$ is $\frac{1}{2}(\theta_1 - \theta_3)$, and the maximum amount of
flowing into the metal in one cycle is

$$\frac{1}{2}(\theta_1 - \theta_3)\sqrt{2kps}/\sqrt{n\pi} - kv'/2nb.$$

It will be seen that I shall make use of the steam-jacket term when I
of the causes, Art. 402, tending to keep the cylinder dry of water. The

continuous flow of heat due to the jacket is very important in this way; but as I shall speak now of the great flow of heat into the metal on admission, this heat coming out again during release and exhaust, I shall neglect the much smaller steam-jacket term in this connection. In a very dry cylinder the steam-jacket term would, however, be important even here.

**228.** Until last year I and others had always assumed that the **range of temperature** of the metal is something approaching half that of the steam; in fact, that $e$ is so large as to lead to the law

$$\text{Heat flow per cycle} \propto 1/\sqrt{n}.$$

I cannot now find the reference, but I am sure that I have seen evidence that the range of temperature in the skin of the metal was about half that of the steam.

The experiments of Professor Callendar have changed my opinion. For example, he found at 0·01 inch depth a range of 4° when the steam range was about 46° at 100 revolutions per minute. He calculated from $k$ and $s$ for iron that the surface range could only have been about 5°. Now I am not sure that I can accept his measurement of the real temperature at the depth 0·01 inch; there is much to be said in opposition to his view, but in deference to his judgment I have altered my notion of the usual value of $e$. If $e$ is small, the heat entering the metal per cycle is proportional to $n^{-1}$. If $e$ is large, the heat is proportional to $n^{-\frac{1}{2}}$. I have often used $n^{-2/3}$ and other powers of $n$ in obtaining empirical formulæ from experimental results. I am now disposed to say that in general I shall assume the heat entering the metal per cycle to be inversely proportional to $\sqrt{n} + cn$, where the $n$ term is more important in dry cylinders and the $\sqrt{n}$ term in wet cylinders. An examination of the results of actual trials of engines, Art. 234, will show that this is reasonable.

**229.** In the above investigation I have taken a simple harmonic change of temperature of the steam. I once sketched out at random a possible indicator diagram for a non-condensing engine with cut off at about half stroke, and one of my students found that the temperature of the steam followed the law

$$\theta = 126{\cdot}3 + 32{\cdot}3 \sin. (2\pi nt + 20^\circ),$$

the time, being measured from dead point, angularity of connecting rod neglected. Usually, of course, it cannot be so simple, but it is evident from the above investigation that the effect of the higher harmonics is small.

My students have taken a variety of hypothetical indicator diagrams, with cushioning, &c.; taking one second as the time of a revolution, they have drawn the curves showing temperature and

time; **they have developed the function in Fourier s**
each term of which is of course treated exactly in the same w
the above. I may say that I have given this exercise to studei
successive years rather as a good practical mathematical exercise
as one which it was worth while to do for the sake of the s
engine. In one year I took account of the fact that some poi
of the barrel surface have a different experience from the clea
surface, but in truth there is not much benefit derivable fron
vague speculative knowledge that we have of the effect oi
piston covering the place, the perpetual change in the surface
the conduction of heat from and to hotter and colder neighbo
places, &c.

Instead of giving the results arrived at so laboriously b
students—results some of which are perhaps incorrectly worke
—I may say that I think the following problem gives a t
suggestion.

**230.** If the infinite block of Art. 227 is all at $\theta_3$, and if suddenly its s
is exposed to steam at $\theta_1$ and kept at that temperature for the time $t$, th
that enters it per unit area is $e(\theta_1 - \theta_3)t$ if $e$ and $t$ are small, and it is

$$2(\theta_1 - \theta_3)\sqrt{\frac{\kappa\rho k}{\pi}}\sqrt{t}$$

if $e$ is large.

I have shown in the note[1] that these two cases lead to the following resu

[1] An infinite block of homogeneous material with a plane face, the tempe
everywhere being 0 till the time $t$ is 0, when suddenly the medium on the oth
of the plane face is kept at constant temperature $v_0$. Let the surface emissiv
$e$; let $v_1$ be the temperature of the skin at time $t$, and $v$ the temperature
depth $x$. Then, as before,

$$\frac{d^2v}{dx^2} = \frac{\kappa\rho}{k}\frac{dv}{dt} = q^2v \quad \ldots \ldots \ldots \ldots \quad (1)$$

I use $q^2$ to represent $\frac{\kappa\rho}{k}\frac{d}{dt}$. Hence

$$v = \epsilon^{-qx}v_1 \quad \ldots \ldots \ldots \ldots \quad (2)$$

$$\frac{dv}{dx} = -q\epsilon^{-qx}v_1 = -qv$$

$$kqv_1 = e(v_0 - v_1),$$

$$\text{or } v_1 = \frac{v_0}{1 + \frac{k}{e}q} \quad \ldots \ldots \ldots \ldots \quad (3)$$

Developing (3) in powers of $q$ or in inverse powers of $q$, we get two s
solutions, one easier to work with when $et$ is small, the other easier to work witl
$et$ is large.

Let $Q$ be the amount of heat which enters the block from $t = 0$, then $Q$
integral of $e(v_0 - v_1)$. This gives an example of the enormous practical value
**Heaviside's operator method** which may be easily understood and used

eat entering the metal per unit area during admission may be represented

$$e(\theta_1 - \theta_3)\left(g + \frac{h}{r}\right) \div n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

: $g$ and $h$ are constants if $e$ is small and to

$$(\theta_1 - \theta_3)\left(g + \frac{h}{r}\right) \div \sqrt{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

s large, if $r$ is the ratio of cut-off. Hence as we are only looking for a ing formula, I shall take it that during admission from $\theta_3$ to $\theta_1$, cut-off being h of the stroke, the heat that enters the metal per unit area is repred by

$$\cdot \ e \frac{g + \frac{h}{r}}{\sqrt{n} + cn} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

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tical tyro to solve problems regarded as insoluble by the very best orthodox ticians (see Mr. Heaviside's *Electro-magnetic Theory*, Chap. V. § 228). The rhich suit small values of $et$ are

$$\left(\frac{t}{a\pi}\right)^{1/2}\left\{1 + \frac{2t}{3a} + \frac{1}{3\cdot5}\left(\frac{2t}{a}\right)^2 \quad \frac{1}{3\,.\,5\,.\,7}\left(\frac{2t}{a}\right)^3 + \&c.\right\} + v_0(1 - e^{t/a}) \quad . \quad . \quad . \quad (4)$$

$$a(e^{t/a} - 1) - \frac{2}{(a\pi)^{1/2}}\left\{\frac{2}{3}t^{3/2} + \frac{4}{15}\frac{t^{5/2}}{a} + \frac{8}{7\,.\,15}\frac{t^{7/2}}{a^2} + \frac{16}{9\,.\,3\,.\,5\,.\,7}\frac{t^{9/2}}{a^3} + \&c.\right\}\Bigg] \quad (5)$$

$= spk/e^2$.

is small enough we see that

$$v_1 = 2v_0\sqrt{\frac{t}{a\pi}} \text{ and } Q = \epsilon v_0 t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

e motion of the piston is simple harmonic, and there are $n$ revolutions ite, if admission is exactly at a dead point, if cut-off is at $\frac{1}{r}$th of the the time of admission $t$, it is evident that $t$ is proportional to os. $^{-1}\left(1 - \frac{2}{r}\right)$. Calling this $t$ I have calculated its value and find that ughly be represented by the function of $r$, which I tabulate. It would be dd a constant to every value of $t$, because admission may be said roughly lace in all cases when the piston is, say, ten degrees from the dead point; cause no change in the character of the formula which I suggest.

| $r$ | $t$ | $20 + 146/r$ |
|---|---|---|
| 16 | 29 | 29 |
| 12 | 33·6 | 32 |
| 9 | 39 | 36 |
| 7 | 44·4 | 41 |
| 5 | 53·1 | 49 |
| 3 | 70·5 | 69 |
| 2 | 90 | 92 |
| $\frac{3}{2}$ | 109·5 | 117 |

ould be easy to obtain a simple function of $r$, which would be in more exact n to $t$, but it is evident that for my purpose even a roughly correct repre-

the $n$ term being more important in dry cylinders and the $\sqrt{n}$ term in cylinders where $e$ is presumably large. Also as $e$ ought to come into the for only when it is small, I shall take it that in this formula, our $e$ increas proportion to the wetness of the cylinder only when small and reaches a maxi value. In fact, if $w$ is the average weight of water present, and $S$ is the ave exposed area of the cylinder surface, I shall consider $e$ to be a function of $w$

$$e \propto \frac{w/S}{1 + mw/S} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Where $m$ is some constant; that is, the heat entering the metal per stroke i

$$\frac{wS}{S + mw} \times \frac{g + \frac{h}{r}}{\sqrt{n} + cn} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If $w$ is the water present at $\theta_3$° C. before fresh steam is admitted, the of heat during admission at $\theta_1$° C. due to the presence of water is $w(\theta_1 - \theta_3)$.

I take $\theta_3$ (the exhaust temperature) as the temperature of the water, pa no attention to the fact that the pressure rises during cushioning, becau maintain that if there is water present it can only be at very low speeds there is equilibrium of temperature between steam and water; the steam is lo superheated. My indicator, Fig. 90, has enabled me to get diagrams at n than 1,000 revolutions per minute, and I find that the cushioning curve al greatly with speed. Cushioning greatly diminishes, in fact, at smaller spe

I shall use $N$ to stand for $\sqrt{n} + cn$; I shall use $S$ to mean the ave surface of metal exposed to the steam. In any type of engine the clear area is proportional to the piston area; the rest of the average surface exp

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sentation will suffice. I shall therefore take it that when $e$ is small the heat ente the metal per unit area during admission may be represented by

$$e(\theta_1 - \theta_3)\left(g + \frac{h}{r}\right) \div n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where $g$ and $h$ are constants.

The solution which suits larger values of $e$ and $t$ is

$$v_1 = v_0\left[1 - \left(\frac{a}{\pi t}\right)^{1/2}\left\{1 - \frac{a}{2t} + 1\cdot3\left(\frac{a}{2t}\right)^2 - \&c.\right\}\right] \quad . \quad . \quad (8)$$

$$Q = ev_0\sqrt{\frac{a}{\pi}}\left(2t^{1/2} + at^{-1/2} - \frac{a^2}{2}t^{-3/2} - \&c.\right) \quad . \quad . \quad (9)$$

where as before $a = spk/e^2$.

Using only the first terms in $t$ we find

$$v_1 = -\frac{2}{\sqrt{t}}\frac{v_0}{e}\sqrt{\frac{spk}{\pi}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$Q = 2v_0\sqrt{\frac{spk}{\pi}}\sqrt{t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Now I find that I get a much more accurate representation of $\sqrt{t}$ than of an expression like $g + h/r$, so that the heat entering the metal per unit area du admission may be represented by

$$(\theta_1 - \theta_3)\left(g + \frac{h}{r}\right) \div \sqrt{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

before cut-off may roughly be taken to be some fraction of the cylindric surface exposed at cut-off, and so we may take it that the exposed surface may be expressed as proportional to

$$d^2 + b\frac{dl}{r}$$

where $d$ is diameter, and $l$ length of cylinder, and $b$ is a constant. The missing heat per stroke is then

$$\left(Se\frac{g + h/r}{\sqrt{N}} + w\right)(\theta_1 - \theta_3) \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

I take it that the amount of steam condensed to provide this heat may be obtained by dividing by $H_1 - \frac{1}{2}(\theta_1 - \theta_3)$.

The indicated steam per stroke is $\frac{1}{4}\pi d^2 l/144 r u_1$, and if $y = \frac{\text{condensed steam}}{\text{indicated steam}}$

$$y \propto \left(Se\frac{g + h/r}{\sqrt{N}} + w\right)\frac{\theta_1 - \theta_3}{H_1 - \frac{1}{2}(\theta_1 + \theta_3)}\frac{ru_1}{d^2 l} \quad . \quad . \quad . \quad . \quad (7)$$

Now I find that if $\theta_3 = 40°$ C. in condensing engines, and $110°$ C. in non-condensing engines, we may take it as roughly true that

$\frac{(\theta_1 - \theta_3)u_1}{H_1 - \frac{1}{2}(\theta_1 + \theta_3)}$ is proportional to $p_1^{-0.6}$ in condensing engines, and is a constant in non-condensing engines. This can easily be checked by a student, and is an interesting exercise. Hence

$$y \propto \left(\frac{S}{ld^2}\frac{gr + h}{\sqrt{N}} + \frac{wr}{d^2 l}\right)f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where $f$ is a constant in non-condensing and is proportional to $p_1^{-0.6}$ in condensing engines.

**231.** If we choose to imagine that in ordinary well-designed engines there is no water at the end of the exhaust, make $w = o$. As the clearance area is much the most important part of $S$, we may roughly take $S \div ld^2$ as the reciprocal of the dimensions of the cylinder, and this is perhaps most usually stated as $1/d$; and we have a working formula, assuming $e$ to be constant.

$$y \propto \frac{r + a}{(\sqrt{n} + bn)d} \text{ non-condensing.} \quad . \quad . \quad . \quad (1)$$

$$y \propto \frac{r + a}{(\sqrt{n} + \beta n)dp_1^{0.6}} \text{ condensing.} \quad . \quad . \quad . \quad (2)$$

**232. Leakage.**—Of the steam missing at cut-off, part is what leaks past valves and piston. This leakage is occurring during the whole cycle, and is probably proportional to $p_1 - p_3$. Messrs. Callendar and Nicholson in studying it, apply the laws of transportation of water through narrow passages (steam condensing on one side of the valve, passing through as water and evaporating on the other side). They find that in one balanced slide valve and three unbalanced examined by them, the leakage in pounds of steam per hour is equal to

$\cdot 02s(p_1 - p_3)/\lambda$, where $s$ is the perimeter of the port and $\lambda$ is what they call the mean overlap. The leakage per second in their experiments seemed to be nearly independent of the speed of reciprocation of the valve. As to their view of the way in which leakage takes place, they say, "So long as the valve is stationary, the oil film may suffice to make a perfectly tight joint; but as soon as it begins to move, the oil film becomes broken up and partly dissipated. Water is being continually condensed on the colder parts of the surface exposed by the motion of the valve. This water works its way through, and breaks up the oil-film under the combined influence of the pressure and the motion. The continual re-evaporation taking place in the exhaust tends to keep the valve and the bearing surfaces of the seat cool, and to maintain the leaking fluid in the state of water. The exhaust steam from the cylinder has the same tendency. The coefficients of viscosity of steam and water at the temperatures which occur in the steam engine are not accurately known. But whereas that of steam increases with rise of temperature, that of water diminishes very rapidly. It is not improbable that the quantity of water which can leak through a given crack under a given difference of pressure, may be from twenty to fifty times greater than the quantity of steam which can leak under similar conditions. This agrees with well-known facts in regard to leakage, and explains how it is that the leakage in the form of water is so great. A few simple experiments were made with regard to the transpiration of water and steam under the conditions in question, and the leakage in the form of water was more than twenty times as great, the water being at a temperature below boiling point. The motion both of the water and the steam, owing to the high velocity, was certainly turbulent or eddying, which would have the effect of greatly increasing the resistance as compared with that due to viscosity, if the motion were steady. For the case of steady motion, comparative tests were made of the relative values of the viscosity of water cold and hot. The measurements were not sufficiently accurate to give the law of the variation of the viscosity with temperature above 212°; but it appeared that the viscosity at 212° F. was only one quarter of that at 62° F., and that it continued to diminish very rapidly. Under the actual conditions of the valve-leak experiments, the water leak is more likely to have been between forty and fifty times the steam leak. An explanation is thus furnished of a possible form of leakage, indirectly due to condensation and re-evaporation, so many times greater than the steam leakage, which, alone, engineers have been in the habit

f contemplating, that it might well claim attention on its own ɔerits, apart from the very limited number of valves on which it has ıitherto been possible to make direct experiments.

"The analysis of a large number of observations, in addition to he few made by the authors, leads to the conclusion that all alves leak more or less when in motion, and that in many cases he greater part of the missing quantity is to be attributed to eakage of this description. Whatever the precise manner in which he leak takes place, it appears to be nearly proportional to the lifference of pressure and to be in most cases independent of the peed. In any case it appears probable that the leakage is connected in some way with the condensation taking place on the valve surfaces. If so, it may evidently be greatly reduced, if not entirely cured, by jacketing, or otherwise heating the valve seat, to minimise the condensation.

"These views have an important bearing on the design of valves. For low-speed engines, separate steam- and exhaust-valves should possess advantages over the ordinary slide valve. The superiority of the compound engine would also appear to be partly due to the great reduction of possible leakage."

**233.** The quantity of water which will pass per second through a capillary passage is proportional to

$$(p_1 - p_3)\frac{a^2}{s\lambda}$$

if $a$ is the cross-sectional area, $s$ the perimeter of the section, and $\lambda$ the length of the passage.

It is practically impossible to guess at the magnitude of these quantities in **a leaking valve or piston.** Very slight local differences of temperature in valves cause great warping, and we have the effects of wear also to consider in estimating the thickness of the water film between faces and seats of valves. Let us take $a$ as proportional to $d^2$ and $s$ and $\lambda$ each proportional to $d$ in similar engines if $d$ is the diameter of the cylinder. This would give us the leakage per stroke $\propto (p_1 - p_3)d^2/n$.

Dividing this by the indicated steam, and assuming roughly that $(p_1 - p_3)n_1$ is constant, we find that the portion of $y$ which is due to leakage is proportional to $r/nd$.

If, then, I am right in this rough generalisation, (1) of Art. 231 ought to be nearly correct as representing both condensation and leakage in non-condensing engines; whereas, in condensing engines, a term proportional to $r/nd$ ought to be added to (2).

*The Missing Quantity—Experimental Results.*

**234.** It has long been known from actual measurement that in a ingle-cylinder engine, $y$ the ratio of **missing steam** at cut-off to the ndicated steam, is greater as $r$ is greater, is greater as the speed is ess, and is greater in small cylinders than in large. Until 1888,

however, there was no experimental investigation the methods of which were sufficiently scientific to withstand criticism. Various formulæ were used to express the results, and they were supposed to be based on theories. For twenty years I have been in the habit of using

$$y = a \frac{r + b}{d(\sqrt{n} + cn)} \quad . \quad . \quad . \quad . \quad (1),$$

where $a$ and $b$ and $c$ are constants which alter with the nature of the engine; $r$ is the ratio of cut-off; $d$ is the diameter of the cylinder in inches. Also $a$ is a constant in non-condensing engines, but varies inversely as the square root of the initial pressure $p_1$ of the steam in condensing engines.

I have used $\sqrt{n} + cn$ and sometimes $n^{\frac{1}{2}}$ in the denominator, telling my students that I could not understand how the theory (Art. 227), admittedly defective otherwise, could be so wrong as I sometimes found it in this particular. I have already pointed out in Art. 228 in what way my old theory was defective.

Professor Cotterill's formula is

$$y = c \frac{\log. r}{d\sqrt{n}} \quad . \quad . \quad . \quad . \quad . \quad (2),$$

where $c$ is sometimes as little as 40 and sometimes as much as 100, both in condensing and non-condensing tests.

Professor Thurston uses

$$y = \frac{c\sqrt{r}}{d\sqrt{n}} \quad . \quad . \quad . \quad . \quad . \quad (3),$$

where $c$ is 30 in a fairly economical engine.

It is easy to show that however (2) and (3) may be made to agree with tests of non-condensing engines, they cannot be made to agree with the tests of condensing engines. Thus, for example, $y$ is supposed to be the same at a given $r$ and $n$, whether $p_1$ is 180 or only 45, whereas in the second case $y$ is usually found to be twice as great as in the first. I do not understand how any one considering the theory of the question can have left out the $p_1$ term in condensing engines. Messrs. Callendar and Nicholson have recently thrown out the suggestion

$$y = r \left\{ \frac{a}{np_1} + \frac{b}{dn^{\frac{1}{2}}} \right\} \quad . \quad . \quad . \quad (4),$$

which is certainly more promising than the others. I have not tried it yet, except on Willans' compound condensing trials, and these it certainly does not agree with, but of course it is only meant for a single-cylinder engine.

t is evident from the more complete theory of Chap. XXXIV. that mple formula can be expected to agree with good experimental ts. The only results which seem to me of scientific value are published by the late Mr. Willans in 1888 and in 1893, in rs read before the Institution of Civil Engineers. Students refer to these classical papers themselves for descriptions of the al valve engines actually employed, and the study of the results e best of all exercises.

*Non-Condensing Trials*, 1888.

**35.** The engine had three cylinders, areas of pistons 34·5, 71·47, 141·3 square inches; stroke, 6 inches. I use $r$ to mean the ratio e greatest volume to which the steam can expand in the engine e volume of steam of its initial pressure at cut-off. This de- on will suit either single, double, or triple expansion engines.

. *Single Cylinder Trials.*—Piston area, 141·34. The initial ure $p_1$ varying from 41 to 109 lbs. per square inch. The ratio t-off $r$ in every trial equal to about $p_1 \div 25$, the speed varying $n = 409$ to $n = 111$ revolutions per minute. I find that $y$ may ken as being fairly well represented by

$$y = 24 \frac{r}{d\sqrt{n}} \quad . \quad . \quad . \quad . \quad . \quad (1),$$

e $d$ is the diameter in inches, although there are some large epancies from the $\sqrt{n}$ law. I assume the law as to $d$, for this not tested in any way. In the trials, $r$ and $p_1$ were not separately d, so that if we had no guidance from theory we might take be equal to $p_1$, divided by $d\sqrt{n}$.

I. *Compound Trials.*—Cylinders 71·47 and 141·3 square inches ea, $y$ being $\dfrac{\text{steam missing at high cut-off}}{\text{indicated at high cut-off}}$. These trials were numerous, and were the most important.

'$d$ is the diameter of the high pressure cylinder, I find that

$$y = 120 \frac{r}{dn} \quad . \quad . \quad . \quad . \quad (2)$$

ies all the trials very well.

these trials of Mr. Willans he kept $r$ always nearly equal to varying $p_1$ and $r$ together, so that the above result may really e $p_1$ and may not be so simple as to $r$. But from the con- tions of Art. 230 I am inclined to think that (2) is correct and is independent of $p_1$; nevertheless we have no proof of this.

III. The triple expansion trials were few, only seven altogether We can only say that $y = 0{\cdot}057$ when $n = 400$ and $p_1$ is from 152 to 172 lbs. per square inch, $r$ varying from 6 to 6·5. If we take $y$ to be of the same form as in the compound trials, and if $d$ is the diameter of the highest pressure cylinder

$$y = 150\,\frac{r}{dn} \quad \ldots \ldots \quad (3).$$

In the above statements I have gratuitously assumed that $y$ is inversely proportional to $d$ in similar engines.

*Condensing Trials*, 1893.

**236.** The central valve engine used was not very different from that of the non-condensing trials, and is like one of the three shown in Fig. 233, except that these are compound only. When all three cylinders are used, steam enters the first cylinder by the ports $AA_1$ when the central valve in it moves up and opens $A$. When the port $A$ is opposite the metallic rings at the end of the cylinder, the steam is cut off: subsequently the central valve closes the port $A$ and places the port $A_1$ in communication with the receiver beneath the piston by the port $A_2$: on the return stroke the steam is transferred from the cylinder to the receiver. Thence the steam enters and leaves the second and third cylinders by the ports $B_1B_1$ and $B_2$ and other ports $C_1C_1$ and $C_2$ not shown in the figure, which is that of a compound engine only. On leaving the low pressure cylinder the steam is transferred in the same way to the under side of the piston [illegible] and on the next down stroke it is allowed [illegible] by the port $D$. In this way the lowest [illegible] the lowest pressure piston [illegible]

[illegible] the crank pin [illegible] tested on their [illegible] 22 [illegible] 14833 square inches [illegible] in the usual sense, [illegible]

[illegible] the friction of the [illegible] low pressure piston [illegible]

[illegible] that for the [illegible] engine as a [illegible] about 50 per cent [illegible] cylinder, and it was

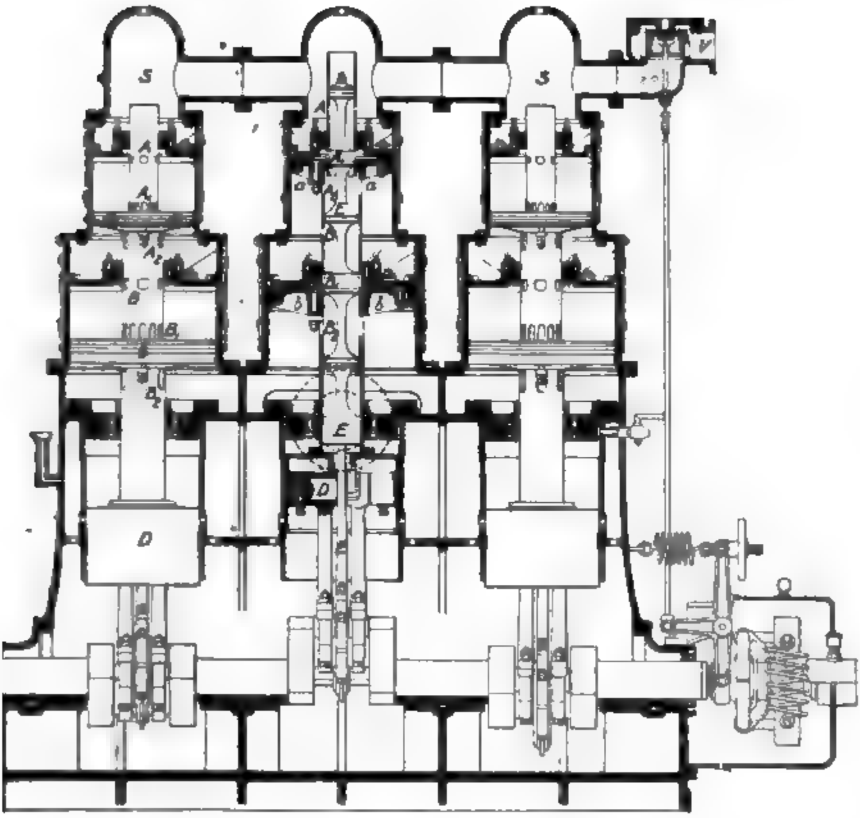



FIG. 233.

a three line Willans compound engine. The double beat throttle valve $V$ (regulated by the governor the shaft) admits steam to all three engines at $S$ $S$ $a$ the high pressure, and $b$ the low pressure, and $D$ or air cushion piston are rigidly connected by tubes or a trunk, and work one crank through a divided rod. Inside the trunk are piston valves worked from an eccentric $E$ on the crank pin. Steam from $S$ enters in the tube at $A$ and $A_1$ into the space above piston $a$ and exhausts from this space into the space From this in the same way next stroke it is admitted above piston $b$, and exhausts into the space hich is the exhaust chamber. The space above $D$ is filled with air to serve as a buffer or cushion (Art. student will do well to make a paper model to see how the piston valves admit, cut off and release, their motion relatively to the trunk The valve rod is always in compression because of the steam bove the top piston valve. The piston connections and connecting rods are also always in compression The Willans' triple engine has another piston between $b$ and $D$ and corresponding valves.

this that induced him to adopt the plan used by Watt in his engine of not allowing the space on the working side of th to communicate directly with the condenser. He attributes the economy of his engines to this method of diminishing the t ture range. I attribute most of it to automatic drainage.

A common error in the measurement of total water sup an engine is due to inaccuracy in measuring the water leve boiler. The engine of Mr. Willans lent itself to great acc measuring the total water, by measuring what left the hot w was of opinion that there was never more than 1½ per cent. present in the steam supplied, as he used a separator.

The following numbers have been taken from Mr. Willan *Proc. I. C. E.*, 1893. I have worried over them for years, t understand their seeming inconsistencies with one another, so thinking these inconsistencies due to errors of experiment; b every one of my failures I have felt that some ingenious stud be able to make a better use of them than I have. I following for what it is worth; it is not good, but I think t better than anything that has been published.

The student ought for each set of tests where $r$ is kep constant, to plot $W$ and $I$ on squared paper, and see if he the linear laws connecting $W$ and $I$ which I give in Art. 148 pounds of steam per hour, and $I$ is the indicated horse-powe

WILLANS' SINGLE CYLINDER CONDENSING TRIALS.

| $p_1$ | $r$ | $n$ | $y$ | $\dfrac{y\sqrt{p_1 n}}{r-0{\cdot}7}$ | $W$ |
|---|---|---|---|---|---|
| 54·91 | 2·538 | 381·5 | ·192 | 15·12 | 811·80 |
| 47·58 | 2·57 | 380·9 | ·205 | 14·77 | 686·1 |
| 37·80 | 2·62 | 381·0 | ·267 | 16·69 | 583·6 |
| 28·93 | 2·65 | 382·1 | ·310 | 16·71 | 465·26 |
| 20·73 | 2·68 | 384·9 | ·326 | 14·70 | 345·4 |
| 16·08 | 2·53 | 378·2 | ·305 | 16·35 | 266·22 |
| 74·12 | 4·04 | 383 | ·336 | 16·94 | 736·5 |
| 64·37 | 4·04 | 382 | ·379 | 17·79 | 676·3 |
| 55·19 | 4·01 | 379·5 | ·382 | 16·69 | 596 |
| 37·94 | 4·10 | 378·3 | ·488 | 17·20 | 440·4 |
| 20·65 | 4·31 | 381·6 | ·645 | 15·86 | 259·1 |
| 16·58 | 4·31 | 379·8 | ·632 | 13·89 | 206·1 |
| In the following tests the steam was superheated. | | | | | |
| 59·24 | 2·44 | 383·7 | ·190 | — | 831·9 |
| 40·27 | 2·58 | 377·6 | ·205 | — | 566·7 |
| 28·80 | 2·63 | 384·6 | ·239 | — | 447·6 |
| 21·37 | 2·64 | 384·0 | ·182 | — | 311·32 |

We see, therefore, that in the Willans' single cylinder condensing trials we may fairly say that

$$y = 16\,\frac{r - 0{\cdot}7}{\sqrt{p_1 n}}$$

or, if the $d$ law is true,

$$y = \frac{217\,(r - 0{\cdot}7)}{d\sqrt{p_1 n}}.$$

Superheating produced no marked improvement at the higher pressures, but there is a marked improvement at the lower pressures.

WILLANS' CONDENSING TABLE II. COMPOUND SERIES.

| $p_1$ | $r$ | $n$ | $y$ | $W$ | $I$ | $\frac{W}{I}$ | $\frac{y\sqrt{p_1 n}}{r-2{\cdot}75}$ |
|---|---|---|---|---|---|---|---|
| 125·91 | 5·68 | 402·16 | 0·119 | 671·44 | 40·14 | 16·72 | 9·2 |
| 106·71 | 5·77 | 405·27 | 0·118 | 564·2 | 33·25 | 16·97 | 8·1 |
| 81·24 | 5·62 | 401·17 | 0·098 | 443·22 | 25·61 | 17·30 | 6·2 |
| 60·50 | 5·69 | 404·44 | 0·132 | 336·13 | 18·69 | 17·98 | 7·0 |
| 37·16 | 6·12 | 398·9 | 0·130 | 219·1 | 10·81 | 20·27 | 4·8 |
| 127·81 | 5·74 | 311·14 | 0·117 | 504·69 | 30·99 | 16·28 | 7·8 |
| 107·19 | 5·74 | 310·99 | 0·118 | 433·10 | 25·69 | 16·85 | 7·1 |
| 81·03 | 5·77 | 301·46 | 0·139 | 344·5 | 19·52 | 17·64 | 7·2 |
| 59·46 | 5·72 | 301·98 | 0·150 | 259·5 | 14·01 | 18·52 | 6·7 |
| 35·07 | 5·84 | 300·06 | 0·199 | 167·1 | 7·61 | 21·96 | 6·8 |
| 126·14 | 5·66 | 203·23 | 0·136 | 340·01 | 19·93 | 17·06 | 7·4 |
| 84·32 | 5·89 | 197·96 | 0·218 | 252·06 | 13·30 | 18·95 | 9·0 |
| 60·57 | 5·73 | 203·0 | 0·219 | 188·9 | 9·42 | 20·05 | 8·1 |
| 35·25 | 6·23 | 196·49 | 0·343 | 125·36 | 5·26 | 23·83 | 8·2 |
| 114·91 | 5·54 | 114·6 | 0·230 | 178·2 | 9·04 | 19·71 | 9·1 |
| 83·44 | 5·86 | 116·07 | 0·264 | 133·56 | 6·66 | 20·05 | 8·4 |
| 39·49 | 5·87 | 112·54 | 0·474 | 78·3 | 2·9 | 27·0 | 10·1 |

WILLANS' CONDENSING TABLE III. COMPOUND SERIES.

| $p_1$ | $r$ | $n$ | $y$ | $W$ | $I$ | $\frac{W}{I}$ | $\frac{y\sqrt{p_1 n}}{r-2{\cdot}75}$ |
|---|---|---|---|---|---|---|---|
| 156·71 | 11·18 | 396·7 | 0·347 | 492·12 | 33·19 | 14·82 | 10·2 |
| 131·25 | 10·81 | 399·02 | 0·294 | 411·47 | 27·11 | 15·18 | 8·3 |
| 110·98 | 10·55 | 402·40 | 0·285 | 349·73 | 22·09 | 15·83 | 7·7 |
| 59·16 | 12·23 | 395·56 | 0·473 | 212·60 | 11·66 | 18·23 | 7·6 |
| 60·41 | 11·04 | 394·23 | 0·397 | 216·50 | 11·86 | 18·25 | 7·4 |
| 137·37 | 11·14 | 295·28 | 0·391 | 335·84 | 22·06 | 15·22 | 9·4 |
| 138·12 | 10·78 | 198·6 | 0·449 | 243·86 | 14·76 | 16·52 | 9·0 |
| 156·86 | 10·59 | 118·08 | 0·527 | 152·43 | 8·84 | 17·23 | 9·1 |

WILLANS' CONDENSING TABLE III. COMPOUND SERIES (*continued*).

| $p_1$ | $r$ | $n$ | $y$ | $W$ | $I$ | $\frac{W}{I}$ | $\frac{y\sqrt{p_1 n}}{r-2.75}$ |
|---|---|---|---|---|---|---|---|
| 168·07 | 15·81 | 402·15 | 0·446 | 392·10 | 27·5 | 14·26 | 8·9 |
| 132·82 | 15·82 | 398·1 | 0·454 | 323·48 | 21·59 | 14·98 | 8·0 |
| 82·56 | 17·50 | 406·7 | 0·665 | 227·28 | 13·18 | 17·24 | 8·5 |
| 161·31 | 16·07 | 303·94 | 0·567 | 302·34 | 19·89 | 15·20 | 9·4 |
| 86·13 | 15·74 | 300·2 | 0·622 | 188·60 | 10·62 | 17·75 | 7·8 |
| 157·91 | 16·05 | 203·26 | 0·736 | 226·04 | 13·46 | 16·79 | 9·9 |
| 73·43 | 17·04 | 198·97 | 1·063 | 129·10 | 6·00 | 21 52 | 9·0 |
| 172·56 | 20·18 | 404·1 | 0·600 | 366·07 | 24·87 | 14·72 | 9·1 |
| 111·03 | 20·24 | 396·02 | 0·649 | 249·4 | 16·03 | 15·51 | 7·8 |

I can make nothing better of the compound trials (condensing of Mr. Willans than this :—

The last column of the table shows the values of

$$\frac{y\sqrt{np_1}}{r-2.75}$$

for all his results except the five trials in which the value of $r$ wa about 5·7, $n = 400$ and $p_1$ varying from 126 to 37, and it is evider that, except for these five trials, we may take

$$y = c\,\frac{r-2.75}{\sqrt{np_1}}, \text{ where } c = 8.20.$$

A study of the numbers will show that there can be no simp formula which is very satisfactory.

**237.** I. As to the effect of **the initial pressure**, it is evide that when

$r = 5.7$, $n = 400$, $y$ slightly increases as $p_1$ is less
$r = 5.7$, $n = 300$, $y \propto p_1^{-\frac{1}{2}}$
$r = 5.7$, $n = 200$, $y \propto p_1^{-\frac{1}{2}}$
$r = 5.7$, $n = 114$, $y \propto p_1^{-\frac{1}{2}}$

---

$r = 11$, $n = 400$, $y$ increases as $p_1$ is less
$r = 16\frac{1}{4}$, $n = 400$, $y \propto p_1^{-\frac{1}{2}}$
$r = 16\frac{1}{4}$, $n = 300$, $y$ increases as $p_1$ is less
$r = 16\frac{1}{4}$, $n = 200$, $y \propto p_1^{-\frac{1}{2}}$
$r = 20$, $n = 400$, $y$ increases as $p_1$ is less.

It is, however, only where $r = 5{\cdot}7$, $n = 400$ that we are perfectly certain that $y$ is not greatly affected by the value of $p_1$, and, indeed, that we may not take it that $y \propto p_1^{-\frac{1}{2}}$.

II. **As to speed.** When $r = 5{\cdot}7$, it is only at the medium pressures, say $p_1 = 60$ to $p_1 = 107$, that we find

$$y \propto n^{-\frac{1}{2}}$$

$$\text{at } p_1 = 36, \quad y \propto n^{-1}$$

$$\text{at } p_1 = 126, \; y \propto n^{-\frac{1}{4}}.$$

The trials for other values of $r$ do not show such large discrepancies from the rule we have given.

I have not quoted any of the numerous other figures of Mr. Willans, but it is to be understood that he tries to trace the amount of steam present at every stage in his compound and triple trials. I find that the following rule is fairly typical. We have seen that the water missing in the high pressure cylinder when $r$ was about $5{\cdot}7$ follows a rather complicated law.

Now if $y_1$ is $\dfrac{\text{missing steam in low pressure cylinder}}{\text{indicated steam in low pressure cylinder}}$, I find the simple rule

$$y_1 = \frac{123}{n + 74}.$$

Certainly the inverse $\sqrt{n}$ law cannot be made to hold.

*Willans' Triple Condensing Trials.*

**238.** In the following trials there is not much variation of $r$ and $n$. The rule

$$y = 110 \frac{r - 7}{n\sqrt{p_1}}$$

will be found to be fairly correct; or assuming the untested law for $d$, if $d$ is the diameter of the highest pressure cylinder in inches

$$y = \frac{600\,(r - 7)}{dn\sqrt{p_1}}.$$

The most important thing to notice is that

$$\text{when } r = 21{\cdot}5, \text{ and } n = 377, \; y \propto p_1^{-\frac{1}{3}}$$

$$\text{when } r = 14{\cdot}2 \text{ and } n = 301, \; y \propto p_1^{-\frac{1}{4}}$$

$$\text{when } r = 14{\cdot}2 \text{ and } n = 380 \;\; y \propto p_1^{-0{\cdot}44},$$

but it is quite possible that more observations would correct the

apparent want of consistency. In all cases, however, $y$ is grea
affected by the value of $p_1$.

WILLANS' CONDENSING TABLE IV. TRIPLE SERIES.

| $p_1$ | $r$ | $n$ | $y$ | $W$ | $I$ | $\frac{W}{I}$ |
|---|---|---|---|---|---|---|
| 177·29 | 13·72 | 379·1 | 0·145 | 383·60 | 29·46 | 13·0[illegible] |
| 175·5 | 14·01 | 383·7 | 0·168 | 380·3 | 29·84 | 12·7 |
| 157·55 | 13·66 | 380·5 | 0·148 | 343·4 | 26·66 | 12·8[illegible] |
| 128·12 | 14·11 | 383·6 | 0·180 | 285·56 | 21·32 | 13·[illegible] |
| 53·97 | 14·10 | 376·6 | 0·269 | 143·02 | 9·28 | 15·[illegible] |
| 51·12 | 15·36 | 381·4 | 0·318 | 131·0 | 8·30 | 15·[illegible] |
| 175·90 | 13·72 | 302·4 | 0·195 | 297·6 | 23·14 | 12·[illegible] |
| 130·33 | 14·23 | 300·2 | 0·241 | 234·57 | 16·79 | 13·[illegible] |
| 52·66 | 14·67 | 301·9 | 0·415 | 113·84 | 6·70 | 16·[illegible] |
| 175·28 | 21·69 | 375·4 | 0·344 | 283·6 | 22·26 | 12·74 |
| 143·8 | 21·31 | 379·5 | 0·371 | 239·2 | 18·28 | 13·00 |
| 74·12 | 21·17 | 375·9 | 0·418 | 139·29 | 9·08 | 15·34 |

## CHAPTER XXV.

### COMBUSTION AND FUEL

**39.** ENGINEERING is really the utilisation of **chemical and** **ical principles**, and yet many men think themselves engineers ave no clear notions of these principles in their fundamental Such men are in truth only capable of doing what other men lone before; they are incapable of foreseeing how any new con-ce will act, but by dint of expensive trial and failure they some-arrive at results which they might have arrived at very inexpen-if they had been better educated. This very general ignorance mentary scientific principles in ingenious men has filled the on our subject with most misleading numbers, arrived at by ntific trials. In other branches of engineering if a man desires ce a new departure he can find figures, the results of scientific from which he can calculate with some accuracy how his new ance will act; in the subject of applied heat, the practical given us in one book contradict each other in the most ex-inary way. In the most authoritative treatises we find on one hat the rate at which heat passes through a square foot of heating surface is practically independent of whether the is copper or iron, and figures that pretend to be right to the n thousandth part are quoted establishing this fact. A few further on we find equally elaborate results showing that ermal resistance of the metal plate is proportional to its thick-nd is ever so much greater in iron than copper. The authors se treatises do not seem for a moment to think that they have the same weight to two **contradictory statements.**

would be easy to quote many examples of this divorce of what rded as practical experience from a knowledge of the most ntary scientific principles. The most unsatisfactory part of junction is this, that although we are sure that the expensive ments were performed, the author in describing them has left of no consequence the very facts which would make them

useful. Usually, however, he has merely paid no attention to what happens to be the most important varying factor in his experiment, and of course his results are inconsistent with one another. All this has made the phenomena in boilers seem to be much more difficult to understand than they really are, and every well-meaning engineer who gives us new figures about heat phenomena from his own measurement, is only adding to a large array of inconsistent looking facts. I am sorry to say that half the writers of papers published by even the highest scientific societies are as ignorant of elementary truths. What is much wanted is a study of combustion and the conduction and other transference of heat in their very simplest forms, in chemical and physics laboratories.

In this book I can only state principles and assume that students have made them part of their mental machinery. I need hardly say that it is impossible to do this by academic absorption from a book.

**240. Chemical symbols** have been cunningly contrived so that they convey a vast amount of information, and by the help of certain tables which have been very carefully prepared they enable us to make exact calculations. To explain fully what follows so that a student shall not get misleading notions is no part of my business; just now I look upon these statements as mere helps to the memory.

A molecule of each of many of the simple gases consists of two atoms. An atom of hydrogen is indicated by $H$; $n$ atoms by $nH$ or $H_n$. An atom of carbon is indicated by $C$, an atom of oxygen by $O$, and of nitrogen by $N$. If the weight of the atom of hydrogen is taken as 1, the atomic weights are $H$, 1; $C$, 12; $O$, 16; $N$, 14.[1]

The following are the symbols of the gases (one molecule of each) with which we are most concerned:—$H_2$; $O_2$; $H_2O$, gaseous water or steam; $CO$, carbon monoxide (commonly called carbonic oxide); $CO_2$ carbon dioxide (commonly called carbonic acid); $CH_4$, methane (commonly called marsh gas or light hydrocarbon); $C_2H_4$, ethylene (commonly called olefiant gas, the best known heavy hydrocarbon).

There are the same numbers of molecules of any gas to the cubic foot, and therefore supposing for convenience we take $H_2$ as indicating two cubic feet of hydrogen, $O_2$ indicates two cubic feet of oxygen, $CO_2$ indicates two cubic feet of carbon dioxide, $CO$ indicates two cubic feet of carbon monoxide, $H_2O$ indicates two cubic feet of gaseous water, &c., the idea being that they are all in the perfectly gaseous state and at the same temperature and pressure. By weight, if $H_1$ indicates 1 lb. of hydrogen, $H_3$ indicates 3 lbs.; $O_3$ indicates 3 times 16 or 48 lbs. of oxygen; $H_2O$ indicates

[1] More exactly, $H$. 1; $C$, 11·92; $O$, 15·88; $N$, 13·94.

2 + 16 or 18 lbs. of water and so on. It is evident that the mere symbol of a gas such as $C_2H_4$ tells us its density; thus $C_2H_4$ has the same volume as $H_2$ or $O_2$ or $CO_2$ or $CO$ or $H_2O$, and therefore $(2 \times 12) + (4 \times 1)$ or 28 lbs. of olefiant gas has the same volume as $2 \times 1$ or 2 lbs. of hydrogen, or as $2 \times 16$ or 32 lbs. of oxygen or $12 + (2 \times 16)$ or 44 lbs. of carbon dioxide or $12 + 16$ or 28 lbs. of carbon monoxide or $2 \times 1 + 16$ or 18 lbs. of gaseous water.

**241.** Consider such an equation as

$$H_2 + O = H_2O$$

We can read this in the following ways:

(1) One molecule of hydrogen combines with half a molecule[1] of oxygen to form one molecule of water.

(2) Two atoms of hydrogen combine with one atom of oxygen to form one molecule of water.

(3) Two cubic feet of hydrogen combine with one cubic foot of oxygen to form two cubic feet of gaseous water.

(4) 2 lbs. of hydrogen combine with 16 lbs. of oxygen to form 18 lbs. of water.

I may add that the total amount of heat derivable from the combustion of 1 lb. of hydrogen is 62,100 Fahrenheit (34,500 centigrade) pound units of heat, the stuff resulting from the combustion being reduced to 62° F.; ordinary differences as to the pressure of the gases beforehand and after being quite insignificant. In what follows, calorific power will be in centigrade heat units unless Fahrenheit is specially mentioned. I have at some length dwelt upon those parts of the signification of the equation $H_2 + O = H_2O$ which are interesting to us. Let the student in the same way write out similar statements concerning each of the following.

$$CH_4 + 4O = CO_2 + 2H_2O$$
$$C_2H_4 + 6O = 2CO_2 + 2H_2O$$

The following equations need not be stated volumetrically because we know nothing of carbon in the gaseous state.

$$C + O = CO$$
$$C + 2O = CO_2$$

**242.** If a pound of **hydrogen** is already in combination with carbon, and the hydrocarbon is burnt in air we assume that the energy required to decompose it is too small to be worth troubling

[1] To speak of half a molecule is a little absurd, but here it saves trouble. The student may if he pleases double everything in the formula and in these four statements.

about. This is partly because we do not know the amount, but also because we know that it cannot be great.

The presence of hydrogen in a fuel is conducive to rapid ignition; the hydrocarbons volatilise below redness and ignite, heating the rest, leaving the fixed carbon porous. This is why wood, peat, and some kinds of brown and gas coals flame so much. When there is little hydrogen, we get flame by using insufficient air so that only carbon monoxide is produced, and this with more air gives flame. Steam conduces to flame production.

**243.** Again, when a pound of **carbon,** say charcoal, is completely burnt, the heat of combination must be somewhat different from what it is if the carbon is part of a hydrocarbon. We assume it to be the same (8,040 units) because we do not know any better. It is urged by some eminent persons that as the combination of 1 lb. of $C$ with $O$ to form $CO$ gives 2,470 units of heat, and the further combination of the so produced $CO$ with $O$ to form $CO_2$ gives 5,600 units of heat, the difference between these numbers represents the latent heat of gaseous carbon. It is a most unscientific statement, as the two cases of combination of $C$ with $O$ have about as much to do with one another as Tenterden steeple and the Goodwin Sands.

In a fuel we distinguish between that portion of the carbon which is called 'fixed' (which would be left as charcoal or coke after destructive distillation) and that which is volatile (being combined with hydrogen as a hydrocarbon like marsh or olefiant gas). Fixed carbon needs to be scrubbed with air as it burns. A hydrocarbon, if mixed at a high enough temperature with a sufficient quantity of air, burns completely into $CO_2$ and $H_2O$ with a blue flame. But if the hydrocarbon not mixed with air is at a high temperature and is suddenly cooled, it becomes decomposed partly into marsh gas and partly free hydrogen, and much of the carbon separates out as solid particles which we call smoke or soot. If, however, there is sufficient air in the atmosphere containing this smoke, and it is heated to a high enough temperature, the carbon becomes consumed forming reddish yellow or white flame.

The burning of carbon seems to be always complete at first, that is, some of the $C$ becomes $CO_2$. If, however, this $CO_2$ comes in contact with white-hot solid carbon, it seems to dissolve the solid and become carbon monoxide, and if the process stops here there is great waste of fuel. The presence of moisture conduces to this action. It is for this reason that when boiler fires are thick it is necessary to admit air *above* the fire as well as below.

**244.** EXERCISE 1. Olefiant gas has the composition $C_2H_4$; in 1 lb. of it, how much carbon and how much hydrogen are there?

*Answer.* $2 \times 12$ or 24 lbs. of $C + 4$ lbs. of $H$ are in 28 lbs. of $C_2H_4$. Hence $\frac{6}{7}$ lb. of $C + \frac{1}{7}$ lb. of $H$ are in 1 lb. of $C_2H_4$.

EXERCISE 2. What is the calorific power of 1 lb. of $C_2H_4$?

*Answer.* $\frac{6}{7} \times 8{,}040 + \frac{1}{7} \times 34{,}500 = 11{,}820$ units.

The experimentally determined number is 11,960.

Experimentally determined, the heat of combustion of 1 lb. of oil of turpentine is 10,850; wood charcoal 8,090; gas coke 8,050; graphite 7,780; sulphur 2,250.

For the following I have taken in each case the means of four careful calorimetric measurements of 1 lb. of each;—carbon monoxide 2,425, marsh gas 13,240, olefiant gas 11,960, benzene $C_6H_6$, 10,100.

**245.** To recapitulate a little. We see then that in considering the combustion of a fuel, 1 lb. of hydrogen needs 8 lbs. of oxygen, and forms 9 lbs. of water. The total heat available is 62,100 Fahrenheit or 34,500 Centigrade units of heat. And when we state calorific power it is preferable not to deduct the latent heat of water. If the water goes off uncondensed as it usually does in our engines, we may roughly say that the total heat available is $62{,}100 - 966 \times 9$ units or 53,400 Fahrenheit or 29,800 Centigrade units. One pound of carbon needs 2·67 lbs. oxygen, and forms 3·67 lbs. of carbonic acid (called by the chemist, carbon dioxide). The heat available is 8,040 Centigrade units. In this case the combustion is said to be complete. But 1 lb. of carbon may unite with 1·33 lb. of oxygen to form 3·33 lbs. of carbonic oxide (called by the chemist, carbon monoxide). The heat available is 2,470 units and the combustion is incomplete. The combustion may or may not be afterwards completed. One pound of oxygen is contained in 4·35 lbs. of air; hence, knowing how much oxygen is needed we know the amount of air needed. When we know the chemical composition of a fuel we can tell the weight of oxygen, and therefore the **weight of air** needed for complete combustion, and we can roughly determine the amount of heat available if we calculate merely from the carbon and hydrogen which are contained in the fuel. Students who know a little chemistry are aware that there is no rule for making this calculation of **the calorific power** which is not likely to be in error as much as, if not more, than 5 per cent. There is no handy instrument which will enable the calorific power to be measured with greater accuracy than this. It is a regular laboratory exercise with my students to measure it with a handy instrument, and it is an instructive lesson to show them the incorrectness of the method. Unless, therefore, we take a very troublesome method of measurement, we cannot do better than to calculate from the chemical composition. Students ought to calculate the calorific power and

the air required for the combustion of some of the fuels of following tables in this way. They may, later, use the formula follows.

*Example.* One pound of each of the following fuels contain following fractions of 1 lb. of carbon and hydrogen. Find weights of oxygen required for complete combustion. As the only ·23 lb. of oxygen in 1 lb. of air, we must divide by ·23 to the weights of air required. From 1½ to twice this amount of a usually admitted to a boiler furnace.

| 1 lb. of Fuel. | lb. of Carbon. | lb. of Hydrogen | lb. of Oxygen needed. | lb. of Air needed. | Heat developed. | Ev at po |
|---|---|---|---|---|---|---|
| Dried wood | 0·40 | 0·05 | 1·467 | 6·38 | 494[illegible] | 9· |
| Brown coal | ·55 | ·01 | 1·547 | 6·726 | 4767 | 8· |
| Bituminous coal | ·70 | ·05 | 2·267 | 9·86 | 7353 | 13· |
| Average British coal | ·80 | ·05 | 2·534 | 11·02 | 8157 | 15· |
| Welsh steam coal (average) | 0·84 | ·05 | 2·64 | 11·48 | 8479 | 15· |
| Anthracite | 0·92 | ·03 | 2·69 | 11·71 | 8432 | 15· |
| Coke | 0·88 | 0 | 2·347 | 10·21 | 7075 | 13· |
| Petroleum | 0·85 | ·15 | 3·467 | 15·08 | 12010 | 22· |
| Coal gas | 0·58 | ·23 | 3·387 | 13·08 | 12600 | 24· |
| 2H + O | | | | | | |

*Example.* One cubic foot of each of the following gaseous contains the following fractions of a cubic foot of the gases st Find the cubic feet of oxygen required for complete combus There is ·208 of a cubic foot of oxygen in one cubic foot of therefore divide by ·208 to find the cubic feet of air needec complete combustion.

| One cubic foot of | Hydrogen. | Carbonic Oxide. | Marsh Gas. | Heavy Hydrocarbons. | Carbonic Acid, Nitrogen, &c. | Comparative values of Calorific powers per cubic foot. | Cubic feet of air requi |
|---|---|---|---|---|---|---|---|
| Average coal gas | ·47 | ·09 | ·34 | ·05 | ·05 | 5·3 | 5· |
| London " | ·506 | ·039 | ·37 | ·055 | ·054 | 5·41 | 5· |
| Scotch " | ·36 | ·068 | ·42 | ·15 | ·036 | 5·56 | 7· |
| Midland " | ·416 | ·044 | ·41 | ·075 | ·072 | 5·27 | 6· |
| Dowson gas | ·187 | ·251 | ·003 | ·003 | ·556 | 1·24 | 1· |
| " " | ·265 | ·182 | ·005 | ·005 | ·423 | 1·34 | 1· |
| Generator gas | 0 | ·34 | 0 | 0 | ·66 | 1 | |
| Siemens gas | ·06 | ·20 | ·01 | ·01 | ·72 | 1 | |
| Water gas | ·50 | ·50 | 0 | 0 | | 2·7 | 2 |
| Generator water gas | ·12 | ·38 | 0 | 0 | ·50 | 1 | |

**246.** It is customary to calculate **calorific powers of fuels** by the following formulæ which ought to be known to students.

When a fuel contains hydrogen and oxygen in the proper proportion to form water, it is assumed that they may be left out of the calculation of the calorific power. Their effect is only to form smoke more easily. We have therefore the following rule:—

Suppose that $c$, $h$ and $o$ are the weights of carbon, hydrogen and oxygen in a fuel. Subtract $\frac{o}{8}$ from $h$, and call the remainder the *available* hydrogen. Consider 1 lb. of hydrogen to have 4·28 times the calorific power of carbon, and thus our pound of fuel has the same calorific power as

$$c + 4{\cdot}28\left(h - \frac{o}{8}\right)$$

pounds of carbon. That is, if $h^1$ is the heat per pound of fuel and if $E$ is its evaporative power (being $h^1$ divided by latent heat of steam at 100° C.), then

$$h^1 = 14{,}500\left\{c + 4{\cdot}28\left(h - \frac{o}{8}\right)\right\} \text{in Fahrenheit units}$$

$$h^1 = 8{,}050\left\{c + 4{\cdot}28\left(h - \frac{o}{8}\right)\right\} \text{in Centigrade units}$$

$$E = 15\left\{c + 4{\cdot}28\left(h - \frac{o}{8}\right)\right\} \text{in evaporation units}$$

Students will calculate $h^1$ and $E$ for each of the fuels of Art. 256. I am sorry to say that this formula, long accepted as giving a fair agreement with calorimetric tests, ought to have 5 per cent. added to its value. It has been found to give from 1·5 to 10·6 per cent. too low a value. A formula now getting into use which is **supposed to be more correct** is

$$h^1 = 8{,}140\, c + 34{,}500\, h - 3{,}000\,(o + n)$$

where $n$ is the weight of nitrogen present in a pound of fuel.

In the table the heat due to the fixed carbon is obtained by assuming that nothing is burnt except that part of the carbon which is fixed; the rest going off unconsumed.

**247.** The student will now work the following easy algebraic exercises. A pound of fuel contains $c$ lb. of carbon, $h$ lb. of hydrogen, $o$ lb. of oxygen, show that roughly;—

1. The pounds of **air needed** for complete combustion

$$A = 11{\cdot}6\ c + 34{\cdot}8\ h$$

2. The **products of combustion** are

$3\frac{2}{3}\ c$ lbs. of carbon dioxide

$9\ h$ lbs. of steam

$8{\cdot}9\ c + 26{\cdot}8\ h$ lbs. of nitrogen.

Work now the following numerical exercises on the above.

3. Taking average coal $c = 0{\cdot}8$, $h = 0{\cdot}05$, $o = 0{\cdot}08$
Calculate $E$, $A$, and the products.
*Answer.* 14·57 ; 11·02 ; 2·933 lbs. of $CO_2$, 0·45 lb. of $H_2O$, 8
of N.

4. By Art. 189 find **the specific heat** of the produc
average coal; given the following specific heats: carbon
·216, nitrogen ·244, steam ·475.

$$\textit{Answer.}\ \frac{2{\cdot}933 \times {\cdot}216 + 0{\cdot}45 \times {\cdot}475 + 8{\cdot}49 \times {\cdot}244}{2{\cdot}933 + 0{\cdot}450 + 8{\cdot}490}$$

$$= \frac{{\cdot}634 + {\cdot}213 + 2{\cdot}071}{11{\cdot}873} = 0{\cdot}246$$

at constant pressure.

5. The specific heat of air is ·238; if, in addition to eve
of necessary air, we admit $a$ lb., what is the specific heat
products? *Answer.*—$({\cdot}246 \times 11{\cdot}873 + 11{\cdot}02\,a \times 0{\cdot}238) \div$
$+ 11{\cdot}02\,a) = {\cdot}238\,(1{\cdot}113 + a)/(1{\cdot}077 + a)$.

6. Find the quantities and specific heats of the produc
30, 70 or 100 per cent. excess air is admitted in the bu
average coal.
*Answer.* 15·2 lbs. of specific heat 0·244, 19·6 lbs. of spec
·243, 22·9 lbs. of specific heat ·242.

In each case part of the total amount of products is
of steam.

7. In cases where the products are 12, 15, 20 and 23 lbs. pe
of fuel; taking the specific heat as ·243 in every case, wha
**necessary loss of evaporative power** in the following cas
outside atmosphere is at 62° F. The boiler water is at
322° F., 382° F., 402° F. *Ans.* If $\theta°$ F. is the temperatur
boiler, and $w$ the weight of products, $0{\cdot}243\ w\ (\theta - 62) \div$

the evaporation which cannot be utilised; the answers are as follow:—

| Weights of products. | Necessary diminution of evaporative power for the following Temperatures of Water in Boiler. | | | |
|---|---|---|---|---|
| | 212° F., or 15 lbs. pressure. | 322° F., or 93 lbs. pressure. | 382° F., or 200 lbs. pressure. | 402° F., or 250 lbs. pressure. |
| 12 | ·453 | ·785 | ·955 | 1·03 |
| 15 | ·566 | ·984 | 1·19 | 1·27 |
| 20 | ·755 | 1·31 | 1·59 | 1·7 |
| 23 | ·867 | 1·41 | 1·83 | 1·95 |

8. One pound of the fuel of Ex. 4 produces 0·45 lb. of steam; the hygroscopic water being 0·05 lb., we have 0·5 lb., whose total heat loss in falling from the temperature of the boiler water to 62° F. ought also to be deducted. We have already considered this loss if it were merely steam vapour. But it loses latent heat 966 units per lb., and if cooled to 62° F., would cool as water and not as steam gas. Hence 966 + (212 − 62) × (1 − 0·475) or, 1,045 is the extra heat per pound, or 522 ÷ 966, or 0·54 is the loss of evaporative power due to this cause.

9. The coal of Ex. 4 is burnt in a boiler whose water is at 322° F. (93 lbs. pressure); 70 per cent. excess air is admitted, what is the **available evaporative power?** *Ans.* 14·57 from Ex. (3) minus 1·3 from Ex. (7) minus 0·54 from Ex. (8) gives us 12·73 lbs. of water evaporated as from and at 212° F. per pound of fuel.

10. If the feed water of the boiler is supplied at 62° F., and evaporated at 322° F. (or 93 lbs. per square inch), and if the steam leaving the boiler is (1) dry steam, (2) has 5 per cent. of wetness, what is the greatest possible amount producible per pound of fuel of Ex. 9? This exercise is in natural sequence with the others, and so I do not like to remove it to Art. 248.

*Ans.* (1) 1 lb. of water from 62° F. to 322° F. needs 260 units of heat; 1 lb. of this kind of steam needs the latent heat 887. Total, 1,147 units. (2) 1 lb. of water from 62 F. to 322° F. needs 260 units of heat; 0·95 lb. of this kind of steam needs the latent heat 887 × 0·95 or 843 units; total, 1,103 units. Hence the standard evaporation of 1 lb. is equivalent to $\frac{966}{1147}$ lbs. dry or $\frac{966}{1103}$ lbs. wet, or 12·73 of standard evaporation is equivalent to 10·72 lbs. of this dry steam, or 11·15 lbs. of this wet steam.

11. The hydrocarbons of the above (Ex. 4) average coal, escape

unburnt; the fixed carbon is only 0·57 lb. per pound of fuel; there i no moisture present; the products of combustion are 23 lbs. pe pound of fuel, specific heat 0·24, water in boiler 322° F. What i the available evaporative power of the fuel (assuming flues s perfect that the gases are reduced to the temperature of th water)?

What is it if it is arranged that the gases must not enter th chimney at a lower temperature than 580° F.?

The feed water being at 62° F., convert the last answer int actual evaporation of dry steam, assuming that 10 per cent. of th heat is lost by radiation from the boiler itself.

12. One pound of average coke contains 15 per cent. moistur and 80 per cent. carbon, the rest being ash and a trace of sulphur.

What is its evaporative power and the amount of air needed fo complete combustion? *Ans.* 15 × ·8 or 12 lbs. of evaporation 11·6 × ·8 or 9·3 lbs of air.

13. For the coke of last exercise. If 70 per cent. excess of air i admitted at 62° F., and if the temperature of the water in a boile is 320° F.; the products of combustion having a specific heat 0·2 what is the really available evaporative power? *Ans.* The quanti of products is (9·3 + 1) 1·70 or 17·51 lbs.; the heat lost because th can cool only to 320° F. is (320 − 62) 0·24 × 17·51 = 1,085 unit Also the latent heat of ·15 lb. of moisture is 966 × ·15 135 units. Our calculation is only roughly correct because tl moisture wastes more heat than this. Taking this as sufficient correct, the unavoidable waste is 1,085 + 135 or 1,215 units, ar this divided by 966 is 1·26 lb. of evaporation. Hence the availab evaporation is only 10·7 lbs.

It is usual to speak of 12 lbs., or 150 **cubic feet of air,** being necessary for the complete combustion of 1 lb. of coal, ar to be quite certain of there being enough we must admit mo than enough.

When the fuel is not thick on the grate as in Cornish ar Lancashire boilers, with a chimney to produce the draught, it difficult to distribute the air properly and so to be sure that ea piece of coal has enough air to scrub it, we admit, on the whol about 24 lbs. of air per pound of fuel. When the fires are thick ai there is forced draught we admit as little as 18 lbs., or even le air per pound of fuel, getting very complete combustion.

This is due to the fact that in thick fires the $CO_2$ formed belo dissolves the carbon higher in the fire forming $CO$, which is bur above the fire.

The student will see from exercises like the above that when a pound of coal is burnt **we have losses** of evaporative power something like the following. First, losses due to hot gases escaping—

25 per cent. with chimney draught.

20 per cent. with forced draught.

Second, about $2\frac{1}{2}$ per cent. because of hot ashes and unburnt solid fuel in the ashes.

Third, 18 per cent. when there is decently good firing but no contrivance for controlling the air admission. This assumes the hydrogen to go off unburnt.

Fourth, 9 to 18 per cent. more when there is also bad firing, so that not only does the hydrogen go off unburnt, but all the carbon of the hydrocarbon part goes off unburnt.

Fifth, 5 per cent. due to radiation of heat from a well covered boiler.

The student may add up these losses as he pleases. He may add further loss due to keeping the furnace door open unnecessarily. Also, if the boiler is not well covered with non-conducting material there is further loss.

**248. How much steam is produced** per pound of coal? When there is no priming [that is when there is no water carried off with the steam] 1 lb. of water needs, to convert it into steam of the following kinds, the following amounts of heat:—

| Temperature of the steam. $\theta$° C. | Absolute pressure in lbs. per square inch. | Total heat per lb. required if feed water is at 40° C. | Pounds of steam corresponding to the heat in 1 lb. of average coal or 8,500 units. | Equivalent of 1 lb. in standard of evaporation. |
|---|---|---|---|---|
| 130 | 39·25 | 604·3 | 14·07 | 1·127 |
| 150 | 69·21 | 610·3 | 13·93 | 1·139 |
| 160 | 89·86 | 613·4 | 13·86 | 1·145 |
| 165 | 101·9 | 614·2 | 13·84 | 1·146 |
| 170 | 115·1 | 616·4 | 13·79 | 1·150 |
| 175 | 129·8 | 617·2 | 13·77 | 1·152 |
| 180 | 145·8 | 619·5 | 13·73 | 1·154 |
| 190 | 182·4 | 622·5 | 13·66 | 1·161 |
| 195 | 203·3 | 624·0 | 13·62 | 1·164 |

The student ought to calculate the numbers of Col. 3 of the above table as an exercise. They are worked out in this way:—Regnault found that to heat a pound of water from 0° C. to $\theta$° C., and then to convert it into steam, required $606{\cdot}5 + {\cdot}305\theta$ units of heat. We start at 40° C., instead of 0° C., and so we merely subtract 40

from what Regnault's calculation gives us. Thus if $\theta$ is 130 as in the table—

$$606{\cdot}5 + {\cdot}305 \times 130 - 40 = 604{\cdot}3$$

Now practical men in comparing their boilers, sometimes said, "My boiler gives 8 lbs. of steam per pound of coal;" another said, "My boiler gives 9 lbs." and the comparison might be very unfair. They saw that they needed a standard. [The standard taken is "An evaporation of one pound shall mean, one pound of water at 100° C., converted into steam at 100° C." This is 536 heat units. So students will please fill in the fifth column of the table, as an exercise.] = 1 Boiler Horse Power. = 34.5 lbs $H_2O$ in one hour.

If they know the actual temperature of the feed-water of a particular boiler they ought to get out a table for it like the above.

EXERCISE 1. How much heat has been given to a pound of feed-water at 40° C., to convert it into what is $\frac{3}{4}$ steam and $\frac{1}{4}$ water at 160° C.? *Answer.* $\frac{1}{4}$ lb. of water needed $(160 - 40) \div 4$ or 30 units. $\frac{3}{4}$ lb. steam needed $\frac{3}{4}\ \{606{\cdot}5 + {\cdot}305 \times 160 - 40\}$ or 462 units: therefore, the pound of wet steam needed 492 units to produce it. Notice that this is 20 per cent. less than what is given in the table.

EXERCISE 2. When an engineer says that his boiler evaporates 10 lbs. of water for every 1 lb. of coal, his feed being at 20° C., and his steam at 190° C.; and another engineer says that his boiler evaporates 11 lbs. of water, his feed being at 60° C. and his steam at 130° C., compare the two numbers. One gets too much into the habit of thinking that 1 lb. of steam just needs as much heat to produce it as 1 lb. of any other kind of steam.

The total heat of 1 lb. of steam at 190° C. is

$606{\cdot}5 + {\cdot}305 \times 190$ or 664 units. Subtract 20 and we get 644.

The total heat of 1 lb. of steam at 130° C. is $606{\cdot}5 + {\cdot}305 \times 130$, or 646, and subtracting 60 we find 586 units. Hence 10 lbs. of the first amounts to 6,440 units, and 11 lbs. of the second amounts to 6,446 units. The two evaporations are then practically the same. Statements then of amounts of evaporation are misleading unless we use a standard of evaporation, and so we always convert any amount of evaporation into an equivalent number of pounds of water at 100° C., converted into steam at 100° C. This unit is of course the latent heat of steam per pound, 536 heat units.

EXERCISE 3. $10\frac{1}{2}$ lbs. of water heated from 40° C., and converted into steam at 180° C., find the equivalent standard amount. *Answer.*

he total heat of 1 lb. of the steam is 606·5 + ·305 × 180, or 661·4 nits. Subtract 40, multiply by 10½, and divide by 536, and we nd the answer 12·2 lbs.

We have the rule "To find the total heat of evaporation;—to 536 dd 1 for every degree that the feed is below 100° C, and ·3 for every legree that the steam is above 100° C.

**249.** In the Fahrenheit scale calculate the following numbers and keep the table by you for reference. The standard of evaporation is the heat required to produce 1 lb. of dry saturated steam at 212° F. from water at 212° F., and is equivalent to 966 Fahrenheit pound heat units.

From Regnault we know that to convert 1 lb. of feed-water at $\theta^\circ_0$ F. into steam at $\theta^\circ$ F. needs $1{,}081 + \cdot 305\theta - (\theta_0 - 32)$ units; this divided by 966 will express the heat necessary to produce a pound of such steam in terms of the standard of evaporation.

| Pressure of Steam. | Temperature of Steam. $\theta^\circ$ F. | Temperature of Feed Water. | | | | | |
|---|---|---|---|---|---|---|---|
| | | 32° F. 0° C. | 68° F. 20° C. | 104° F. 40° C. | 140° F. 60° C. | 176° F. 80° C. | 212° F. 100° C. |
| 14·7 | 212 | 1·19 | 1·15 | 1·11 | 1·08 | 1·04 | 1·00 |
| 28·8 | 248 | 1·20 | 1·16 | 1·13 | 1·09 | 1·05 | 1·01 |
| 52·5 | 284 | 1·21 | 1·18 | 1·14 | 1·10 | 1·06 | 1·02 |
| 89·9 | 320 | 1·22 | 1·19 | 1·15 | 1·11 | 1·07 | 1·03 |
| 145·8 | 356 | 1·23 | 1·20 | 1·16 | 1·12 | 1·08 | 1·04 |
| 225·9 | 392 | 1·24 | 1·21 | 1·17 | 1·13 | 1·09 | 1·06 |
| 336·3 | 428 | 1·25 | 1·22 | 1·18 | 1·14 | 1·11 | 1·07 |

Thus when we say that the standard evaporation of 1 lb. of coal is 10·3, we mean that 1 lb. of coal will produce 10·3 ÷ 1·20, or 8·6 lbs. of steam at 356° F. from feed-water at 68° F.

**250.** Exercise. The Nixon's Navigation coal used by Donkin and Kennedy in their boiler tests (Art. 261) had a total evaporative power 16·47 [as from and at 212° F.], if measured by the chemist, nd needed 10·5 lbs. of air for complete combustion. Taking the pecific heat of the gases as ·243, and that ·37 lb. of water goes off ith the gases;—

1. Show that if $\theta^\circ_0$ F. is the temperature of the boiler-room, and the temperature of the steam, the heat necessarily wasted in urning this coal in a *perfect boiler* with just the right amount of air $\dfrac{11\cdot 5 \times \cdot 243(\theta - \theta_0)}{966}$ in evaporation units.

2. Taking the temperature of the air in the boiler-room as 60° F

show that for the following pressures of steam in the boiler we have the following results :—

| Absolute Pressure. | Heat (in evaporative units) necessarily going off in gases. | Greatest possible evaporation from and at 212° F. | Percentage of Chemist's determination. |
|---|---|---|---|
| 14·7 | ·43 | 15·67 | 95 |
| 30 | ·54 | 15·56 | 94·5 |
| 50 | ·63 | 15·48 | 94 |
| 75 | ·70 | 15·4 | 93·5 |
| 100 | ·76 | 15·34 | 93 |
| 150 | ·84 | 15·26 | 92·5 |
| 200 | ·91 | 15·19 | 92·25 |
| 250 | ·96 | 15·13 | 92 |

3. Make out tables for $1\frac{1}{2}$ and twice the amount of air necessary for complete combustion.

| Absolute Pressure. | Evaporative power in a perfect Boiler as from and at 212° F. | | |
|---|---|---|---|
| | Air 1. | Air $1\frac{1}{2}$. | Air 2. |
| 14·7 | 15·67 | 15·46 | 15·24 |
| 30 | 15·56 | 15·29 | 15·02 |
| 50 | 15·48 | 15·16 | 14·85 |
| 75 | 15·4 | 15·05 | 14·70 |
| 100 | 15·34 | 14·96 | 14·58 |
| 150 | 15·26 | 14·84 | 14·42 |
| 200 | 15·19 | 14·73 | 14·28 |
| 250 | 15·13 | 14·65 | 14·17 |

It will be seen that if $\theta°$ F. is the temperature of the steam; if $\theta°_0$ F. (taken usually as 60° F.) is the temperature of the supplied air; if $E$ is the evaporative power of the fuel as found in the fuel tester; if $A$ is the weight of air supplied per pound of fuel, if $w$ is the weight of water going off, the true evaporative power in a perfect boiler is

$$E' = E - w - \cdot 238(\theta - \theta_0)(A + 1)$$

and this is what I call $a$ in my formula, Art. 262.

**251. Coal Tester.** Some coal from different parts of each sack being taken from many sacks, it is spread out evenly on a clean floor, and again and again sampled from different parts, till a small quantity is obtained which may be regarded as an average sample. It may be tested chemically,[1] and the calorific power calculated as in Art. 246.

[1] Thorpe's Dictionary gives the following as the process in use by students at the Royal College of Science.

may be burnt with oxygen in a calorimeter, and the heat directly ured. In a simple form, the Thomson calorimeter, a small hed quantity of powdered coal is placed in a small platinum ble inside a glass vessel, surrounded by about two quarts of r in another glass vessel; oxygen is admitted by a brass tube, plays on the surface of the coal which is ignited by a fuse. The ucts of combustion which do not stay inside, escape by holes in bottom of the vessel and pass up as bubbles through the water, g broken up by wire gauze, so that all the heat of combustion s the temperature of the water by an amount which may be sured with a thermometer. I have seen spiral tubes immersed he water provided for the escape of the products instead of by bling.

In a calorimeter in common use the powdered fuel is mixed with ifficient quantity of a mixture of the chlorate and nitrate of ssium to generate enough oxygen for the combustion. My lents have used this for twenty years, and they know quite well the results of such a test are not particularly valuable.

**252.** The **Gas Tester of Mr. Dowson**, as used in my ratory, burns the gas; the hot products are cooled nearly to the perature of the room by contact with metal kept cool by flowing er. There being a steady state of flow of gas and water, the rate ow of the gas is measured in a gas meter; the rate of flow of the er is occasionally tested by measuring the time taken to fill a ked vessel. The difference of temperature of the entering and ing water (usually about 15 or 20 centigrade degrees) is measured two thermometers, and this enables the calorific power to be ulated.

**253. Temperature of Combustion.** It is difficult to know t a man means when he says that he has **measured** the temature of a flame. No doubt he may measure the temperature of ething immersed and struck by the flame. Our difficulties inse when he says he has **calculated** the temperature of a flame. ordinary method of calculation is infantile in its simplicity.

EXERCISE 1. Wood charcoal (calorific power 8,080). is burnt in the right amount of air for complete combustion. What is the of temperature? *Answer.* We have 12·6 lbs. of products, of ific heat, say 0·24, and $8{,}080 \div (12{\cdot}6 \times {\cdot}24)$ is 2,672 degrees tigrade.

I refrain from giving the usual exercises by which the comtion of charcoal in oxygen gives 10,183° C., or of hydrogen in gen 6,743° C. Calculations like this can give no notion even of

the *relative* temperatures produced in the several cases. It is evident that the existence of dissociation will not allow us to assume a constant capacity for heat in the products of combustion. Coke in a furnace produces much more intense heat than coal. In glass-making it is found that 8 or 9 lbs. of coke is equivalent to 12 lbs. of coal in usefulness from this cause. When coke is used in a boiler the fire-box part receives much more heat relatively to the flue part than when coal is used. More intense heat is producible by gaseous fuel than by solid, mainly because there need be no excess air.

**254. Fuels.** The numbers of the following tables give some idea (really a very rough one and sometimes misleading) of the usual composition of one pound of each of the fuels by weight. Substances which occur in mere traces, such as sulphur, are not mentioned. The oxygen is mainly combined with an eighth of its weight of hydrogen as water, and it would be worth while for a student to merely give two columns of numbers, as in the table of Art. 245, one of carbon, the other to be headed "hydrogen," and to be really the hydrogen of the table, with one eighth part of the oxygen subtracted from it, because the combustion of this hydrogen may be thought complete already in the fuel. $h - o/8$ is usually called the *available hydrogen*. I had carefully prepared a column showing the amount of fixed carbon in each, but I have had to discard it. It is interesting to note that our knowledge about the composition and properties of fuels is practically the same as what was available forty years ago. In preparing this book I have made strenuous efforts to increase what was known to me in 1870, but I find **no new reliable information.**

**255. Dried wood** is nearly all of much the same chemical composition; air-dried wood has usually 20 per cent. of hygroscopic water. 50 per cent. carbon, 6 per cent. hydrogen, 42 per cent. oxygen. Some has very little ash; some from 2 to 5 per cent. Sometimes cotton-stalks, brushwood, straw, the residue of sugar cane and other **vegetable refuse** are used as fuels. **Peat** is of very varied density. It is woody tissue changed more or less by oxidation, $CH_4$ and $CO_2$ being given off. It has in boilers about half the evaporative power of coal. In our imperfect calorimeter, peat in its usual state has an evaporative power 4·7; dried 6·0. The older peats are not very different from recent brown coal. The fuels obtained artificially from wood, charcoal, and liquid and gaseous hydrocarbons are not in use for boilers.

**Coal** varies in specific gravity from 1·2 to 1·8. The recent or young coals, lignite and brown coal, retain some of the woody stru

ture which has disappeared in ordinary or older coal, in which the elements of the woody fibre have escaped as carbon-di oxide marsh gas and water, the decomposition being due not so much perhaps to oxygen of the air as to mouldering and internal action.

The **gradual change** from woody tissue is shown in the following table of vaguely correct numbers:—

| | Carbon. | Hydrogen. | Oxygen. |
|---|---|---|---|
| **Wood** . . . . . . . . . . | 100 | 12·18 | 83·07 |
| **Peat** . . . . . . . . . | 100 | 9·85 | 55·67 |
| **Lignite** . . . . . . . . . | 100 | 8·37 | 42·42 |
| **Bituminous coal** . . . . | 100 | 6·12 | 21·23 |
| **Anthracite** . . . . . . . | 100 | 2·84 | 1·74 |

It is evident that the change which a few millions of years' burial, probably under great pressure and some increase of temperature, produces in wood, is to increase the proportion of carbon and diminish that of oxygen; observe that the change is gradual, brown earthy coal and lignite being younger than bituminous, and this being usually younger than anthracite.

From this point of view the following table of composition of brown coal is interesting.

| | Carbon. | Hydrogen. | Oxygen and Hydrogen. Neglecting Water and Ash. |
|---|---|---|---|
| **Fibrous brown coal** . . . | ·63 | ·05 | ·32 |
| **Earthy** ,, ,, . . . | ·72 | ·05 | ·23 |
| **Pitch** ,, ,, . . . (conchoidal in fracture and evidently becoming bitumen). | ·77 | ·075 | ·155 |

**256. Lignite** burns with a long, smoky flame; calorific power 4000 6000; does not cake. **Ordinary coal** is vegetation, such as water-gged drift-wood, or standing trees, which has been covered up with nd and clay, and is found in beds from $\frac{1}{4}$ inch to 4 feet thick, these metimes forming much thicker beds, with thin partings of sand clay between. It is found in the most ancient and most modern ological formations, with every variety of colour and appearance, m the brown Scotch cannel to the velvet black Newcastle caking

coal. With no lustre as in some cannels, to the shining bituminous caking coal and to the semi-metallic iridescent anthracite. There is the softness of Newcastle coal, and the hardness of anthracite coal which yields no bitumen to any re-agent.

The **carbon** varies from ·70 to ·94 in anthracite.
„ „ ·57 to ·84 in cannel.
„ „ ·75 to ·83 in splint coal,

but there are not these extremes in all bituminous coals. *Bituminous* is the name, not wisely but well given, to the coals whose properties are between those of lignite and anthracite. Probably *flaming* is a better title.

The **hydrogen** varies from 1·5 per cent. in anthracite to 9 per cent. in some Scotch coals. In fact it is obvious that coals are of the most varied chemical constituents. They have from 1 to 34 per cent. of **ash.** All yield solid, liquid, and gaseous products. A **shale** has almost no fixed carbon; anthracites have much. **Good coke** is only obtained from caking coals, in which the volatile parts are 25 to 40 per cent. of the whole, so that the coke is 75 to 60 per cent. The fixed carbon varies from 18 to 52 per cent. in caking coals.

**Anthracites** are almost all carbon, and have a short flame, easily extinguished unless kept at high temperature and scrubbed with air. They are difficult to ignite. The specific gravity varies from 1·4 to 1·6. Hard, brittle, submetallic lustre, conchoidal fracture, smokeless flame. Free burning, **bituminous coals** have about 15 per cent. of their weight of volatile hydrocarbons (marsh gas, olefiant gas, tar, naphtha, &c.); when heated they swell and become porous, so that air gets well into every part. If dry there is not much tendency to smoke. Specific gravity 1·25 to 1·3. **Bituminous caking** coals as from Newcastle (usually velvet or grey-black in colour, uneven in fracture, soils the fingers, and fractures in little cubes) have sometimes as much as 30 per cent. of volatile hydrocarbons in them. They burn with a long yellow flame, soften with heat, portions tend to adhere, and they do not become porous when heated, so that they give more trouble in furnaces. Specific gravity 1·2 to 1·25. The bituminous coals have more calorific power as they have more volatile constituents in them, but this renders it more difficult to burn them in boiler grates. Even with the best hand-firing they are apt to give rise to smoke and soot deposited in the flues. **Welsh coal** gives least trouble to the stoker

nting smoke, but mixtures of Newcastle and Welsh coal are difficult to deal with. We may, however, say that even in rials with the most careful hand-stoking some fuel goes away . The presence of water or of much oxygen in the fuel seems ice to the **formation of smoke.** However bituminous a y be, if its volatile constituents are mixed at a sufficiently nperature with enough air, they burn completely with a blue f heated first and cooled before mixing with air, they decom- o marsh gas and hydrogen and carbon, and deposit the carbon e and soot, and the higher the temperature and the more the cooling the more soot will be formed. This is the most nt kind of imperfect combustion. Thus one pound of average coal has 0·80 lb. of carbon and 0·05 lb. of hydrogen, and we that its evaporative power is 15·2. But if we drive off the parts unconsumed (because we do not mix them with air at nough temperature), we have only 0·57 lb. of fixed carbon d the heat due to this, even if we capture it all, can oduce an evaporation of 8·6. As for the fixed carbon, , forming carbon dioxide. But if sufficient air is not sup- s combustion is imperfect, it forms carbon monoxide only, is another kind of imperfect combustion due to there not sufficient supply of air. In the burning of coke, or the fixed in shallow fires especially, it will be found that the fuel needs tually **scrubbed with air.**

he bituminous coals, we have *splint or hard coal*, black shaded own in colour; slaty curved principal fracture; cross fracture and splintery; not easily broken; does not kindle easily; gives ire, high temperature; is much prized. *Cherry or soft coal*, in Staffordshire, is more abundant than the last; has a black or slightly grey appearance, sometimes with shining does not cake; is easily broken, with a slaty fracture, so ere is considerable waste; it is easy to ignite, and burns *Cannel* coal, common near Glasgow and at Wigan and y; so called because it burns with a flame like that of a candle, called *parrot* because its flat pieces are apt to fly off, making noises. Dark grey or brown in colour; takes a polish (jet is y of it); has a flat conchoidal fracture, frequently slaty; does the fingers, is not easily broken: it yields in distillation olatile products and less coke, more ash and more sulphur dinary coal. It has probably been derived from mosses, seaweed, &c., rather than from tree vegetation like other

| One pound of Fuel. | Pounds of Carbon. | Pounds of Hydrogen. | Pounds of Oxygen. | Heat developed. Cent. units. | Evaporative power. |
|---|---|---|---|---|---|
| **Charcoal—** | | | | | |
| Wood . . . . . . . . | 0·93 | — | — | 7487 | 13·95 |
| Peat . . . . . . . . | | | | | |
| **Coke—** | | | | | |
| Good . . . . . . . . | 0·94 | | | | |
| Medium . . . . . . | 0·88 | 0·038 | ·01 | 8351 | 15·56 |
| Bad . . . . . . . . . | 0·82 | | | | |
| **Coal—** | | | | | |
| (Anthracite) . . . . . | 0·94 | 0·02 | 0·01 | 8196 | 15·29 |
| ,, . . . . . | 0·90 | 0·03 | 0·03 | 8148 | 15·18 |
| Dry, bituminous . . . | 0·90 | 0·04 | 0·02 | 8582 | 15·99 |
| ,, ,, . . . | 0·77 | 0·05 | 0·06 | 7663 | 14·28 |
| Caking . . . . . . . | 0·88 | 0·05 | 0·05 | 8594 | 16·03 |
| ,, . . . . . . . | 0·81 | 0·05 | 0·04 | 8069 | 15·03 |
| Cannel . . . . . . . | 0·84 | 0·06 | 0·08 | 8375 | 15·61 |
| Dry long flaming . . . | 0·77 | 0·05 | 0·15 | 7266 | 13·54 |
| Brown earthy . . . . | 0·74 | 0·06 | 0·20 | 7266 | 13·54 |
| Lignite . . . . . . . | 0·65 | 0·06 | 0·25 | 6232 | 11·61 |
| **Peat—** | | | | | |
| Kiln dried . . . . . | 0·60 | 0·07 | 0·30 | 5967 | 11·12 |
| Air dried . . . . . . | 0·46 | 0·05 | 0·24 | 4392 | 8·185 |
| **Wood—** | | | | | |
| Kiln dried . . . . . | 0·50 | 0·06 | 0·42 | 4300 | 10·09 |
| Air dried . . . . . . | 0·40 | 0·05 | 0·33 | 3530 | 6·58 |
| Mineral oil (refined) . . | 0·84 | 0·16 | 0 | 12280 | 22·87 |
| ,, ,, (refined) . . | 0·85 | 0·15 | 0 | 10940 | 20·39 |

| | | Carbon. | Hydrogen. | Oxygen and Nitrogen. |
|---|---|---|---|---|
| Splint coal | Wylam Banks, Newcastle . . . | 74·823 | 6·180 | 5·085 |
| | Glasgow coal-field . . . . . . | 82·924 | 6·491 | 10·457 |
| | Wigan, Lancashire . . . . . . | 83·753 | 5·660 | 8·039 |
| Cannel coal | Parrot coal, Edinburgh . . . . | 67·597 | 5·406 | 12·432 |
| Cherry coal | Jarrow, Newcastle . . . . . . | 84·846 | 5·048 | 8·430 |
| | Chief coal from Glasgow . . . . | 81·208 | 5·452 | 11·923 |
| Caking coal | Garsfield, Newcastle, deep bank | 87·952 | 5·239 | 5·416 |
| | South Hetton, Durham . . . . | 83·274 | 5·171 | 3·036 |

**Average Composition of Bituminous Coals from Different Localities.**

| Locality, average of. | Specific gravity of coal. | Carbon. | Hydrogen. | Nitrogen. | Sulphur. | Oxygen. | Ash. | Percentage of coke left by each coal. |
|---|---|---|---|---|---|---|---|---|
| 36 samples from Wales . . . | 1·315 | 83·78 | 4·79 | 0·98 | 1·43 | 4·15 | 4·91 | 72·60 |
| 18 ,, ,, Newcastle | 1·256 | 82·12 | 5·31 | 1·35 | 1·24 | 5·69 | 3·77 | 60·67 |
| 28 ,, ,, Lancashire . | 1·273 | 77·9 | 5·32 | 1·30 | 1·44 | 9·53 | 4·88 | 60·22 |
| 8 ,, ,, Scotland . . | 1·259 | 78·53 | 5·61 | 1·00 | 1·11 | 9·69 | 4·03 | 54·22 |
| 7 ,, ,, Derbyshire . | 1·292 | 79·68 | 4·94 | 1·41 | 1·01 | 10·28 | 2·65 | 59·32 |

The average weight of coal in heaps is 50 to 60 lb. per cubic foot. **Coke** is what remains when the volatile constituents (10 to 65 per cent.) are driven off from coal at a high temperature (in a coke oven is best; in a gas-making retort is next best; in open heaps is least best and least economical). Coke tends to absorb 15 to 20 per cent. of its weight of moisture, even when protected from rain. There is usually 10 to 15 per cent. of ash.

Waste or small **coal pulverized** is often injected with regulated amounts of air from fans into furnaces giving good results.

**257.** Crude **petroleum** consists of about ·85 of carbon and ·15 of hydrogen. Its calorific power as measured varies from 9950 to 10,830, being greater than that of refined oil. For the same production of heat the volumes of coal and crude petroleum are about as 50 to 33. Where good means have been employed for injecting petroleum with a proper supply of air against fire brick in marine furnaces, there were found a reduction of 40 per cent. in weight of fuel carried, and 6 per cent. in bulk, and 75 per cent. in labour. There is prompt lighting and extinguishing of the fire, and great ease of regulation and perfect combustion, with less excess air than when coal is used. There is greater cost of fuel.

Air passed over the surface of the **lighter oils** (with low flashing points, and therefore dangerous) takes up sufficient vapour to become explosive, and this mixture is used in some engines. The law makes it troublesome now to carry such oils. **Oil gas** may be made from the ordinary petroleums, and used in gas engines. A good oil engine mixes its supply of oil with air, explodes it with perfect combustion, and utilises its energy without there being any tarry deposit left to clog the valves. Properties of oils important in oil engines are described in Art. 280.

The buttery-looking crude **residue left** after extracting oil from shale will not burn on applying a light. When melted and forced in jets with superheated steam and air against fire-clay furnace sides, it burns well.

**258. Gaseous fuel** is easily conveyed by pipes; there may be great economy and higher temperature in its combustion than with a solid fuel, not much more air being supplied than what is just necessary; there is no dust, no cinder, no ash. **Coal gas** manufactured for lighting purposes is also employed in gas engines. Coal for gas-making is expensive, gives off about 30 per cent. of volatile stuff, which yields about 5 cubic feet of gas per lb., leaving about 0 per cent. of coke.

**Water gas** is produced by alternately passing air up through a

thick fire to make the coke white hot, and then passing up steam. The $H_2O$ in presence of hot carbon becomes $H_2$ and $CO$. Some forms of water-gas are good in boiler and metallurgical furnaces, but contain enough tar to clog the valves of a gas engine. **Dowson's** gas seems to contain more of the heating power of the fuel (it must be anthracite or coke, else the tarry products give trouble) than any other. A fire more than 18 inches thick is maintained in a fire-brick lined furnace. Fuel is admitted through a hopper and valve. There is a little boiler producing superheated steam, which blows by a nozzle into a closed ashpit, carrying air with it. The action is continuous. The oxygen of the air combines with carbon to form $CO_2$. The $CO_2$ dissociates as it rises in the fire into $CO$ and $O$; this oxygen later combines with more $C$. Again the $H_2O$ in presence of white hot $C$ becomes $H_2$ and $CO_2$, and some of the $CO_2$ dissociates as before. Coming from the top surface of the fuel we have $H$ and $CO$, and some $CO_2$ with nitrogen, also we find ashy dust and tarry matters. The gases are cooled and passed through water spray and wet coke into a gas holder, and drawn off for use in gas engines or to be burnt in furnaces.

**Natural gas** is found in districts where oil is found; it rises from a depth of 500 to 2,000 feet, and at the surface has a pressure of from 150 to 200 lb. per square inch, being at first at 1,000 lb. per square inch. Calorific power 14,000 to 15,600 per pound. It is found that 1,000 cubic feet of gas is equal to from 80 to 133 lb. of coal in boiler heating power. These gas wells are rapidly getting exhausted, as is shown by the great diminution of pressure.

# CHAPTER XXVI

## THE EFFICIENCY OF A BOILER

**259.** THE most important of steam engine processes are the **giving of heat to water in the boiler**, and the taking of heat from **water (steam) in the condenser**. In an exceedingly good engine we may say that we give energy represented by the number 10 in the boiler, take away 9 in the condenser, converting 1 into indicated work.

Our greatest trouble is in the boiler. The fuel is white hot, and **radiates heat** very rapidly to the fire box. In a marine boiler or stationary boiler about half the total heat reaches the water through the sides of the fire box. But if the fire on the grate is made thicker, as it is in a locomotive boiler or in a marine boiler under forced draught, it radiates only a little more per square foot of grate, because its exposed surface and temperature are not much increased by mere thickness, and consequently only about a quarter, or even less, of the total heat reaches the water through the fire box. We may take it that the fire box part of the heating surface is very much more efficient per unit area than the flue part, and this is one reason why the Thornycroft boilers are so efficient. But if in a particular boiler we burn twice as much more fuel in the hour, although the flues will give more than twice as much heat to the water, the fire box will increase its supply only a little. And here we have a very curious property of flues which ought to be well studied. Whether a tube is made of copper or iron or brass, is of no consequence, except as to convenience and oxidation by the flame. The real resistance to the passage of heat is not due to the bad conductivity of the metal; it is due to the fact that the particles of hot gases will not come up fast enough to the surface to get cooled, and the particles of water will not come up fast enough on the other side to get heated. See Art. 377.

page 588

EXERCISE 1. If the average difference of temperature between flue gases and boiler water is 500 Fahrenheit degrees; assume that a $\frac{3}{8}$-inch metal plate has this temperature difference between its sides, what heat passes per hour? Express the heat in evaporation units.

*Answer.* Taking the conductivity $k$ of copper as ·004, and of iron as ·00088 in inch second pound units, the heat per square foot per second is $144 \times 500\,k \div \frac{3}{8}$. In evaporation per hour it is $144 \times 1333\,k \times 60 \times 60 \div 966$ or $7{\cdot}16 \times 10^5\,k$, and is 2864 lbs. in the case of copper, 630 lbs. in the case of iron.

EXERCISE 2. The largest result from an actual boiler is seen in the table, page 426, to be 10 lbs. of evaporation per square foot of heating surface per hour, what fraction of the total resistance to the passage of heat is made up of mere resistance of the metal?

*Answer.* 0·016 in the case of iron, 0·0035 in the case of copper.

**260.** Given water and steam at a certain temperature and hot furnace gases at a very much greater temperature; to get the heat from the one to the other quickly and without too great an extent of heating surface. **This is the problem** to be solved by boiler makers. There is a plate of metal through which the heat has to pass. The resistance to the heat passing seems to be very greatly at the two surfaces. The actual thickness of metal (if less than $\frac{3}{8}$-inch thick), and even the nature of the metal (that is, whether it is copper or whether it is iron or brass), do not seem to matter very much. If we sometimes object to iron tubes being used, it is not because iron is a much worse conductor of heat than copper; it is because the iron suffers more from the flame. What makes a good deal of difference is this: gases and liquids give up or take up heat by convection. They are really bad conductors of heat. The student ought to hang a red hot ball in water near its surface, and he will find that although the surface water boils, a thermometer placed an inch below does not show any rise of temperature. Watch a flask of water heating over a Bunsen burner; throw in some fine solid particles, say of potato, to look at, and note how the hot water rises and the cold water replaces it.

We want the surfaces of the metal wall to be scrubbed, the one with hot gases and the other with circulating water, and the student who pays most attention to simple experiments on convection is most likely to invent the best boiler.

We have much the same problem in getting heat away from the steam in a surface condenser. Joule, who studied convection, was able to condense 100 lbs. of steam per hour per square foot of surface. Practical engineers are happy if they get one-twentieth of Joule's

rformance. He let cold water in a tube surround steam in a con-ntric tube; they flowed in opposite directions. Probably the best iler will be one in which a flame or hot gas tube surrounds or surrounded by a water tube, the gas and water flowing fast in posite directions.

In Fig. 198 I show a vertical boiler with vertical *Field* tubes ig. 201) filled with water surrounded by flame. If these were linary tubes, the water in them would get red hot and would casionally burst out with violence, and this would form one of the ry worst possible contrivances for heating water. In truth, how-er, there is an inner tube, fixed as shown in Fig. 201, so that hot ter rises in the outside space and cold water comes down the itral tube, the circulation being very rapid. Till Thornycroft rented his boiler, this was the most expeditious contrivance for ating water.

**261.** I have examined a great number of experimental results m boilers. Many of them are troublesome to deal with, because els and states of metal surfaces differ. Draught is not always ecified. The student ought carefully to study the following results. :. Donkin's book has just been published, in which he describes all e experiments on boilers which a student is likely to find time study. The information in the table on the following page is from older publication.

It will be noticed that with all good types of boiler, working at eir best, we find that we seldom have less than $9\frac{1}{2}$ lbs. of steam tandard) per pound of fuel and seldom more than 13, although the e varies from 9 lbs. of steam per square foot of heating surface to , and the fuel per square foot of grate varies from 60 to 8.

We have seen, Art. 250, that the total evaporative power of a elsh coal may be taken as 16·47. In the following, I know that :. Donkin rejects of this 0·37, because of latent heat of the hygro-pic and formed steam. I think he has no more right to do this in to reject other losses of heat which are absolutely certain in all lers; however, let us take 16·1. The efficiency 75 per cent. in the owing table means that $16{\cdot}1 \times 0{\cdot}75$ lb. of *Standard* evaporation produced per pound of coal.

Mr. Donkin gives the following summary of 405 boiler tests anged in order of merit.

I said, in Art. 134 that the boilers in the London electric supply tions produce, almost without exception, on an average, $8\frac{1}{2}$ lbs. of am per pound of coal. They may be taken as steam of 165 lbs. square inch from feed water at 100° F. It is easy to show that

# BOILER EXPERIMENTS BY DONKIN AND KENNEDY, 1887—90.

| | Vertical tubular | Lancashire | | Locomotive type wet firebox and with large tubes | | Water tube | | Three Cornish boilers, Galloway tubes | Lancashire boilers with Galloway tubes in each flue | Green's economiser and mechanical stokers | Portable boiler, locomotive type | Elephant boiler, economiser generally used with 2 boilers | Steel locomotive boilers on tank goods engine with copper fireboxes: not running | running | running | Merryweather's fire-engine vert. multitubular | Cornish multitubular | Portable boiler, locomotive type | Cornish, Galloway tubes | Cornish, multitubular corrugated flue | Locomotive (stationary) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Heating surface (boiler) sq. ft. | 771 | 567 | 567 | 778 | 778 | 470·5 | 470·5 | 2100 | 980 | 980 | 316 | 430 | 850 | 850 | 850 | 120 | 101·5 | 295 | 324 | 804 | 1177 | 1 |
| Economiser heating surface (sq. ft.) | 0 | 0 | 0 | 0 | 0 | 470·5 | 470·5 | 0 | 1142 | 1142 | 0 | 1033 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| Total heating surface (sq. ft.) | 771 | 567 | 567 | 778 | 778 | 941 | 941 | 2100 | 2078 | 2078 | 316 | 1463 | 850 | 850 | 850 | 120 | 101·5 | 295 | 324 | 804 | 1177 | 3 |
| Area of fire grate (sq. ft.) | 10·5 | 15 | [illegible] | 10·5 | [illegible] | 14·95 | 14·95 | 63 | 21 | 21 | 9·06 | 19·15 | 12·4 | 12·4 | 12·4 | 5·70 | 3·55 | 10·5 | 16·75 | 20 | 31·43 | 4 |
| Steam pressure (above atmosphere) | 40 | [illegible] | [illegible] | 14·0 | [illegible] | 100·8 | 90·2 | 61·7 | 93·8 | 92·7 | 75·7 | 60·2 | 145 | 133·7 | 130·7 | 115 | 70·2 | 80·7 | 66·3 | 88·7 | 104·7 | 5 |
| Feed temperature (Fahr.) | 42·0 | [illegible] | [illegible] | 45 | [illegible] | 49 | 49·3 | 116·2 | 88·5 | 80·8 | 56·3 | 65·3 | 56·5 | 50·7 | 52 | 52 | 58·7 | 60 | 98·1 | 47 | 59·2 | 6 |
| Total feed water per hour (lbs.) | [illegible] | 177 | 2000 | [illegible] | [illegible] | 5730 | 2013 | 4401 | 2552 | 2760 | 1087 | 2100 | 4454 | 3020 | 4000 | 1310 | 444 | 920 | 2104 | 4020 | 7006 | 7 |
| Total coal per hour (lbs.) | 130 | [illegible] | [illegible] | [illegible] | [illegible] | 214 | 220 | 482 | 334 | 357 | 120 | 211·5 | 441 | 348 | 303 | 107·6 | 48 | 114 | 262 | 429 | 801 | 8 |
| Thickness of fire (ins.) | 13 to 15 | [illegible] | [illegible] | [illegible] | | 3·5 | 3 | 9 to 12 | 10 to 12 | | 4·5 | — | 5 to 6 | 4 to 5 | | — | 4 | — | 8 | 3 to 4 | 2½ | 9 |
| Number of times fire was fed per hour | 7·1 | [illegible] | [illegible] | [illegible] | | 12 | 12 | 0·7 | continuous | | 3·73 | 8·0 | 8 | 5 | 5·3 | — | 2·2 | — | - | 8 | 8 | 10 |
| Temp. (Fahr.) of furnace gases after leaving boiler | 367 | [illegible] | [illegible] | [illegible] | [illegible] | 330 | 324 | 495 | 372 | 270 | 635 | 264 | | 570 | 617 | — | 408 | 575 | 533 | 732 | | 11 |
| Temp. (Fahr.) of gases between boiler and economiser | | | | | | 640 | 620 | | 500 | 401 | | | | — | | — | — | — | | — | | 12 |
| Ratio of air used to that theoretically required | 1·56 | 2·4 | 1·60 | 2·01 | 4·26 | 2·62 | 2·70 | 1·96 | 3·0 | | 1·81 | 2·02 | 1·56 | 2·05 | — | — | 2·8 | — | 2·07 | 1·56 | | 13 |
| Lbs. of coal per sq. ft. of grate per hour | 17·0 | 10·3 | 16·5 | 7·03 | 0·25 | 14·3 | 14·7 | 6·96 | 16 | 16·8 | 12·4 | 11·0 | 35·5 | 28·1 | 31·7 | 34·3 | 13·7 | 10·8 | 16·7 | 21·6 | 25·5 | 14 |
| Lbs. of coal per sq. ft. heating surface (boiler only) | 0·24 | 0·27 | 0·40 | ·34 | ·20 | ·46 | ·47 | ·21 | ·30 | ·36 | ·20 | ·40 | ·51 | ·42 | ·40 | 1·65 | ·25 | 0·4 | ·81 | ·53 | ·68 | 15 |
| Evaporation per lb. of coal (as from 212° F.) | 9·57 | 9·92 | 10·02 | 9·55 | 8·46 | 9·9 | 9·50 | 11·4 | 10·99 | 11·16 | 10·70 | 10·06 | 12·15 | 12·51 | 12·20 | 7·96 | 9·91 | 9·67 | 9·16 | 10·15 | 10·64 | 16 |
| | | | | | | 72·6 | 70 | - | 79·7 | 70·2 | – | 76·8 | * These values where economiser is included. | | | | | | | | | |
| Percentage heat usefully spent | 62·6 | 60·0 | 66·8 | 64·9 | 55·7 | 68·9 | 66·7 | 70·3 | 66 | 60·7 | 68·3 | 64 | 78·6 | 80·7 | 79·5 | 56 | 61·8 | 68·0 | 60·5 | 64·3 | 71·0 | 17 |
| Percentage heat escaping in furnace gases | 11·4 | 16·0 | 18·9 | 24·5 | 28·0 | 16·9 | 16·5 | 16·0 | 21·8 | — | 20·0 | 6·5 | 17·0 | 20·6 | 22·0 | — | 22·7 | — | 22·4 | 20 | | 18 |
| Percentage radiation | 4·4 | 6·0 | 5·0 | 6·1 | 0·9 | 6·4 | 6·3 | 4·0 | 7·1 | — | 5·0 | 5·4 | 3·0 | 5·9 | 3·4 | — | 4·0 | — | | 7·0 | 3·0 | 19 |
| Percentage lost by imperfect combustion | 12·4 | 0·0 | 1·2 | 0 | 0 | 3·0 | 0·8 | 1·7 | 0 | — | 1·3 | 0 | 4·3 | 0 | 0 | — | — | — | 0 | 1·6 | - | 20 |
| Percentage heat unaccounted for | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | — | [illegible] | — | 17·1 | 8·8 | 20·0 | 21 |

ans an *average* efficiency under very varying load, and resting nked up fires, of 61 per cent.

| Type. | Stoking. | No. of experiments made. | Mean efficiency of two best experiments on each type per cent. | Lowest efficiency of each type, one experiment only. | Mean efficiency of all the experiments. |
|---|---|---|---|---|---|
| tube (1½-inch tubes) | hand | 6 | 84·1 | 66·6 | 77·4 |
| tive . . . . . . . | ,, | 37 | 83·3 | 53·7 | 72·5 |
| hire (2 flue) . . . . | ,, | 10 | 74·4 | 65·6 | 72·0 |
| orey . . . . . . . . | ,, | 9 | 76·1 | 57·6 | 70·3 |
| ,, . . . . . . . . | hand and machine | 29 | 79·8 | 55·9 | 69·2 |
| ck . . . . . . . . | hand | 24 | 75·7 | 64·7 | 69·2 |
| smoke tube . . . | ,, | 11 | 81·2 | 56·6 | 68·7 |
| . . . . . . . . . | ,, | 25 | 81·7 | 53·0 | 68·0 |
| . . . . . . . . . | ,, | 9 | 81·0 | 55·0 | 67·7 |
| ck . . . . . . . . | ,, | 6 | 69·6 | 62·0 | 66·0 |
| nt . . . . . . . . | ,, | 7 | 70·8 | 58·9 | 65·3 |
| tube (4 inch tubes) . | ,, | 49 | 77·5 | 50·0 | 64·9 |
| hire (2 flues) . . . | machine | 40 | 73·0 | 51·9 | 64·2 |
| . . . . . . . . . | hand | 3 | 65·9 | 60·0 | 62·7 |
| hire (2 flues) . . . | ,, | 107 | 79·5 | 42·1 | 62·4 |
| ck . . . . . . . . | ,, | 6 | 73·4 | 54·8 | 61·0 |
| hire (3 flues) . . . | ,, | 6 | 66·7 | 52·0 | 59·4 |
| nt . . . . . . . . | hand and machine | 8 | 65·5 | 54·9 | 58·5 |
| hire (2 flues) . . . | hand | 8 | 74·3 | 45·9 | 57·3 |
| l . . . . . . . . | ,, | 5 | 76·5 | 44·2 | 56·2 |

**2. A Working Theory.** On the whole, the following prin- eem sufficiently well established to be worth the consideration who design boilers.

**a fire-box**, the grate area of which is $G$ square feet, when $F$ uel are burning per hour, the evaporation is roughly repre- by $bG + cF$, where $b$ and $c$ are greater as the ratio of the the fire-box surface above the grate exposed to radiation to ite area is increased; $b$ and $c$ are diminished by admitting ir per pound of fuel than is necessary. For both these reasons are greater in a locomotive than in a Lancashire boiler. iore complicated formula [1] may easily be framed to suit better

[1] ve sometimes used the following:—The evaporation from the fire-box is

$$\frac{aF}{bF+1} \text{ or } \frac{aF}{b^1f+1}$$

is $\frac{A}{45H}$, or $b^1$ is $A \div 45k$, $A$ being pounds of air per pound of fuel, $H$ being ce of the fire-box which may receive heat by radiation, $f$ being fuel per hour re foot of grate, and $k$ being radiation surface per square foot of grate. this is a better formula in some cases, but applied to the French e boiler, Art. 264, $b$ is very far from being a constant.

the notions which have been given us by our knowledge of combustion, but we are looking only for a general formula which shall be fairly correct within the ordinary limits, and which shall lend itself to easy algebraic work, and this one will do.

If the evaporation per pound of fuel when perfectly burned in a coal tester, is $a$ lb. If $a - \boldsymbol{a}$ represents the loss of heat per pound of fuel, because more than the exactly right amount of air is admitted, and because there is not a sufficiently large and well arranged combustion chamber; because of hot ashes, &c.; the available heat is represented by $\boldsymbol{a}$ and not $a$. Thus, when there is natural draught, and especially when the fire is thin, the fuel is not scrubbed with air sufficiently, unless we admit twice the absolutely necessary amount; we saw in Art. 247 that much of the value of the fuel was lost. When there is such poor draught and such a small combustion chamber that ¾ths of the hydrocarbons go off unconsumed, we saw that much more of the value of the fuel is lost. In fact, we may say, that by bad or good stoking, bad or good size of combustion chamber, bad or good draught, $\boldsymbol{a}$ may be anything from ·95 to ·60 of $a$.

The total available evaporation is then $\boldsymbol{a}F$, and if we effect the amount $bG + cF$ in the furnace, there remains $\boldsymbol{a}F - (bG + cF)$ or $(\boldsymbol{a} - c)F - bG$ to be dealt with **in the flues.** Now it seems that the efficiency of a flue may very well be represented by

$$e = 1/(1 + \mu/l)$$

where $l$ is the average length of the flues and $\mu$ is proportional to the hydraulic mean depth of the flue on the flame side, and also to the hydraulic mean depth or badness of circulation on the water side. (See Chap. XXXIII.)

The **hydraulic mean depth** on the flame side of a straight tube of any kind of uniform section is the area of its section divided by perimeter touched by the flame or gases. The hydraulic mean depth or badness of circulation on the water side is not easy to specify exactly in mathematical language, but is quite easy to understand; it is greatly diminished by artificial stirring. The efficiency of a flue is greatly diminished by the deposition of soot on the gas side, or by deposition from the water on the other side, and either of these may be said to increase $\mu$. When the flue is not a mere straight tube, the hydraulic mean depth may be said to be diminished by all obstructions which are such that the hot gases are made to impinge on heating surface. It is not altogether

rrect. and yet is nearly correct to say that anything which in-eases friction in a flue bounded by heating surface, increases ficiency. A feed-water heater may either be taken as increasing $l$ or minishing $\mu$. The gases give their heat to the metal because they e hotter than the metal, and because they are in turbulent motion, hich continually replaces the cooled layers close to the metal with esh, hot stuff. I have worked out a rough theory of how the heat given up, and it is given in Chap. XXXIII. It suggests that hen fluid friction is quadrupled, the rate of giving up of heat is oubled.

We find in Art. 259 that the resistance to the passage of heat om gases to water is usually about 300 times as great as the mere sistance of a copper tube $\frac{3}{8}$ inch thick. I have found by experi-ient that the temperature of a cooling ball of stone close to its urface remains for a long time very much higher than the cold water hich violently scrubs the surface, and this curious phenomenon ust be greatly exaggerated in the difference in temperature between ases in a flue and the metal of the flue. Men who write elaborate reatises on steam boilers, will quote Isherwood's experiments, which howed that the heat passing through flues $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{3}{8}''$ thick was ractically the same, so that it did not depend to any appreciable xtent upon the thickness of the metal, and yet they will also, to he confusion of a thoughtful reader, dilate somewhere else upon the nportance of doubling evaporation by halving the thickness of the ubes.

It is usual to speak of the heating surface $S$ of a boiler, counting s heating surface the total surface of metal which is touched by he hot gases. In boilers, the evaporation ranges from $1\frac{1}{4}$ to 9 lbs. f steam per hour per square foot of heating surface, and from 00 to 1000 lbs. per hour per square foot of grate. I consider that lmost nothing has retarded improvement in boilers so much as ich statements about area of heating surface. In many ex-eriments on multitubular boilers when half the tubes have been losed up, the boiler was found to be just about as efficient as efore.

We cannot expect to have the same efficiency of a flue for very reat differences in the amounts of gas passing. If this were so, ie efficiency of a set of tubes would depend only on their length d diameter, and not on the number of them. But it is interest-g to notice that within certain limits there is no great loss of ficiency in plugging up half the tubes. Thus, for example, the

efficiency of the Wigan boiler[1] was tested. Every alternate diagonal row of tubes was plugged and the heating surface was thus reduced by 206 square feet.

| | All tubes open. | Half of tubes closed. |
|---|---|---|
| lb. of coal per sq. ft. of grate per hour | 25 | 24 |
| Evaporation per lb. of coal | 12·4 | 12·2 |
| Very light smoke, duration in minutes per hour | 2·8 | 8·0 |

So that with about twice the velocity of gases in a tube, we have about twice as much evaporation from the tube.

**263.** The subject is so important that I will describe here some famous **experiments made in France** upon a locomotive boiler, about twenty years ago. Grate area 9 square feet; 125 tubes 148 inches long, 1⅞ inches inside diameter. The boiler was divided into five sections, the tubes running through, but the sections kept distinct. Each length of the barrel was 3 feet long, there being 3½ inches of tubes attached to what is called the fire-box section. The draught was produced in the chimney by a blast of steam from another boiler. There seems to be pretty much the same evaporative power in 1 lb. of coke as in 1 lb. of briquettes, and I usually take it that 1 lb. of fairly dry coke has a total evaporative power of 14 lbs. Now, in the experimental boiler the pressure was 80 lbs. per square inch, the feed-water was probably at 62° F.; this needs 1149 units per lb. as against 966 units, which is the standard of evaporation. That is, we may take the evaporation of 1 lb. in the table as equivalent to 1·2 lb. as from and at 212° F. Hence, of the evaporation in the table, 1 lb. of coke would produce a total evaporation of 11·76 lbs. This is reduced because of the moisture in the coke.

Also, neglecting the fact that possibly more than 50 per cent. excess air was admitted, we have probably 18 lbs. of gases of specific heat about 0·25; the most perfect flues cannot reduce these gases to a lower temperature than 320° F., that of the water, and so we must deduct $18 \times 260 \times 0{\cdot}25$ or 1170 units of heat. This reduces the total available evaporation to 10·88 lb. It is not unreasonable to

[1] *The Steam Engine*, by D. K. Clark, in four volumes, p. 114. I shall often refer to this book in the following pages.

gine that 10 per cent. of the whole evaporative power or lb. is absent on account of incomplete combustion, because this uite usual even with good stoking, and we arrive at 9·7 lbs. as the st probable total available evaporation of 1 lb. of the fuel. My dents have tried and we have seen that whether we take 11, or , or 9, there is no great difference in the following deductions.

The observations reduced to English units are given in the ble. The evaporations are in pounds per hour.

| | Draught-inches of water. | Fuel in pounds per hour. | Evaporation from firebox, $E_0$. | Evaporation from first section of flue, $E_1$. | Evaporation from second section of flue, $E_2$. | Evaporation from third section of flue, $E_3$. | Evaporation from fourth section of flue, $E_4$. | Total evaporation of boiler. | Water evaporated per pound of fuel. |
|---|---|---|---|---|---|---|---|---|---|
| | | | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. |
| ke | ·79 | 436·5 | 1530 | 996 | 430 | 228 | 128 | 3312 | 7·59 |
| | 1·57 | 654·7 | 2018 | 1408 | 671 | 380 | 231 | 4708 | 7·19 |
| | 2·36 | 727·5 | 2222 | 1789 | 931 | 528 | 337 | 5806 | 7·98 |
| | 3·15 | 793·7 | 2229 | 1921 | 997 | 572 | 396 | 6115 | 7·71 |
| | 3·94 | 771·6 | 1810 | 1892 | 1030 | 614 | 440 | 5786 | 7·50 |
| riquettes | ·79 | 476·2 | 1806 | 964 | 445 | 240 | 147 | 3602 | 7·56 |
| | 1·57 | 743·0 | 2356 | 1368 | 738 | 687 | 264 | 5110 | 6·88 |
| | 2·36 | 923·7 | 2933 | 1969 | 1025 | 645 | 425 | 6997 | 7·58 |
| | 3·15 | 1025·0 | 2291 | 1778 | 900 | 579 | 422 | 6990 | 6·82 |
| | 3·94 | 978·8 | 2961 | 2499 | 1228 | 774 | 502 | 7984 | 8·16 |
| Briquettes. Half the bes closed y plugs at re-box end. | ·79 | 386·0 | 1811 | 803 | 356 | 191 | 117 | 3278 | 8·45 |
| | 1·57 | 610·7 | 2057 | 1138 | 550 | 308 | 187 | 4240 | 6·94 |
| | 2·36 | 707·7 | 2710 | 1448 | 722 | 449 | 290 | 5619 | 7·94 |
| | 3·15 | 793·6 | 2979 | 1624 | 845 | 475 | 334 | 6252 | 7·88 |
| | 3·94 | 848·8 | 3058 | 1874 | 948 | 580 | 425 | 6886 | 8·11 |

**264.** Exercise 1. Plot $E_0$ (the evaporation **from the fire-box**) d $F$ (lb. of fuel) on squared paper, and see if you obtain some such les as these—

$E_0 = 700 + 2\ F$, Coke,

$E_0 = 700 + 2{\cdot}4\ F$, Briquettes,

$E_0 = 700 + 2{\cdot}8\ F$, Briquettes with half the tubes plugged up.

e shall now consider **the flue part.**

In whatever way I have manipulated the figures of these famous ts, and I have done this in many ways at many times, I have always ld the fifth set for briquettes abnormal, and I think the reason in the tests not having lasted long enough. As a matter of fact, vever, it is evident that there are no discrepancies from constancy

in $\epsilon_4$, the efficiency of the flues of the next table, which may not be due to errors of measurement. It is interesting to notice that there was actually a greater flue efficiency when half the tubes were plugged up. I make it out to be ·612 when all the tubes were open, and ·655 when half were plugged up, the mean being ·634. If we reject the fifth test with briquettes, as I have always been greatly inclined to do, there is even a greater increased efficiency due to plugging up the tubes.

EXERCISE. If 10 is the greatest possible evaporation per pound of fuel, $10\,F - E_0$ is the heat entering the first section of the flue; $10\,F - E_0 - E_1$ is the heat entering the second section of the flue, and so on. If we divide the evaporation in a part of the flue by the heat entering it, we get its efficiency. I have calculated $e_1$, $e_2$, $e_3$, $e_4$, the efficiencies of the four flue parts. Also I have calculated $\epsilon_1$, $\epsilon_2$, $\epsilon_3$, $\epsilon_4$, where, for example, $\epsilon_3$ means the efficiency of the first three sections taken together, and $\epsilon_4$, the efficiency of the whole of the flue part.

It is to be noticed with briquettes that, whether we take the boiler as a whole or any portion of the flue part, the efficiency was actually greater when half the tubes were closed up. Notice that in any of the three sets, the flues, or any section of them, has about the same efficiency, whether there is much or little fuel being burnt.

FLUE EFFICIENCIES.

| Fuel, lb. per hour. | | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $\epsilon_1$ | $\epsilon_2$ | $\epsilon_3$ | $\epsilon_4$ |
|---|---|---|---|---|---|---|---|---|---|
| *Coke* . . . . | 436·5 | ·351 | ·234 | ·162 | ·108 | ·351 | ·502 | ·582 | ·630 |
| | 654·7 | ·311 | ·215 | ·155 | ·112 | ·311 | ·459 | ·542 | ·593 |
| | 727·5 | ·354 | ·286 | ·226 | ·187 | ·354 | ·539 | ·642 | ·708 |
| | 793·7 | ·336 | ·263 | ·205 | ·178 | ·336 | ·510 | ·610 | ·680 |
| | 771·6 | ·320 | ·256 | ·206 | ·204 | ·320 | ·493 | ·600 | ·680 |
| Means . . . . | | ·334 | ·251 | ·191 | ·158 | ·334 | ·501 | ·595 | ·658 |
| *Briquettes* | 476·2 | ·326 | ·223 | ·154 | ·112 | ·326 | ·476 | ·558 | ·606 |
| | 743·0 | ·270 | ·198 | ·132 | ·103 | ·270 | ·418 | ·495 | ·548 |
| | 923·7 | ·311 | ·236 | ·195 | ·160 | ·311 | ·475 | ·578 | ·644 |
| | 1025·0 | ·255 | ·177 | ·136 | ·114 | ·255 | ·387 | ·470 | ·531 |
| | 978·8 | ·368 | ·284 | ·252 | ·217 | ·368 | ·547 | ·661 | ·731 |
| Mean . . . . | | ·306 | ·224 | ·174 | ·141 | ·306 | ·461 | ·352 | ·612 |
| Mean of first four | | ·290 | ·208 | ·179 | ·122 | ·290 | ·439 | ·525 | ·580 |
| *Briquettes* Half the tubes closed by plugs at firebox end. | 388·0 | ·389 | ·281 | ·210 | ·163 | ·389 | ·560 | ·650 | ·708 |
| | 610·7 | ·281 | ·188 | ·130 | ·091 | ·281 | ·417 | ·494 | ·539 |
| | 707·7 | ·332 | ·247 | ·205 | ·117 | ·332 | ·495 | ·598 | ·665 |
| | 793·6 | ·326 | ·254 | ·190 | ·164 | ·326 | ·497 | ·592 | ·661 |
| | 848·8 | ·344 | ·266 | ·222 | ·191 | ·344 | ·519 | ·625 | ·702 |
| Means . . . . | | ·334 | ·247 | ·191 | ·145 | ·334 | ·498 | ·592 | ·655 |

Now in the first and second series of tests each section consists of 5 tubes, $1\frac{3}{4}$ inch internal diameter, 3 feet long; taking dimensions feet, the hydraulic mean depth $m$ of a tube (and therefore of any imber of tubes) being sectional area $\div$ perimeter, is ·039 feet. We id that the above averages for briquettes, all the tubes being in use tisfy very well the law for flues made of round tubes

$$e = \frac{\cdot 87\, l}{153\, m + l}$$

For coke with all the tubes, and for briquettes when half the ibes are closed, it is very strange but we find that the results agree ith wonderful exactness giving the rule—

$$e = \frac{\cdot 97\, l}{145\, m + l}$$

We must then in no case depend upon the area of heating surface; it we take it that without great error we may assume that there is iuch the same fraction of their total heat taken from the gases by a ie, however quickly they run through, and that the efficiency of the ues is—

$$e = \frac{l}{hm + l} \quad . \quad . \quad . \ (1)$$

'here $l$ is the length of a tube and $m$ its hydraulic mean depth or $\frac{1}{4}$ f its diameter, and $h$ is a constant, which is less as the circulation of 'ater is better. We may in general take $h$ to be 150 with the sort f circulation of water common in locomotives.

**265.** Of the total evaporative power $aF$, the furnace takes the mount $bG + cF$; and the amount $aF - (bG + cF)$ enters the flues. he total evaporation $W$ is therefore made up of

$$bG + cF, \text{and } e \left\{ (a - c)\ F - bG. \right\}$$

We take $a$ equal to about 0·9 of the real evaporative power of the el after we have subtracted the energy which the gases would still ive if they were reduced to the temperature of the water in the iler. Also we must have deducted something for want of perfect mbustion. Even if there is absolutely no smoke there is probably ' per cent. to deduct; black smoke means a reduction by another 17 r cent. Thus $a$ will depend upon the character of the fuel, size of mbustion chamber, &c. $b$ and $c$ depend upon the area exposed to liation per sq. foot of grate, and to a small extent on the character he fuel, and the amount of air admitted. It is to be noticed that

when the size of grate is altered without altering anything else, really alter the values of $b$ and $c$, and to some extent $a$ also, because increase radiation surface and volume of combustion chamber p square foot of grate. We have arrived at the result

$$W = b(1-e)G + \left\{c + (a-c)e\right\}F \quad . \quad . \quad . \quad (2)$$

If now $l/hm$ be called $\lambda$, a term proportional to the length divided diameter of tubes and greater as there is better circulation of wate

$$W = \frac{bG}{1+\lambda} + \frac{c+a\lambda}{1+\lambda}F \quad . \quad . \quad . \quad . \quad . \quad (3)$$

I shall usually write this

$$W = AG + BF \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Where $A$ and $B$ are constants of much the same value in goo specimens of any class of boiler.

Or I may use $w$ for $W/F$, the evaporation per pound of fuel, and or $F/G$, the fuel per square foot of grate, and so have

$$w = \frac{A}{f} + B \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

When there is as good circulation on the water side as in a loco motive boiler in motion (the motion helps circulation), we may tak $\lambda$ as the length of one of the tubes divided by forty times it diameter.[1]

**266.** My roughly correct speculations are such as befit th subject. They have led to an expression, (3) or (6), of some valu Those who only *use* boilers will probably be satisfied with th expression

$$W = AG + BF$$

$W$ being total evaporation as from and at 212° F., and $G$ = area grate in square feet, $F$ = lb. of fuel per hour, and $A$ and B a nearly constant for a particular boiler. If the boiler has very lon

[1] We ought to use the formula of the note, page 427, in any case where we kn that the air supplied per pound of fuel is constant. Instead of (3) above we have

$$W = aF\left(1 - \frac{\lambda}{1+\lambda}\,\frac{bF}{1+bF}\right) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This may be written $W = aF\dfrac{1+cF}{1+bF}$ where $c$ is a constant whose value is $b/(1+\lambda$ In using (3) or (6) it is to be remembered that $b$ is nearly of the value $A/45H$ (s note, page 427) and $\lambda$ is length of a tube divided by about 40 times its diameter.

flues the term $A$ is small; for example, in the **French** **otive boiler** of Art. 263 I find that

$$W=9{\cdot}1F$$

s all the observations better than any other simple formula; more, I find that whether coke was used or briquettes, and r or not half the tubes were closed we have much the same ie errors in assuming this law to be true are very much of the rder as the discrepancies in the actual measurements that were

ain in the experiments made with the famous **Newcastle** **e boiler** in 1857, the grate was altered from 22 square feet square feet, and the firing from $3\frac{1}{8}$ cwt. on the larger grate to i the smaller. The heating surface was 749 square feet of : and flues, together with a feed-water heater of 320 square There was nearly perfect combustion. The results when using d-water heater agree with

$$W=34G+9{\cdot}74F.$$

not using the feed-water heater the results (not over so great :) agree with

$$W=70G+7{\cdot}48F.$$

the way in which we expect poorer efficiency of flues to affect mula; diminishing $B$ and increasing $A$.

Mr. Isherwood's experiments described in Mr. D. K. Clark's the curious results obtained are easily explainable if one bers the significance of the various terms of (3) or (6). For e, how when we diminish grate area, we really increase radi- urface and combustion chamber volume per square foot of In his first series where $G=10{\cdot}8$, and the heating surface $S$ 0·3, his results satisfy

$$W=18{\cdot}5G+5{\cdot}65F.$$

ierwood, like many other people, noticed the uncertainty as to ct of $S$. We know now that it is length of flue divided by lic mean depth and circulation of water that are important ; $S$. Readers of Mr. Clark's book about this place will notice trials of **feed-heaters** (table, page 283) how they may produce ease of efficiency of as much as 15·7 per cent. In practice it 15 to 20 per cent., but this would probably not be so great if e stoking and automatic regulation of the draught were ed. Clean tubes give about 6 per cent. more efficiency than ibes.

At page 299 of Mr. Clark's book we may notice what a great amount of unconsumed gas escapes even in the best hand stoking trials, and in page 300 that the supply of air which gave the best results with careful stoking of a French boiler was only about 33 per cent. in excess of what was absolutely necessary.

The table of page 302 (Clark) is worth study, it shows 61 per cent. of the total heat going into the water, the unconsumed gases taking 5·5 per cent.; clinker and ash 1·5 per cent.; heat in gases taking off 5·5 per cent.; smoke and carbon 0·5 per cent. The hygrometric and formed water took 2·5 per cent., and the heat carried off by the brick work was 23·5 per cent. of the whole. These numbers ought to be compared with those of the table, Art 261.

It is worth while considering the trial at page 308 of Mr. Clark's book of what is now an out-of-date boiler—a locomotive boiler once used by Thornycroft in torpedo boats. There is neither sufficient radiation surface nor combustion chamber space above the grate. The gauge pressure was 117 lbs. per square inch, feed 55° F.

| | | | | |
|---|---|---|---|---|
| Air pressure in stokehole (inches) | 2 | 3 | 4 | 6 |
| $F$=coal per hour (lbs.) | 925 | 1177 | 1472 | 1815 |
| $f$= ,, ,, per square foot grate | 49 | 62 | 78 | 96 |
| $W$=steam per hour (lbs.) | 6530 | 7770 | 9320 | 10840 |
| $w$=evaporation units per pound of coal | 8·31 | 7·81 | 7·45 | 7[illegible] |

Here it will be found that

$$w=\frac{122}{f}+5{\cdot}86$$

or

$$W=122G+5{\cdot}86F$$

**267.** The average results of experiments from the best boilers of the following types are pretty much the same. Assuming that there is proper provision for mixing air with the gases at a high enough temperature; that the provision for draught is more than is actually needed, so that the stoker has perfect control of it (a condition which is far too often neglected); then the fuel being any kind of good coal, either Welsh (whose superiority really consists in behaving well even when the stoking is bad and giving little trouble), or Newcastle, or Lancashire, or Derbyshire, or a mixture of Welsh with any of these, we find

$$w=\frac{A}{f}+8{\cdot}5$$

if $w$ is the evaporation in pounds of water as from and at 212° F

ound of coal. $f$ is the fuel per hour per square foot of grate. 'alues of $A$ are as follows:

| | $A$ | limits of $f$. |
|---|---|---|
| **Lancashire with Galloway tubes, French or other good stationary boilers, with feed-water heaters** . . . . . . . . . | 36 | 30 and 8 |
| **Marine cylindric with return tubes** . . | 54 | 40 and 12 |
| **Railway locomotive** . . . . . . . . . | 135 | 140 and 30 |
| **Water tube** . . . . . . . . . . . . . | 45 | 100 and 10 |

**68.** I have long used **another empirical formula**, which is nient. $W$ is the total evaporation per hour as from and at F. $F$ is fuel per hour, $G$ is grate area, $\lambda$ is length of flue divided s hydraulic mean depth. $a$, $b$, and $c$ are constants. $a$ is about for any good Welsh coal, or indeed any other bituminous if the stoking is good; but if the stoking is not very good, we still use $13\frac{1}{2}$ for Welsh, but smaller values for other coals. $b$ is nber which is less as there is a better arrangement for water ation; $c$ is a number which is less as there is more surface ceive radiation in the fire-box.

$$W = \frac{aF}{1 + bF/\lambda + cF/G}$$

udents have obtained values of $b$ and $c$ for many types of boiler. e whole, perhaps, the extra complication is not atoned for by ch greater accuracy but what it is better to keep to the simpler

**69. New Type of Boiler.** Fig. 234 is a diagrammatic sketch oiler made really to enable my students to keep our theory in . Ever since it was first drawn some years ago I have seen that tural alterations are necessary—but these would suggest them- to a practical engineer; for example, dust from the fire ought allowed to settle in such a way as to allow of frequent re- l, so that the passage from $F$ ought to be above and the flow t gases through the tubes be a downward flow. There is no giving heat by gases until after combustion is complete. $B$ is a fire- furnace strongly cased. Coal is fed in, preferably automatically, ; the whole space $C$ is filled with fuel, which is white hot at $C_1$; shes are raked from $A_2$. Air is sucked in at $A_1$ and at $A_2$, and it to see that the combustion may be perfect in the chamber $F$

from which the flame passes through the tubes $T$ to the uptal
The draught may be produced in $U$, or preferably by a fan drivi
in at $A_1$ and $A_2$. The draught required is very great, as the c
tubes $T$ are only about $\frac{1}{4}$ inch in diameter. These tubes are p
(not quite touching) in a cylindric vessel $D$, the spaces between
being filled with water, which is kept in rapid circulation b

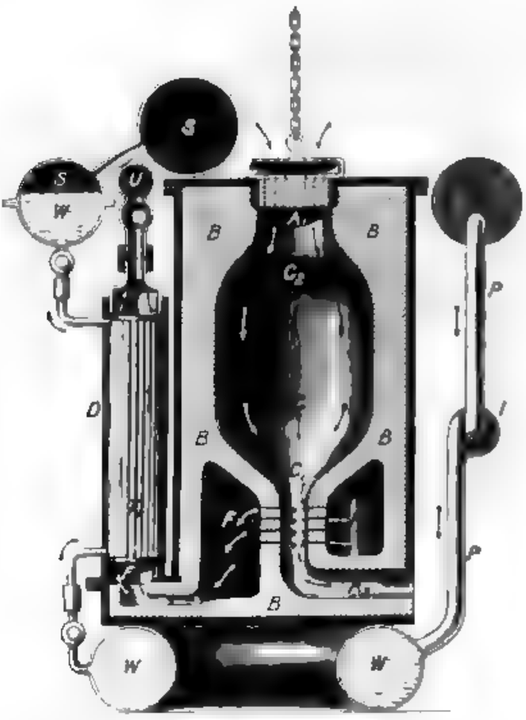



Fig. 234.

injector $I$, although a pump may be used instead, to drive
from the upper ring pipe $W$ to the lower one by $P$ and back
by the tubes. I think it a mistake to have the water circul
often: it ought to scrub the tubes so much in passing once thr
$D$ as to become all steam in the upper part of $D$, or rather i
ring pipe $WS$, which communicates with the steam pipe $S$. T
are a number of cases, $D$, any one of which is easily detachable
the tubes are frequently cleaned inside and out. Instead of cyli
cases, $D$, every flue tube may be concentric with a water tube
it is easy to make them detachable in large or small group
cleaning purposes.

## CHAPTER XXVII

### GAS AND OIL ENGINES

**270.** Air would be almost the best stuff to use in a heat engine ly that it is so difficult to give it heat and to take heat from it as ggested by the Carnot and other cycles. This difficulty has aused the want of success of all the **ingenious air engines** which rere invented 50 years ago, except perhaps for very small powers. as and oil engines may be looked upon as air engines in which the ifficulty has been got over; it is possible to give to a mass of air (or ather a mixture which is mainly air) in a cylinder, so much heat as o make it white hot in so short a time as $\frac{1}{100}$ to $\frac{1}{80}$ part of a second. Ve let the stuff escape with a great deal of heat instead of trying to xtract the heat again, because we need not use the same air over nd over again, and hence we have a practical form of air engine hich fulfils all the fine predictions made about it 50 years ago. We w in Art. 189 that the usual coal gas mixture **contracts 3 per cent. n combustion,** the Dowson gas mixture 5 per cent., and oil engine ixtures 3 per cent. ~~The richer the mixture the less the contraction~~. 'e have seen in Art. 245, **how much air** is needed for the complete mbustion of a cubic foot of coal gas or Dowson gas or for a quantity of l. ~~The violence of the explosion,~~ that is, the **rapidity of the com- ustion,** is less and less as we depart from the exactly right propor- ns of air and gas, and it has up to the present time been found nvenient for this and other reasons to admit from ~~100 to 50 per~~ ~~more air, which from one point of view means a loss of energy.~~ e considered in Art. 189 the usual mixture, before and after combus- n, and we found that the **specific heats and their ratio** were not atly altered. This is mainly due to the great quantity of nitrogen sent and indeed of inert gases generally. There is more difference en Dowson Gas is used, but in all cases we may take it that for

rough calculations which are indeed all that we can make, the behaves just as if it were a perfect gas which itself undergoe chemical change, which has no energy more than its pressure, vol and temperature tell us of, and which receives its heat from other source than itself. In fact we may regard the stuff as were air, only that its specific heat ratio $\gamma$ is 1·37. This is number which we use, in default of a better, in all gas an engine calculations. We may have doubt as to whether combu is or is not complete at any point in the indicator diagram, bu are in no doubt of our power to calculate temperature on assumption of behaviour as a perfect gas.

The student is recommended to read carefully Mr. Dugald Cl book on gas and oil engines. I know of nobody so capable wh given as much thought to the whole subject. He measured carefull rise of pressure with time, in a closed vessel containing various mix of explosive gases, which were, however, giving up heat to the v all the time. His curves are well worth study, although t was no compression before explosion. **The most important r** known to me derivable from his experiments is this and it a with the results of Hirn, Bunsen and several others: *only abo to 60 per cent of the heat energy of the stuff seems ever to be deve in the explosion.* This is probably due to dissociation, but if any thinks that he has explained the matter by calling it by this nan will be undeceived if he examines the following figures.

**271.** Mixtures of air and Oldham gas exploded.

| Volume proportions of mixture. | Maximum pressure absolute lb. per square inch. | Highest temperature, Centigrade. | Time to reach highest pressure, seconds. | Calculated highest pressure, absolute. | Calculated highest temperature, Centigrade. | Heat indicated. |
|---|---|---|---|---|---|---|
| 1 gas 14 air | 55 | 806 | 0·45 | 105 | 1786 | ·447 |
| ,, 13 ,, | 67 | 1033 | 0·31 | 111 | 1912 | ·537 |
| ,, 12 ,, | 75 | 1202 | 0·24 | 118 | 2058 | ·582 |
| ,, 11 ,, | 76 | 1220 | 0·17 | 127 | 2228 | ·545 |
| ,, 9 ,, | 93 | 1557 | 0·08 | 149 | 2670 | ·582 |
| ,, 7 ,, | 102 | 1733 | 0·06 | 183 | 3334 | ·518 |
| ,, 6 ,, | 105 | 1792 | 0·04 | 217 | 3808 | ·446 |
| ,, 5 ,, | 106 | 1812 | 0·06 | | | |
| ,, 4 ,, | 95 | 1595 | 0·16 | | | |

I find that Mr. Clerk's results are fairly well represente highest gauge pressure $p = 136 - 6{\cdot}57\ x$, if $x$ is the volun air per cubic foot of gas; and hence as the available heat in a

volume is inversely proportional to $1 + x$, the indicated heat is proportional to $(136 - 6{\cdot}57.x)/(1 + x)$. This is a maximum when $x = 8{\cdot}85$. But between $x = 7$ and $x = 11$ I find only a difference of about 2 per cent from a mean value.

When heat is given to a gas at constant volume the amount of heat is proportional to the change of pressure. In the present case therefore it is proportional to the highest gauge pressure (neglecting the small differences in specific heat). Hence, what I call indicated heat, is the ratio of the highest gauge pressure to what it would be if all the heat were indicated.

I have no doubt that dissociation is the explanation, but why should we always get about the same amount of dissociation at such different pressures and temperatures? It is not explainable by loss of heat to the cold vessel. Bunsen used a very small vessel, a few cubic centimetres capacity; Berthelot used a vessel 4000 cubic centimetres capacity, and they found much the same results with mixtures of hydrogen and oxygen. Clerk asserts that he has obtained practically the same result with mixtures of coal gas and air compressed before ignition. Twice the pressure before (and therefore twice the heat available) gives twice the pressure after explosion for the same kind of mixture, so that we still have only from 50 to 60 per cent. of the heat developed. These results are in agreement with what we find in Gas Engine Indicator diagrams. The calorific power of gas is always measured by reducing the products to the temperature of the room, and it may be that we never do get this heat developed unless we reduce the products to a temperature less than that of these explosion experiments. The latent heat of the steam formed is one part which must always be wanting, and it may be true, albeit not quite easy for an electrician or a chemist to believe, that there is considerable dissociation at even the lowest of the explosion temperatures tried. But why should the unindicated energy always be of about the same fractional amount? I see no solution of the difficulty.

It has been shown that, starting ignition with a small spark, the time of ignition increases as the volume of the vessel is larger—but by mechanical disturbance or artificial projection of a flame, the ignition may be made almost as rapid as we please even in weak mixtures and large vessels.

There is still much to be done, but it is fairly well proved that we may look upon such calculations as those of Art. 287 as giving us fully the efficiency of the gas engine of any of the types there mentioned if we assume that only 60 per cent of the heat of combustion is really given to the working stuff as heat.

I am sorry to say that the only other set of published experiments on the explosion of mixtures of coal gas and air in an iron vessel gave much smaller pressures than those of Mr. Clerk. From these experiments I make out the formulæ :—

$$p = 104{\cdot}7 - 5{\cdot}71\,x$$
$$p = 83{\cdot}3 - 3{\cdot}2\,x'$$

Where $p$ is the highest gauge pressure in pounds per square inch; $x$ is the volume of air added to one cubic foot of gas; $x'$ is the volume of air together with products of previous combustions added to one cubic foot of coal gas before ignition. It is tolerably certain that this result cannot be more than roughly true for other sizes of vessel than the one employed, and that we have no right to use it for any case in which the pressure is that of several atmospheres before ignition. If we treat the first of these results of Mr. Grover's as we treated that of Mr. Clerk we find that the most heat will be indicated when $x = 9{\cdot}65$.

Mr. Grover's most interesting result is that better effects are obtained when some of the extra air is replaced by the products of previous combustions. It will be interesting to know what effects he obtains when he uses high initial pressures, for his results so far are in disagreement with our gas engine experiences. The common practice of having nearly as great a volume of old [illegible] it is difficult to say exactly what the [illegible] actual measurements of air admitted [illegible] the use of a scavenging [illegible] old products before [illegible] produced good results; [illegible] replacing the old products [illegible] because they are too [illegible] Possibly also lower [illegible]

[illegible]

**[illegible] Engine** [illegible] an ordinary [illegible] specimen in [illegible] inches in diameter, [illegible] to keep it cool [illegible] information. [illegible] of gas and air was [illegible] and when the

admission valve closed, was ignited by an electric spark. The diagram, Fig. 235, showing three explosions ($AB$ is the atmospheric line) has a highest pressure of 48 lbs. per square inch. The consumption was about 95 cubic feet of coal gas per hour per indicated horse-power. When the speed increased, the governor acted by increasing the proportion of air from 6 volumes to 12 per cubic foot of gas. The air and gas were admitted by two slide valves worked by two eccentrics.

The **Hugon Engine** differed from the Lenoir only in having better mechanical construction; ignition was by a flame instead of the badly

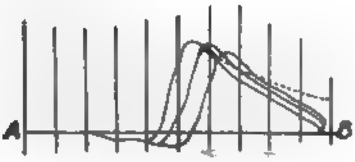



FIG. 235.—LENOIR DIAGRAMS.
Cylinder 8 inches, diameter 16 inch stroke

arranged electric spark. The consumption of gas was reduced to 85 cubic feet per hour per indicated horse-power.

Much the same principle was employed in the **Bischoff** engine and indeed Fig. 236 fairly well represents the sort of diagram obtained from any of the three.

An enormous improvement was effected in the use of the **Otto and Langen** free piston vertical cylinder engine. In this the charge

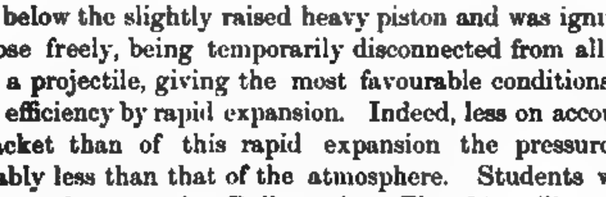



FIG. 236.—HUGON DIAGRAMS.
Cylinder 8 inches, diameter 10 inch stroke. 75 revs. per minute Scale 1 inch to 36 lbs.

came in below the slightly raised heavy piston and was ignited. The piston rose freely, being temporarily disconnected from all gearing; rose like a projectile, giving the most favourable conditions possible for good efficiency by rapid expansion. Indeed, less on account of the water jacket than of this rapid expansion the pressure became considerably less than that of the atmosphere. Students who have worked exercises on the Bull engine, Fig. 21, will understand the matter. The piston began to fall, acted on by its own weight and

by some outside pressure due to the atmosphere, and in beginning to fall became geared to a shaft which therefore now received mechanical energy. I am astonished that the gas per hour per brake horse-power was not even lower than the 44 cubic feet actually found by experiment, because there was coolness before ignition and

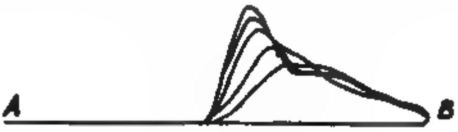



FIG. 237.—BISCHOFF DIAGRAMS.
Cylinder 3½ inch diameter, 11½ inch stroke. 112 revs. per minute. Scale 1 inch to 36 lbs.

therefore probably good combustion with plenty of time for it, and these were combined with rapid and large expansion.

Many thousands of these engines were in use. Their noise enabled the less noisy 'silent' Otto Engine to be rapidly introduced.

In spite of these favourable conditions, a careful examination of the diagram shows, and especially when the more dilute mixtures were used, that there must have been combustion going on to the end

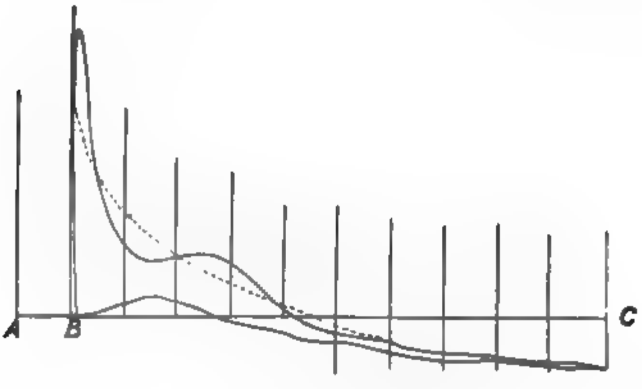



FIG. 238.—OTTO AND LANGEN DIAGRAM.
Cylinder 12½ inch diameter, observed stroke 40½ inches; 28 explosions per minute. Scale 1 inch to 36 lbs. per square inch.

of the stroke. We need not say that this is altogether due to dissociation. Ignition goes on more slowly in a more dilute mixture at a lower pressure. Possibly a less rapid starting of the piston, the use of a heavier piston, would have increased the efficiency. I am told that the consumption in some of the larger forms of these engines was as low as 30 cubic feet of gas per hour per indicated horse-power. In the **Brayton** Engine, the mixture of gas and air was compressed to 75

or 95 lbs. per square inch absolute, and ignited as it passed into the working cylinder through a metal grating. Combustion occurred at nearly constant pressure; there were cut off, expansion, release and fresh admission, just as in a steam engine. An engine of 4 brake or 5 indicated horse-power seemed to consume about 280 cubic feet of gas per hour.

The first good oil engine was a Brayton gas engine using oil, pumped in and burning with the compressed air just like the gas. It was wonderfully steady and to be relied upon for not getting out of order. It consumed about 2 lbs. of oil per hour per indicated horse-power. The *Simon* gas engine was a modified Brayton engine.

The **Otto engine**[1] has four operations in one cycle of two revolutions. It looks much like a single acting steam engine with


FIG. 239.

trunk piston with sturdier frame and parts than usual. Cold water is kept circulating through the cylinder water jacket *WJ*. The volume of the clearance space has gradually been diminished from two-fifths of the volume when greatest, to one seventh. I use with students the large lecture model fig. 239, which has the old slide method of regulation now discarded; discarded because of the greater pressures used now. The exhaust (conical seat) valve *E* is closed by a spring and is opened by the lever *L* worked by the cam *T* on the shaft *S* which makes one turn for every two of the engine. A crank *D* on this shaft gives a reciprocating motion to the slide *S*. The flame *F* is used for ignition. When the slide is properly placed, the passage *B*

[1] Dr. Otto in 1876 made the engine, patented by Beau de Rochas in 1862, a practical success, and so it is always called the Otto engine.

allows air to be sucked from $A$ and gas from $G$ in proper proportions into the cylinder, and they mix with the products of previous combustions left in the clearance space. The usual mixture is 1 of coal gas to about 11 of air and products.

This goes on during the whole forward motion of the piston. In the back stroke the mixture is compressed sometimes to more than 100 lbs. per sq. inch. It is interesting to note on the model how the small chamber is filled with gas, and coming opposite the flame $F$ is ignited, and how this chamber full of ignited gas comes opposite the cylinder passage just before the dead point position. There was a certain amount of complication in the way in which it communicated with the passage especially in large engines, so that ignition might really be effective; and it is one of the most interesting things in connection with gas and oil engines, that although the ignition chamber might and often did communicate with the explosive mixture before the end of the stroke, yet ignition did not really occur until the end of the stroke. The piston moves slowly near the ends of its stroke and this conduces to effective ignition. Ignition occurs with remarkable rapidity, the pressure rising 100 lbs. per sq. inch (usually accounting for about half the total heat of the gas supplied), and as the piston moves forward the stuff expands, and the pressure falls. Before the end of the forward stroke the exhaust valve opens, the stuff rushes away through an exhaust chamber and the exhaust pipe. At the end of the back stroke, products remain in the clearance space, and one cycle is complete. The usual governor closes the gas supply when the speed is too great, so that an explosion is missed. There is another kind of governor which throttles the gas supply so that there is some kind of mixture exploded, rich or poor in gas, every cycle. There are some curious kinds of governor in use, but the ordinary centrifugal form is as good as any. The engine is started by lighting the gas jet $F$, turning on the gas supply, and giving a few turns by hand to the fly wheel until an explosion occurs. In large engines a second cam keeps the exhaust open for part of the compression stroke during the starting of the engine. About half the total heat energy was usually carried away by the water of the jacket; about 30 per cent. went off in the exhaust and about 16 per cent. was accounted for by the indicator work. The exhaust gases were at about 400° to 450° C.

In the diagram, Fig. 240, $D$ is the drawing in, $C$ is the compression, $I$ is the ignition, $E$ the expansion, $RA$ is the exhaust. The use of a planimeter is the easiest way of getting the true area of the diagram.

hough it is not difficult to recollect what are positive and what e negative breadths. After a missed explosion, the ignition essure is usually higher, because the passages are cooler, and, erefore, a greater weight of gas enters; also the clearance space as air in it rather than products to mix with the new charge. In 381, an engine giving 9 brake horse-power and 11·5 indicated, sed 250 cubic feet of Glasgow gas per hour (Glasgow gas is much

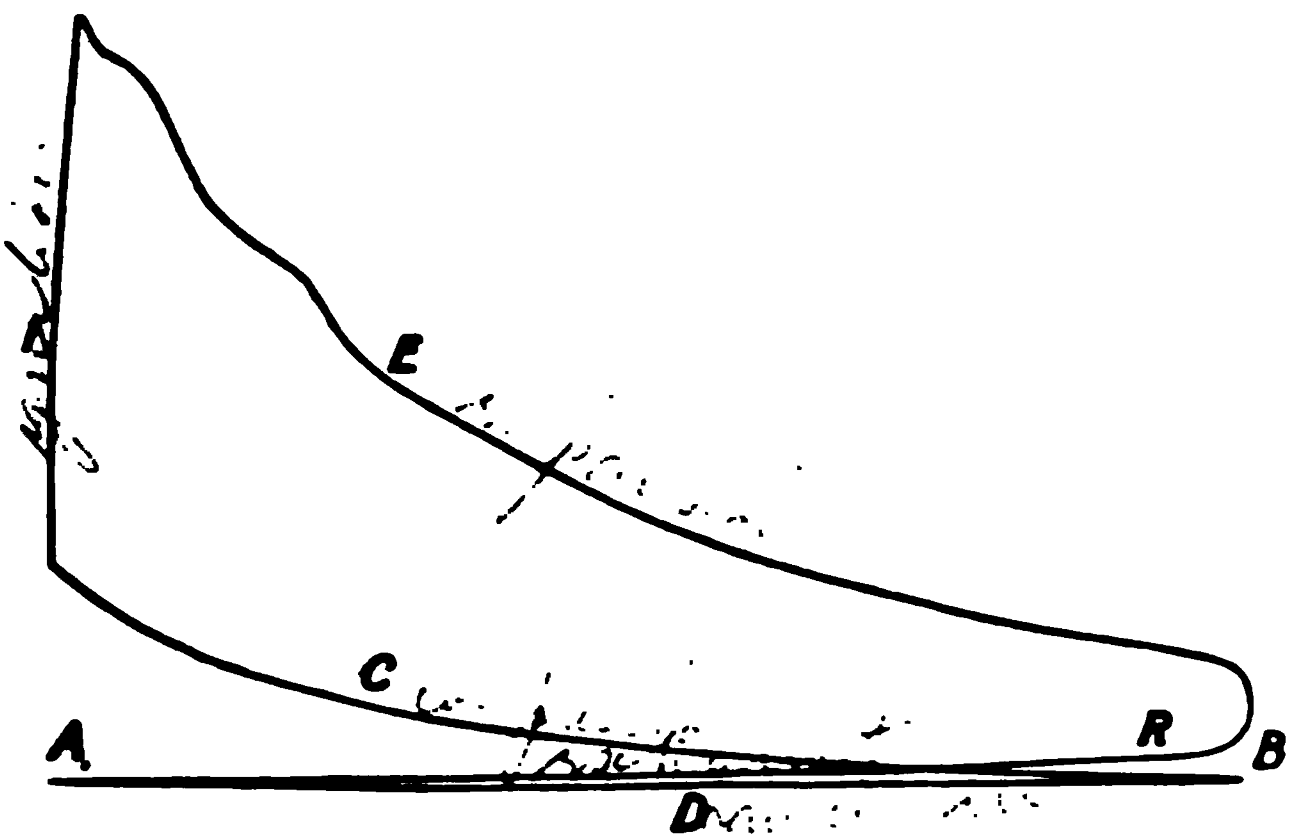



FIG. 240.

better than London gas). It was not uncommon to find that the indicated work was 18 per cent. of the total calorific energy of the charge.

**Large engines** are more efficient than small ones, probably because of the relatively smaller cooling surface. This is very evident from the following trials of Otto engines of different sizes. Efficiency here means the ratio of indicated work to the calorific energy of the gas in one charge.

| | Diameter of cylinder. | Stroke. | Efficiency by (1). Page 448. | Indicated efficiency. |
|---|---|---|---|---|
| Nearly same compression or value of *r* . . . . . . . . | 7" | 15' | ·428 | ·25 |
| | 11½" | 21' | ·428 | ·275 |
| Nearly same compression or value of *r* . . . . . . . . | 9½ | 18 | ·40 | ·21 |
| | 14 | 25 | ·41 | ·277 |

If ignition occurred at absolutely constant volume, and if all the heat were accounted for, and the compression and expansion were

adiabatic, and if the clearance were in the ratio 1 to $r$ of the greatest volume, it is easy to show, as in Art. 473, that the efficiency is

$$\epsilon = 1 - \left(\frac{1}{r}\right)^{\gamma - 1} \quad . \quad . \quad . \quad (1)$$

and as $\gamma = 1{\cdot}37$, the greatest efficiency possible when using the Otto cycle, and $r$ is 5, is 45 per cent. For good reason we cannot expect to get an efficiency approaching this, but it is interesting to note from the following tests that as $r$ increases the efficiency increases.

**Trials as to Compression.** The same engine was used in the two tests except that the size of the clearance space was altered.

| | | |
|---|---|---|
| Pressure (abs) before ignition | 75 | 105 |
| Cubic feet of gas per *I H P hour* | 19 | 17·6 |

This result was surely to be expected. I have put this strongly to students for the last eighteen years, and I am inclined to think that the superiority of the modern gas engine is mainly due to the better recognition now of the importance of small clearance or large $r$. The Table, Art. 277, will bring this out strongly.

The actual diagram of an Otto engine is in no way very different from the hypothetical diagram of ignition and release at constant volume and two adiabatics, except in only about 55 per cent. of the heat of the charge being given in the ignition.

**273.** In **Atkinson's differential engine** a curious mechanism was employed to give a very rapid motion to the piston just after ignition so that cooling should be more the effect of expansion than be due to the water jacket. This principle is the most important thing to remember, but the mechanism by which it was carried out was complex and troublesome.

The Atkinson engine was very efficient, mainly due, I think, to this rapidity of expansion, but also for the following reason. Suppose that we have the Otto cycle, as shown in Fig. 241, *A B C D*, and we have settled the best compression pressure. Now, instead of letting the stuff escape at *C*, let it continue to expand to *E*, by making the cylinder larger, without altering the clearance space or volume of charge admitted, we get the extra work *C, E, F, D,* with no further expenditure of energy. To be strictly correct we ought to say $C\ E\ F^1\ D^1$, where $FF^1$, or $DD^1$, is twice the pressure which represents the friction of the engine *due to this increased part of the cycle.*

The curious mechanism used gave trouble, and **Atkinson**

ınted another curious form of engine (called **the cycle**) to carry the same idea, the four strokes made by a piston in one revo-on being all unequal. This engine has also been given up in e of the wonderfully good results obtained, and Mr. Atkinson structed another engine with only the ordinary piston and con-ting rod mechanism, whose action is probably likely to be copied the future, when *impulse-every-revolution* engines will probably largely used; although I think that it is not now being made. side of the piston pumps air into a chamber at 20 lbs. per square

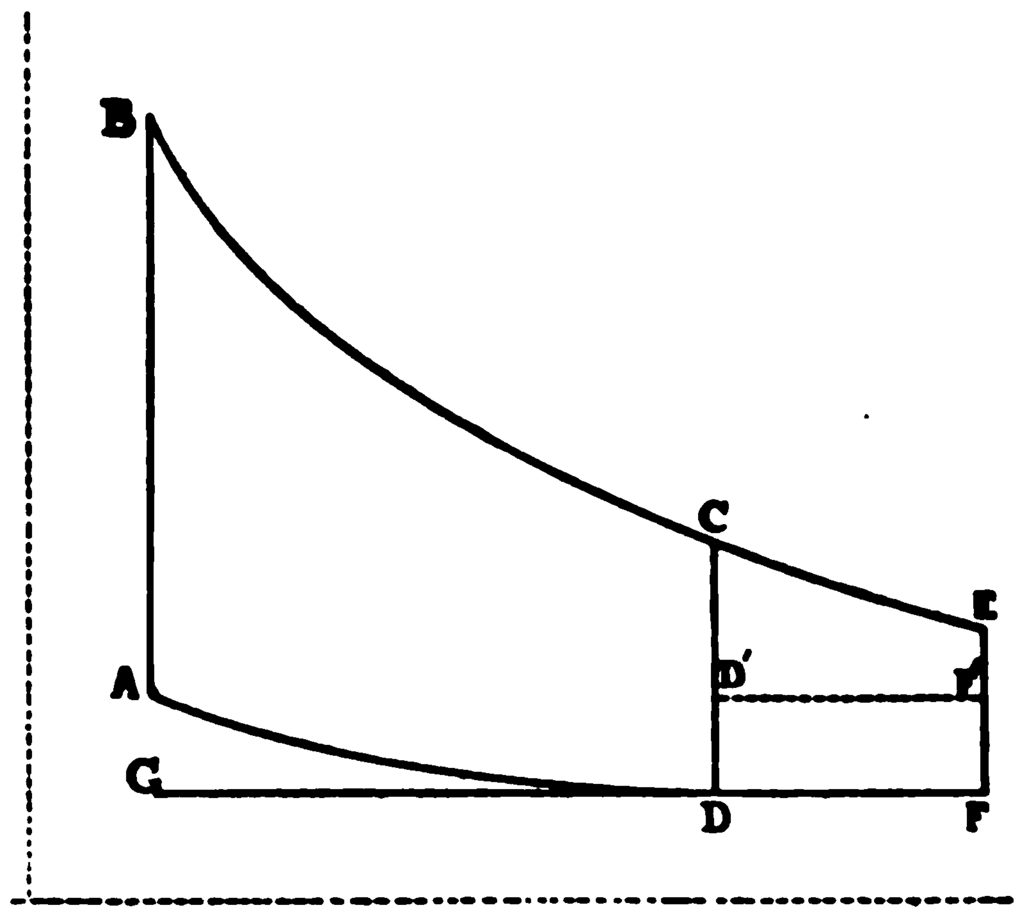



FIG. 241.

h (absolute). The air flows through a valve to the other side of piston and causes exhaust gases to escape faster; this air is now ıpressed, receiving a mixture of gas and air at sufficient pressure n a pump.

Since 1884 I have urged the importance of the two ideas bodied by Mr. Atkinson in his engines. The exercises in Art. 287 w the gain due to increased expansion. The rapid expansion ses less heat to be given to the metal. Unfortunately, in practice found that although there is less heat given to the water jacket n in the Otto cycle engines, more heat goes away in the exhaust.

There are other impulse-every-revolution engines which are more ess based on the principle first worked out by Mr. Clerk.

There have been many suggestions to use spray, or wet steam de the cylinder, to carry off the heat and to utilise part of n a six-stroke cycle. They have failed through difficulties of tion.

**274.** On the expiry of the Otto patent in 1890, there was a great fall in the price of gas engines. Engines on the Otto cycle were so well developed that few other engines are made. Dowson gas has become extensively employed. The principal improvement effected since 1886 consists in the diminution of the **clearance space.** It has always been known from formula (1) of 272 that this would effect increased economy. It is now being carried out the increase of economy exceeds anticipations, and it is to compression more than to anything else that the increased economy is due. But besides increased compression, the improved design and size of **valves and ports** lets the fresh charge in at higher pressure, and lets the exhaust gases escape more freely; in fact, the old throttling has been done away with. In the old slide the openings had to be small, otherwise the pressure on the slide became very great, and, indeed, this risk of pressure on the slide used to make it difficult to use high compression. Slides are now no longer used and an **ignition tube** is used instead of a flame. The ports now present less area to the incoming charge, and other sources of absorption of heat during ignition, such as contractions where flame passes, are done away with. The student will see from our theory of flues, Art. 377, that there must be extraordinarily more heat given to the metal by throttling action, for example at the exhaust valves than in any other way. It is sometimes thought to be very convenient, for several reasons, to have all the ports, valve seats, &c., in one casting which may be bolted on to a cylinder, but this convenience is often gained by having narrow ports, a great source of loss of efficiency absent in the best modern engines. There are also changes to increase strength and diminish cost of manufacture. The cross-head guide is now in one casting with the cylinder, or, rather, there is no cross-head, merely a long trunk piston. The bevil wheels driving the side shaft are now screw gear, and this has made the engine bed more symmetrical.

Figs. 243-5 show one form, and Figs. 247-9 show a smaller form of the **modern Crossley Otto** engine cylinder; gas enters the air passage by a conical valve, lifted by a lever and cam, and controlled by the governor which either admits gas well or not at all. These are well shown in Figs. 247-9. Gas and air enter the cylinder by a conical valve *A*, Fig. 248, opened by a lever, acted on by a cam.

The exhaust *E*, is also a conical valve actuated by a lever and cam.

Ignition occurs when port of the compressed stuff enters the tube

'ig. 242, kept hot by a Bunsen burner. Thus admission occurs ugh the double-seated valve *V*, which is worked by a lever and . The valve *V* allows the tube *T* to be open to the atmosphere l it lifts from one seat, and then a small amount of inflammable f displaces the previous products, so that there may be certainty gnition.

There are many small engines made in which there is **no valve** reen the hot tube and the cylinder. It is found that we can nd upon the ignition not taking place till the end of the com-

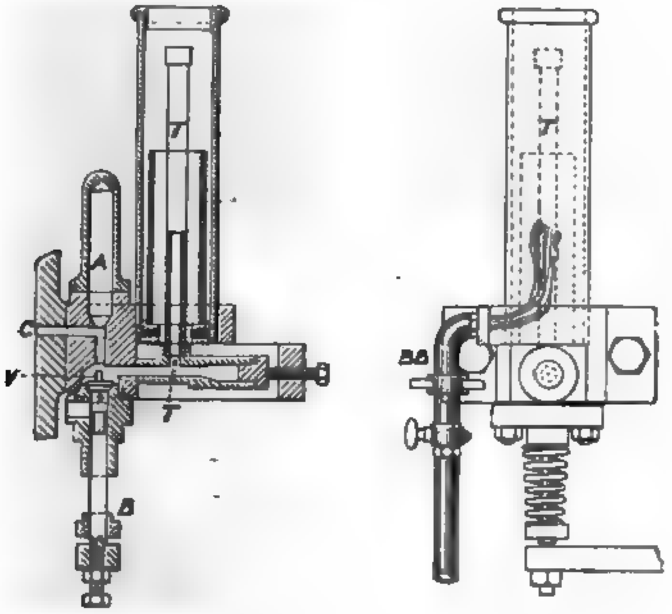
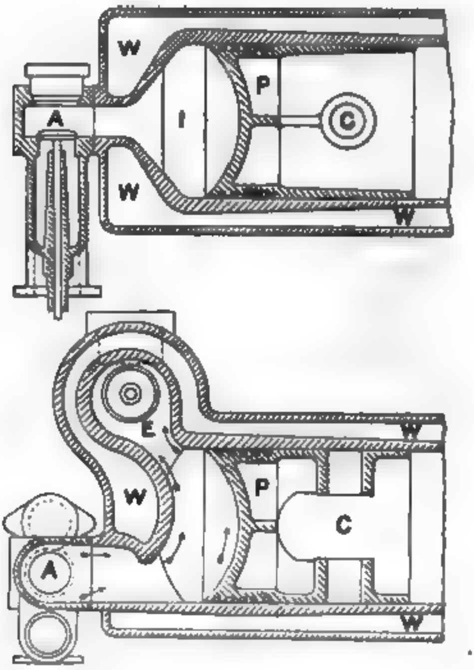



FIG. 242.—TUBE IGNITER.
Tube kept hot by Bunsen Burner *B B*.

ion stroke, and then it is certain to occur; surely one of the curious of phenomena! It is probably related to the fact that ng fluid seems more unstable when expanding, so that there be a great starting of eddying motion at the beginning of troke.

**:78. Scavenging** seems to be undoubtedly beneficial. It is ally valuable in engines using Dowson gas. Among other its we may notice the greatly diminished chance of an explosion e incoming charge through meeting the hot exhaust gases. after one or more missed explosions, an explosion is much more

violent and hurtful to the engine when scavenging is not emp
Hence scavenging enables larger and cheaper engines to be bui
these engines might use much hotter jacket water. It is effec
the Modern Otto Cycle engine in the following ingenious way l
Atkinson. The stuff in the long exhaust pipe (65 feet long) ge
a state of vibration like the air in an organ pipe, and by givin
proper length we get the cylinder to be partially vacuous (2 lbs.
atmospheric pressure) at the end of the exhaust stroke; consequ




FIGS. 243 AND 244.—CROSSLEY GAS ENGINE.
*W*, water jacket. *P*, piston. *C*, cross head. *A*, admission. *E*, exhaust.

the exhaust valve being kept open, a valve is able to adm
which drives out most of the remaining gases and indeed ser
cool the passage through which the incoming charge now
This contrivance acts better for well loaded engines than
load is variable. Scavenging is effected in the **Wells (Pre
engine** by pumping air into the cylinder.

Figs. 243 and 244 show the shapes of the passages and pisto
&c., which facilitate this scavenging action. The common Cr
Otto engines range from 100 brake horse-power at 230 revol

per minute, to 1¼ horse-power. The brake power is usually 2 to 3 times the nominal power. The makers now guarantee 1 indicated horse-power for 16½ cubic feet of gas per hour, or 17 for the smaller engines.

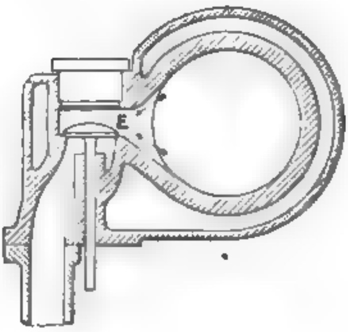



FIG. 245.

There are many forms of engine using the Otto Cycle now manufactured. Art. 277 shows the improvement effected since 1881.

There is an engine used for electric lighting which gives 170 brake horse-power on full load, using a governor which reduces the supply of gas and air simultaneously, but misses no explosions. The compression pressure varies from 20 to 75 lbs. (absolute). The result is a speed fluctuation of only three per cent.

I am told that Messrs. Tangye use a curious method of cooling the Jacket water by the atmosphere. The water is sprayed to the roof of the engine room, and is caught again in gutters.

**276. Self-Starting Gear.** The form most in use is Mr. Clerk's, as improved by Mr. Lanchester and shown in Fig. 246. When the engine is stopping, the valve $V$ is opened so that the cylinder $C$ and

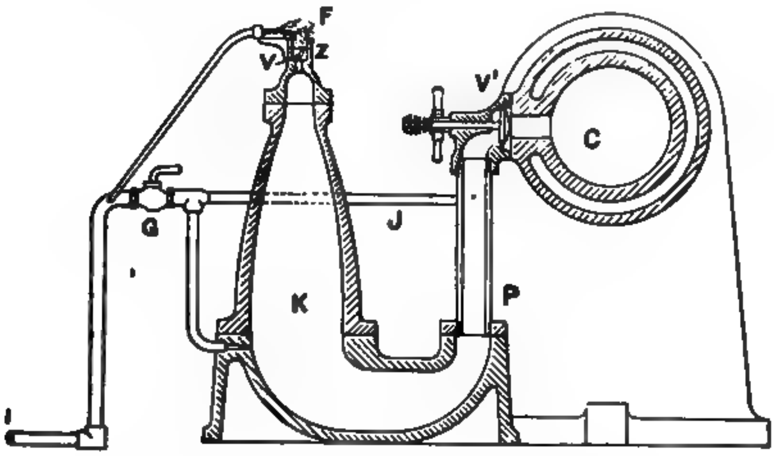



FIG. 246.

pipe $P$ and chamber $K$ get filled with air sucked in through $Z$. To start the engine, the gas cock $G$ lets gas flow into $K$ and $P$, and

either by another cock or the exhaust valve into the cylinder *C*. The flame *F* is lighted and presently gas escapes through *Z* and burns at the flame *F*. *G* is now closed and the flame at *Z* shoots back (in a way familiar to people who use gas stoves) igniting the stuff in *K*, closing the valve *Z* against an upper face, and the ignition proceeding along *P* reaches the cylinder. A maximum pressure of 200 lbs. per square inch is reached in the cylinder, quite sufficient to start the engine. When lower starting pressures are sufficient, a much simpler starter is used in which there is no vessel like *KP* to supplement the volume of the cylinder itself. [January, 1899. I have just tested engines with a later form of starter.]

**277.** In the following table the four Crossley engines marked * will give the best illustration of the **improvement going on.** Hence I give also their efficiencies as calculated by the formula (1), Art. 272. It may be conjectured that in the last case the efficiency might probably only be ·22 without scavenging. Other facts indicate some such gain (say 12 per cent.) due to scavenging. It must be remembered that the calorific power of gas in London has altered a little since 1881.

| | Cubic feet gas per I. H. P. hour | Cubic feet gas per brake H. P. hour | Compression pressure, absolute | Indicated power | Revolutions per minute | Highest pressure | Energy of charge indicated | Efficiency by (1) of Art. 272. |
|---|---|---|---|---|---|---|---|---|
| Crossley * Otto 1881 | 25·3 | 34·0 | 47 | 9 | 164 | 125 | ·17 | ·33 |
| Atkinson cycle 1887 | 19·78 | 22·50 | — | 5·56 | | — | ·206 | |
| " " 1888 | 19·22 | 22·61 | 181 | 11·15 | 131 | — | ·228 | |
| Crossley * Otto 1888 | 20·33 | 23·87 | 76·6 | 17·22 | 160 | 212 | ·21 | ·40 |
| Ignition tube Crossley,* 1892 | 21·2 | 25·9 | 61 | 19·25 | 160 | 215 | ·20 | ·37 |
| Lift valves / Ignition tube / Scavenging } Crossley * 1894 | 14·5 | 17·0 | 102·5 | 14 | 200 | 250 | ·25 | ·43 |
| Crossley . . . | 13·33 | — | | 46·5 | | | | |

In the latest type of **Griffin** two-cylinder engine, admission and compression occur on one side of each piston and ignition and expansion on the other side, alternately, so that there are two explosions per revolution. Worked with Dowson gas there is a specimen indicating 600 horse-power at 120 revolutions per minute. The **Stockport** engines from 1 to 200 brake horse-power run at from 240 to 150 revolutions. For larger powers two cylinders are used, tandem or side by side. In a 400 horse-power (nominal) at Godalming, the governor usually controls the gas supply to only one of the cylinders. The **Tangye** engines are single cylinder from ½ to 125 brake horse-power

and two-cylinder from 86 to 292 brake horse-power. With large cylinders a large and small exhaust valve are used, the smaller having a slight lead. Also the cam and lever open the gas valve through a secondary lever and tumbler to prevent wear. The Acme or Burt engine is said to be "compound," but it merely carries out the Atkinson Cycle principle. It is said to use (the 6 horse-power nominal size) 18½ cubic feet of gas per brake horse-power hour. I might greatly extend this catalogue, but indeed there is nothing specially interesting in the 30 or 40 types of gas engine now being manufactured in this and other countries. Mr. Donkin in his book

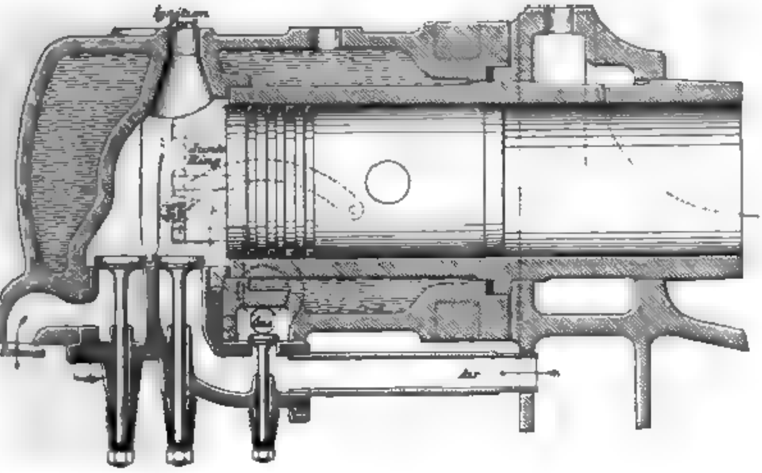


FIG. 247.

(1896) gives the results of a great many tests, with the names of the experimenters and references.

**278.** Prof. Burstall recently read before the Institution of Mechanical Engineers a preliminary **report of experiments** on a small gas engine, Figs. 247–9, in which various things might be altered separately. He could alter the clearance by removing a Junk ring on the end of the piston. He could also alter the length of the connecting rod. He measured the air as well as the gas [the numbers for air are corrected for air in clearance space]; he used a special electric method of ignition, and a timing valve. But what he did is evident from the table, page 457. The brake horse-power may be calculated from the indicated power by the formula $B = \cdot 72\,I + 0\cdot 2$.

The following is the composition of the gas by volume; ·045 of heavy hydrocarbons (taken to be $C_3H_6$), ·007 of $O$, ·059 of $CO$, ·353 of $CH_4$, ·463 of $H$, ·073 of $N$.

The results seem to indicate that economy greatly depends upon the ratio of air to gas, and that more air ought to be used when more compression is used. It is worth while noting how complete the

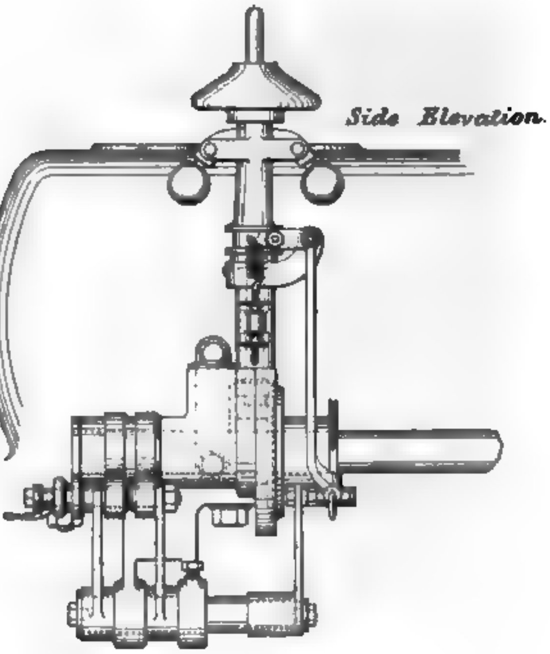



FIG. 248.

combustion may be before exhaust, and I believe that it generally is very complete in ordinary gas engines.

Teachers will find materials for a great number of exercises for elementary students in the table. For example:

1. Find in every case the greatest possible efficiency, by the formula (1) of Art. 272.

2. Find in every case the oxygen in the exhaust if combustion is perfect.

3. Taking pressure at the beginning of compression as 14 lbs. per square inch, and assuming that the law, $pv^n$ constant, is true, find $n$ in every case.

4. Find what the highest pressure would be in every case if all the heat entered the stuff at constant volume.

5. Check the numbers in column 14 from those in 12 and 13.

**279. Oil Engines.** Gas may be produced from safe burning oils and used like coal gas in a gas engine. An oil engine is supplied

| Number | Clearance by workin of cyll | Air per foot o | A exh per | | Exh calc assu I con | | Revolu per mi | Ratio numb explosi possible per ce | Pressure igniti | Maxim pressu | Indica horse-p | Gas per in horse-pow cubic f | Indicated. | To jacket. | In exhaust. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $CO_2$ | $O$ | $CO_2$ | $O$ | | | | | | | | | |
| 1 | 0·57 | 9·1 | 6·0 | 10·0 | 6·3 | 8·1 | 118 | 89 | 52 | 118 | 1·72 | 33·0 | ·126 | ·286 | ·558 |
| 2 | 0·49 | 8·0 | 7·6 | 6·4 | 7·2 | 6·4 | 161 | 50 | 62 | 211 | 2·65 | 25·6 | ·162 | ·350 | ·488 |
| 3 | 0·49 | 8·36 | 7·2 | 7·4 | 7·1 | 7·8 | 158·3 | 58 | 64 | 171 | 2·69 | 25·9 | ·155 | ·318 | ·527 |
| 4 | 0·49 | 7·13 | 8·0 | 5·2 | 8·2 | 5·0 | 164·3 | 52 | 64 | 203 | 2·79 | 27·3 | ·154 | ·415 | ·431 |
| 5 | 0·46 | 8·35 | 7·2 | 7·6 | 7·0 | 8·0 | 204·4 | 67 | 65 | 167 | 3·79 | 26·9 | ·156 | ·371 | ·473 |
| 6 | 0·46 | 8·48 | 7·2 | 7·6 | 7·0 | 7·96 | 204·2 | 88 | 65 | 105 | Rejected ignition too late. | | | | |
| 7 | 0·41 | 9·48 | 6·0 | 9·6 | 6·0 | 8·6 | 156·2 | 64 | 72 | 171 | 2·87 | 23·3 | ·180 | ·372 | ·448 |
| 8 | 0·41 | 7·18 | 8·2 | 5·6 | 8·1 | 4·7 | 158·2 | 53 | 74 | 237 | 2·44 | 29·7 | ·144 | ·452 | ·404 |
| 9 | 0·36 | 7·08 | 8·2 | 5·8 | 8·2 | 5·6 | 157·7 | 55 | 76 | 230 | 2·61 | 27·7 | ·152 | ·428 | ·420 |
| 10 | 0·36 | 10·01 | 5·2 | 10·8 | 5·1 | 9·0 | 155·5 | 69 | 80 | 161 | 2·61 | 24·1 | ·177 | ·406 | ·417 |
| 11 | 0·37 | 8·75 | 6·5 | 7·0 | 6·6 | 7·8 | 154·2 | 69 | 83 | 203 | 2·50 | 26·6 | ·164 | ·418 | ·418 |
| 12 | 0·37 | 5·28 | 9·3 | 0·4 | — | — | 154·9 | 61 | 83 | 286 | 2·52 | 35·1 | ·117 | ·309 | ·574 |
| 13 | 0·37 | 10·4 | 6·2 | 9·4 | 5·6 | 10·5 | 117·6 | 75 | 86 | 179 | 1·84 | 28·2 | ·150 | ·415 | ·435 |
| 14 | 0·25 | 9·25 | 5·4 | 10·6 | 6·4 | 9·1 | 117·7 | 78 | 102 | 210 | 1·60 | 31·1 | ·136 | ·426 | ·438 |
| 15 | 0·25 | 8·6 | 6·8 | 8·6 | 6·9 | 8·1 | 197·2 | 92 | 103 | 201 | 5·10 | 20·35 | ·210 | ·354 | ·436 |
| 16 | 0·25 | 7·18 | 7·4 | 6·6 | 8·3 | 5·4 | 157·5 | 70 | 105 | 278 | 2·48 | 30·28 | ·135 | ·328 | ·537 |
| 17 | 0·25 | 10·8 | 3·6 | 11·6 | 5·4 | 10·9 | 148·9 | 85 | 105 | 135 | 2·56 | 23·8 | ·180 | ·290 | ·530 |

with oil, not with oil gas. It is not usual to include among engines the vapour engines which use dangerous light oil. In th the oil is sprayed so as to be in the state of finely divided liq particles in air; the air easily vaporises the liquid drops, and explosive mixture is used as in a gas engine. Or the engine dr

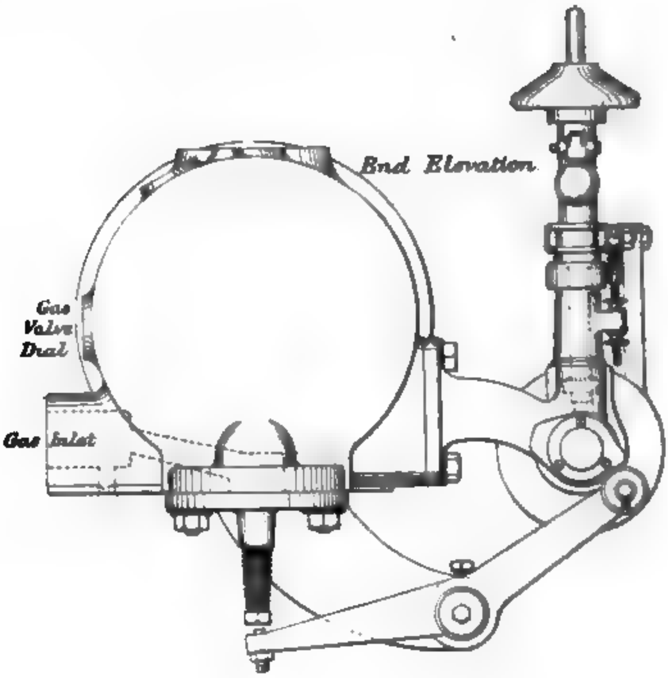



FIG. 249.

air through a liquid "gasoline," and this mixes with more air the cylinder, the Otto cycle being followed.

Safe burning oils with flashing points (Abel test) above 73° F. used in engines in the following ways.

**1.** *Priestman.* The oil is sprayed so as to consist of fi divided liquid particles in air, and when this is heated to 260° by the exhaust gases, the liquid particles become vapour, leaving residue. this vapour is drawn into the cylinder with more air, ju gas is drawn into a gas engine. The theory of the action and cycle of operations are exactly those of gas engines. A defect of method is that during compression we are dealing with a vapour, a gas, and high pressures tend to produce liquefaction; this is r marked when the heavier and cheaper kinds of oil are used. liquefied oil lubricates the cylinder.

2. *Hornsby-Ackroyd.* The oil is injected into the cylinder, or rather into a very hot recess at the end of it, and vaporised there.

3. The oil is vaporised in a small gas or vapour producer kept very hot, external to the cylinder, and introduced as vapour.

Gas and oil engines in England use the tube ignition, or what comes to much the same, ignition by the hot surface of the combustion chamber; but in America and Germany, ignition by the electric spark is quite common, probably because mechanical engineers have some electrical knowledge in those countries. The flame igniter is never used now. In England, the electric igniter is used only in the Priestman oil engine, I believe. Ignition by the hot surface of the combustion chamber seems to be finding greater favour with the makers of gas and oil engines.

**280. Mineral Oil.** If I were devoting my attention to the invention or improvement of an oil engine, I would make a careful experimental study of the physical and chemical properties of oils. In use there are American and Russian petroleum, and Scotch paraffin oils. Crude petroleum is a mixture of gaseous liquid and solid hydrocarbons. American oil consists mainly of the paraffin series of hydrocarbons, $C_nH_{2n+2}$, but there are also some olefines, $C_nH_{2n}$, whereas the Russian oil consists mainly of olefines, or rather naphthenes, $C_nH_{2n-6}H_6$. The student will do well to go to Mr. Clerk's book for a few elementary notions on the complex chemistry of these oils. It is not generally known that Mr. Clerk early in life paid great attention to the subject. The most interesting thing is that if a heavy member of the paraffin series distils off from an oil, and, after liquefying, drops back on the hotter oil, it *cracks* or decomposes into a lower paraffin and an olefine and carbon. This fact is of importance in the refining of oils. At a high enough temperature, we may get any of them decomposing to marsh gas and carbon, possibly with hydrogen. Merely heating an oil in a closed vessel does not seem to decompose it; for effective decomposition, it is necessary to distil. The volatile liquids "petroleum ether," and "petroleum spirit or naphtha," which are easily distilled from American petroleum, are called dangerous. The common burning oils have a flashing point not lower than 73° F., as tested by the *Abel* apparatus, which every student ought to practise the use of.

On very gradually heating American Royal Daylight oil, Prof. Robinson found that it begins to boil at 144° C. At 215° C., 25 per cent. of the stuff has distilled; at 230° C., 35 per cent. has been distilled at 300° C., 76 per cent. has come over; at 340° C., 82 per

cent., and at 358° C., his highest temperature, he still had a residue. The colour gradually darkens during the heating. All oils evaporate in this gradual way because they are mixtures, and at high temperatures the constituents decompose and leave a residue of carbon or tar. But it is easy to charge air with their vapour at much lower temperatures, leaving no residues. Some of these constituents which cannot be driven off by direct heat are easily and completely distilled by blowing superheated steam through the liquid. Even the bubbling of air through oil will allow it all to go off in vapour without leaving a residue. The surfaces in combustion chambers need not be nearly red hot, either for vaporising the oil or igniting mixtures of oil and air, and this is specially true in the case of richer mixtures and heavier oils.

**281.** In the **Priestman engine**, Fig. 252 is the spray producer. Oil comes along *OL* from a tank with air pressure above its oil of 5 lbs. per square inch (gauge), and its fine jet at *L* meets air from *J* (coming from the same tank) in such a way that the spray cloud is produced passing into the vaporiser, Fig. 253, at *K*. The spray vaporises here at about 260° F., the temperature of the surrounding exhaust passage being about 600° F. Air passes through the valve *L* and past the throttle valve *G* and by many holes *a*, *b* to the vaporiser, and the explosive mixture is sucked into the cylinder by the inlet valve *I*, Fig. 251 (the explosive stuff in the vaporiser is a source of danger). The piston *D* compresses the charge and at the end of the stroke an electric spark passes between two platinum points at the end of *K*. It costs about a penny per day to maintain the bichromate battery used to work the induction sparking coil. It would be much better to use a couple of small secondary cells. The spark is timed by contact pieces *K*, Fig. 250, operated by the eccentric rod *E*, which works the pump *P*, driving air into the oil tank. The eccentric shaft has half the speed of the crank shaft: the oil tank has a relief valve.

The rest of the cycle is like that of a gas engine. *E*, Fig. 251, is the exhaust valve, *F* the cross head. The governor turns the throttle spindle *H*, Fig. 253, which has an oil passage through it to *K*, so that both air and oil are regulated in quantity. To start the engine the hand pump is used to send oil through the spraying nozzle and oil spray is formed in the heater *H*, Fig. 250, which mixes with air and gives a blue flame (only needed in starting) to heat the vaporiser. When this is hot enough the fly wheel is turned and the engine starts off.

Prof. Robinson made some tests in 1892 using *Imperial* Oils with a

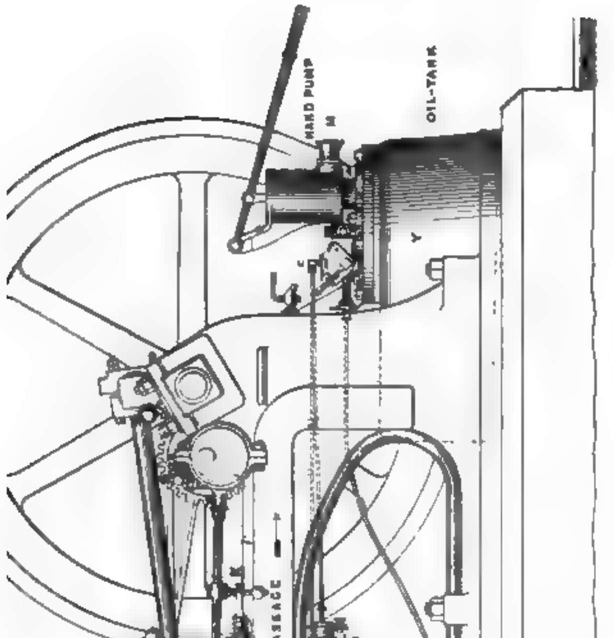



Fig. 250.

*Priestman Oil Engine.* Each test lasted 2 to 3 hours; the engine ran at about 212 revolutions per minute. The indicated power is about 12 to 15 per cent. greater than the brake power. Air pressure in oil tank 9 to 12 lbs. per square inch above atmosphere. The heat carried off by the water jacket was from 40 to 50 per cent. of the whole. Vacuum at end of suction stroke, 5 lbs. below atmosphere or 7 lbs. running light.

The combustion at half load was very slow, the highest pressure being reached about quarter stroke. Probably there is a best clearance and best size of engine for each kind of oil.

| | Russian ordinary or Russolene. | | Broxbourne Lighthouse. (a Scotch paraffin). | | Royal Daylight (American). | | Russian Lustre. | |
|---|---|---|---|---|---|---|---|---|
| Specific gravity | ·824 | | ·810 | | ·796 | | ·825 | |
| Specific heat | ·43 | | ·44 | | ·47 | | ·45 | |
| Flashing point (Abel) | 82° F. | | 152° F. | | 76° F. | | — | |
| Boiling point | 304° F. | | 329° F. | | 291° F. | | — | |
| Hydrogen in 1 lb. | ·1407 | | ·1390 | | ·1486 | | ·1395 | |
| Carbon in 1 lb. | ·8588 | | ·8601 | | ·8462 | | ·8600 | |
| Price per gall. (London) in pence | $3\frac{3}{8}$ | | $4\frac{1}{8}$ | | $4\frac{1}{2}$ | | $3\frac{1}{4}$ | |
| Calorific power, Fahr. units | 21200 | | 21000 | | 21500 | | 21100 | |
| | Full load | Half load. | Full load. | Half load. | Full load. | Half load. | Full load. | Half load. |
| Brake power | 6·76 | 3·54 | 7·5 | 3·9 | 7·05 | 3·7 | 6·9 | 3·7 |
| lb. of oil per brake h. p. hour | ·958 | 1·32 | 0·94 | 1·216 | ·912 | 1·37 | ·989 | 1·32 |
| Fraction of total heat represented by brake energy | ·127 | ·095 | ·130 | ·101 | ·135 | ·090 | ·124 | ·083 |
| Temperature of vapour entering cylinder, Fahr. | 258° | 270° | 258° | 267° | 270° | 276 | 282 | 300 |
| Pressure before ignition (abs.) | 43 | 27 | 45 | 27 | 40 | 27 | 40 | 25 |
| Highest pressure | 135 | 65 | 155 | 65 | 135 | 65 | 135 | 64 |

Prof. Unwin, some of whose illustrations I have taken the liberty to reproduce, Figs. 250–3 (*I.C.E. Proc.*, 1892), using less clearance and getting compression pressures of 50 and 42·6 lbs. (abs.), obtained in 1892 better results from Daylight and Russolene; ·842 and ·946 lbs. of oil per brake horse power hour. He used 33 lbs. of air per pound of oil at full power. When he used 30 per cent. more air he got 4 per cent. less efficiency. In reading his important paper, the student will remark that he deducts the latent heat of the water formed from the full calorific power of the fuel, and I do not think this right for statements of efficiency, although very important in a study of the engine. The incoming mixture of the Priestman engine is at a high tempera-

ture, and this causes the power to be less for a given size of cylinder and the temperatures and loss of heat to be greater than in the gas engine. Also it prevents the use of great compression

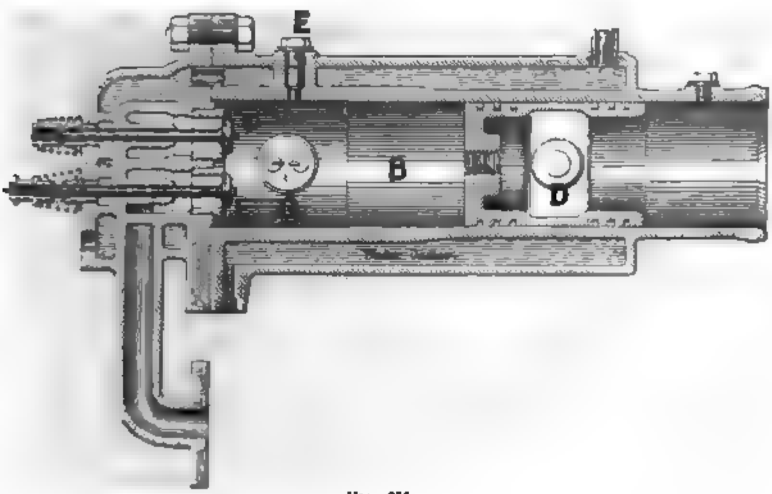



FIG. 251

because of the danger of ignition in the compression stroke. We can use more compression with lighter oils. As in all the other oil engines in the market, the diagram does not differ in appearance from that of a gas engine using the Otto cycle, and the values of the specific

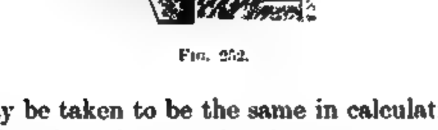



FIG. 252.

eats, and $\gamma$, may be taken to be the same in calculations or possibly little nearer what the values are for air.

**282.** The Samuelson or **Griffin engine** is like the Priestman in principle, but the governing is by missing explosions altogether and a tube igniter is used. There is an ingenious lamp for keeping the tube

hot. A wire keeps covered with oil by capillary action, and an air jet playing on it carries off spray which forms a fierce blue flame. There are several good oil lamps now, the *Etna*, for example, of Messrs. Crossley, and since these have been invented the hot tube igniter has shown its superiority to the electric spark.

**283.** Of the second class, the best known is the **Hornsby-Ackroyd** engine. It uses heavy, cheap oils as well as ordinary burning oils. It is shown in Fig. 254, which is a section through the valves $E D$, and also through the water jacketed cylinder $A$ and combustion chamber $C$, the one section hiding the other. Oil is pumped into the hot vaporiser $C$ through a water jacketed valve box which has a bye pass back to the oil tank opened by the governor when the speed is too great. Thus the pump keeps

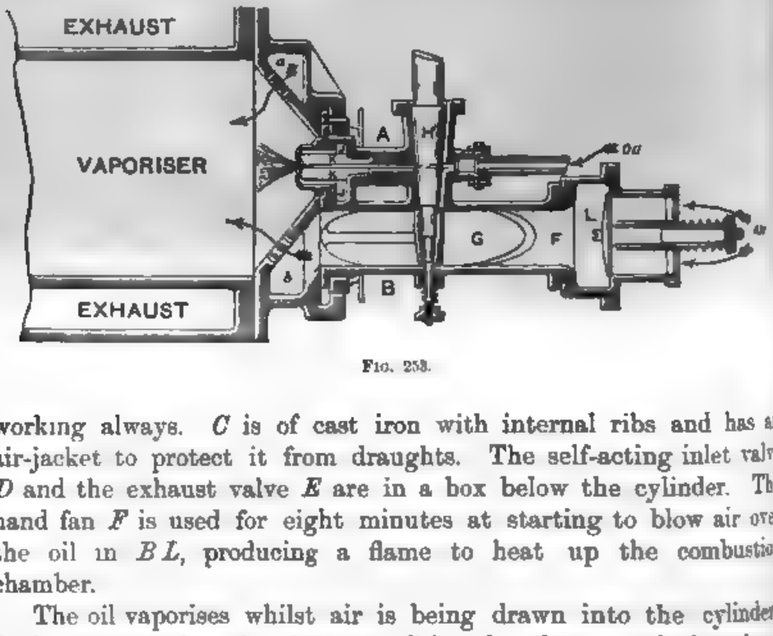



FIG. 253.

working always. $C$ is of cast iron with internal ribs and has an air-jacket to protect it from draughts. The self-acting inlet valve $D$ and the exhaust valve $E$ are in a box below the cylinder. The hand fan $F$ is used for eight minutes at starting to blow air over the oil in $B L$, producing a flame to heat up the combustion chamber.

The oil vaporises whilst air is being drawn into the cylinder, during compression, the air enters $C$ by the throat, and the mixing and pressure are just sufficient at the end of the stroke to produce ignition. It is probably vaporisation that always takes place. Gaseification would probably leave a black residue, and tarry stuff would clog the valves. It is a very wonderful thing that we can depend upon ignition not taking place till the end of the stroke, and indeed we are beginning to rely upon this and to do away with ignition valves in small engines using a tube igniter. It seems that

n when we attempt to ignite by electric spark or flame before the of the compression stroke, the actual ignition waits for the dead nt to be passed. Anyhow, this is securely relied upon in many ines. There is always a little adjustment of the volume of the rance space needed. It seems that with heavy oil the ignition asier at lower temperatures than with light oils, and Mr. Clerk iks that this is due to the greater stability of composition of

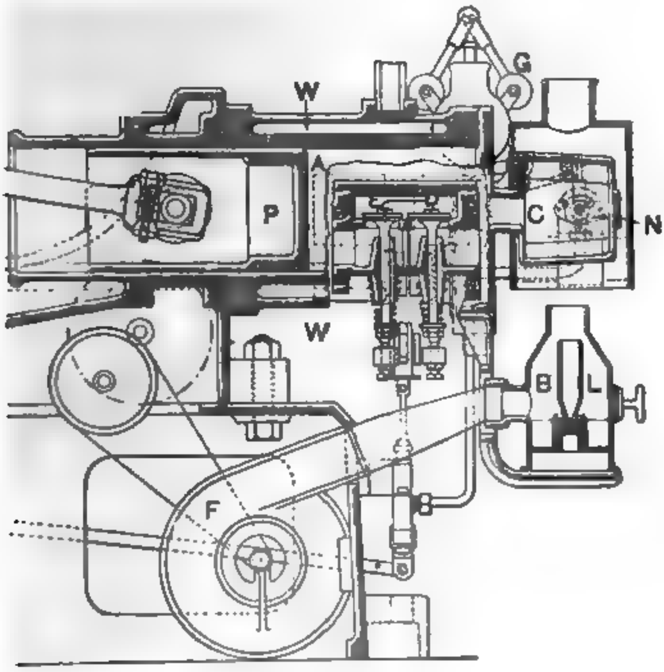



FIG. 254.

t hydrocarbons, the heavy ones separating their carbon so that ogen in a nascent state is set free.

The cylinder, unlike that of the Priestman engine, requires ication as in gas engines.

In careful tests, 0·153 of the energy was indicated, 0·268 went he water jacket; 0·579 went off in the exhaust

**384.** Of the third class, there are many types. There is more for the vaporisation of the oil for each charge, because it rs in a separate vessel and is drawn in through a valve just as would be drawn in.

The numbers of the following table, prepared by Mr. Clerk, g some idea of the results now obtained from oil engines :—

| | Class I. | Class II. | | Class III. | | | |
|---|---|---|---|---|---|---|---|
| | Priest-man. | Horns-by. | Cross-ley. | Camp-bell. | Britan-nia. | Wells. | We-ma |
| Lb. of oil per brake h. p. hour | ·95 | ·98 | ·82 | 1·12 | 1·68 | 1·04 | 1· |
| Compression pressure . . . | 27 | 50 | 65 | 40 | 45 | 32 | 38 |
| Maximum pressure . . . . | 130 | 112 | 225 | 200 | 155 | 135 | 145 |
| Brake horse power . . . . . | 7 | 8 | 7½ | 4·8 | 6·2 | 6·5 | 4· |
| Weight of engine in cwt. . . | 36 | 40 | 32½ | 27 | 33 | 36½ | 26 |
| Weight cwt. per brake h.p. . | 5·1 | 5 | 4·3 | 5·6 | 5·3 | 5·6 | 5· |

I give this table with a little misgiving, because some of numbers are derived from public trials and others are from trials interested persons. I take it that the best efficiency yet obtai according to this table 0·82 lb. of oil per brake horse power hou 14·7 per cent. The Diesel oil engine is said to have an efficie nearly half as great again as this (Art. 291).

**285. Calculations.** We have already seen, Art. 192, how following useful rules are derived. They are the rules most used the designers of new engines.

I. In any change from state $p_1, v_1$ to $p_2, v_2$ of a mass of gas t heat *received* is

$$H=\frac{1}{\gamma-1}\{p_2v_2-p_1v_1\}+W \quad . \quad . \quad . \quad . \quad (1)$$

where $W$ is the work done by the gas in expanding against a vacuu

II. If there is expansion according to the law $pv^s$ constant, t work done is

$$W=\frac{p_1v_1}{s-1}\left\{1-\left(\frac{v_1}{v_2}\right)^{s-1}\right\} \text{ or } \frac{p_1v_1}{s-1}\left(1-\frac{t_2}{t_1}\right) \quad . \quad . \quad (2)$$

and

$$H=W\frac{\gamma-s}{\gamma-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In fact,

$$\frac{dH}{dv}=h=\frac{\gamma-s}{\gamma-1}p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This rule is easily kept in mind if we remember that $p$ is t rate of doing foot-pounds of work per unit change of volume, an is the rate of *receiving* foot-pounds of energy per unit change volume.

If gas is *compressed* from volume $v_2$ to volume $v_1$, and if
istant, the work done *upon* the gas is

$$W=\frac{p_1 v_1}{s-1}\left\{1-\left(\frac{v_1}{v_2}\right)^{s-1}\right\} \quad . \quad . \quad . \quad . \quad (5)$$

; $H$ *given out* by the gas is

$$-H=\frac{\gamma-s}{\gamma-1}W \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

n compression according to the law $pv^s$ constant, the rate at
ork is being done *upon* the gas is $p$. The rate at which the
ing energy as heat is

$$-h=\frac{\gamma-s}{\gamma-1}p \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

. **Exercise.** I do not know if a modern engine would give
e sort of results which I used to obtain in 1881 from a gas
which had an electric light governor; that is, it had an
ı every cycle, sometimes from a weak mixture, sometimes
strong one. I found that if $v$ is the volume corresponding
ighest pressure $p$, we might say roughly that $pv$ was con-
Also I found that the work done in reaching this point
the pressure before ignition, was also nearly constant. If
e strictly true, show that it means that $H$, the heat which
f shows that it has received during ignition (if it were a
gas and did not receive heat from itself), is the same for
d strong mixtures.

$H=\frac{1}{\gamma-1}(pv-p_0v_0)+W$, and if $W$ is constant and $pv$
nt, $H$ must be constant.

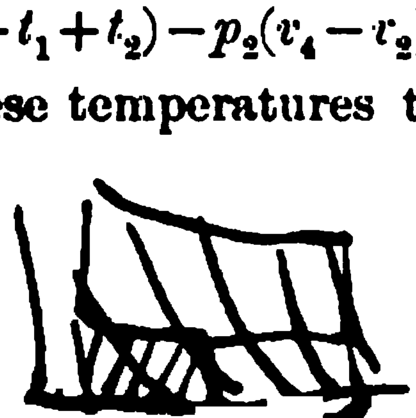
. **Important** *Numerical* **Exercise.** There is a cylinder
reatest volume including clearance space $v_1$ is $v_4$. I take it
cost of the engine is proportional to $v_4-v_1$. A volume $v_2$
d gas at atmospheric pressure $p_2$ and absolute tempera-
is compressed adiabatically to $v_1$; it receives heat $H$ at
volume so that it gets to $v_1$, $p_3$, $t_3$. It expands adiabatic-
and is released.

255 shows the diagram. The compression part may be Cow1
either in a pump or the working cylinder. If $C$ is the
for heat at constant volume of the amount of stuff with
e deal, the work done in compression is $C(t_1-t_2)$; in ex-
$C(t_3-t_4)$: and the nett work is evidently

$$W=C(t_3-t_4-t_1+t_2)-p_2(v_4-v_2) \quad . \quad . \quad . \quad (1)$$

we must change all these temperatures to functions of the

H H 2



$p_2(v_4-v_2)$

volumes, and as in adiabatic operations $tv^{\gamma-1}$ is constant; if we $\gamma-1$ be called $a$

$$t_1 = t_2\left(\frac{v_2}{v_1}\right)^a,\ t_4 = t_3\left(\frac{v_1}{v_4}\right)^a$$

Also $$H = C(t_3 - t_1).$$

Also $$t_3 = \frac{H}{C} + t_1 = \frac{H}{C} + t_2\left(\frac{v_2}{v_1}\right)^a$$

These enable us to express $W$ in terms of $v_2$, $v_1$, $v_4$ and $t_2$.

$$W = H\left\{1-\left(\frac{v_1}{v_4}\right)^a\right\} + ct_2\left\{1-\left(\frac{v_2}{v_4}\right)^a\right\} - p_2v_2\left(\frac{v_4}{v_2}-1\right) \quad . \quad .$$

dard Gas Eng.

$$\epsilon = \frac{W}{H} = 1-\left(\frac{v_1}{v_4}\right)^a + \frac{ct_2}{H}\left\{1-\left(\frac{v_2}{v_4}\right)^a\right\} - \frac{p_2v_2}{H}\left(\frac{v_4}{v_2}-1\right) \quad . \quad .$$

$v_2$

If we take the mixture to be (by volume) 9 of air to 1 of coal gas take it that a cubic foot of it will weigh

$\left(1+\left|\frac{v_1}{v_4}\right|^a\right)$

$$\frac{\cdot 076 \times 274}{t_2} \text{ lb. and in foot-pounds } C = 263\,\frac{\cdot 076 \times 274}{t_2}\,v_2$$

$$H = 52600\,\frac{274}{t_2}\,v_2 \text{ foot-pounds.}$$

Hence $$\frac{Ct_2}{H} = \frac{263 \times \cdot 076}{52600}\,t_2 = \frac{t_2}{2630}$$

$$\frac{p_2v_2}{H} = \frac{2116\ t_2}{52600 \times \cdot 274} = \frac{t_2}{6810}$$

If we take $t_2 = 290$, or 16° C., we have

$$\frac{Ct_2}{H} = 0{\cdot}11 \text{ and } \frac{p_2v_2}{H} = \cdot 0426$$

and $H = 49710v_2$, so that

$$\epsilon = 1-\left(\frac{v_1}{v_4}\right)^a + 0{\cdot}11\left\{1-\left(\frac{v_2}{v_4}\right)^a\right\} - \cdot 0426\left(\frac{v_4}{v_2}-1\right) \quad . \quad .$$

If $\frac{v_1}{v_2}$ be called $x$ and if $\frac{v_4}{v_2}$ be called $y$ and if we make the volume swept through by the piston in every case to be 1 cubic foot

$$v_4 - v_1 = 1, \text{ or } v_2(y-x) = 1$$

Standard $y=1$

$$\text{or } v_2 = \frac{1}{y-x},\ \frac{v_1}{v_4} = \frac{x}{y},\ H = \frac{49710}{y-x}$$

$$\epsilon = 1 - x^a y^{-a} + 0{\cdot}11(1-y^{-a}) - 0{\cdot}0426(y-1).$$

I shall take $a = 0{\cdot}37$.

In the Otto cycle $y = 1$ and $\epsilon = 1 - x^a$, $H = \dfrac{49710}{1-x}$.

CIENCY AND WORK DONE IN ONE CYCLE (AS FIG. 255), FOR VARIOUS AMOUNTS PRESSION $v_2/v_1$ AND FOR VARIOUS EXPANSIONS BEYOND THAT OF THE OTTO.

| of $y$ $r_2$. | Values of the compression $v_2/v_1 = 1/x$. | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 5 | 7 | 10 |
| ycle | 0<br>0 | ·2261<br>22479 | ·3341<br>23408 | ·4484<br>27863 | ·5132<br>29676 | ·5733<br>31666 |
| | | | | | | ·6268<br>22300 |
| 2 | ·208<br>10340 | ·3834<br>12706 | ·4660<br>13898 | ·5554<br>15539 | ·6056<br>16212 | ·6521<br>17060 |
| | | | | | | ·6637<br>13750 |
| | ·370<br>7085 | ·4363<br>8676 | ·5082<br>9475 | ·5853<br>10391 | ·6274<br>10917 | ·6675<br>11440 |

ve the efficiency and $W$ for each case.

2, $x=1$ represents the Lenoir, Hugon, and Bischoff cycle, but
ider has a volume
ubic feet.
see that although
(for all compres-
considerable gain
efficiency in ex-
; as much again
ie Otto cycle, the
of the engine is
r its size. Also
w that the me-
l efficiency is less.
ry case greater
sion produces, not
gain in efficiency,
ain in the output.

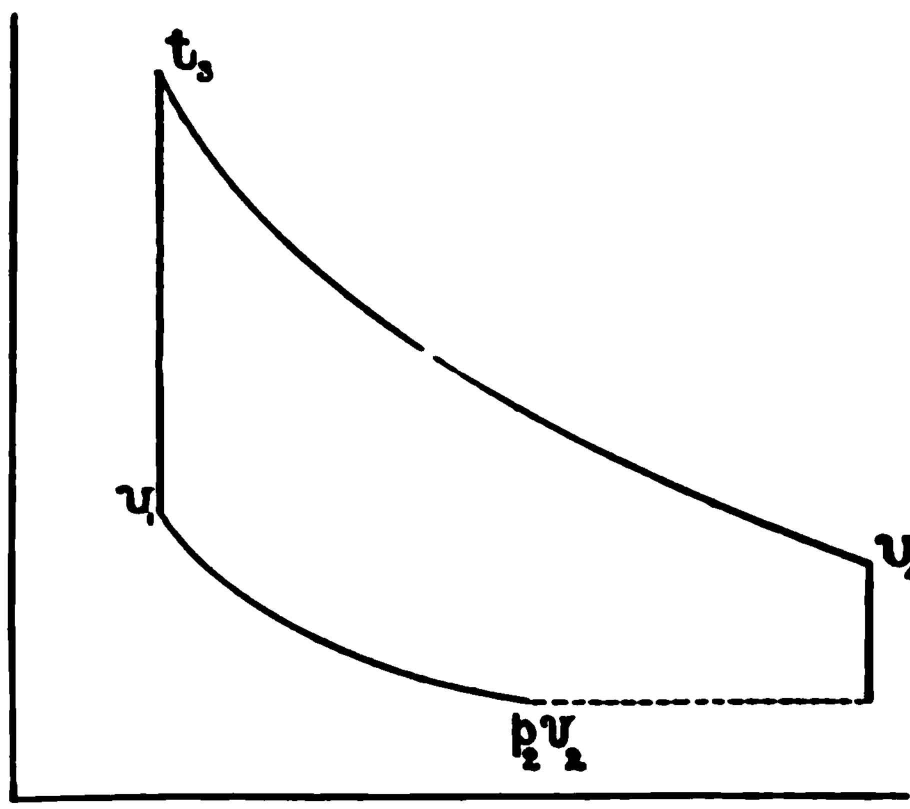


FIG. 255.

ve no doubt that this is now the most important consideration engine makers. To what extent ought we to take advantage expansion than occurs in the Otto cycle? I believe that

compounding is remote from us. There is no such necessity as exists in the steam engine for keeping a cylinder hot, rather the reverse, and I take it that this is the most important reason for compounding in the steam engine. Again there is the ever-present difficulty with valves to admit hot stuff from one vessel to another.

There is every reason to believe that we shall in the Otto reach such compression as $x = \frac{1}{10}$, and in view of the future I have considered this case more fully than the others. It is noticeable that the increase of efficiency is considerable when we expand $1\frac{1}{2}$ times or twice as much as in the Otto, and the diminution of work from a given size of cylinder is not so great (at all events for half as much more expansion) but what we may expect to see this improvement introduced. The fact that cooling tends to occur through mere expansion rather than the water jacket is another matter of great importance. At present we have not enough information to enable us to settle the right ratio of $v_4$ to $v_2$, but if there had been more space at my disposal I should have been glad to consider the question more fully. In using such a table we must recollect that there is more relative loss by friction when we have a large engine of less power. Also there is more frictional loss with greater compressible pressures.

I have sometimes endeavoured to get a notion of the effect of this, and have used the formula

$$\text{Brake power} = I(\cdot 86 - \cdot 02r),$$

where $r$ is the ratio of greatest volume to the clearance volume.

My students have $\theta\phi$ sheets (Art. 205) ready for the working of any exercises on perfect gases, series of lines of equal $v$, $p$, and $E$ being drawn as well as the $\theta$ and $\phi$ lines. On such a sheet it is easy to draw the $\theta\phi$ diagram for the hypothetical cases discussed here. It is also easy to convert a real $pv$ diagram into a $\theta\phi$ diagram.

**288.** I find that beginners may learn more from exercises worked like 7, 8 and 9 of the following sheet than through algebraic expressions, like those just given. I select this sheet from many others, which I have year by year or week by week put before evening students at the Finsbury Technical College, and I give it as a specimen of the exercises which students ought to do.

*Finsbury Technical College, October 20th,* 1892

1 This first question concerns a number of conversions of units of energy, such as are given in Chap. XV. I find that in 1892 I was

rious to know what all the other costs in the production of energy re as compared with the mere cost of the fuel.

2. In a gas engine cylinder at one point in the diagram where = 2, $p$ = 14·7, the temperature is known to be 150° C. What is temperature where $p$ = 136 and $v$ is 1·2 ?

3. On the expansion curve of an oil engine diagram the following asurements are made. The scales are of no consequence. Find law of expansion approximately. [Plot log. $v$ and log. $p$ on squared er.]

| $v$ | 1·1 | 1·4 | 1·7 | 2·1 |
|---|---|---|---|---|
| $p$ | 168 | 120 | 88 | 65 |

4. Calculate the energy obtainable from 1 lb. of liquid fuel, which ntains 0·8 lb. of carbon and 0·135 of hydrogen. Give it in Cent. at units, in foot-pounds and in evaporative poundage, from and 100° C. What volume of air is required for its complete combustion ?

5. Calculate the energy obtainable from 1 cubic foot of gas, conining 0·2 cubic foot of hydrogen, 0·5 of marsh gas, 0·2 of olefiant s. What volume of air is required for its complete combustion ?

6. Power is distributed by shafting to small shops at £30 per num per horse power. A shop uses power for 54 hours per week. hat is the cost per horse power hour ? If the engine uses 3 lbs. coal per hour for each horse power delivered to customers, and al is at seventeen shillings per ton, compare the cost of the coal th the total cost.

7. A cubic foot of a mixture of coal gas and air is taken (1 : 9 by l.) at 100° C. and pressure 2,116 lbs. per square foot. How much ergy is given to it in compressing it adiabatically to 0·5 cubic foot ? ake $\gamma$ = 1·38.) Find also its pressure and temperature at the end. w give it 40,000 foot-pounds of heat, keeping its volume constant. hat are its new pressure and temperature ? Now let it expand iabatically to 1 cubic foot ; how much energy does it lose (absolute rk done by it upon a piston, say) ? What is the nett work done ? vide by 40,000 for the efficiency. [Students were expected to do s by the formulæ of Art. 192.]

8. Repeat all the calculations of (7), but let the smaller volume 0·4 or 0·3 or 0·2 or 0·1 cubic foot. If all cases are worked out ow the results in a table.

9. Prepare a new table, but let the last expansion be to 2 cubic feet, and subtract 2,116 foot-pounds from the balance of work done.

**289.** I made sure that students did these exercises after the lecture. I refrain from giving a sheet in which a complete set of exercises was to be worked out from a given indicator diagram, and the information that accompanied it. I refrain because this book is getting to be much too large, but I cannot help giving a few exercises from another sheet which lies before me. It is evident that I had a

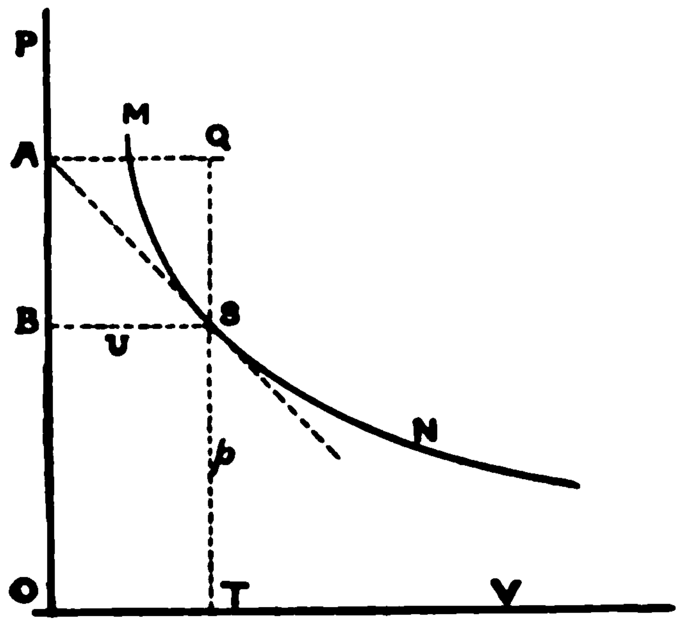



FIG. 256.

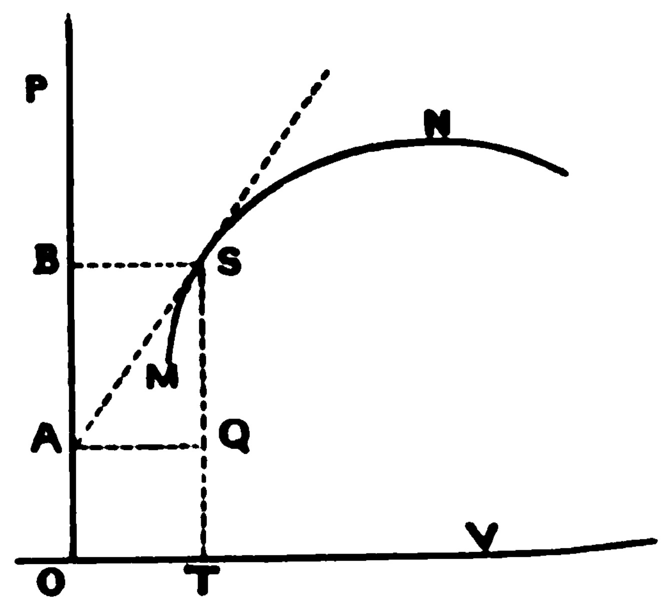



FIG. 257.

hard-working set of evening students that year, and I wonder how they have used their knowledge. This sheet is dated 3rd November 1892.

1. Some of you have taken a gas engine and some an oil engine diagram, and you have drawn curves showing $p$, $t$, $h$ [$h$ is $\frac{dH}{dv}$ of Art. 285], and combustion [our $\frac{dH^1}{dv}$ of Art. 294], the volume being abscissa. Prepare another sheet in which *time* is the abscissa. You may assume an infinitely long connecting-rod.

2. [I find that I here gave some rules already given in Art. 285, the following one is a new statement of an old rule. I do not, however, like to draw tangents to curves.]

Draw tangent $SA$ to a $p, v$ curve $MSN$ at the point $S$. Prove that

$$(1) \text{ In Fig. 256, } h = \frac{\gamma \cdot ST - SQ}{\gamma - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$(2) \text{ In Fig. 257, } h = \frac{\gamma \cdot ST + SQ}{\gamma - 1}$$

3. On October 20th I asked you to find the useful work done in various cases of clearance, and of total volume of cylinder. Now, the

value of a given type of engine may be stated as depending somehow upon, 1st, The fact that we obtain $x$ foot-pounds of energy usefully from one explosion. 2nd, The cost of the engine and its maintenance and attendance, which may be taken as proportional to $v_4$, the volume of the cylinder. 3rd, The pressure after ignition, which depends upon clearance; because if the pressure is great the engine costs more money, and is more of a nuisance. What is your idea of a figure of merit made up of $v_2$, $v_4$ and $v_1$?

**290.** Exercise. A cubic foot of gas engine mixture at atmospheric pressure $p_1$ and absolute temperature $t_1$ is compressed adiabatically to the pressure $p_2$ and temperature $t_2$. It is then ignited at constant pressure $p_2$ to the volume $v_3$ and allowed to expand adiabatically to the atmospheric pressure again and temperature $t_4$. Find the work done and the efficiency. This is the Brayton engine principle.

*Answer*. The heat given is $H = K(t_3 - t_2)$. The heat that would be taken out to begin a new cycle with the same stuff is $K(t_4 - t_1)$. Hence the work $W$ done is $K(t_3 - t_2 - t_4 + t_1)$, and the efficiency is $\epsilon = \frac{W}{H}$ or $1 - \frac{t_4 - t_1}{t_3 - t_2}$, but as compression and expansion are adiabatic, $\frac{t_4}{t^3} = \frac{t_1}{t_2}$, so that

$$\epsilon = 1 - \frac{t_1}{t_2}$$

Along an adiabatic $tp^{\frac{1}{\gamma} - 1}$ is constant, and hence

$$\epsilon = 1 - \left(\frac{p_1}{p_2}\right)^{1 - \frac{1}{\gamma}}$$

Thus as $p_1$ is one atmosphere, if we take $\gamma = 1{\cdot}37$, we have the following values of $\epsilon$, and $W$ is the same as $\epsilon$ if $H$ is 1.

| $p_2$ in atmospheres | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 17 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\epsilon$ | ·1708 | ·3123 | ·3835 | ·4297 | ·4630 | ·4888 | ·5095 | ·5345 | ·5547 | ·5709 | ·6008 |

**291.** It is now fourteen years since I first gave exercises like those of Arts. 287–290 to my students, pointing out the gain of efficiency due to increased compression. The first engineer who has tried to carry out the idea has met with wonderful success in the **Diesel motor**. The best account of its performance which I have seen, is in *The Engineer*, October 15, 1897. Careful experiments

have been made, but I believe that the only published accounts of them are written by Mr. Diesel himself. The consumption of oil was 0·56 lb. per hour per brake horse power, so that the efficiency is 46 per cent. better than the best results given in the table, page 466. Mr. Diesel, in his 20-horse power engine at about 160 revolutions per minute, pumps air into a receiver at 700 lbs. per square inch. This very hot air enters with oil in a state of combustion into the motor cylinder (9·8 in. diameter, 15·7 in. stroke) at the beginning and for about one quarter of the stroke, the pressure falling; it is then cut off and the stuff goes on expanding to the end of the stroke, when it is exhausted; cushioning brings the pressure to 400 lbs. per square inch in the motor cylinder before a fresh admission takes place. It is then the Brayton cycle except that during the combustion the pressure is not kept constant. A water jacket has been found necessary. It is said that there is no great falling off in efficiency when working at half load.

**292.** EXERCISE. In the Atkinson gas engine, at a famous trial in 1888, the expansion and compression curves followed the laws $pv^{1\cdot 264}$ constant and $pv^{1\cdot 205}$ constant. Taking $\gamma = 1\cdot 367$ in the expansion and $\gamma = 1\cdot 385$ in the compression (see Art. 189), what is the rate at which the stuff, as a gas, shows that it is receiving or losing heat?

*Answer.* If $h$ is rate of heat reception per unit volume, so that it may be represented to the same scale as the pressure,

$$h = \frac{1\cdot 367 - 1\cdot 264}{1\cdot 367 - 1} p = 0\cdot 28\, p \text{ in the expansion}$$

$$-h = \frac{1\cdot 385 - 1\cdot 205}{1\cdot 385 - 1} p = 0\cdot 467\, p \text{ in the compression.}$$

In the compression heat is being lost to the cylinder nearly half as fast as work is being done upon the stuff.

**293.** In 1885, with Prof. Ayrton, I published a paper in the *Proceedings* of the Physical Society in which I pointed out how the gas engine diagram ought to be studied. I took a diagram which I had obtained from a 6-horse engine at Finsbury, and from my own and other measurements of temperature, showed how we might find the rate of combustion of the gas going on in the ignition and expansion, and how the whole chemical energy was disposed of. The exercises of Art. 189 illustrate how I showed that we might speak of the stuff in a gas (using coal or Dowson gas) or oil engine cylinder before and after combustion, as if it were the same perfect gas with $\gamma = 1\cdot 37$, which had undergone no chemical change, and had received heat from an outside furnace. [The alteration is small, but it may be

is sufficient to account for the fact that we never get more
0 per cent. of the chemical energy in an explosion.] I then
ly showed that the compression and expansion parts of the
m follow a law like $pv^m$ constant and how to find $m$, but that in
ine controlled by an electric light governor there was an easy
stating the law for the whole ignition and expansion parts
he diagrams (bad and good explosions) on a card.
is is

$$p = Mv^{-m}\left\{\kappa^1 + nu - \sqrt{(\kappa - nu)^2 + s}\right\}$$

$u = v$ the volume − the clearance volume; $\kappa^1$ and $\kappa$ are
nts. $s$ is also a constant, but any very small number will do
$n$ is a constant which depends upon the point in the stroke
the maximum pressure occurs, and this really, for a given
of engine and method of ignition, depends upon the richness
mixture. $M$ is a constant which depends upon the recentness
last explosion. $m$ is the ordinary index of $v$ in the expansion
I showed how inexact all calculations from the diagram
unless we used an empirical formula like this. It was, how-
ufficiently accurate to represent the whole of a curve by two
sions

n part $\quad p = (a + bu)\,\kappa v^{-m}$ . . . . . . (2)

sion part $\quad p = \kappa v^{-m}$ . . . . . . . . . (3)

$\kappa$, $a$ and $b$ are constants.
ing the formula (see Art. 285) for reception of heat energy
it change of volume

$$h = \frac{dH}{dv} = \frac{1}{\gamma - 1}\left(\gamma p + v\frac{dp}{dv}\right) \text{ where } \gamma = 1{\cdot}37 \quad . \quad (4)$$

a diagram of $h$ to the same scale as $p$. I then showed with
approach to accuracy how the total energy of a change was
d of. We have so greatly improved on those results of 1885
shall not venture to give them here, and I will now use
ram (Fig. 258) sent me from King's College (one of Mr.
ll's tables, Art. 278) to illustrate my method of finding the rate
bustion.
nd, $p$ being in lb. per square foot and $v$ in cubic feet,

ession curve $\quad p = 1884\,v^{-1{\cdot}13}$ . . . . . . (5)

sion curve $\quad p = 4877\,v^{-1{\cdot}23}$ . . . . . . (6)

n curve $\quad p = ({\cdot}336 + 100u)\,4877\,v^{-1{\cdot}23}$ . . . (7)

$u = v - 0{\cdot}247$.

I am afraid that the rise of pressure on ignition is too rapi us to be able to speak accurately of its law in the present excepti

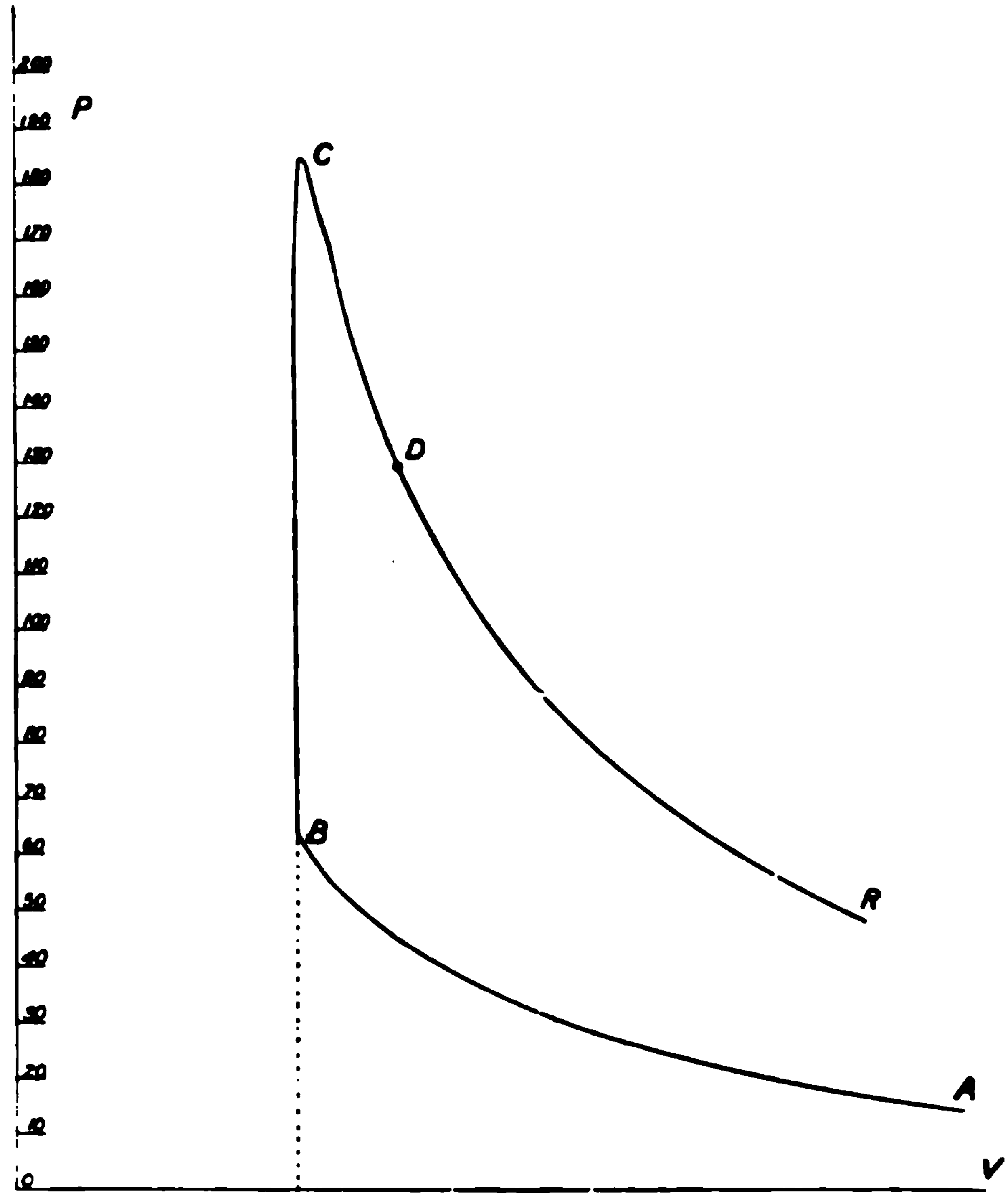



FIG. 258.

Average effective pressure, 44·6; diameter of cylinder, 8½ inches; stroke, 18 inches; 170 revolutions and 83 explosions per minute; gas, per explosion, ·081 cubic feet; clearance, ·1247 cubic feet.

case. From these we find, if $h$ is rate of *gain* of heat by the stu

**Compression** $h = -\ \cdot 65\ p.$

That is, the stuff is *losing* heat at a rate which is $\frac{2}{3}$rds of pressure

**Expansion** $h = \cdot 378\ p$

**Ignition** $h = \cdot 378\ p + 2{\cdot}110 \times 10^6\ v^{-\cdot 23}$

For the reason given I feel that there is an unnecessary pret at accurate statement for the ignition part.

**294.** *Rate of loss to* **Water Jacket.** We usually know the loss per explosion to the water jacket if the engine is kept on ful for a few hours. In this case it was 35 per cent. of the total he the charge, the indicated energy being 16 per cent. **During the**

only 50 per cent. of the possible explosions took place, or one explosion in four revolutions or eight strokes. Hence, if $w$ is the indicated work on the diagram, we have $\frac{35}{16}w$ given in eight strokes. We shall not be far wrong if we take $w$ as being equal to the heat given in the ignition and expansion stroke up to release.

In my paper in 1885, I assumed that the rate of loss of heat per second by the stuff to the jacket is proportional to $\theta - 60$ if $\theta°$ C. is the temperature of the stuff. As more area is exposed when temperatures are lower, I thought that this was a good enough rule for rough calculation. I might now use a rule deduced by Mr. Wimperis from some experiments by Mr. Petavel, on the loss of heat $\epsilon$ per square cm. per degree by bright platinum in an atmosphere of carbon dioxide, at temperatures ranging from 200° C. to 1,200° C., and at pressures ranging from 6 cm. to 228 cm.

$$\epsilon = 1{\cdot}55 \times 10^{-8}\, p\,(1000 + \theta) + 1{\cdot}67 \times 10^{-6}\, \theta$$

where $p$ is in pounds per square inch and the temperature is $\theta°$ C. I have not found this altogether satisfactory, however, nor is it right to assume that such a law can hold for our high pressures and an iron surface. Let the student work for the present according to my old rule. Calculate $\theta$ at every point of the diagram, assuming that at $A$ it is 120° C. Assume that $\frac{dH^1}{dt}$ (where $t$ is time) is represented to some scale by $\theta - 60°$ C. We want $\frac{dH^1}{dv}$, where $v$ is volume, and as

$$\frac{dH^1}{dv} = \frac{dH^1}{dt} \cdot \frac{dt}{dv}$$

we have to divide $\frac{dH^1}{dt}$ by the velocity of the piston to get $\frac{dH^1}{dv}$ to an unknown scale.

Now make the average height of the $\frac{dH^1}{dv}$ curve equal to the average pressure of the indicator diagram, because the loss of heat $H^1$ to the jacket is equal to $w$, and so we get the true value of $\frac{dH^1}{dv}$ to the same scale as the pressure.

The values of $\frac{dH}{dv}$ as calculated from (4) Art. 293, are given.

Add $\frac{dH}{dv}$ and $\frac{dH^1}{dv}$ to find the total rate of development of heat by combustion.

This is on the same scale as the pressure, and is very interesting.

If it is desired to know the **rate of combustion per** multiply by the velocity of the piston.

Our present knowledge only allows us to make ver approximations. In all probability there is very rapid con in the stuff, just as it is throttled in passing the exhaust val

| $v$ | $p$ Lb. per square inch. | $\theta$° C. | Velocity of piston. | $\theta - 60$ Divided by velocity. | $dH^1/dv$ Lb per square inch. | $dH/dv$ Calculating from (Art. 293) lb. per square inch. | Total rate of combustion per unit volume. |
|---|---|---|---|---|---|---|---|
| ·247 | 63·5 | 264 | 0 | ∞ | ∞ | 1260 | ∞ |
| ·248 | 170·0 | 1179 | — | — | — | 1260 | — |
| ·253 | 183·9 | 1329 | 30 | 42·30 | 211·5 | 69·5 | 281·0 |
| ·279 | 167·9 | 1341 | 67·5 | 18·98 | 95·0 | 63·48 | 158·5 |
| ·336 | 129·1 | 1226 | 113 | 10·32 | 51·6 | 48·79 | 100·4 |
| ·394 | 107·1 | 1186 | 137 | 8·219 | 41·1 | 38·67 | 79·8 |
| ·452 | 90·9 | 1146 | 151 | 7·193 | 36·0 | 34·37 | 70·4 |
| ·512 | 76·8 | 1076 | 159 | 6·389 | 32·0 | 29·03 | 61·0 |
| ·672 | 68·1 | 1068 | 158 | 6·377 | 31·9 | 25·74 | 57·6 |
| ·630 | 60·0 | 1031 | 153 | 6·346 | 31·7 | 22·69 | 54·4 |
| ·697 | 53·1 | 1006 | 137 | 6·905 | 34·5 | 20·07 | 54·6 |
| ·750 | 47 7 | 958 | 114 | 7·877 | 39·4 | 17·99 | 56·4 |

The velocity of piston is nearly proportional to the ord a semi-circle on the stroke as diameter, and may be to an so that $(\theta - 60) \div$ velocity of piston $= c\,\dfrac{dH^1}{dv}$. This is plott $v$ on squared paper, as a curve whose average height but the average value of $\dfrac{dH^1}{dv}$ = average effective press $44{\cdot}33 \times 144$ lbs. per square foot, and hence $c = \dfrac{8{\cdot}95}{44{\cdot}63} \times 144$. this $c$, I get true $\dfrac{dH^1}{dv}$ to be $\frac{1}{5} \times 144$, or I multiply the fifth by 5 to get the sixth column of numbers. Rate of combust second in Column 9 is to an unknown scale, being the column multiplied by velocity of piston.

Every time I have made this interesting calculation on oil engine diagram, I have found, as here, that the rapidity combustion (per second) reaches a maximum some tim ignition begins. In the present case, it is some time a pressure has begun to fall, about $D$, Fig. 258. The result to be plotted and shown in a curve.

**295.** Mr. Wimperis worked this problem more elaborate

found $\theta$ everywhere in compression, ignition and expansion, and calculated $\epsilon$ as above. In compression, he had $-\frac{dH}{dv}=0\cdot65\,p$, and dividing by velocity of piston as given above, he had numbers proportional to $\frac{dH}{dt}$ or $\frac{dH}{dv}\cdot\frac{dv}{dt}$. He divided by $\epsilon$ and $\theta-60$, and took the quotient to represent $A$, the area of metal exposed to radiation in each case. It was interesting to note that these values of $A$ were in pretty much the right proportions. He now took these found values of $A$ to calculate from $\epsilon$ and $\theta$ the values of $\frac{dH}{dt}$, and, therefore, $\frac{dH}{dv}$, &c., in the ignition and expansion.

I do not give his results here, because I think that the method is too good to be illustrated by a case in which we are in doubt of the starting temperature. Also I think that the skin temperature of the metal may not be so nearly constant as it seems to be in the steam engine, and, besides, an exercise like what I have given will serve better to start a student in thinking about this subject.

**296. Exercise. Rate of Combustion.** The following exercise will show how a student may obtain some information as to the combustion going on in one of Mr. Clerk's experiments. The information is not very exact, but it is worth something. Mr. Clerk (Art. 271) took a mixture of 1 of Oldham gas to 9 of air at 14° C. and atmospheric pressure, and obtained a curve showing the pressure at various times after ignition. This is one of his many results, and I chose it at random. I made the following measurements of $t$ (time in seconds after ignition) and $p$ (the increase of pressure in pounds per square inch). The rise of temperature $\theta$ ought to be almost 20 times $p$.

I thought that after $t = \cdot 7$ the combustion had probably ceased, and that, thereafter, I might take the rate of loss of heat to the cylinder as being represented by

$$q = a\theta + b\theta^2$$

I found that with considerable accuracy this seemed to be the case, and, indeed, that I might take

$$\theta = \frac{10{,}000}{1 + 34t}$$

So that $a$ is practically nothing, for

$$-\frac{d\theta}{dt} = \cdot 0034\,\theta^2$$

The observations are not so very regular as to allow us to ·0034, rather than ·0033, and I thought it well to get help in t following way. I plotted $\theta^2$ from $t = o$ to $t = 1$,

$$\text{and found } \int_0^1 \theta^2\, dt \text{ to be } 692800$$

Now we know that the total heat was 2670$c$, and the heat 30 remains, so that $b \int_0^1 \theta^2 .\, dt$ ought to be 2370$c$, and hence $b = {\cdot}003$ This wonderful agreement with the previous result gave me so satisfaction. I take the capacity of the stuff to be $c$, a constant;

$$q = {\cdot}0034\ c\ \theta^2$$

Hence we may take

$$\text{rate of combustion} = c\left(\frac{d\theta}{dt} + {\cdot}0034\ c\ \theta^2\right)$$

I smoothed the curve for $\theta$, and found the values of $\frac{d\theta}{dt}$ given in t table; the addition of the numbers in the fourth and fifth colum gives those in the sixth, which seem to me very interesting. Witho making too much of the result, we may say that it gives a rough correct sort of indication of how the combustion takes place.

Combustion going on in a Closed Vessel.

| $t$ Seconds. | Observed gauge $p$. | $\theta$. | $d\theta/dt$. | Rate of loss to vessel divided by $c$. $\cdot 0034\,\theta^2$. | Rate of combustion divided by $c$. |
|---|---|---|---|---|---|
| 0 | 0 | 0 | ? | 0 | ? |
| ·05 | 45 | 900 | 20000 | 2754 | 22754 |
| ·1 | 77·5 | 1550 | 0 | 8168 | 8168 |
| ·15 | 64 | 1280 | − 4500 | 5572 | 1072 |
| ·2 | 53·5 | 1070 | − 3100 | 3894 | 794 |
| ·3 | 42 | 840 | − 1900 | 2400 | 500 |
| ·4 | 34·5 | 690 | − 1350 | 1618 | 268 |
| ·5 | 28·75 | 575 | − 1000 | 1124 | 124 |
| ·6 | 25·0 | 500 | − 750 | 851 | 101 |
| ·7 | 21·0 | 410 | − 574 | 597 | 23 |
| ·8 | 18·0 | 368 | − 461 | 461 | 0 |
| | 16·25 | 329 | 268 | 368 | 0 |
| 1·0 | 15·0 | 300 | 306 | 306 | 0 |

# CHAPTER XXVIII.

## VALVE MOTION CALCULATION.

**297.** A SLIDE VALVE worked by an eccentric or crank on a iiformly rotating shaft gets very nearly a **simple harmonic otion.**[1] There are **various ways** of studying this motion.

1. Counting time $t$ in seconds from the dead point position of e engine, if $y$ is the distance of the slide to the right of the middle its stroke at the time $t$; if $r$ is the half travel of the valve, or the ngth of the crank working it, or the eccentricity of the eccentric; if revolves at $q$ radians per second, and if $a$ is the angle of advance, angles are measured really in radians (although I shall sometimes rite them as degrees), then

$$y = r \sin (qt + a)$$

hether or not the crank goes round uniformly, if $\theta$ is the angle hich it makes with the inner dead point (nearest the cylinder), and $x$ is the distance of the piston from the end of its stroke (most mote from the crank), the crank being $R$ and connecting rod $L$

$$y = r \sin (\theta + a)$$

$$x = R(1 - \cos \theta) + \frac{R^2}{4L}(1 - \cos 2\theta)$$

ry nearly. If we take $\theta = qt$ and if the crank makes $q$ radians r second, the valve has a simple harmonic motion, and the piston s a fundamental simple harmonic motion with its octave or other such motion of twice the frequency.

[1] Simple harmonic motion is regarded now as a badly chosen term. Some such m as "simply periodic motion," suggested by Professor Schuster, would be ter. Simple vibration ought to be used instead of simple harmonic vibration. nploy the usual term unwillingly.

2. In Fig. 259 if $OE = r$ the half travel of the valve, or
the eccentric crank working the valve which slides in a c
parallel to $DOF$; in the position shown in the figure, the va
the distance $OH$ from the n
its stroke. If this is c
with (1), and if $GOE^1$ =
angle of advance, $OE^1$ is t
tion of the eccentric cran
the main crank is in th
point position $OD$.

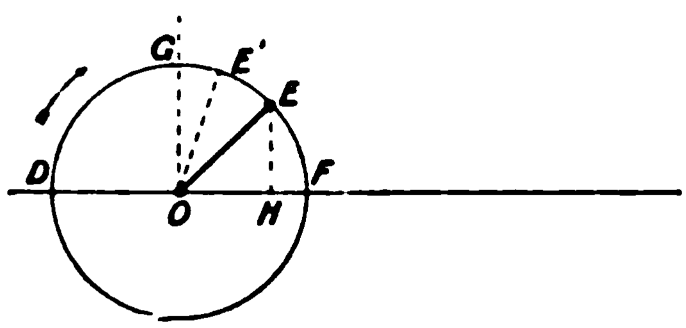



FIG. 259.

3. In Fig. 260 if $DO$
line of centres and $GOG^1$ a line at right angles to $DOF$
$COG = C^1OG^1 = a$ the angle of advance. Make $OC = OC^1$
half travel of the valve and describe the circles shown, on
$OC^1$ as diameters. If the main crank is in any angular
$OB$ the intercepts $OB^1$ cut off by the circle $OC$ show $y$ the
of the valve to the right of its mid stroke; the intercepts

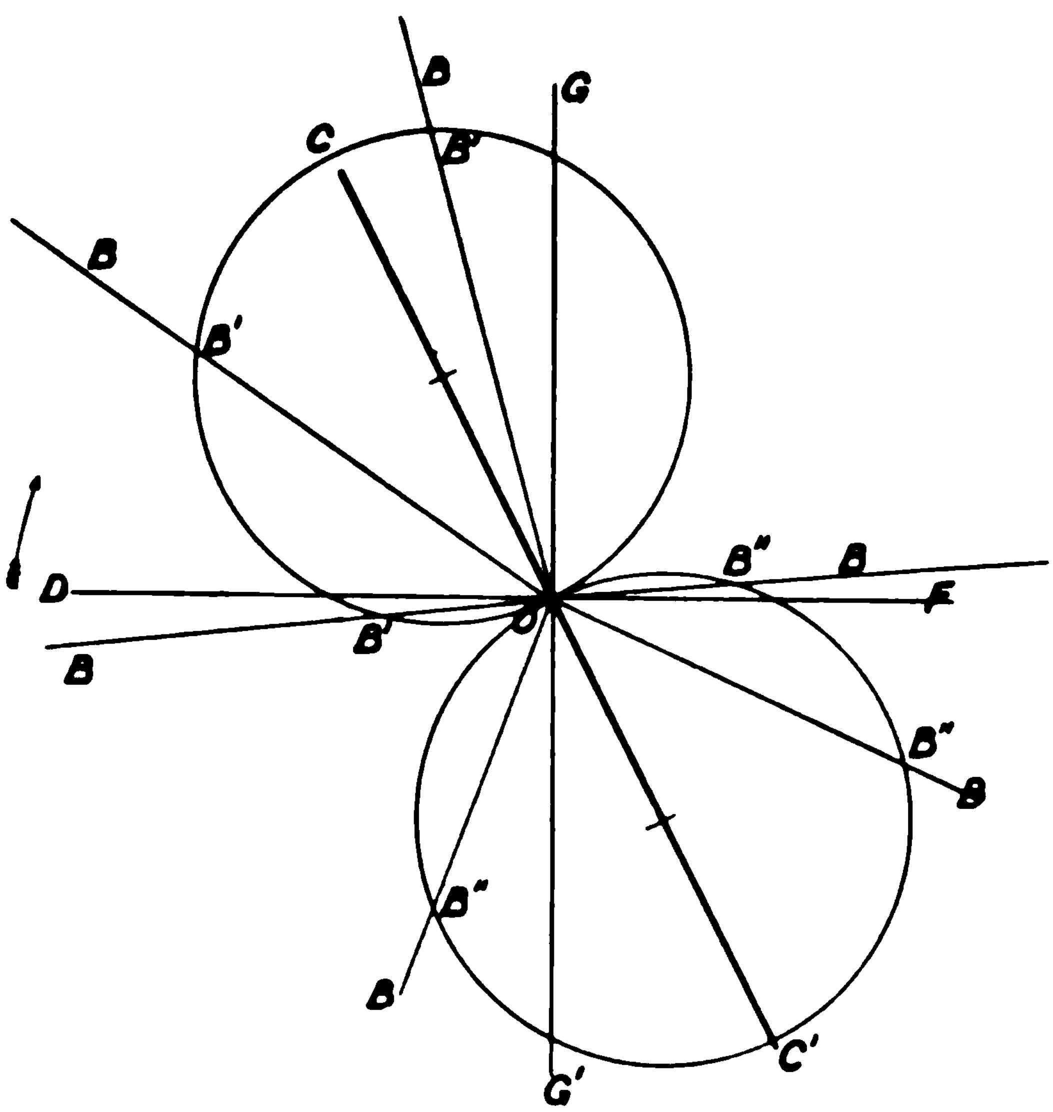



FIG. 260.

off by the circle $OC^1$ show the distance of the valve to
of its mid stroke. This method of study has already bee
upon. It is the one that I myself prefer in spite of t
that the angle of advance is always set out as if it were n
Should the velocity of the valve be wanted as an intercep

nk position, it is only necessary to draw two new circles whose meters are at right angles to $COC^1$.

4. In Fig. 261, $BM$ represents, the time of one revolution, from $B$ ich represents one dead point or $t = o$, to $M$ which represents the ne dead point again or $t = T$ the time in seconds of one olution.

A point $P$ shows by $PQ$ the distance $y$ of the valve to the right its mid-position at the time indicated by $BQ$. It is easy to see t the sine curve $EFPIKAE$ is drawn, just as the projection the spiral edge of a screw thread is drawn. Starting with $E^1$ $OE^1$ is the angle of advance) divide the circumference of the cle $E^1FPG$ into any number of equal parts numbering the points

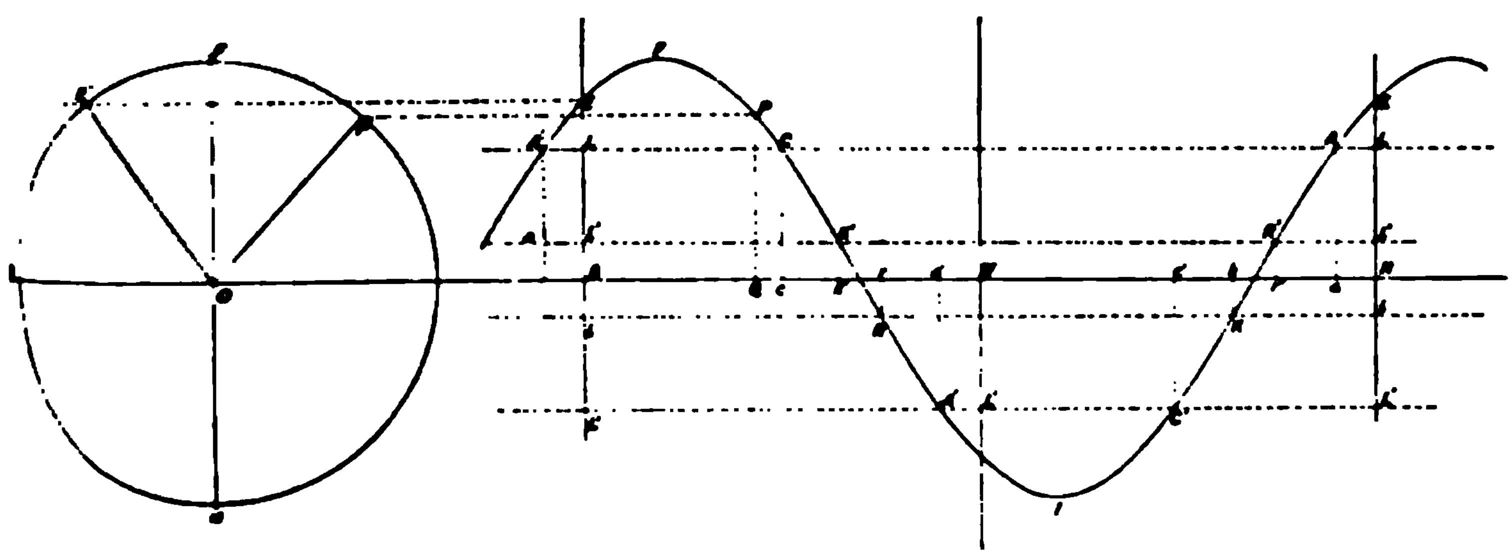


FIG. 261.

division 0, 1, 2, &c. Divide $BM$ into the same number of equal rts, and starting with $B$, number the points of divisions 0, 1, 2, &c., oject horizontally and vertically. Or again, it may be very ickly drawn on squared paper, using a table of sines of angles. It important to note $BE$ the distance of the valve from the middle the dead point; this is the lap + the lead, or $r \sin a$.

As in Art. 73 if from $y$ we subtract the lap $BL$ we get $LE$ the ning of the port to steam, drawing $LCAL$ parallel to $BM$ is the t way of making this subtraction and we see that $BC$ repre- t the time or the angle passed through by the crank when cut takes place. If $Bl$ is the inside lap and $ll$ is drawn, we $Br$, the angle passed through by the crank when release takes e and $Bk$ when cushioning takes place. $W$ bisects $BM$ and $W$ ws a dead point. If $BL^1$ and $Bl^1$ are the outside and inside laps the other side of the valve, we find $A^1$, $C^1$, $R^1$, and $K^1$ for the rn stroke.

The value of this method of study, which is really very clumsy n motions are all simple harmonic, lies in this, that it is almost

the only method of study, and is certainly the simplest method, when the motions are not simple harmonic—that is in practically every case in which the slide valve is worked from link motions or radial valve gear. My students have practised it, as shewn in Fig. 296, for twenty-four years, but as applied here, it is now published I think for the first time

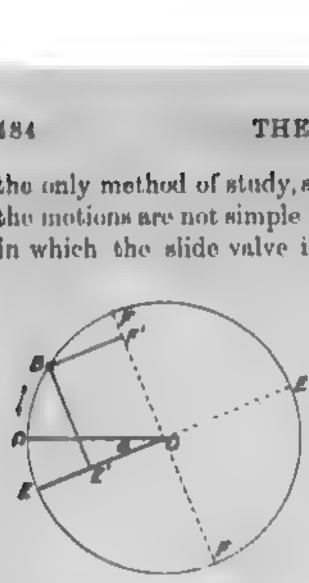


FIG. 262.

The curve showing the displacement of the valve is sometimes put upon the same sheet as a curve showing the position of the piston. Thus in Fig. 296 $Cc$ shows the distance of the valve from its mid stroke and $CE$ shows the distance of the piston from its mid stroke for any position of the crank. I myself prefer to compare valve position with that of the crank, and having found the crank positions when the four events occur, to use a template method of getting the piston position as in Fig. 93.

5. If a valve is worked from a crank shaft. Let $OD$ (Fig. 262) be a dead point position of the main crank. Make $DOE$ equal to the advance $a$ of the valve; then for any position $B$ of the main crank, draw perpendiculars to $EOE$ and to $FOF$, which is at right angles to $EOE$. The valve is at the distance $BE^1$ to the right of its mid position, and its velocity is represented by $BF^1$, and its acceleration is represented by $BE^1$. The scales of such measurements ought never to give any trouble. For example what is the scale of the displacement $BE^1$? It is evidently to the scale to which $OE$ represents the half travel. The scale of the velocity $BF^1$ is the scale to which $OE$ represents the greatest velocity. The scale of the acceleration $BE^1$ is the scale to which $OF$ represents the greatest acceleration, this is the same as the centripetal acceleration of the crank-pin which would give the slider its motion.

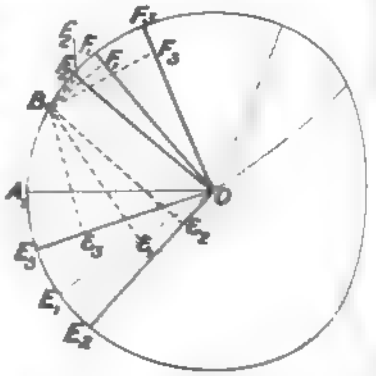


FIG. 263.

If a number of slides are worked from the same shaft with different half-travels and angles of advance [advance means angle

xceeding 90°, by which slider crank is ahead of main crank], the istance of each and all of them to the right of its mid stroke for ny position of the crank-pin, or any other rotating point of reference, ay be shown on one diagram. Let $OA_1$ (Fig. 263) be the dead oint position. Make $A_1OE_1$ the advance $a_1$ of one slider, make $_1OE_2$ the advance $a_2$ of the second slider, make $A_1OE_3$ the advance the third slider, and so on. Then for any position $B$ of the crank-n the sliders are at the distances $BE_1$, $BE_2$, $BE_3$, &c., to the right their mid positions. Furthermore if $OF_1$, $OF_2$, &c., are perpen-cular to $OE_1$, $OE_2$, &c., the perpendiculars $BF_1$, $BF_2$, &c., represent e velocities of the sliders.

6. Let points on $A_1OA_2$ (Fig. 264) represent the positions of the iston. Describe the circle $A_1GA_2G^1$. Let the main crank be in he position $OB$. Make $A_1OE^1$ the angle of advance, then if to

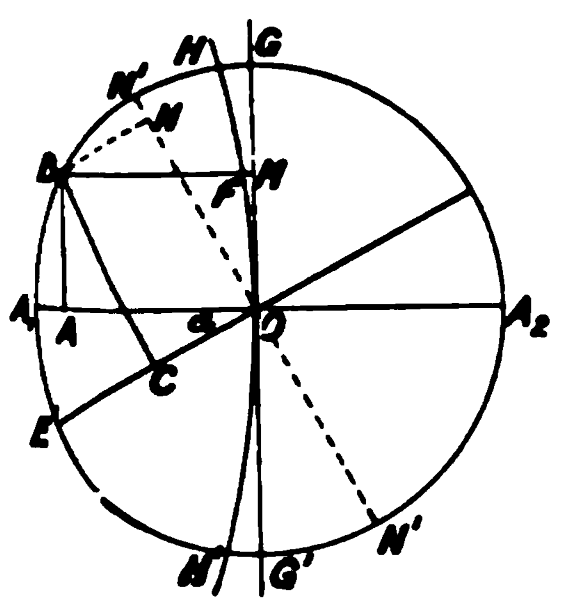



Fig. 264.

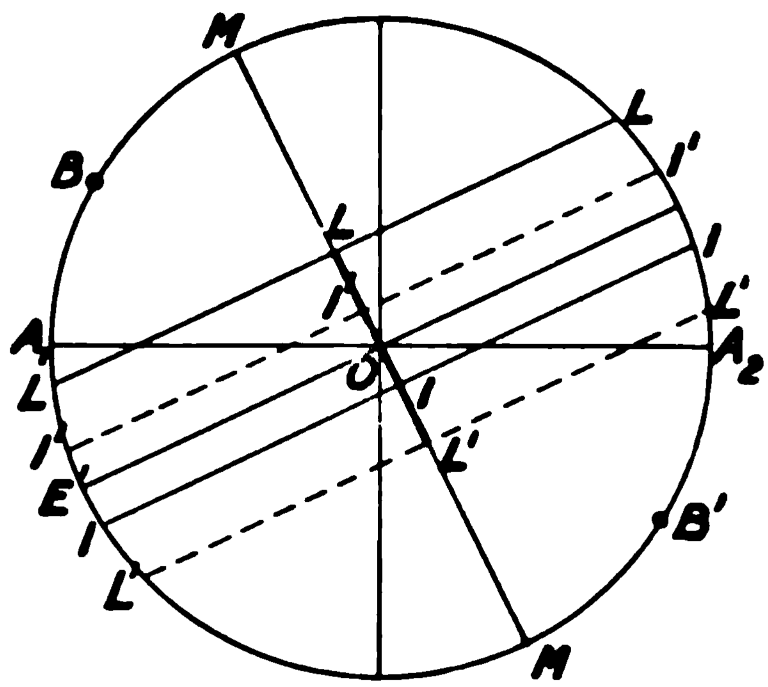



Fig. 265.

ne scale $A_1A_2$ is the travel of the piston, and if to another scale it epresents the amount of the travel of the valve; drop the per-endiculars $BA$ and $BC$, and $BM$, then when the piston is $OA$ or $BM$ rom the middle of its stroke; the valve is $BC$ from the middle of s stroke. This is evidently easy to prove. Also if $q$ is the angular elocity of the crank, the speed of the valve is $q.BN$, $BN$ being easured on the scale on which $ON^1$ is the half travel of the valve. f we wish to take into account the angularity of the connecting-d, we draw $HOH^1$, a circular arc with radius that of the con-ecting-rod, centre in the line of centres; then the distance of e piston from the middle of its stroke is not $BM$ but $BF$. Drawing nes (Fig. 265) parallel to $E^1O$ at distances from it, $OL$ = outside p; $OI$ = inside lap; $OL^1$ = outside lap on the other side of the lve, $OI^1$ = other inside lap; we see that the distances of any int like $B$ or $B^1$ from these lines show the amounts of opening

of the valve to steam and exhaust on the two sides of the piston. The positions of the main crank at admission, cut off, release and compression in both forward and back strokes, are evidently given by the ends of these lines, and the velocities of the valve when these events occur are presented by $\frac{1}{2}q$ times the lengths of these lines.

7. Let $A_1O$ (Fig. 266) be the line of centres. Make $A_1OE^1$ the advance; $OE^1 =$ the half travel. About $E^1$ describe circles whose radii are the outside and inside laps. Draw four tangents from $O$ to these circles. Prove that these tangents, produced if necessary, are the positions of the main crank when the four important events occur. $Oa^1$ admission, $OC$ (or $C^1O$ produced) when cut off occurs. $OR$ (or $r^1o$ produced) when release occurs, $OK$ when cushioning occurs. Of course the proof is easy as soon as one shows that the perpendicular distance of $E^1$ from any radial line drawn from $O$ is the valve displacement for that position of the crank.

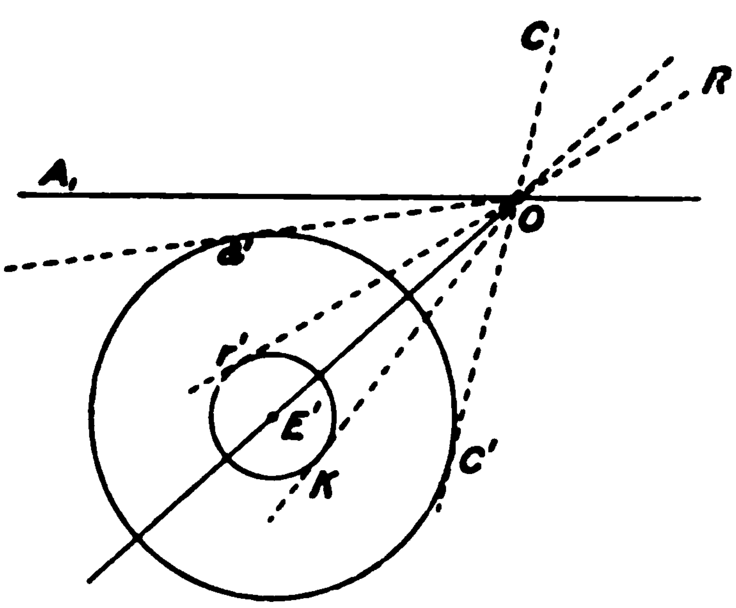


FIG. 266.

**298.** It is quite easy from what has been given, and using the methods either of 1, 2, 3, 4, 5, or 6, for any student accustomed to easy practical geometry to work such **problems** as:—

1. Given travel and port openings to steam for two positions of the main crank, to draw the hypothetical diagram.

A particular case of this is, given half travel, cut off and lead.

2. Given travel and ratios of amounts of port openings for three positions of the crank, draw the diagram.

3. Given travel, advance and ratio of lap to lead.

4. Given amounts of port opening for three positions of the crank. A special case of this is:—given the lead, the position of the crank at cut off, and the opening of the port in some other position of the crank.

5. Given the maximum opening of the port and given the openings for two given positions of the crank. A special case of this is:—Given the position of the crank at cut off, the lead, and the maximum opening.

**299.** On a diagram (Fig. 267) let a point $P$ show by its distance $PE$ from a line $B_1OB_2$ the distance of the piston from the middle of its stroke, and by its distance $PD$ from a line $O_1OO_2$ the distance of

valve to the right of its mid stroke. These distances need not o the same scale. If $O_1L$ and $O_1L^1$ are the laps (to the same : as the valve motion) on the two edges of the valve, and if $O_1l$ $O_1l^1$ are the two inside laps, the horizontal lines from these points the curve at admission, cut off, release and compression in for- l and backward strokes. If the valve has a simple harmonic motion

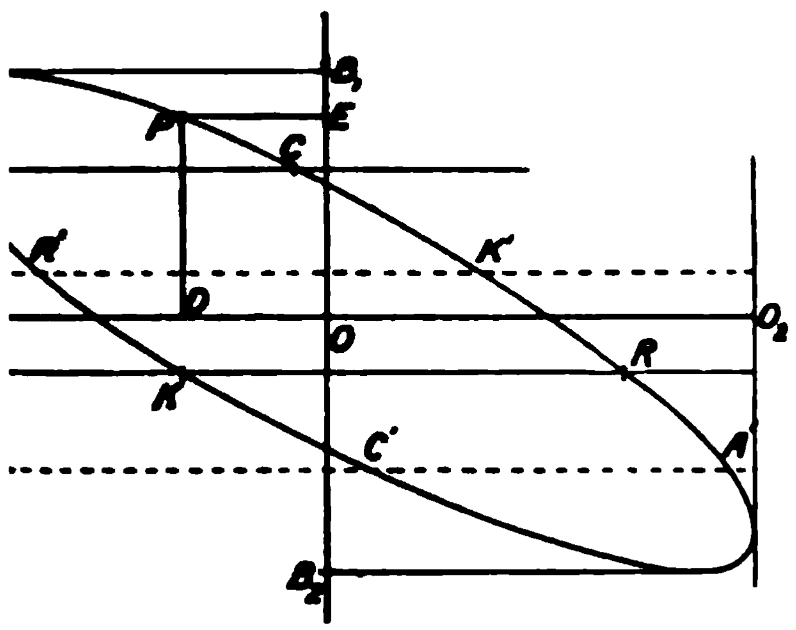


Fig. 267

DIFFERENT SCALES.

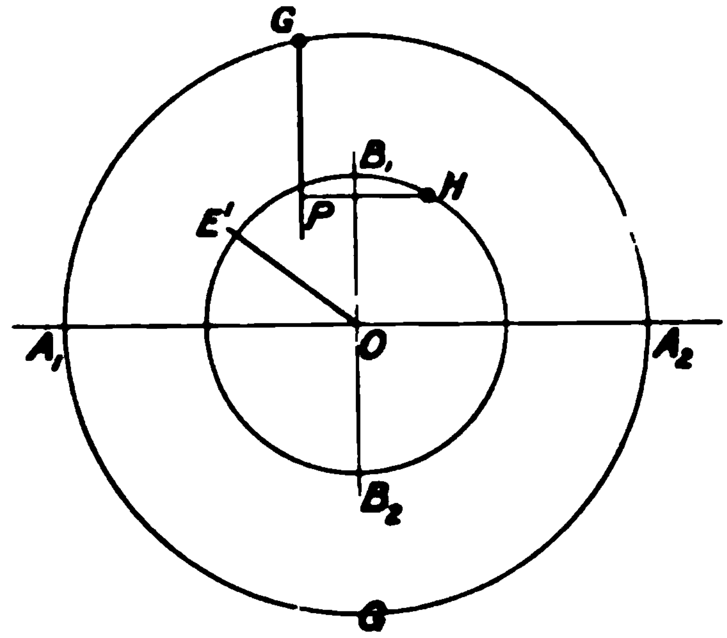


Fig. 268.

the piston also, the curve is evidently an ellipse, and the student do well to draw it by projection as in Fig. 268. Let $OB_1$ repre- the half travel of the valve, and let $A_1A_2$ to any other convenient e represent the travel of the piston. Describe the circles. Let angle $E^1OA_1$ be made equal to the advance. Divide both circles the same number of equal parts, and number the points of sion, beginning with $A_1$ and $E^1$ as 0, 1, 2, &c. Project vertically points on the larger circle, and horizontally from corresponding its on the smaller circle, and we evidently obtain the curve re-

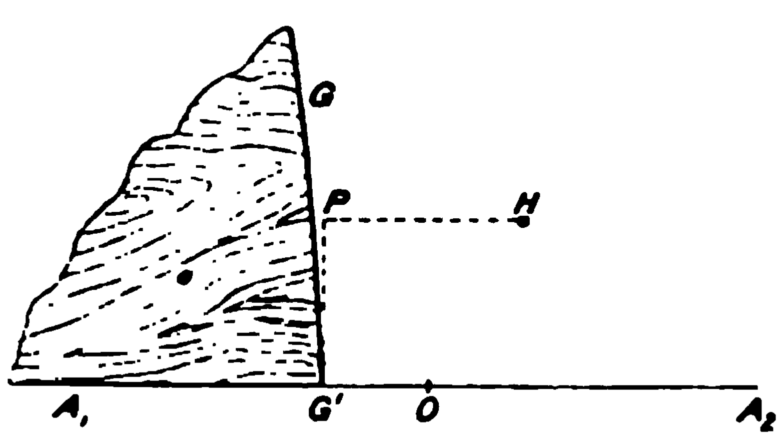


Fig. 269.

:d. Thus if $G$ is a point on the larger and $H$ on the smaller, $P$ point on the curve.

To take into account shortness of connecting rod, we proceed as e; but Fig. 269 shows how we use $G$ and $H$ to find $P$. We ct from $G$ to $A_1OA_2$ by our curved template of Art. 67 to find $G^1$,

and the vertical from $G^1$ meeting the horizontal from $H$ gives the true $P$.

Or again we may take the diagram Fig. 296, and in Fig. 297 we plot $Cc$ as ordinate and $CE$ as abscissa.

**300.** Mr. Macfarlane Gray's (or Müller's) method of showing the displacement of the piston for any position of the crank is interesting to look at, but is not easily applied in practice because of the great size of the drawings needed. Prove its correctness.

Let $AOF$ (Fig. 270) be the line of centres, the piston being on the side $A$. Let $G$ be the crank pin and $O$ the centre of the crank

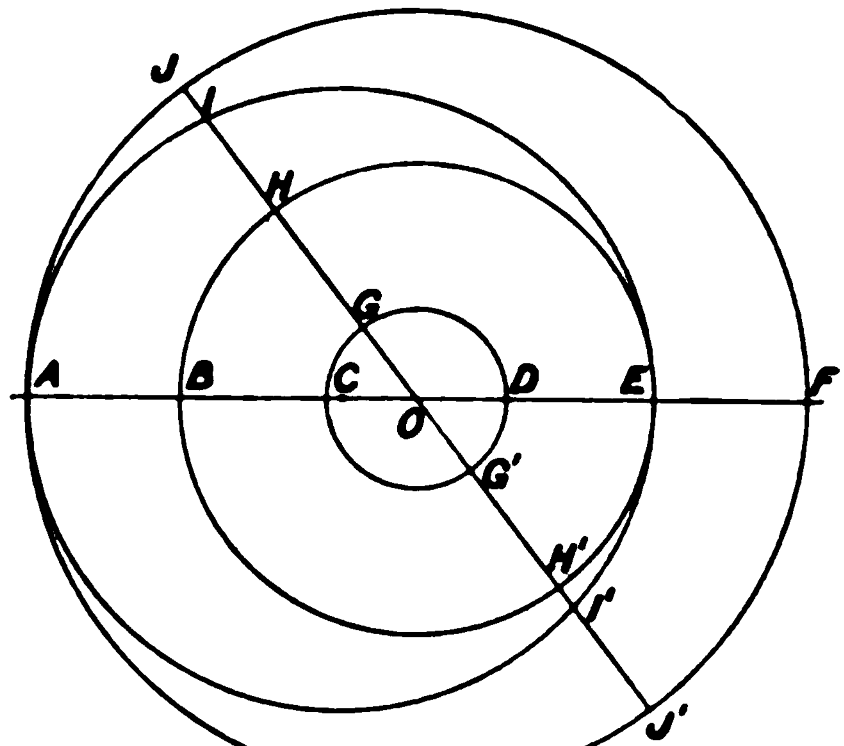



Fig. 270.

shaft, $OG = r$, $AC = l$, the length of connecting rod. Describe the circle $AJF$ about $O$ with $l + r$ as radius. Describe $BHE$ with $l-r$ as radius. On $AE$ as diameter describe $AIE$. Then for the position $G$ of the crank pin, the displacement of the piston from the left hand end of its stroke is $JI$; displacement from the right hand end being $HI$. For the position $G^1$ of the crank pin we have $H^1I^1$ the displacement of the piston from the right hand end of its stroke.

**301. Combinations of Motions.** All cranks or eccentrics working sliders are given as to position when we say that they have so much advance $a$, that is the amount in excess of 90°, by which they are ahead of the main crank; the half travel $r$ being also given. The motion is defined by

$$y = r \sin (\theta + a) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$y$ being distance of slider to right of mid position when main crank makes an angle $\theta$ with dead point. Suppose that one crank can give motion (1) and another crank can give

$$y^1 = r^1 \sin (\theta + a^1) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Suppose that a slider could get both these motions at the same time, what would the total motion be?

Draw the two cranks in their proper positions relatively to the main and of their proper lengths. Thus if $OD$ (Fig. 271) is the main crank and $MON$ is the direction of motion of the slider, draw $OF$ at right angles to $OD$. If $FOA = a$ and $OA = r$; if $FOB = a^1$ and $OB = r^1$, complete the parallelogram $OAEB$. Then $OE = R$ is the length of a crank and $FOE = \alpha$ is its angle of advance, which would give to the slider a motion which is the sum of the motions (1) and (2).

To prove this, drop perpendiculars from $A$, $B$ and $E$ on $ON$. The slider would be at the distance $OA^1$ to the right of its mid position

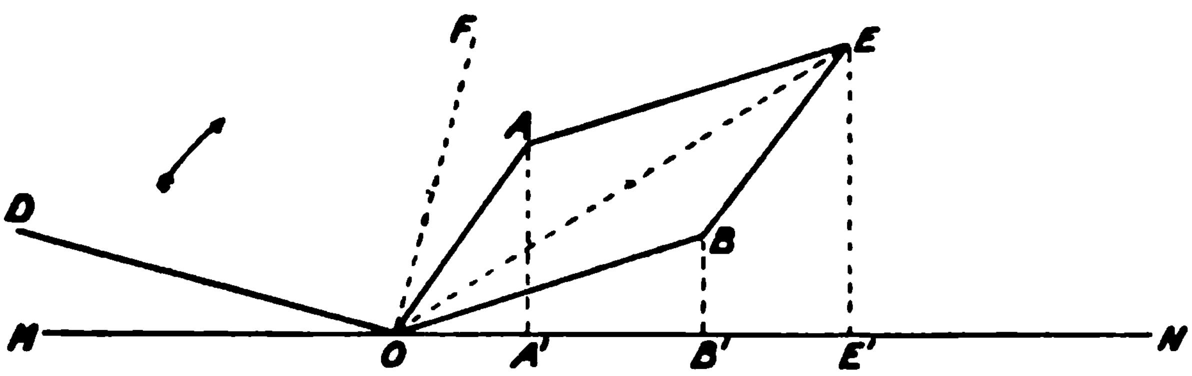



FIG. 271

if $OA$ alone worked it; it would be at the distance $OB^1$ to the right of its mid position if $OB$ alone worked it; it would be at the distance $OE^1$ to the right of its mid position if $OE$ alone worked it. But it is obvious that as $OA$, $OE$, and $OB$ are supposed to go round with the same angular velocity; in any position whatsoever $OE^1 = OA^1 + OB^1$, and hence the proposition is proved.[1]

In particular let the student notice that if $a = 0$ and $a^1 = 90$, (1) and (2) become

$$x = r \sin \theta, x^1 = r^1 \cos \theta$$

and

$$x + x^1 = \sqrt{r^2 + r'^2} \sin (\theta + \alpha)$$

where $\alpha$ is such that $\tan \alpha = \dfrac{r^1}{r}$.

He had better draw the figure that corresponds to this.

**Exercise.** Suppose that a slider gets the motion (1) and that another slider moving on or near the first gets the motion (2), what is the motion of the second slider **relatively** to the first? That is, suppose a fly to be on the first slider and not to know that it was in motion, looking at the second slider, what would the motion of this second slider *appear* to be?

[1] A student who knows a little trigonometry sees how to express $FOE$ and the length of $OE$ in terms of $a$, $a'$, $r$ and $r'$.

aw $OA$ and $OB$ (Fig. 272) as before. Make $OA$ the rallelogram $OBAE$ of which $OB$ is a side. Then $OE$

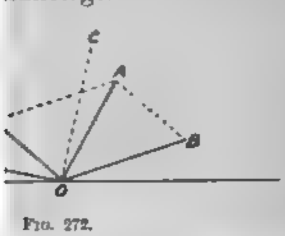



FIG. 272.

is a crank which would give to the second slider, if the first slider were at rest, the motion which the fly thinks that it has when both sliders are really in motion.

This proposition is ed in cases where one valve works on the back of another as . 150.

**402.** In what way is it p o give to a slider a combination two motions?

Lord Kelvin in his Tide ; Machine has shown us how to to an inkbottle a combin many simple harmonic motions various frequencies, am , and epochs. But he used a flexible connection. Usually the mechanical engineer requires a rather rigid connection.

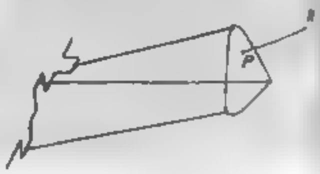



FIG. 273.

Let three cranks work, nearly at right angles to it, the three corners of a plate (Fig. 273). Then from any point $P$ in that plate a slider may be worked by a rod $PN$ which would get a combination of the three, depending on where the point is placed.


FIG. 274.

In this way by again combining such motions we can give a slider a combination of many crank motions.

**Use of a Link.** Usually we only want to combine two crank motions, and a link is commonly used.

If $A$ and $B$, Fig. 274, are two points getting small displacements (or velocities, or accelerations) $a$ and $b$ at right angles to $AB$, then the displacement (or velocity or acceleration) $c$ of $C$, a point in the same straight line is

$$c = \frac{BC}{AB}a + \frac{AC}{AB}.b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Notice the fraction of each of the two motions that $C$ gets and study the proposition well. It is quite easy to prove by drawing $AA^1 = a$ and $BB^1 = b$, and calculating $CC^1$ or $c$.

I shall here speak of the link as being vertical and its displacements as horizontal.

If in the plane of the paper the end $A$, Fig. 275, of the straight link $AB$ gets a horizontal S. H. motion from the crank $OA'$, whose centre is on the level of $A$ and the end $B$ gets a S. H. motion from the crank $OB'$, whose centre is on the level of $B$ and which goes round with the same angular velocity, then any point $C$ in the link gets a motion which is just the same as if it were worked by an ideal crank rotating at the same angular velocity about a centre $O$ on the level of $C$. To find this ideal crank: Let $OA''$ and $OB''$ show the relative angular positions of the given cranks at any instant, and let $OA''$ and $OB''$ represent their lengths to scale. Join $A''B''$. Divide $A''B''$ in $C''$ so that $A''C'':C''B''$ as $AC:CB$. Join $OC''$. Then $OC''$ is an ideal crank of the proper length and properly related angularly to the given

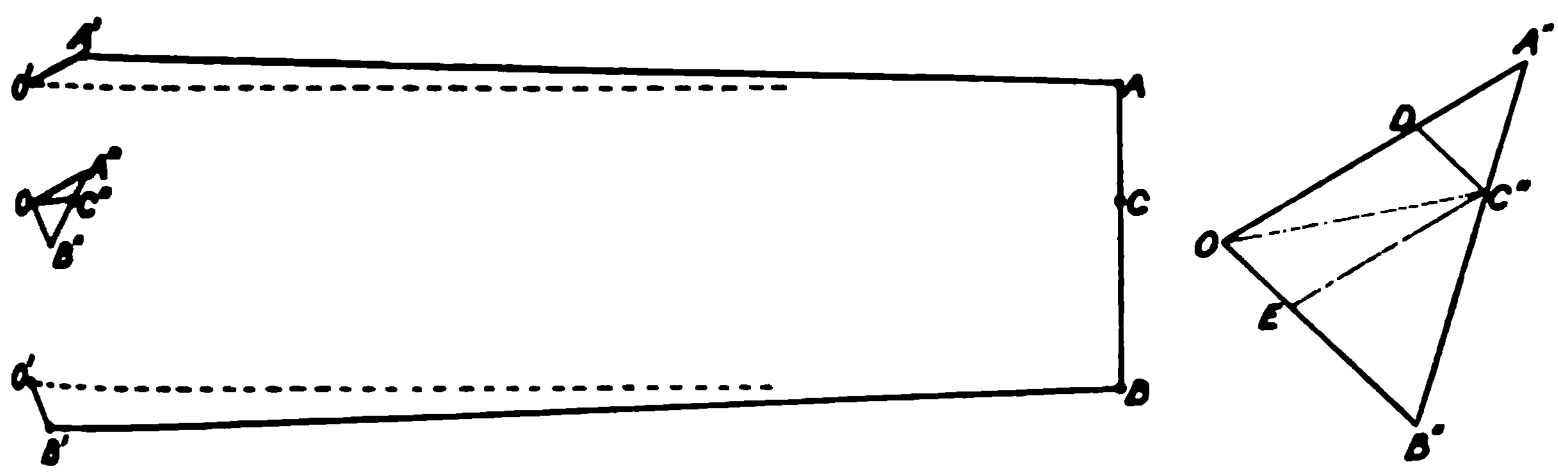



FIG. 275.

cranks to give to a point $C$ exactly the same motion which it gets in quite a different fashion. $C$ really gets its motion because it is in a link $ACB$; but the crank $OC''$ would give it that motion if it were not constrained by the motions of $A$ and $B$.

To prove this. Draw $C''D$ and $C''E$ parallel to $B''O$ and $A''O$. Magnify the figure as shown here. $C$ has a fraction of $A$'s motion; the fraction $\frac{BC}{AB}$. Now $A$ is moved by a crank like $OA''$. Hence $C$ would get its proper fraction of $A$'s motion if it were moved by a crank $\frac{BC}{AB} \cdot OA''$. But $A''C''B''$ is divided proportionately to $ACB$, and we see that $OD$ is just the crank which would give to $C$ its proper fraction of $A$'s motion. Similarly $OE$ is just the crank which would give to $C$ its proper fraction of $B$'s motion, and it is evident from Art. 301 that the ideal crank $OC''$ would just give the sum of the motions which $OD$ and $OE$ would give.

We always assume the motions to be simple harmonic and to be very small and at right angles to the length of the link,

as our rules are not exactly true when these conditions are not fulfilled.

Instead of speaking of the length of the ideal crank we say "the half travel," and in a steam engine the angle minus 90° that the ideal crank makes with the dead point position on the side remote from the cylinder is called the "advance." These terms are used because we use them in Art. 74.

Fig. 276.

EXERCISE 1. In a link the point A gets a half travel a and an advance α, the point B gets a half travel b and an advance β, what are the half travel and advance of C?

*Answer.* Draw O [illegible] represent the main crank and O [illegible] at right angles to it. [illegible] and $OA' = a$. Make $GOB' = \beta$ and $OB' = b$. Join $A'B'$ [illegible] it in $C'$ as the link $AB$ is divided in C; the half travel of C [illegible] its angle of advance is $GOC'$.

EXERCISE 2. Suppose the half travel of A is 3 inches and of B is 2 inches, suppose the advance of A to be o and the advance of B to be 90 degrees, find the half travel and advance of C if C is shifted along from A towards B. 1st, when C is at A; 2nd, when $AC = \frac{1}{4}AB$; 3rd, when $AC = \frac{1}{2}AB$; 4th, when $AC = \frac{3}{4}AB$; 5th, when C is at B.

EXERCISE 3. Suppose the half travel of A to be 3" and its advance 0, the advance of B to be 90° and its half travel to be altered from 0 to 1 to 2 to 3 to 4 inches; find the half travel and advance of C in each case, if C is always midway between A and B.

It is evident that if C is not in the straight line joining A and B but is with A and B in a flat arc of a circle the relative distances of C in the arc from A and B may be taken pretty much in the same way as in the straight line.

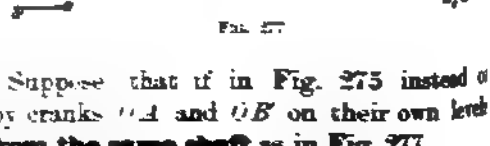


Fig. 277.

**303.** EXERCISE 4. Suppose that if in Fig. 275 instead of A and B being worked by cranks OA and OB' on their own levels they are **both worked from the same shaft** as in Fig. 277.

Find by skeleton drawing or in **any other way the limits of A's motion, say A'' and A'''.** Now note that if a crank **O'a' on the same**

level as $A$ worked it in the direction $O'A$, $A$ would be at $A''$ when $O'a'$ or $O'a''$ is horizontal; but truly $Oa$ would then be in the direction $OA''$ as shown in the dotted line; in fact $O'a'$ must be ahead of $Oa$ by an angle equal to $AOO$ and its length must be half of $A''A'''$. On looking into the matter more carefully it will be seen that the angle ought to be something between $A''OO$ and $A'''OO$. I usually take $OA_0$ the length of the rod $aA$ to find the point $A_0$ and I take the angle $A_0OO$ to be the correct amount by which $Oa'$ is ahead of $Oa$. Also, I think that it will be easily seen that the length of $Oa'$ is best obtained, not by skeleton drawing and finding the positions of $A''A'''$, &c., but by making $aOa = A_0OO$, and drawing $aa$ at right angles to $Oa$; thus $Oa$ is found parallel and equal to $O'a'$. In the same way we find $O\beta$ a virtual crank which on the level of $B$ would give it the motion which it gets from $Ob$. Now of course if we join $a\beta$ and divide in $\gamma$ as the link $AB$ is divided in $C$ we get $O\gamma$ a virtual

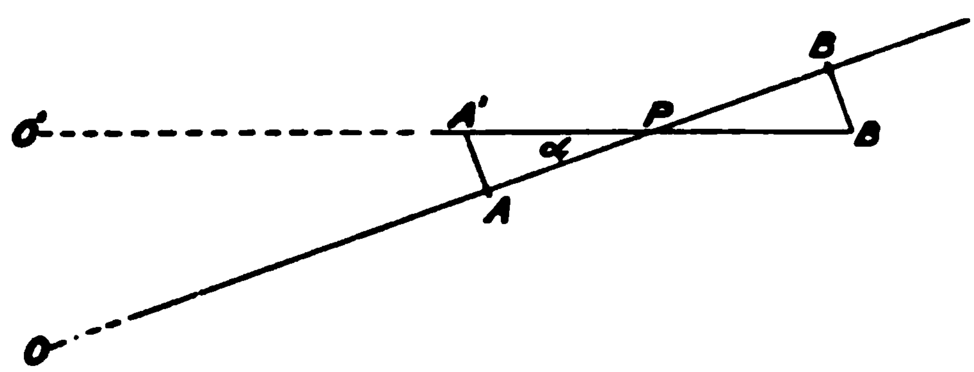



FIG. 278.

crank which if on the level of $C$ would give to $C$ the motion that it really gets.

**304.** Although the above way of putting this matter will commend itself to the practical man, the following **more mathematical method** may also be welcome. In any case, however, we are compelled to leave out small terms and to an experienced man the above method is just as good as this that follows.

A crank $Os = r$ works a slider $Q$ in the direction $QC$, what is the nature of the motion of $Q$, Fig. 279?

Perhaps it is more simply seen in Fig. 278. $P$ is the middle of $Q$'s path $A'B'$. If the motion were in the direction $BPAO$, in the line of $O$, $PA = PB = r$. By drawing $BB'$ and $AA'$ perpendicular to $OAB$ we have the ends of the path of the slider, if the rod were infinitely long. Now if a crank at $O'$ gave the motion $A'PB'$, it would be of length $PA' = PB' = r \sec a$ and when the $O$ crank was directed towards $OB'$ (the same as $OP$ if rod is very long) the $O'$ crank would be directed in the line $O'P$. In fact the $O'$ crank would be $a$ ahead of the $O$ crank.

Let $OC$, Fig. 279, be at right angles to $CPQ$.
Let $PQ=x$. Let $OS=r$, $SQ=l$, $COS=\theta$, $OP=l$, $OC=a$.
Let $CP=k=\sqrt{l^2-a^2}$.

$$\left.\begin{array}{l} r\sin\theta+l\cos\phi=k+x \\ r\cos\theta+l\sin\phi=a \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1$$

$l\sin\phi=a-r\cos\theta$, therefore $l\cos\phi=\sqrt{l^2-a^2+2ar\cos\theta-r^2\cos^2\theta}$.

$$l\cos\phi=k\sqrt{1+\frac{2ar\cos\theta-r^2\cos^2\theta}{k^2}}.$$

Treat $\frac{1}{k^2}(2ar\cos\theta-r^2\cos^2\theta)$ as a small quantity; that is, like $\sigma$ where $\sqrt{1+\sigma}=1+\frac{1}{2}\sigma$ very nearly.

Having done this, use $2\cos^2\theta=\cos 2\theta+1$, and we find

$$l\cos\phi=k+\frac{ar\cos\theta}{k}-\frac{r^2}{4k}-\frac{r^2}{4k}\cos 2\theta.$$

Hence

$$x=-\frac{r^2}{4k}+r\sin\theta+\frac{ar}{k}\cos\theta-\frac{r^2}{4k}\cos 2\theta \quad . \quad . \quad . \quad . \quad (2).$$

Now $r\sin\theta+\frac{ar}{k}\cos\theta=r\sqrt{1+\frac{a^2}{k^2}}\sin\left(\theta+\tan^{-1}\frac{a}{k}\right)$,

and the meaning of this is :—

Set off $OT$ ahead of $OS$ by the angle $SOT=OPC$; make $ST$ perpendicular to $OS$; then the motion of $Q$ is what a crank of length $OT$, and parallel to $OT$

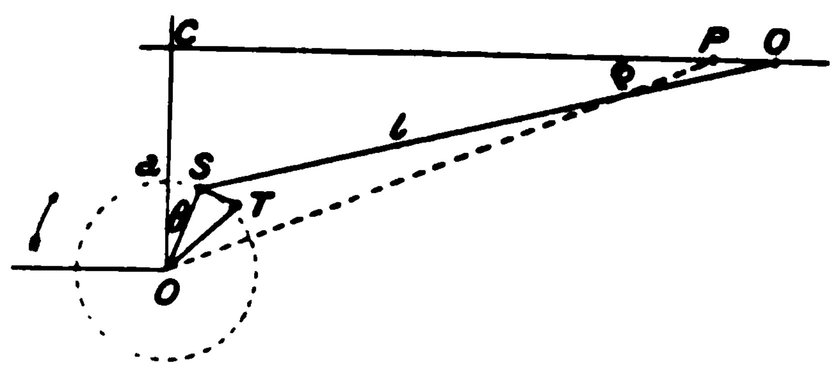



FIG. 279

with an infinitely long connecting rod, would give it, if its centre were on the level of $PC$; but there is also an octave of the amount $-\frac{r^2}{4k}\cos 2\theta$. It will be seen that this octave is exactly in the same proportion to its fundamental as if the crank $r$ with a rod $k$ worked a slider directed towards $O$ in its motion.

**305.** Now notice that if instead of $Q$ having a horizontal motion from $Q$ to $Z$, Fig. 280, it moves from $Q$ to $R$ or $Q$ to $N$; $QM$ being in the direction of the centre of the shaft. Instead of the direct displacement $QM$ we have (drawing $MNRZ$ at right angles to $QM$ and $RS$ and $NT$ vertical) displacements $QN$ or $QR$, instead of $QZ$, and the horizontal components $QV$ or $QT$, or $QS$ or $QZ$ are by no means equal. Hence the horizontal motion of any point in a link depends upon the direction of its path and this depends upon its **method of suspension**. Given the horizontal motions of the ends we easily find the horizontal motion of any point in the link, but we have no easy rule

as yet for finding the horizontal motions of the ends for every method of suspension. There is another important matter. The valve is not worked from an invariable point $C$ in a link whose distances from the ends $A$ and $B$ are constant. It will be shown in Art. 329 that the

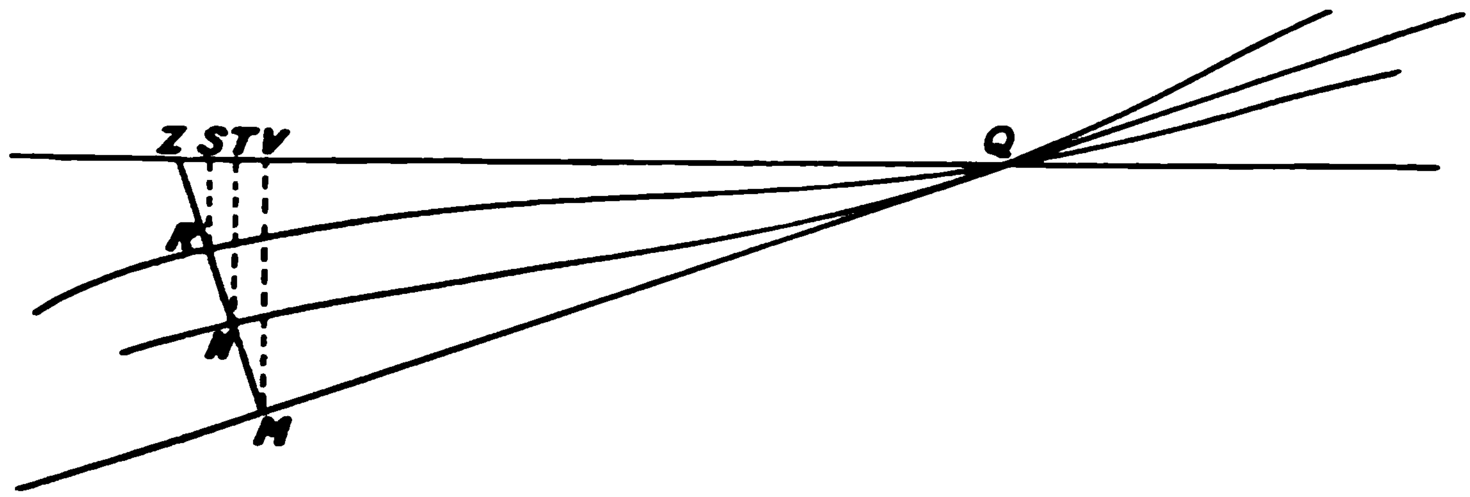



Fig. 280.

complication introduced by the **sliding of the block** in the link is not such a serious matter as the above one.

I hope that students will bear in mind the above considerations when studying the following rough and ready rules (the only ones known) for link motions in general, so that they may not imagine these rules to be more accurate than they really are.

**306.** Fig. 281 shows the **Gooch link motion.** If the forward $Oa$ and backward $Ob$ eccentric cranks are each 3 inches and each

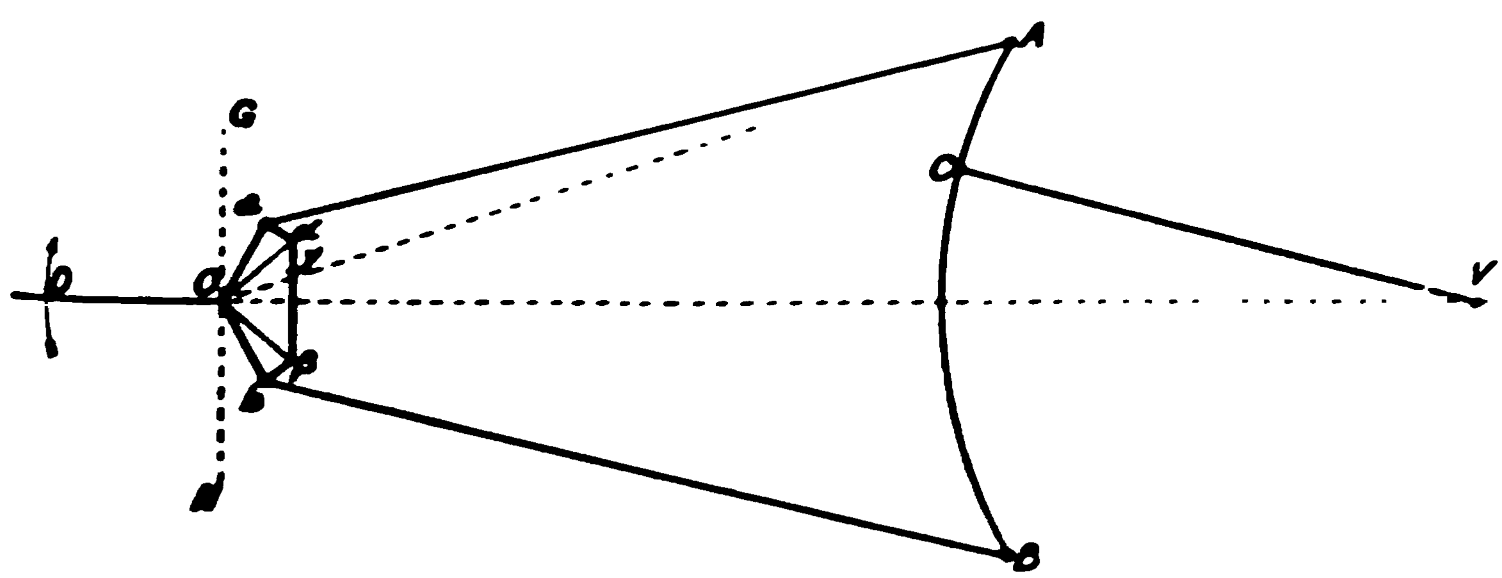



Fig. 281

has 30 degrees of advance, that is each is 120 degrees from the main crank. Each of them is said to have 30 degrees of advance $GOa$ or $HOb$, because in full forward gear the valve may be said to be worked by the forward eccentric alone and in full back gear the valve may be said to be worked by the back eccentric alone, the engine going the reverse way.

**Draw the gear in** ***dead point position*** with the crank away from

the cylinder. In this position see if the rods $aA$ and $bB$ are crossed or open. The figure shows open eccentric rods. Imagine the link $AB$ to be so supported and the rods to be so long relatively to the link that $A$ and $B$ move horizontally; $A$ gets motion from the eccentric $Oa$; $B$ gets motion from $Ob$; the block $C$ may be raised or lowered. It works the valve rod $V$ in the direction $OV$ and indeed

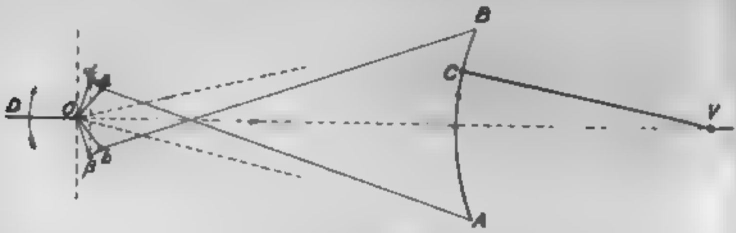



FIG. 282.

$V$ gets the horizontal motion of $C$ if the radius rod $CV$ is held at a constant slope by the other parts of the gear.

$Oa = Ob = 3$ inches; $GOa = HOb = 30°$ $A\alpha$ is perpendicular to $Oa$; $aO\alpha$ is made equal to $AOV$. $b\beta$ is found in the same way. Join $\alpha\beta$ and divide in $\gamma$ in the proportion in which $AB$ is divided by $C$, then $O\gamma$ is the half travel of the valve in the present position of the gear and $GO\gamma$ is the angle of advance.

Notice that if the gear is shifted $C$ being altered in position, our figure $\alpha O\beta$ is not altered; we only alter the position of $\gamma$.

In Gooch gear with crossed rods, see Fig. 282, the student will find that $O\alpha$ is now *behind* $Oa$ by the amount $AOV$. There is no difficulty in seeing the reason for the rule used in the following example.

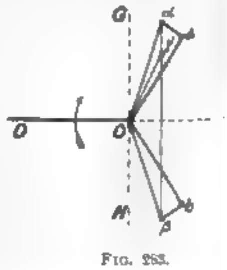



FIG. 283.

Let full fore and back half travels each be 3″, advances 40 degrees. Draw the gear *in the dead point position*. Set off $oa = ob = 3''$, $Goa = Hob = 40°$. Make $aoa = bo\beta = AoV = BoV$. Draw $a\alpha$ and $b\beta$ at right angles to $oa$ and $ob$ and so get $\alpha$ and $\beta$. Join them and divide $\alpha\beta$ in $\gamma$ the proportion in which $C$ divides the link. Then $o\gamma$ is the half travel and $Go\gamma$ is the advance of the valve.

**307. Stephenson Link Motion.** Show it in dead point position and at mid gear.

I. *Open eccentric rods.* EXAMPLE, Fig. 284. Forward eccentric half travel 3″, advance 30°. Same for back. Make $oa = ob = 3''$, $Goa = Hob$

. In full forward or backward gear when $A$ is lowered to $C$ or $B$
d to $C$ the half travel and advance are really as represented by
l $ob$. Let the student satisfy himself that this is so [and that it
be so also if the rods were crossed].
is only then in intermediate positions, say half gear or mid
hat we have to use our rule of Art. 303, which is of course a

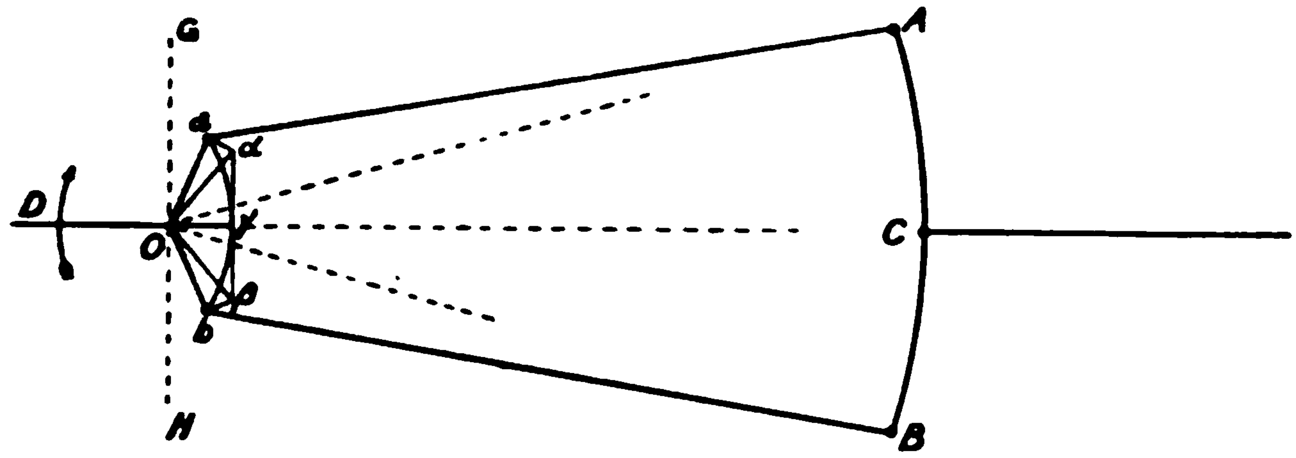



FIG. 284.

tedious. Now we find it necessary to work out an answer
ly only in one intermediate position, say the mid gear position as
Our rule of Art. 303 comes evidently to this. Make $aoa = bo\beta$
$C$. Draw $aa$ and $b\beta$ at right angles to $oa$ and $ob$. Join $a\beta$ and
in $\gamma$. Then we have the following:

*swers.* Full forward gear $oa = \frac{1}{2}$ travel, $Goa$ = angle of advance
Full back gear $ob = \frac{1}{2}$ travel, $Hob$ = angle of advance.
Mid gear $o\gamma = \frac{1}{2}$ travel, 90° angle of advance.

aw a curve, an arc of a circle say, through $a\gamma b$ and *assume* what
e nearly true, that if this *arc* $ab$ be divided at any time in a
$c$ in the proportion in which the link $AB$ is divided by the
? then $oc$ is the half travel and $Goc$ is the angle of advance.
Work out in the same way the same example but with crossed
ric rods.
both cases test the following rule invented by Mr. Macfarlane
We have given $oa$ and $ob$, Fig. 284; join $a$ and $b$ by an arc of a
whose radius is—

$$\frac{ab \times aA}{2AB}$$

ng the straight distance from $a$ to $b$. If the rods are open, the
concave to $o$. If the rods are crossed the arc is convex to $o$.
m not sure that this or any other so-called easy rule is really
than the above correct rule.
t the student beware of refining too much on any of these
actions. He ought to remember that they are all approximate.

ten to talk learnedly about whether the arcs through
e above figures are arcs of circles or arcs of parabolas
.... dealing with rules of infinite exactness.

**n Link.** This is more troublesome than either of the
the gear at dead point position, and in full forward gear
each case by our rule of Art. 303 $o\alpha$, Fig. 285, the half travel
$a$ the angle of advance. In the same way find $o\beta$ for full

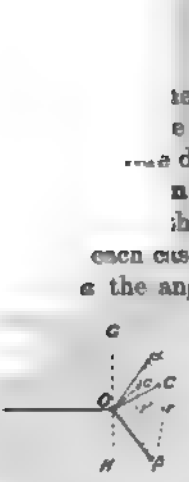


Fig. 285.

back gear and $o\gamma$ or $o\gamma^1$ for mid gear. The rule
has to be followed out in each case but having so
we know $o\beta$ unless the link is an unsymmetrical
one. $o\gamma$ is the sort of mid gear answer one obtains
if the rod... ...pen and $o\gamma^1$ if crossed. Now
draw an a... ...rcle through $a\gamma\beta$ or $a\gamma^1\beta$ and
for any i... ...te position divide the arc as
the block divides the ... ...$oc$ is the half travel of the
e and $Goc$ is the a... ...ce.

... Exercises of Art. ... every young student ought to
will have taught what ... hange of distribution of steam
nay expect by shifti... ...above link motions. The man
studies his results b... ...of Art. 80 will possess what is
sometimes thought to be very ad...ced knowledge of this subject.

**309. General Remarks on Link Motions.** Any arc of the link in the Stephenson motion has as its radius the distance to the centre of the eccentric disc. Any arc in the Gooch link has as radius its distance to the worked pin on the valve rod. There is a geometrical construction to find exactly the positions of the points $G$, $K$ and $I$, Fig. 118, in the Allan Link. In all valve gears there is some such essential rule. Such rules need not be remembered: any man who can reason a little will work out such rules for himself if he will only remember the reason for them, which is this: When a gear is shifted there is one thing that must remain nearly unaltered—namely, the position of the valve at mid travel.

I do not think that anybody has yet given attention to the possibility of getting a good result by arranging that when the gear is shifted the valve's mid travel shall not be a symmetrical position over the ports, and yet careless constructors of gears sometimes, accidentally, benefit by the method. This is only one of many interesting problems which ought to be worked out with simple models.

The rules which I have given will only enable us to state rather roughly what occurs when the gear is shifted. We use them to help in designing the gear, and then we make a model or a skeleton drawing to show what the exact motion of the valve is. The following is a

drawing office method of making a **skeleton drawing.** It is desired to draw the motion of the valve full size or two or three times full size without showing the crank shaft and drawing arcs with radii so great as the eccentric rod lengths. Let $ED$, Fig. 286, be a position of the reversing link in which we wish to study the motion of $V$ on the valve rod $VR$: we really move the main crank into say 24 positions and for each of these we find the position of the link and, therefore, the horizontal displacement of $V$. This means that having the arc $DD'$ with $E$ as centre, for each position of $a$ and $b$ ($oC$ is the main crank, $oa$ and $ob$ the forward and back eccentrics) we strike out the two arcs $AA'$, $BB'$ with $a$ and $b$ as centres and the lengths of the eccentric rods as radii, and having these three arcs $AA'$, $BB'$, $DD'$, we

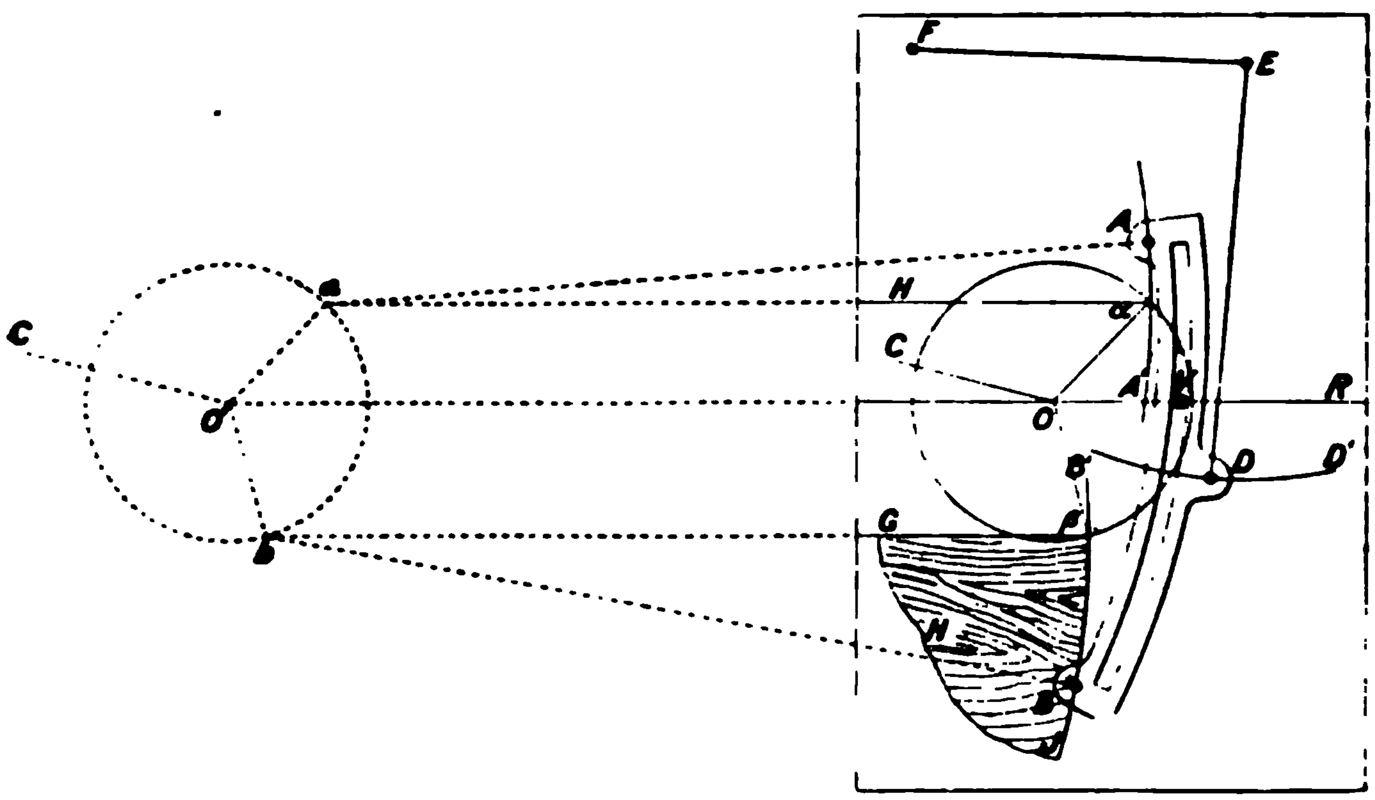



FIG. 286.

get a tracing of the link and place it on our drawing so that the points $A$ and $B$ and $D$ are on the three arcs, and now we mark the position of $V$. The dotted part of Fig. 286 may be dispensed with. We draw $oC$, $oa$, $o\beta$ as if $o$ were the crank shaft centre; $oC$ the main crank and $oa$ and $o\beta$ the two eccentrics in any one of say 24 positions. Now prepare a template like $GHJ\beta$ with an arc $\beta J$ drawn with a radius $bB$ or $aA$ and with a line on it $G\beta$ normal to the arc at $\beta$. Apply this template so that the point $\beta$ on it is at $\beta$ or $a$ on the drawing, and $G\beta$ is horizontal, and evidently the arc $B'BJ$ or $AA'$ may be drawn.

I may say that I do not like the method unless a better kind of template is prepared, because there is always great chance of error when we are asked to draw a line from a point that must be the corner of a set square or template.

**310.** We shall see in Art. 327 that the sliding motion of the block

in the link slot does not practically introduce any octave into the valve motion and therefore does not tend to produce inequality of distribution on the two sides of the piston. Although, therefore, its study is of no great importance in connection with steam distribution, it is important to try to diminish the sliding on account of wear and tear. The student ought to make skeleton drawings showing the actual motions during the revolution of an engine, of points at the ends and middle and half way between the ends and middle; 1st, when the link is suspended at the middle of the slot; 2nd, at the usual point near the middle; 3rd, at one end; 4th, half way between middle and one end. He will find that the first is best for engines expected to go equally well in both directions, and the 4th is best for an engine which is mainly expected to go in one direction. In fact the point of suspension ought to be near the position in which the link block most commonly works. This is what is usually said. It is on the

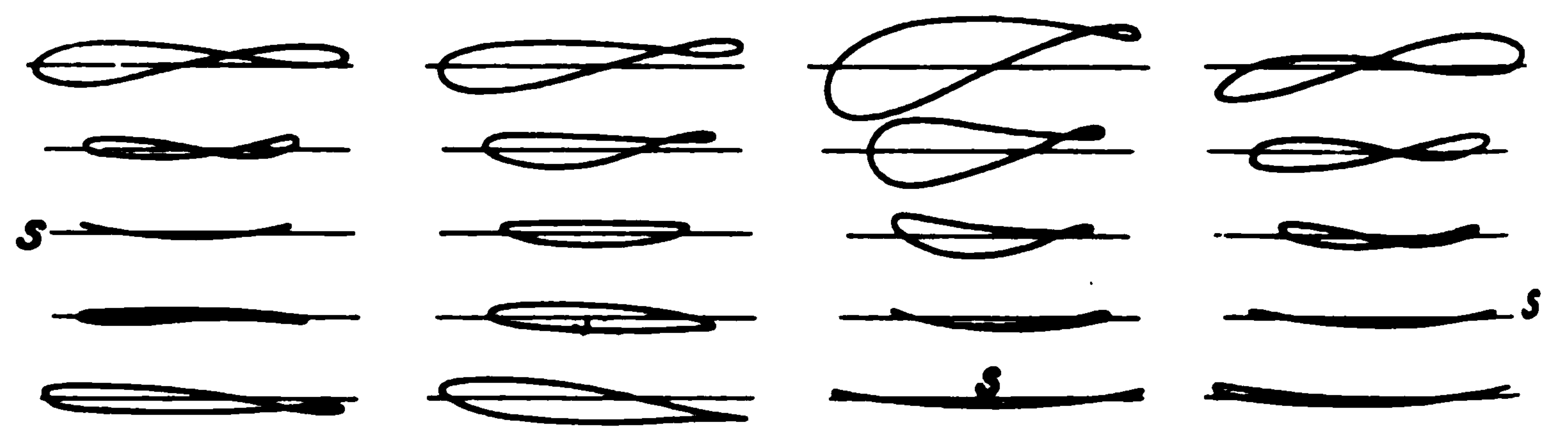



FIG. 287.

assumption that we want the valve to have a nearly pure simple harmonic motion, but I am not sure that this is what we ought to aim at.

Students must make their own drawings, but Fig. 287 illustrates my meaning. The paths of the suspension point $S$ and four other points are shown for each method of suspension of a Stephenson link with open rods. In the second set the suspension point is really the middle of the chord of the link.

It is easy to draw the link when in mid gear in its positions for the two dead points. The line bisecting at right angles the distance between these two positions of the suspension pin is the **average position** of the suspension (or reversing) link for all positions of the gear, so that it is easy to get the best arrangement of $G$, Fig. 117, Stephenson, or of $C$, Fig. 119, Gooch. I am afraid that I do not believe much in giving more exact rules than these, nor do I believe in the mathematics with which we sometimes try to disguise the fact that we are trying to get rid of octaves in the motion, and these octaves

may be **very useful helps** and ought to be cultivated in the right direction rather than destroyed.

Exercise. Show that with the Stephenson link if the engine runs only one way we can greatly equalise the lead for different amounts of expansion by using unsymmetrical eccentrics. I give this exercise because it is a common exercise for students and will do no harm, but I cannot see why people should be anxious to effect this object.

**311. Radial Valve Gear.** In link motions and radial valve gear we have a link $AB$ whose average position is at right angles to the direction of motion of the valve. The great difference between them is this; in link motions $A$ and $B$ get motions in the direction of the valve rod whose half travels are generally the same, their angles of advance being acute angles and equal for the two directions of motion; the point $C$ which works the valve, shifting relatively to $A$ and $B$ when the gear is shifted; whereas in radial valve gears the advance of $A$ is either $\pm$ 90°; that is, $A$ is in $+$ or $-$ synchronism

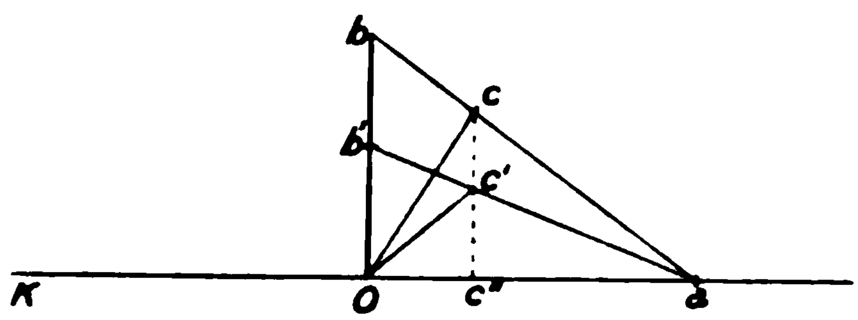



Fig. 288.

with the crank pin; $B$ has an advance 0 but a variable half travel; the point $C$ which works the valve never alters its position relatively to $A$ and $B$.

In link motions angles must be carefully measured before we can even roughly approximate to the conditions, whereas in radial valve gears by an easy inspection we see the half travels, we know that the advances are 90° or 0, and we have no difficulty in making a rough approximation to the motion of the valve.

Thus for example, let $C$ be between $A$ and $B$. Then we know that $A$ will have 90° advance, $B$ has no advance.

Let $KOa$, Fig. 288, be the line of centres; let $Oa$ be the half travel of $A$ with its 90° advance; let $Ob$ be the half travel of $B$ with its no advance. Join $ba$ and divide $ba$ in $c$ in the proportion in which $C$ divides the link $BA$. Join $Oc$; then $Oc$ is the half travel of the valve and $boc$ is the angle of advance.

Let the gear be shifted so that $Ob'$ is the half travel of $B$. Nothing else is altered. Join $b'a$ and divide as before in $C'$. Then

$Oc'$ is the half travel of the valve and $boc'$ is the angle of advance. Evidently all points like $c$ $c'$, &c., lie in a line at right angles to $OA$.

It is evident by drawing Zeuner Circles that in all radial valve gears, as in the Gooch link, the lead is constant.

**312.** I have given the general definition of all radial gears. There are several forms of radial valve gear which satisfy this definition:—

"There is a link $AB$, Fig. 289, whose average position is at right angles to the valve rod; $A$ describes a closed curve more or less

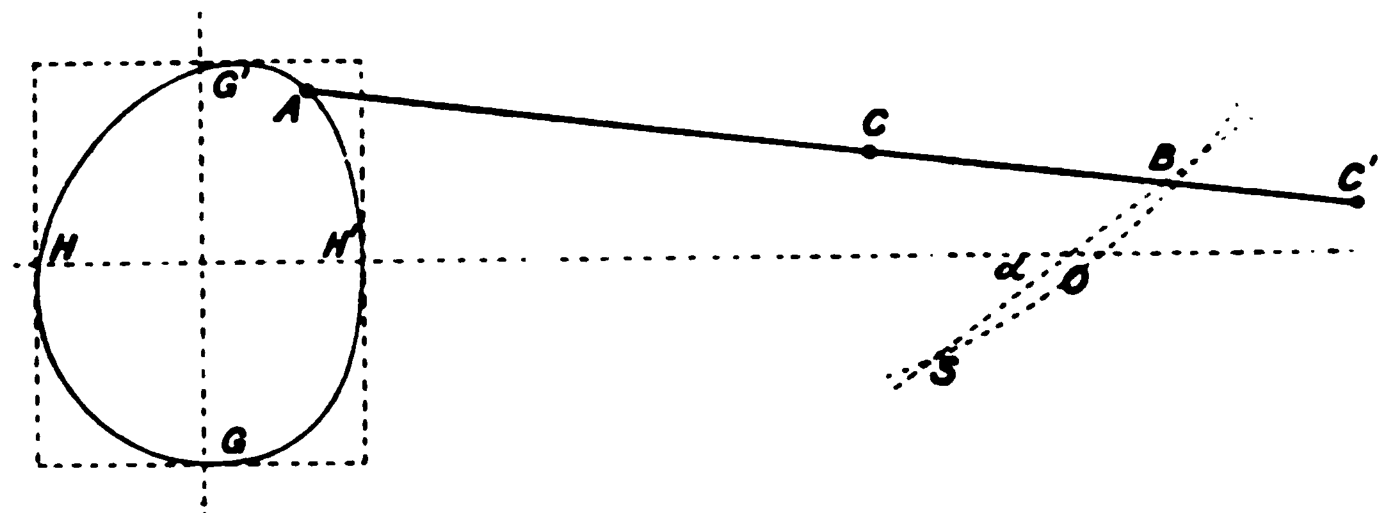



FIG. 289.

nearly circular or elliptic and $B$ has a reciprocating motion; a point $C$ between $A$ and $B$ or in $AB$ produced works the valve."

Let the centre line of the engine and valve rod be vertical. Let $G'H'GH$ be the path of $A$; let the straight line $BOS$ represent the average slope of $B$'s path. Let it make an angle $a$ with the horizontal.

Let $A$ be at $G'$ when the crank pin is in its lowest position and then $C$ is between $A$ and $B$.

Let $A$ be at $G$ when the crank pin is in its lowest position, then $C$ is in $AB$ produced.

It is evident that the following construction comes from the above rule.

(1) As in the **Hackworth, Marshall** and other gears where $C$ is between $A$ and $B$.

Draw lines $hoa$, Fig. 290, and $bo$ at right angles. Make $Oh$ equal to half the greatest horizontal dimension of the figure $G'HGH'$, say half of $HH'$ (that is the distance between the extreme vertical tangents). Make $Oa$ equal to half the greatest vertical dimension of $G'HGH'$, say half of $GG'$. Divide $ao$ in $c''$ so that $oc'' : c''a = BC : CA$, and draw $c''c'c$ at right angles to $Oa$.

The gear is changed by altering the angle $a$. Set off $ohb = a$

cutting $c''c'c$ in $c$. Join $Oc$; then $Oc$ is the half travel and $boc$ ngle of advance.

As in the **Joy** and other gears where $C$ is in $AB$ produced. ɔve description is correct, only that $c$ is in $ab$ produced: but it is better to write a new description.

w lines $hoc''$, Fig. 291, and $bo$ at right angles. Make $oh$ equal the greatest horizontal dimension of the figure $G'HGH'$, f of $HH'$. Make $oa$ equal to half the greatest vertical ion of $G'HGH'$, say half of $GG'$. Divide $ao$ produced in $c''$ so $' : c''a :: BC'' : C''A$ in Fig. 289 and draw $c''c'c$ at right angles The gear is changed by altering the angle $a$. Set off $ohb = a$. and produce to cut $c''c'c$ in $c$. Join $oc$, then $oc$ is the half nd $boc$ is the angle of advance.

en one sees a new radial valve gear for the first time, say at n railway station on a locomotive, one ought to look out for a

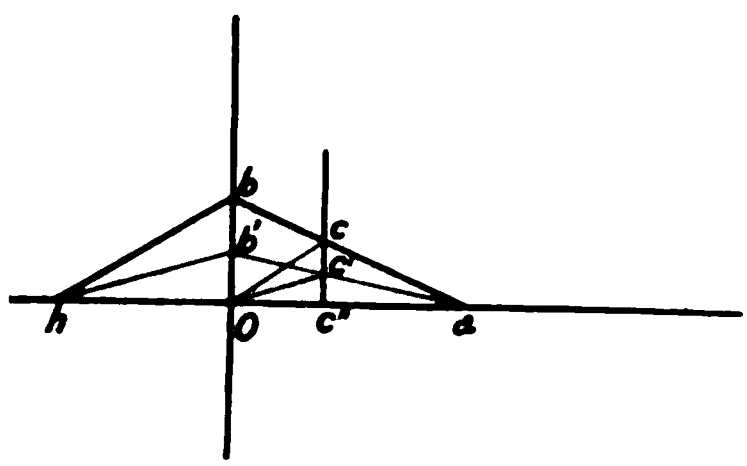


FIG. 290.

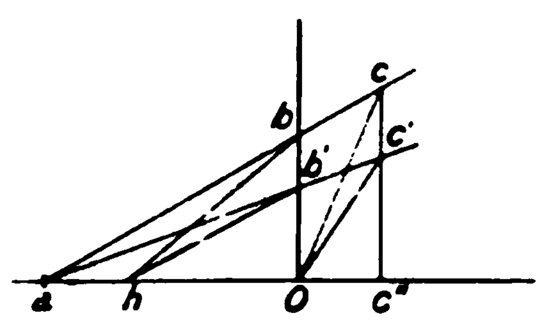


FIG. 291.

$'B$ or $ABC$ with the above mentioned characteristics. When nd there is no difficulty in studying the motion.

**3.** In some engines in which the valve is worked by one c there is a governor on the crank shaft which alters the vel and the angular advance.

ne form, Fig. 292, the eccentric disc consists of two parts c to one another and to the shaft; one is keyed on the shaft, ɔnd is loose on the first. They are connected by links to masses restrained by springs and a dash pot. When the goes too fast the masses move out from the centre and cause motion of the two parts of the disc so that the outer part a disc of less eccentricity and more advance. It is easy to that the change shall be much what it is in the Gooch link dial valve gear, giving a constant lead. In another form only one disc with a slot in it at right angles to the main nd the disc moves bodily relatively to the shaft when the r masses move. In yet another form the eccentric disc $A$ is ttached to the straps $B$ of a ring on another eccentric disc $C$ the crank shaft. The masses of the governor cause a

rotation of $B$ relatively to the shaft, and consequently the total eccentricity of $A$ is altered, the effect being a change of travel and

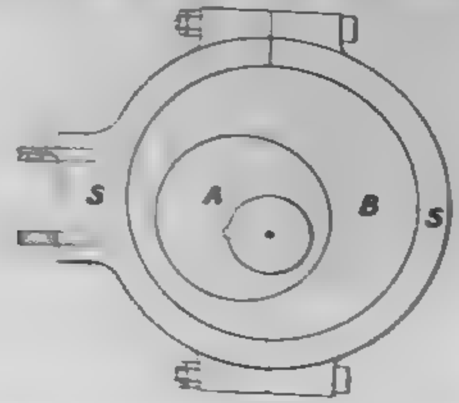
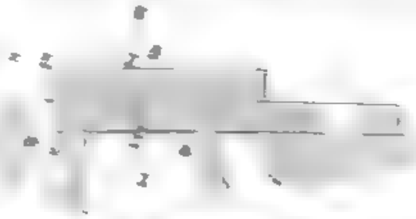



Fig. [illegible]

advance something like what is produced by a Stephenson link motion with open rods.

**314. Independent Cut Off Slide** as in Fig. 150. Let $A$ and $B$, Fig. [illegible] be the middle points of the two slides. When at their mid positions $A$ and $B$ are on the line $ODCO$. Let $AC = y$ be the distance of the main or distribution valve to the right of its mid position and let $BD = x$ be the distance of the cut off slide from its



[illegible]

[illegible] position of the main crank of the engine [illegible]

[illegible]

[illegible] measuring $EF$ and $y - x$ [illegible] the displacement [illegible] from the known amount [illegible]

[illegible] the value of $y - x$ [illegible]

Let the distribution valve have a half travel $a$ and an angle of advance $\alpha$. Let the lap be $L$ and the inside lap $l$. Find the positions of the main crank at admission, cut off, release and compression as if no cut off valve existed. This is our easy example of Art. 76. We let $AOA'$, Fig. 294, represent the centre line of the engine; $DOD'$ is at right angles to $AA'$. $DOB$ is the angle of advance. $OB = OC$ (in $BO$ produced) is the half travel. $Z$ and $Z$ are the Zeuner circles on $OB$ and $OC$ as diameters; $LL'$ and $ll'$ are arcs with $O$ as centre

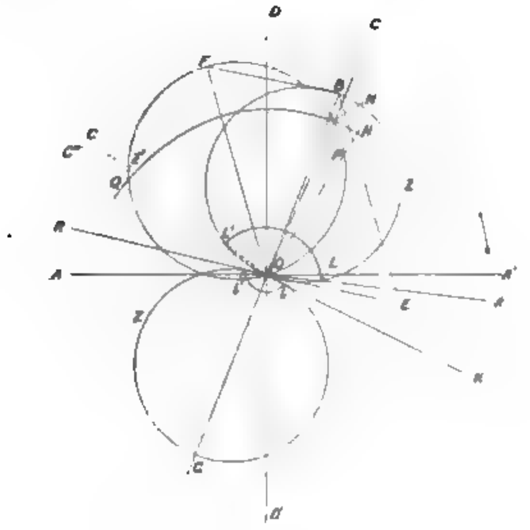


FIG. 294.

and radii the two laps. $OLA$, $OL'C$, $Ol'R$, $OlK$ are the positions of the main crank at admission, cut off, release and compression.

Now we have to see how the cut off valve cuts off steam from the space $I$, Fig. 293, by $EF$ becoming $o$, before the crank reaches the position $OC$, Fig. 294.

$OB$ with the advance $DOB$ works the distribution valve. At any instant the displacement produced by it was called $y$. Let $OE$ with the advance $DOE$ work the cut off valve; at any instant the displacement produced by it was called $x$. We want to find a crank which would produce a displacement $y-x$. Art. 301 tells us to make $OB$ the diagonal of the parallelogram $OEBF$ of which $OE$ is one side and $OF$ is the crank required. On $OF$ describe a Zeuner circle; we know that when the main crank is at $OP$ moving in the direction of the arrow from the dead point $OA'$, the distance $OP$ represents $y - x$. Now describe the circular arc $NMQ$ with a radius equal to $AI - BE$

of Fig. 293. It is evident that $PN$ represents $AI - BE - (y - x)$ or $EF$ and when this is $O$ we have the real cut off at $OMC'$.

From $OMC'$ to $OQC''$ the passage $IG$ of the valve, Fig. 293, can receive no steam and so the cut off effected at $OC$ by the distribution valve itself is a thing of no importance.

Note that $OC''$ must occur later than $OC$ else we shall have a fresh rush of steam when the passage is uncovered, before the main cut off occurs.

The result arrived at is then that $OA$, $OC'$, $OR$ and $OK$ are the positions of the main crank when the four important events occur.

The two usual ways of varying the cut off are (1) altering the throw $OE$; this is easily effected by a governor as is shown in Fig. 143; (2) altering the distance apart of the two blocks $M$ and $N$, Fig. 150, which form the cut off valve (shown as one piece in Fig. 293), that is, altering the distance $BE$. If $BE$ is lessened, $AI-BE$ or the radius $ON$, Fig. 294, is increased and $OMC'$ is later.

When an engine is to reverse it is usual to work the distributing valve from a link motion either in full forward or backward gear, and for equal cut off in either direction, $OE$ ought to have 90° advance.

It is quite easy for any student who is fond of elementary practical geometry to work ordinary exercises on this valve motion, if he really understands what I have here given.

Exercise. Distributing value, half travel 3 inches, advance 32°, lap 1·32 inches, inside lap 0·6 inch; show that the crank makes the following angles with the dead point, at admission ($-$ 5°¼), release (161°·5) and compression (45°) and if no cut off valve existed, at cut off (121°·3). The cut off valve is worked by an eccentric with 90° advance and 3·12 inches half travel. The distance $AI$, Fig. 293, is 13 inches and the distance $BE$ may be varied in the following way: show that we get the following as the positions of the main crank at true cut off.

| Distance $BE$. | $AI-BE$. | Crank at real cut off. | Fraction of stroke before cut off, connecting rod infinitely long. |
|---|---|---|---|
| 12·5 | 0·5 | 40° | ·115 |
| 12·0 | 1·0 | 50°·5 | ·18 |
| 11·5 | 1·5 | 61°·5 | ·258 |
| 11·0 | 2 | 73°·5 | ·35 |
| 10·5 | 2·5 | 88° | ·487 |
| 10·25 | 2·75 | 98° | ·57 |
| 10·15 | 2·85 | 103° | ·613 |
| 10·05 | 2·95 | 111° | ·675 |
| 10 | 3 | 121°·3 | ·756 |

he distribution and cut off valves may be worked from two blocks ferent positions in a Gooch link.

is unnecessary to make here a special study of the motion of a worked from an eccentric, when the motions of valve and are not parallel, as this requires only a knowledge of elemen- ractical geometry.

ouble ported and trick and other valves are easily seen to need ecial study. There are cylinders with two exhaust and two ports, each pair having a slide valve worked by an eccentric ink motion, or preferably the two steam ports have two slides, so ach slide when opening or closing its port shall be moving at ghest speed. These also require no special study.

**15. Motion not Simple Harmonic.** The motion of a valve d by an eccentric is not exactly a simple harmonie motion; but very seldom indeed that the discrepance is of the slightest tance. If it were not pedantic I would say that we have y to replace the straight lines $LL^1{}_1II^1$, &c., of Fig. 265, by rcs of circles.

hen the valve is worked from a link motion or radial valve he discrepance may be so well marked as to be very beneficial rtful. It is interesting to know that motion of a valve worked y of the gears is usually a simple harmonic motion, to which is added on another of twice the frequency, an octave as the ians call it. On this subject I must ask my readers to consult ok on the calculus. I have tried many ways of representing otion, but I am afraid that there is none more instructive ier than by drawing the two sine curves as in 4, Art. 297.

ius if the distance of the valve to the right of its mid stroke the main crank makes the angle $\theta$ with its dead point is $y$,

$$y = a_1 \sin(\theta + \epsilon_1) + a_2 \sin(2\theta + \epsilon_2)$$

ses the motion. There is usually also a small constant term I have not included. In well-designed gears this term is ally 0 in all positions. What is usually studied and what ve studied in Arts. 297–313 is the first part, where $a_1$ is the ravel and $\epsilon_1$ the angle of advance if the motion were simple nic. But there is the octave term with a small half travel $a_2$ angle of advance $\epsilon_2$. In all radial valve gears studied by me 0°, and $a_2$ can be found by easy inspection of the gear in any n. But in any case we can find $a_1$, $\epsilon_1$, $a_2$, $\epsilon_2$ from skeleton g measurements. I give some examples of this later on. suppose that we know the results.

Draw circles with radii $OE_1 = a_1$, $OE_2 = a_2$; make the angles $GOE_1 = \epsilon_1$ the angle of advance, and $GOE_2 = \epsilon_2$. Divide the first circle at $E_1 a_1 b_1$, &c., into 24 equal parts, and $BM$ into 24 equal parts and project horizontally and vertically to get the sine curve. Divide $E_2 a_2 b_2 c_2$, &c., into 12 equal parts, and $BM$ into 24 equal parts. In both cases begin with 0, and number the points 0, 1, 2, 3, &c., and projecting horizontally and vertically get the sine curve. Now add the ordinates of $ACPIA^1$ and $DDD$ together to get the curve, whose ordinate is the true displacement $y$, distances from $B$ meaning angles or positions of the crank. We can now draw the outside and inside lap lines as in Fig. 261, and get the positions of the main crank when admission, cut off, release, and compression take place. When we use this sine curve method of working, the exact effect of the octave is at once evident. Thus let a student having drawn $ACPIA^1$, as in Fig. 295, now draw $DDD$ on a piece of tracing paper,

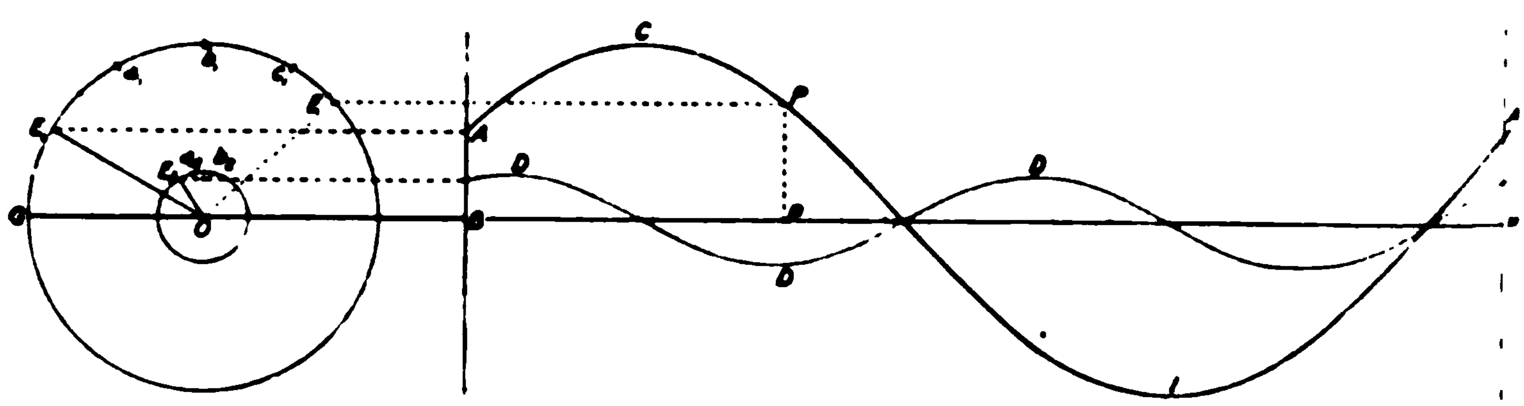


FIG. 295.

and let him notice the different effects produced by sliding the tracing paper (in fact altering $\epsilon_2$) on the compound curve, and on the cut off at either end of the stroke. In most radial valve gears $\epsilon_2$ is nearly $\frac{\pi}{2}$. Hence the octave comes as in Fig. 296.

If we have no octave as in Fig. 261, or here in the dotted curve $F$, Fig. 296, it will be seen that the crank is in the same positions in both strokes when the valve is at the same distance from mid stroke. The existence of the octave changes this, and this is the reason why all link motions and radial valve gears tend to cut off earlier in one stroke than the other. Terms in $3\theta$ or $5\theta$ would have no such effect; the effect is due to terms in $2\theta$ and $4\theta$, but practically we need only consider the fundamental term in $\theta$, and the octave or term in $2\theta$. This will become clearer if we consider a radial valve gear, in which I have found the motion for a certain grade to be given by

$$y = 3 \sin (\theta + 57) + 0{\cdot}3 \cos 2\theta$$

In Fig. 296 $BM$ represents $\theta$ from 0 to $2\pi$ the ordinate from $BM$

sine curve $FF$ represents $3 \sin (\theta + 57)$. The sine curve $GG$ ents $0{\cdot}3 \sin 2\theta$. The ordinates of $FF$ and $GG$ being added

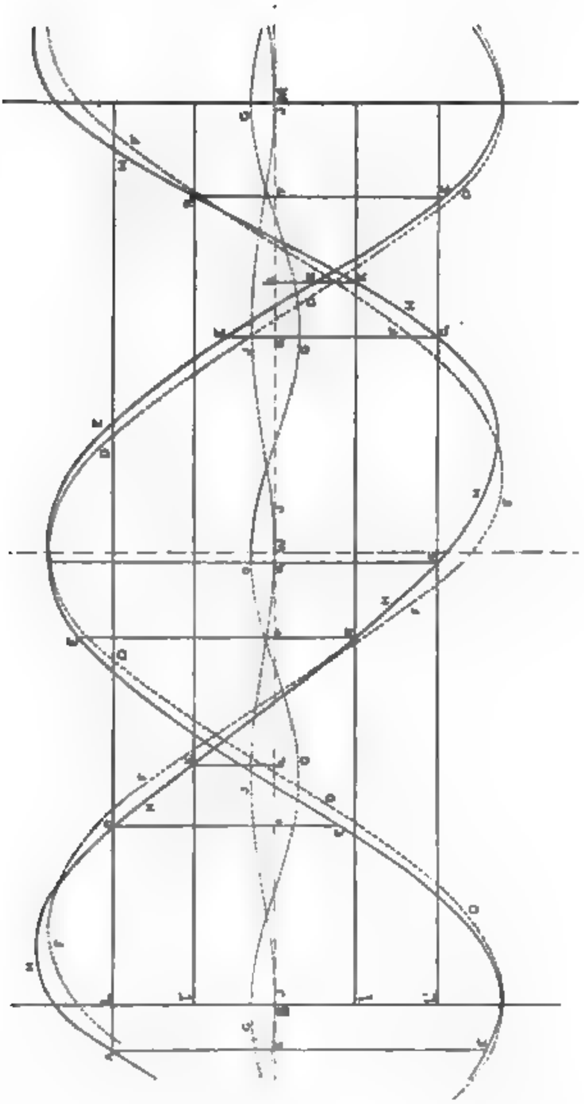


FIG. 206.

er, we have $y$ represented as the ordinate of the curve $H$.

e distances $BL$ and $Bl$ represent the lap and the inside lap ively, and $BL^1$, $Bl^1$ are the laps for the other end of the

cylinder. In this case I have made $BL = BL^1$, and $Bl = Bl^1$. I ing lines as shown, we see the effect of the octave in causin admission $a$, and cut off $c$ to occur earlier, and the release $r$ and pression $k$ later for one side of the piston, whereas for the othe $a^1$ and $c^1$ are later, and $r^1$ and $k^1$ earlier than when there is no o

On the same figure the ordinate of the curve ***EEE*** show displacement of the piston from the middle of its stroke for

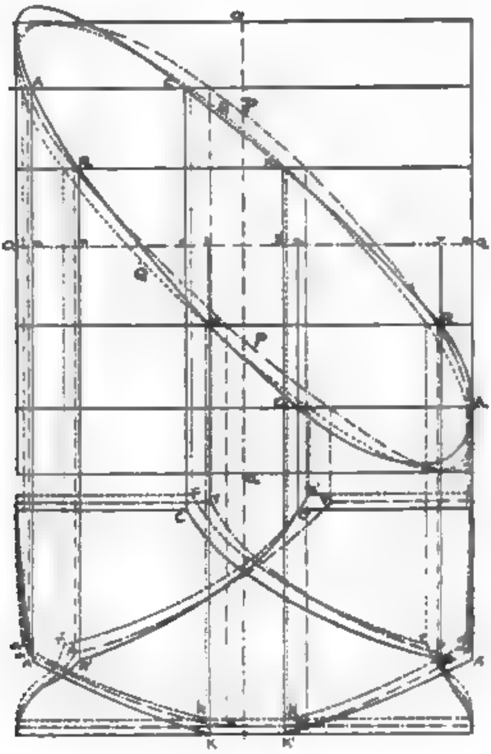


FIG. 297.

position of the crank (connecting-rod five cranks long). The d line $DDD$, which we shall not use, represents what the piston placement would be if the connecting-rod were infinitely long. T displacements are $aE$, $cE$, $rE$ and $kE$ for the admission, cut off, re and compression on one side of the piston, and $a^1E$, $c^1E$, $r^1E$, $k^1$ the other side of the piston.

I have shown the same results by the **oval diagram me** Fig. 297. The ordinates and abscissæ of the curve $CK^1RC^1$ present displacements of the valve to the right of its mid st

nd of the piston from the end of its stroke, and they are easured from Fig. 296. The distance $Aa$ or $Cc$ is the lap, nd $rR$ or $kK$ is the inside lap. The dashed letters are for the ther side of the piston. The student sees how we arrive at the ypothetical diagrams $ACRK$ and $A^1C^1R^1K^1$ for the two ends of he cylinder. He will do well, however, to see what diagrams lrawn here as $acrk$ and $a^1c^1r^1k^1$) he obtains if he uses the ellipse $Q$, which represents the valve and piston motions as simple harmonic motions, and also the diagrams (drawn here as $a\gamma\rho\kappa$ and $a^1\gamma^1\rho^1\kappa^1$) if he uses the oval curve $PP$, which represents the valve motion as simple harmonic, but the true motion of the piston. In the present case he will see that the octave in the valve motion produces inequality of distribution on the two sides of the piston of much the same kind as that due to the shortness of the connecting-rod, and he will note that we usually have power to cause these to coalesce or to oppose one another. In the present case, if both motions are simple harmonic, there is symmetry, see $c$ and $c^1$; but if the valve motion is simple, the shortness of the connecting-rod makes $\gamma^1$ earlier than $\gamma$; to counteract this and get $C^1$ later than $C$ the octave in the valve motion is very useful. The effect of angularity of the connecting-rod is sometimes opposed by giving different amounts of lap and of inside lap to the two sides of the valve.

EXERCISE. Show that when we equalise the points of cut off and of release or compression by inequality of the lap and inside lap, we do not equalise the other two important events for the two ends of the cylinder; or, that if the leads are made equal, the points of cut off are unequal. It is not difficult, however, to show that a good approximation to equality in both may be produced if we drive the valve through a bell crank lever.

**316. Fourier Analysis.** I have in Art. 302 shown how we combine simple harmonic motions. Suppose that by a skeleton drawing method or by means of a large model we get the displacement of a slider for each of many positions of a crank; it is, in my opinion, essential for a scientific study of a valve motion, to express the displacement in terms of a fundamental simple periodic motion and its harmonics, the octave being the most important. I here give an example to illustrate how this may be done in any case. The whole of the work is shown in the table, page 513, although the example is one in which we are looking for a fundamental term and its three harmonics, each with an amplitude and a lead or lag.

In the table the displacement $y$, of a valve from a fixed point, is given for 24 different positions of the crank.

To obtain the displacements of the valve from its mean position, find the average of all the 24 values of $y$ (in this case I find 5), and subtract this from each.

The resulting values, $y'$, are given in column *A*. Our aim is to express the valve's motion in terms of the position of the crank $\theta$ by a Fourier series. We really never need more than two terms, but I shall here consider four.

$$y' = a_1 \sin(\theta + \epsilon_1) + a_2 \sin(2\theta + \epsilon_2) + a_3 \sin(3\theta + \epsilon_3) + a_4 \sin(4\theta + \epsilon_4)$$

Let the student imagine $\theta$ and $y'$ to be plotted (from $\theta = 0$ to $\theta = 360°$) on squared paper. Then if one half of the curve from 180° to 360°, is superposed on the other half from 0° to 180°, the 1st, 3rd, 5th, &c., components in the above expansion will be eliminated [this is easy to see if these components be drawn separately], and the resulting curve will be :—

$$y' = 2[a_2 \sin(2\theta + \epsilon_2) + a_4 \sin(4\theta + \epsilon_4)]$$

Similarly, if the original curve be divided into three equal portions by lines perpendicular to the axis of $\theta$, and the three parts superposed on each other, the 2nd, 4th, 6th, &c., components will be eliminated, and the resulting curve will be :—

$$y' = 3a_3 \sin(3\theta + \epsilon_3).$$

It is an easy exercise for the student to prove this either graphically or analytically. If he has difficulty let him consult Mr. Wedmore's paper in the *Proceedings* of the Institution of Electrical Engineers, 1896, or General Sir R. Strachey's paper in the *Proceedings* of the Royal Society, May, 1886.

The table shows how the above method is employed without actually drawing the curves. For instance, columns *A*, *I*, and *J* are the three equal parts superposed, and when added give column *K* which is three times component 3. In this case zero.

An examination of the table easily shows how it is all produced.

*Component* 1. Imagine column *N* to be continued to the top of the table; ordinate 0 will be + 2·520; average of ordinates, from ordinate 0 to 11 inclusive, treating all as positive, is 22·900 ÷ 12 = 1·908. We use this method of finding $a_1$ because of the rule ;—

$$\text{Maximum ordinate } a_1 = 1{\cdot}908 \times \frac{\pi}{2} = 2{\cdot}997, \text{ say } 3.$$

$$\text{To get } \epsilon_1, \qquad \frac{\sin \epsilon_1}{\sin 90°} = \frac{2{\cdot}5161}{2{\cdot}997} = {\cdot}8395$$

$$\therefore \epsilon_1 = \sin^{-1} {\cdot}8395 = 57°9', \text{ say } 57°.$$

*Component* 2. By inspection of column $H$, the maximum ordinate

$$a_2 = \cdot 50 \text{ and } \epsilon_2 = 90^\circ.$$

*Component* 3. Zero. See Column $K$.

*Component* 4. Average of ordinates from 0 to 2 inclusive $= \cdot 1725 \div 3 = \cdot 0575$.

Maximum ordinate $a_4 = \cdot 0575 \times \frac{\pi}{2} = \cdot 091$.

By inspection $\epsilon_4 = 0$.

Hence $y = 5 + 3 \sin(\theta + 57^\circ) + \cdot 5 \cos 2\theta + \cdot 09 \sin 4\theta$ the required expression.

| | No. of ordinate. | $y$ | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $y$ | $y'$ or $y-5$. | This is A superposed on itself. | A+B. | Or half of C being sum of comp. 2 and 4. | Being D superposed on itself. | Being D and E. | Being half of F = comp. 4 | Being D − G or comp. 2. |
| 0 | 0 | 8·02 | 3·02 | −2·02 | 1·00 | ·50 | − ·50 | 0 | 0 | ·50 |
| 5* | 1 | 8·37 | 3·37 | −2·33 | 1·04 | ·52 | − ·345 | ·175 | ·0875 | ·4325 |
| 0 | 2 | 8·33 | 3·33 | −2·66 | 0·67 | ·335 | − ·165 | ·170 | ·085 | ·250 |
| 5 | 3 | 7·93 | 2·93 | −2·93 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 4 | 7·34 | 2·34 | −3·01 | −0·67 | − ·335 | + ·165 | ·170 | − ·085 | − ·250 |
| 5 | 5 | 6·71 | 1·71 | −2·75 | −1·04 | − ·52 | + ·345 | ·175 | − ·0875 | − ·4325 |
| 0 | 6 | 6·13 | 1·13 | −2·13 | −1·00 | − ·50 | | | 0 | − ·50 |
| 5 | 7 | 5·58 | 0·58 | −1·27 | −0·69 | − ·345 | | | ·0875 | − ·4325 |
| 0 | 8 | 4·99 | − ·01 | −0·32 | −0·33 | − ·165 | G continued downwards. ☞ | | ·085 | − ·250 |
| 5 | 9 | 4·38 | − ·62 | 0·62 | 0 | 0 | | | 0 | 0 |
| 0 | 10 | 3·80 | −1·20 | 1·53 | +0·33 | + ·165 | | | − ·085 | + ·250 |
| 5 | 11 | 3·34 | −1·66 | 2·35 | +0·69 | + ·345 | | | − ·0875 | + ·4325 |
| 0 | 12 | 2·98 | −2·02 | | | | | ·50 | −2·520 | |
| 5 | 13 | 2·67 | −2·33 | M continued upwards ☞ | | | | ·52 | −2·850 | |
| 0 | 14 | 2·34 | −2·66 | | | | | ·335 | −2·995 | |
| 5 | 15 | 2·07 | −2·93 | | | | | 0 | −2·930 | |
| 0 | 16 | 1·99 | −3·01 | − ·01 | 3·02 | 0 | | − ·335 | −2·675 | |
| 5 | 17 | 2·25 | −2·75 | − ·62 | 3·37 | 0 | | − ·52 | −2·230 | |
| 0 | 18 | 2·87 | −2·13 | −1·20 | 3·33 | 0 | | − ·50 | −1·630 | |
| 5 | 19 | 3·73 | −1·27 | −1·66 | 2·93 | 0 | | − ·345 | −0·925 | |
| 0 | 20 | 4·68 | −0·32 | −2·02 | 2·34 | 0 | | − ·165 | −0·155 | |
| 5 | 21 | 5·62 | 0·62 | −2·33 | 1·71 | 0 | | 0 | 0·620 | |
| 0 | 22 | 6·53 | 1·53 | −2·66 | 1·13 | 0 | | + ·165 | 1·365 | |
| 5 | 23 | 7·35 | 2·35 | −2·93 | 0·58 | 0 | | + ·345 | 2·005 | |
| mean ordinate | } 5 | | | being A superposed. | being A superposed. | A+I+J being 3 times component 3 which is 0. | | D repeated. | A − D being components 1 and 3 or comp. 1 only. | |
| | | | A | I | J | K | | M | N | |

Now in a valve motion we have no elaborate work like this. For example, with the numbers measured from a skeleton drawing: Subtracting the average, superposing and dividing by two we get at once the results given.

**317.** Exercise. Make a skeleton drawing of a **Hackworth** (or Angstrom) gear (straight slot) as in Fig. 298. $OA = 3$ inches; eccentric rod $AB$ 18 inches; $BC = 7\frac{3}{4}$ inches, and show that, for the following values of $a$ (when $a$ is negative the rotation is against the hands of a watch) we have the following results obtained by my students for the vertical motion of $C$. I may say that the octave advance is difficult to find exactly when the octave is small; errors of 15 or 20° are easy to make.

| $a$ | Fundamental Motion. | | Octave. | | Usual Rule of Art. 312. | |
|---|---|---|---|---|---|---|
| | Half travel. | Advance. | Half travel. | Advance. | Half travel. | Advance. |
| Full forward 25° | 1·51 | 59° | ·04 | 90° | 1·47 | 56° |
| Full backward −25° | 1·51 | 59° | −·04 | 90° | | |

Following the rule of Art. 323 it is easy to see that we ought to get the following results. An octave with a + amplitude is one like

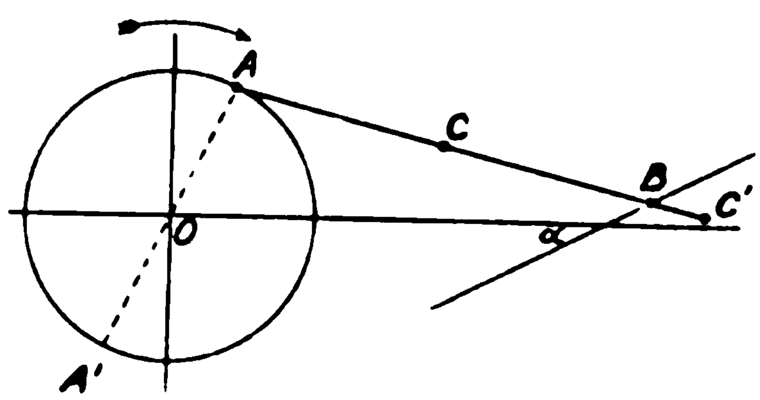



Fig. 298.

what is shown in Fig. 296, which would produce earlier cut off on the side of the piston remote from the crank. In every case the amplitude of the octave is, roughly

$$\frac{AC}{AB}\,\frac{r^2}{4l}\tan a \text{ or } 0{\cdot}071 \tan a$$

It is evident that this is practically negligable. If $a$ is 25°, $\tan a = {\cdot}4663$ and the amplitude of the octave is $\pm$ ·033 inches. It will be noticed that my students get $\pm$ ·04, but it is so small that discrepance was certain to occur.

**318. Exercise.** Hackworth (or **Marshall** or Bremme) with slot of radius $13\frac{1}{2}$ inches, its centre being at the end $N$ of an arm of $13\frac{1}{2}$ inches long, the other end of which is fixed at $B'$, Fig. 124. The other dimensions as in the first case with straight slot. Obtain the following information from a skeleton drawing. $a$ is the angle which the arm $NB'$ carrying the centre $N$ makes with the vertical in Fig. 124.

| $a$ | Fundamental Term. | | Octave. | |
|---|---|---|---|---|
| | Half travel. | Advance. | Half travel. | Advance. |
| Full forward 25° | 1·54 | 57° | ·15 | 90° |
| Full backward −25° | 1·53 | 59° | ·11 | 90° |

Following the rule of Art. 323 we find the nearly negligable octave to be $0\cdot071 \tan a$, as in the last case, together with $+\frac{AC}{AB}\frac{1}{8}\sec^3 a$ of Art. 322 or altogether

$$\frac{1}{8}\sec^3 a + 0\cdot071 \tan a$$

If $a$ is 25°, the amplitude of the octave in full forward gear is

$$\cdot128 + \cdot033 \text{ or } \cdot161 \text{ inches.}$$

Whereas in full backward gear it is

$$\cdot128 - \cdot033 \text{ or } \cdot095$$

Here again the discrepances from actual results are negligable.

**Exercise.** The student will do well to take $a = \pm 25°$ in full forward and back gear, making the curvature of the slot convex to

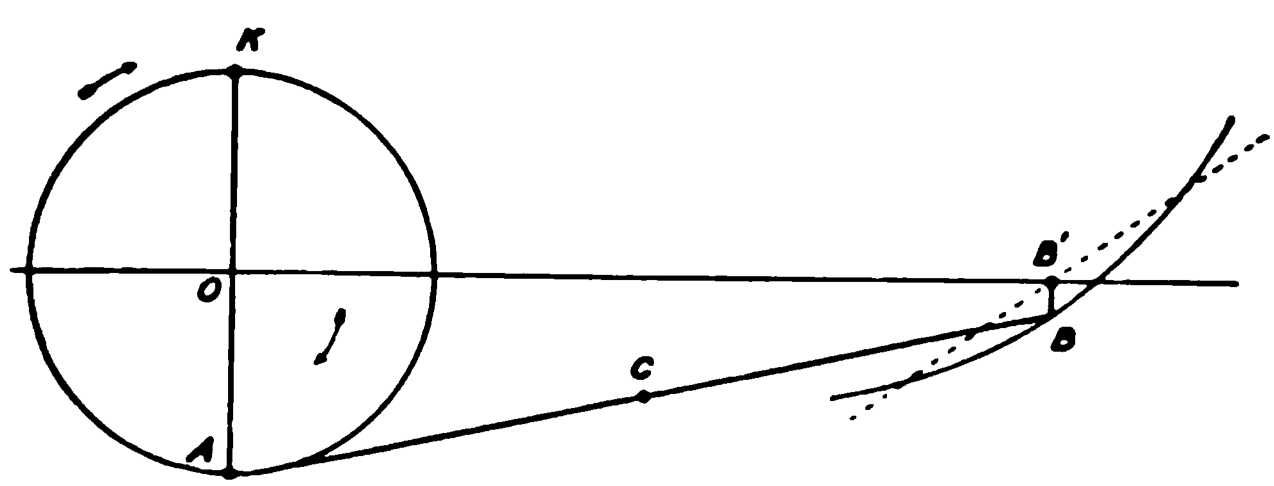



Fig. 200.

the cylinder and keeping to the above dimensions, but letting the eccentric be with the crank instead of being 180° ahead of it:

working the valve from a point $C'$ in $AB$ produced, and finding y downward displacement of the value in full forward and back g for any angle $\theta$ passed through by the crank from the dead p position nearest the cylinder. Should he by mistake leave curvature of the slot concave to the cylinder he will be intereste noting the very different way in which his octave occurs.

**319.** EXERCISE. **Joy gear,** Fig. 300.

Crank $OP$ 8 inches, connecting rod $KP$ 40 inches, $DP$ 14 inc

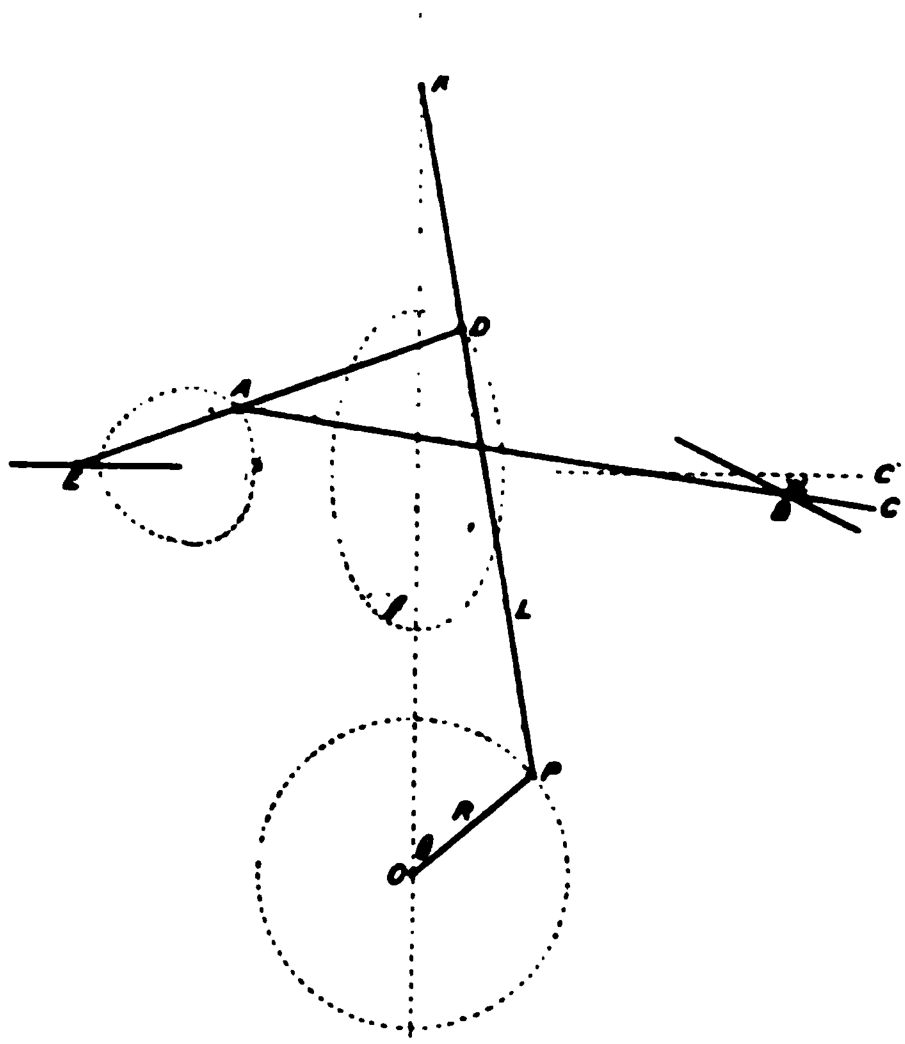


FIG. 300.

$AB$ 16 inches, $BC$ 8 inches, $DE$ 24 inches, $DA$ 8 inches. Radiu path of $B$ 12 inches.

Taking $a = 25°$ as full forward gear, find $y$ the downward (I 300) displacement of $C$ when the crank makes $\theta$ with the de point nearest the cylinder.

*Answers.* Answers obtained by my students:—

1. When the curved slot is concave towards the valve

$$y = 2{\cdot}55 \sin(\theta + 36°) + {\cdot}35 \cos 2\theta$$

2. When the slot is straight

$$y = 2{\cdot}3 \sin(\theta + 35°) - {\cdot}15 \sin 2\theta$$

3. When the curved slot is convex towards the valve

$$y = 2{\cdot}18 \sin(\theta + 36°) - 0{\cdot}35 \cos 2\theta$$

The student will note that the curved slot must be concave towards the valve to give an earlier cut off in the down stroke. The other form aggravates the evils due to the weights of moving parts and angularity of connecting rod.

I will now proceed to give some rules as to the production of the octaves in valve motions.

**320.** Propositions concerning the **Creation of Octaves.**

I. Prove that if there are three points $ACB$ in a straight line keeping their distance apart; if $a$ and $b$ are the displacements of $A$ and $B$ resolved in any particular direction, the displacement of $c$ in the same direction is

$$c = a\frac{CB}{AB} + b\frac{CA}{AB}.$$

In my book on Applied Mechanics, I show that if from a point $O$ we draw $OA''$ and $OB''$ to represent in clinure and magnitude the displacements of $A$ and $B$; join $A''B''$ and divide $A''B''$ in $C''$ in the

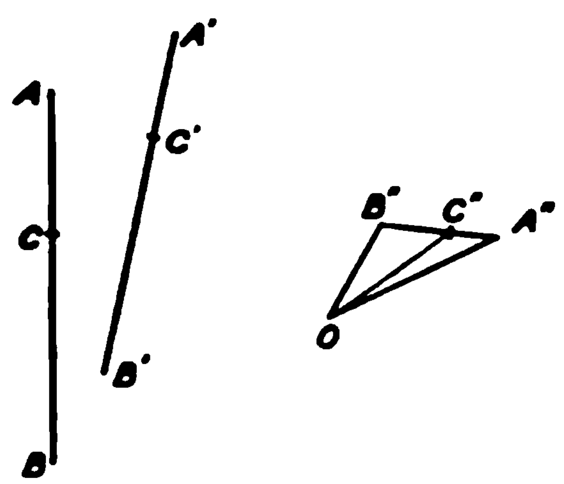



FIG. 301.

proportions in which the link is divided in $C$; then $OC''$ shows the clinure and magnitude of $C$'s displacement.

By projecting these displacements on a line in any direction from $O$, the above proposition is proved.

II. In any standard position of $ACB$ let parallel rectangular coordinate axes be drawn through $A$, $C$ and $B$, and let these points at any time be at the distances $x_1, y_1$; $x, y$; $x_2, y_2$ from their respective axes

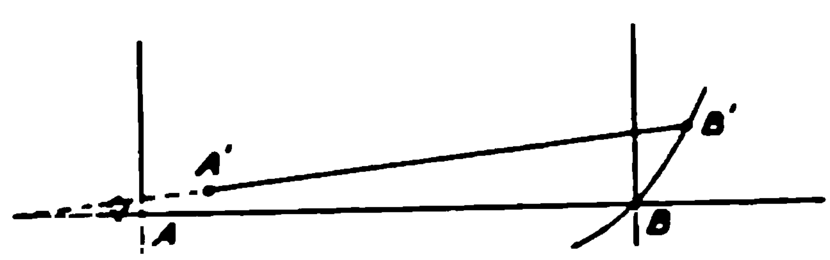



FIG. 302.

the proposition I. may be used to find $x$ and $y$ from $x_1$, $x_2$ and $y_1$, $y_2$.

III. If the motion of $A$ is known and if the path of $B$'s motion is known we can find the motions of $B$ and of $C$.

Choose the initial position of $AB$ as the common axis of $x_1$ and $x_2$ and let $AB = l$. Let $\phi$ be the angle which the link $A^1B^1$ makes with the standard position $AB$.

$$l \sin \phi = y_2 - y_1$$

$$l \cos \phi + x_1 = l + x_2$$

$$y_2 \text{ is a known function of } x_2$$

Then
$$\left(\frac{y_2 - y_1}{l}\right)^2 + \left(1 + \frac{x_2 - x_1}{l}\right)^2 = 1$$

and from this $y_2$ and $x_2$ may be found in terms of $y_1$ and $x_1$.

We shall in future neglect small terms.

IV. If $A$ the end of a long rod, $AB$ of length, $AB = l$ has a simple harmonic motion, in what I shall call the vertical direction $AOA^1$.

Such that
$$OA = y = a \sin qt.$$

where $a$ is small compared with $l$; and if $B$ has motion at right

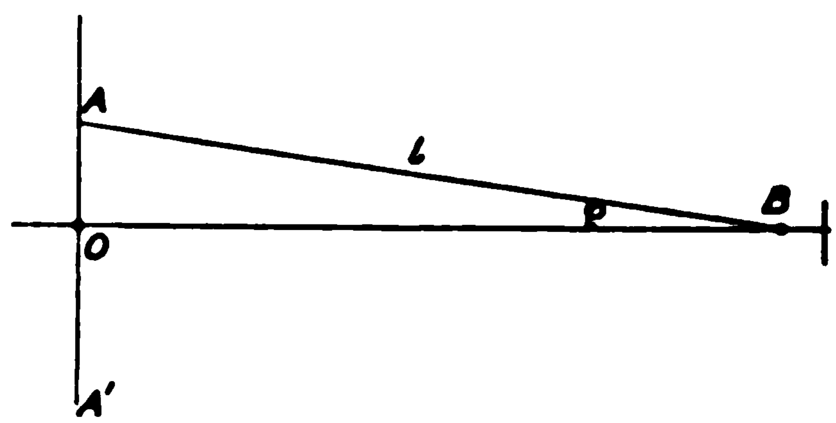



FIG. 303

angles to $AOA^1$ in the direction $OB$, which I shall call horizontal what is $B$'s motion?

$$OB = l \cos \phi$$

$$OA = a \sin qt = l \sin \phi$$

$$\cos \phi = \sqrt{1 - \frac{a^2}{l^2} \sin^2 qt}$$

$$OB = l \sqrt{1 - \frac{a^2}{l^2} \sin^2 qt} = l - \frac{a^2}{2l} \sin^2 qt$$

since $\frac{a^2}{l^2} \sin^2 qt$ is supposed to be always small; now

$$\sin^2 qt = \tfrac{1}{2} - \tfrac{1}{2} \cos 2qt$$

$$\text{so that } OB = l - \frac{a^2}{4l} + \frac{a^2}{4l} \cos 2qt$$

which is a S. H. motion of amplitude $\frac{a^2}{4l}$, the middle point being at a distance from $O$ equal to $l - \frac{a^2}{4l}$.

A very little thought will show that however the S. H. motion of $A$ may be stated (that is, from whatever instant we may count time) $B$ is at the ends of its stroke when $A$ is at either end or the middle of its stroke.

If we take $OA = a \sin (qt + \epsilon)$ it is easy to see that

$$OB = l - \frac{a^2}{4l} + \frac{a^2}{4l} \cos 2(qt + \epsilon)$$

Hence if $\epsilon = 90°$ so that if $OA = a \cos qt$

$$OB = l - \frac{a^2}{4l} - \frac{a^2}{4l} \cos 2qt$$

Notice that if the motion of $A$ has a small harmonic, the effect of this is a very greatly reduced octave of it in $B$'s motion, and it may usually be neglected.

If motion of $B$ to the right of its mid position be called positive when is the positive displacement of $B$ greatest? *Answer.* When $A$ is at its mid stroke; half way up or half way down.

If $O$ is not the middle point in $A$'s motion, it will be found that $B$'s motion has the frequency of $A$ with an octave.

V. In the case of IV.; every point in $AB$ has a vertical simple harmonic motion synchronous with $A$'s motion and proportional to its distance from $B$.

VI. Any kind of periodic motion of the same period as $A$'s may be given to $B$ by letting the path of $A$ be a curved path.

VII. Whatever be the actual path of $A$, if it has a symmetrical simple harmonic vertical motion, so that $y = a \sin qt$; the vertical motion of any point in $AB$ follows the rule V.

VIII. If $A$ has a small horizontal periodic motion, $x_1 = f(t)$, $B$'s motion is what it was before; but in addition it has the horizontal motion of $A$, or $B$'s displacement is

$$OB = l - \frac{a^2}{4l} + \frac{a^2}{4l} \cos 2qt + f(t),$$

IX. If $A$ describes a circular path of radius $r$ with uniform speed. Let the angle that $OA$ makes with the upward drawn vertical from $O$ at any instant be called $\theta$.

$$x_1 = r \sin \theta, y_1 = r \cos \theta$$

or counting time from when $\theta$ is $O$

$$x_1 = r \sin qt,\ y_1 = r \cos qt.$$

Therefore $$OB = l - \frac{r^2}{4l} + r \sin qt - \frac{r^2}{4l} \cos 2qt.$$

Of course $OB$ expresses the motion of a piston if the connecting rod is of length $l$ and the crank is $r$.

X. If $A$ describes a path such that

$$x_1 = a_1 \sin qt + m \sin (2qt + \epsilon_1)$$

$$y_1 = b_1 \cos qt + n \sin (2qt + \epsilon_2)$$

$$OB = l - \frac{b_1^2}{4l} + a \sin qt + m \sin (2qt + \epsilon_1) - \frac{b_1^2}{4l} \cos 2qt.$$

XI. If in X. instead of $B$'s path being straight and horizontal it is still straight, but makes an angle $a$ with the horizontal; its mid point being as before in the horizontal from $O$. Neglecting small terms, the horizontal motion of $B$ is the same as before, and if $x_2$ is its horizontal distance from the mid point and $y_2$ its vertical distance,

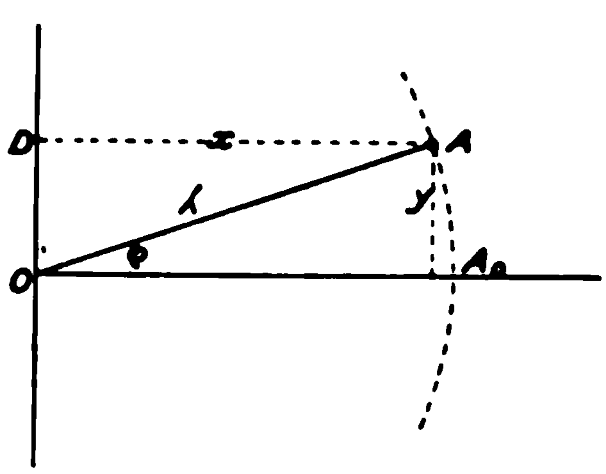


FIG. 304.

$$y_2 = x_2 \tan a.$$

Thus the octave in $y_1$ does not play any part in $B$'s motion, a most important fact to remember.

XII. If $A$ has a vertical displacement from $A_0$ equal to $y = a \sin qt$ and is centred about the point $O$, $OA$ being $\lambda$ which is great compared with $a$ and $OA_0$ is horizontal, find $x$ or $AD$. $\lambda \cos \phi = x$, $\lambda \sin \phi = y = a \sin qt$.

$$x = \lambda \sqrt{1 - \frac{a^2}{\lambda^2} \sin^2 qt} = \lambda - \frac{a^2}{4\lambda} + \frac{a^2}{4\lambda} \cos 2qt$$

That is, the horizontal motion of $A$ is simple harmonic of half the period or twice the frequency of the vertical motion.

Hence, if instead of $B$'s path being horizontal it is in the arc of a circle whose average direction is horizontal as in Fig. 305, it is evident that in moving from $M$ to $P$, this up and down motion is very nearly a simple harmonic motion which will be exactly reversed if the dotted path is followed.

XIII. If instead of $B$ moving in the straight path of XI. it moves in an arc of a circle, Fig. 306, with the average slope $a$ and radius $\lambda$

he upward motion of $B$ is what it was in XI. together with what it ould be if the average slope of the arc of the circle were $O$ multilied by sec. $a$. That is, if the fundamental part of the horizontal iotion has an amplitude $a$, the vertical motion is $x_2$ tan $a$ + an :tave of amplitude $\frac{a^2}{4\lambda}$ sec. $a$, if $\lambda$ is great compared with $a$.

XIV. To illustrate how octaves may be created or destroyed by a :versing lever. The ends of a link $A$ and $B$, Fig. 307, move in the raight lines $CB$, $CA$; if $B$ has a simple harmonic motion along $BC$ hat is $A$'s motion? The easiest way of putting this is:—If $AC$ be alled horizontal, $B$ has a simple harmonic motion of amplitude $b$, ay, which is of amplitude $b$ cos $a$ horizontally (and this motion $A$

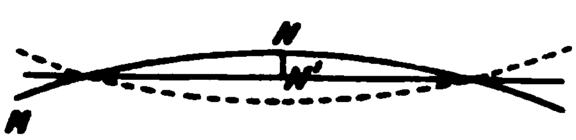


Fig. 305.

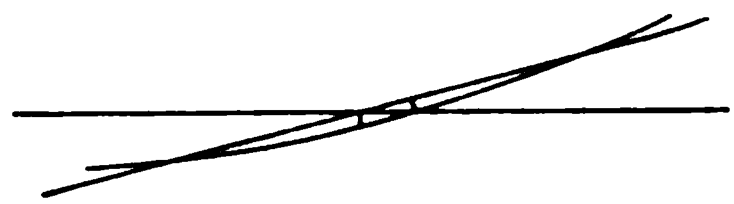


Fig. 306.

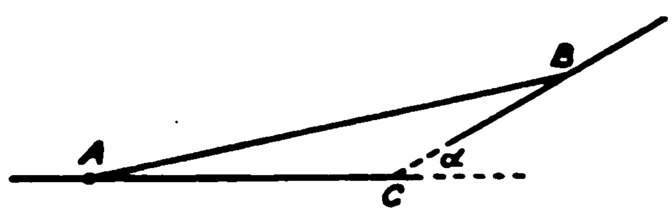



Fig. 307.

has also) together with one of amplitude $b$ sin $a$ vertically. Now uch a vertical motion (see Proposition IV.) of $B$ produces an octave n $A$ whose amplitude is $\frac{b^2 \sin^2 a}{4 \cdot AB}$ together with a fundamental which shall here neglect. We see therefore that a simple harmonic motion ı either $A$ or $B$ produces a simple harmonic motion in the other, gether with an octave whose amount depends upon the angle $a$. If ie path of $A$ or of $B$ is a short arc of a circle we have practically the ıme effect. So that either $A$ or $B$ may be the end of a bell crank ver.

**321.** It is of no use paying particular attention here to the actual gns of the terms. No student can remember them, but it is evident ıat in all vertical and horizontal motions of the guided pins in links hose average directions are parallel to or at right angles to the line ' centres, being driven by a uniformly rotating crank, we have ındamentals of the same period which are in + or − synchronism or e $\frac{1}{4}$ period apart, that is, they can be expressed all as $\pm$ sin $qt$ or $\pm$ cos with amplitudes quite easy to find, together with octaves which ach their positive or negative maxima when $qt$ or $\theta$ is $o$, that is

when the driving crank is in the direction of the line of centres. Any point intermediate therefore between two guided pins has a vertical or horizontal motion intermediate between $\pm$ sin $qt$ and $\pm$ cos $qt$. Say $a \sin (qt + e_1)$ together with an octave $\pm a_2 \cos 2qt$. If this is the motion of a slide valve we see from Fig. 296 the nature of the distribution of steam caused by it; we see that the gear may be arranged to admit steam longer at one end of the stroke than at the other, that we may cause it not merely to counteract the effect of the angularity of the connecting rod, but to more than counteract it. We saw that a single eccentric on a modern vertical engine where the cylinder is above the crank, gives more admission in the down stroke because of the angularity of the connecting rod, whereas we want just the opposite effect on account of the downward acting weights of moving parts. This may be counteracted to some extent by giving more lap on the upper side of the valve, but it may also be done by getting a proper octave in the valve gear.

I have here given **the general principles** which guide us in the study of any such gear. Unless as part of one's routine drawing office work it is hardly necessary to apply these principles to the detailed study of any particular gear. I am inclined to think that instead of solving puzzles in this way, it is better to make a skeleton drawing, to measure the displacement of the valve for equal angles passed through by the crank; calculate the fundamental and octave by the rule of Art. 316, now alter the gear and repeat, thus seeing how the alteration affects the octave.

Although I dislike the study as a misuse of one's faculties, I will indicate here how some such gears may be taken up.

**322. Radial Valve Gear. The Octave.** If the above principles are remembered it will be found that an easy (although possibly a slightly tedious) inspection of a radial valve gear gives the octave. In the Hackworth, Fig. 299, with **curved slot** at $B$, or its equivalent with the swinging link or in the Joy gear, the vertical motion of $C$ is practically that of the valve, and in so far as the octave part is concerned it is the fraction $\frac{AC}{AB}$ of the vertical octave in $B$.

The crank being at $OK$ and piston in highest position the eccentric is at $OA$, let us say, and the valve would be in the condition which we studied in Art. 312 (neglecting the octave which the straight slot would also have), only for $BB^1$ which is evidently the amplitude of the octave; this then gives us at once Fig. 296, and when the engine is reversed we have the same effect. Whereas, if the slot had been

curved with the concavity downwards, the other way, we should have had just the opposite effect, the octave being negative in the top position of the piston.

The octave is a maximum at a dead point. It is easy to see in the same way that if $C$ is in $AB$ produced, as it is in the Joy gear and in varieties of the Hackworth (called often Marshall or Bremme) gear, when the eccentric is with the crank and not 180° ahead of the crank, we have just the opposite rule as to curvature. A slot convex towards the cylinder gives an octave like that of Fig. 296, giving earlier admission and cut off on the side of the piston remote from the crank.

In all cases we may take it that roughly the octave produced by the curvature is $\frac{AC}{AB} \cdot \frac{r^2}{4R} \sec^3 a$, if $r$ is the eccentric radius and $R$ is the radius of the slot or curved path of $B$ and $a$ is its average inclination to the line $OB'$, Fig. 299. This rule is not very exact if $a$ is much greater than 20° as new harmonics then come in, but these are easy enough to study. It will be found that this is practically the whole of the octave to be studied in the Hackworth gear. Of course we can create another octave by using a short rod connecting $C$ with the valve rod. Indeed I feel that I ought to have said more about this rod, but the octave produced by its shortness is easily stated by Proposition IV. to be $\frac{r^2}{4\lambda} \sin(2\theta - 90°)$ if $\lambda$ is the length of the rod. It counteracts the effect of a slot concave to the cylinder.

When we know that a slider in an engine has a simple harmonic motion in any direction, we settle on what we shall call the positive side of the motion: 1st, we find $a$ the amplitude; 2nd, we find what is the displacement when the crank is at dead point (I always take the inner or cylinder side dead point). If we call this $a \sin \alpha$ then the displacement is

$$a \sin(\theta + \alpha)$$

in the positive direction.

Instead of the second measurement above, I sometimes find as my second measurement the position $\theta$ of the main crank when the positive displacement first reaches its highest value. This is often a much easier thing to do if we have the engine before us and we can turn it round; if this is $\theta'$, then what I have called $\alpha$ above, is $90 - \theta'$.

**323.** Now let us consider any gear, say the Hackworth, Fig. 298, with straight slot.

I. In vertical or horizontal motion, $A$ has no octave.

II. Horizontal motion of $B$; positive distances are measured to the left of mid position.

1st. $B$ has $A$'s horizontal motion considered in VII., Art. 320.

2nd. $B$ has a horizontal motion due to $A$'s vertical motion. See IV. of Art. 320. Its amplitude is $\frac{r^2}{4l}$ where $l = AB$, and it reaches its maximum value when $A$ is at the top or bottom.

III. Downward vertical motion of $B$. This is the horizontal motion multiplied by tan $a$. For the octave part its amplitude is $\frac{r^2}{4l}$ tan $a$ and it reaches its maximum downwards when $A$ is either at the top or the bottom.

IV. The downward fundamental motion of $C$ we have studied in Art. 312. The octave has an amplitude $\frac{AC}{AB} \cdot \frac{r^2}{4AB}$ tan $a$, and reaches its maximum when $A$ is highest or lowest, that is, whether $A$ is 180° from the main crank or is synchronous with the main crank. In the one case $C$ is between $A$ and $B$. In the other case it is in $AB$ produced, but this is of no consequence. In both cases we evidently have the octave coming as in Fig. 296.[1]

**324.** Let us take the **Joy gear with straight slot.** I assume that students know the Joy gear, Fig. 300, that the path of $D$ is like an ellipse, the lower end of which is blunter than the top, and also that they have noted the character of $A$'s path. The study of $D$'s motion is the best preparation for the study of $A$'s.

The centre line of engine in the figure is vertical. Positive vertical displacement is downwards. Positive horizontal displacement is to the right.

1st. Let $E$ have only a horizontal motion and let $B$ move in a straight slot. We seek for the octave only. What is the amplitude of $C$'s vertical motion, and when is it a maximum downwards? Or when $P$ is at its dead point, nearest $K$, what is $C$'s displacement downwards?

I. (1) $D$ horizontally has no octave. Vertically its fundamental motion is that of $P$; vertically downwards $D$ has $K$'s octave diminished, or an amplitude $\frac{DP}{KP} \frac{K^2}{4L}$, and it is at its maximum downwards when $\theta$ is 90.

[1] Mr. Harrison, whose excellent paper (*Proc.* Inst. C.E., 1893), ought to be referred to, has pointed out to me that the octave due to the shortness of rod $AB$, Fig. 299, is really $\frac{AC}{AB} \cdot \frac{r^2}{4AB}$ tan $a$ (cos $2\theta$ − tan $a$ . sin $2\theta$).

II. (2) $E$ has $D$'s horizontal motion, with an octave of amplitude $\frac{R^2}{4ED}$ which reaches its maximum to the right when $D$ is most up and down, that is at $P$'s dead points.

III (1) $A$ has $D$'s horizontal motion with the addition of an octave, a fraction of $E$'s, or one with an amplitude $\frac{AD}{ED} \cdot \frac{R^2}{4ED}$. This gives to $B$ a vertical octave of amplitude

$$\frac{AD}{ED} \frac{R^2}{4ED} \tan a$$

which reaches its maximum downwards at the dead points.

IV. (2) $A$ has, vertically, a fraction $\frac{AE}{ED}$ of $D$'s whole vertical motion and of course of its octave; that is, the vertical octave of $A$ has an amplitude $\frac{AE}{ED} \cdot \frac{DP}{KP} \frac{R^2}{4L}$ and it is at its maximum downwards when $\theta$ is 90. Also $A$'s vertical fundamental motion of amplitude $\frac{AE}{ED}$ $R$ produces a horizontal octave of $B$, of amplitude $\left(\frac{AE}{ED} R\right)^2 \div 4\,AB$ and multiplying this by tan $a$ we get a vertical octave in $B$ which reaches its maximum when $\theta = 90°$.

Now an octave which is at its maximum positively when $\theta = 90$ is at its maximum negatively when $\theta = 0$.

Considering the vertical octaves of $A$ and $B$ we see that $A$ has an octave whose amplitude is $\frac{AE}{ED} \frac{DP}{KP} \frac{R^2}{4L}$ and reaches this value negatively when $\theta = 0$.

$B$ has octaves $\frac{AD}{ED} \frac{R^2}{4ED}$ tan $a$, max when $\theta = 0$.

$\frac{AE^2 . R^2}{4 . ED^2 . AB}$ tan $a$, negatively max when $\theta = 0$.

Hence $C$ has an octave of amplitude.

$$\frac{AC}{AB}\left(\frac{AD}{ED} \frac{R^2}{4ED} - \frac{AE^2}{4ED^2} \frac{R^2}{AB}\right) \tan a - \frac{CB}{AB} \frac{AE}{ED} \frac{DP}{KP} \frac{R^2}{4L}$$

which reaches this value downwards at the dead points. Therefore, this gear will produce a motion like what is shown in Fig. 296. If the rod working the valve is of length $\lambda$ and if the half travel of $A$ horizontally is $r$, there is another octave $\frac{r^2}{4\lambda}$ sin $(2\theta - 90°)$.

It is evident that this sort of work is more tedious to read than to work out by oneself.

In either the Marshall or Joy gear we have already seen the **effect of curving the slot.**

What is the effect of $E$ moving in an arc instead of a straight line? Evidently $E$ has a vertical octave; $A$ has the fraction $\frac{AD}{ED}$ of this, and $C$ has the fraction $\frac{CB}{AB}\frac{AD}{ED}$ of it. We can make it reach either a + or − maximum at the dead point by having the swinging link which carries it, centred above or below $E$.

When the point $C$ is not exactly in the straight line connecting $A$ and $B$ or in $AB$ produced we get an effect to which I have not referred, but which it is quite easy to study by skeleton drawing and the method of Art. 316.

**325. Octaves in Link Motions.** Probably tens of thousands of skeleton drawings have been made showing the motion of a valve worked from linkages, but we have had no systematic study of valve motions leading to easy rules. I venture to think that my method of studying the *octave* will yield good results. Unfortunately I have never yet taken up the subject thoroughly; every session when I have been on the point of obtaining simple generalisations from my students' work, other matters have claimed my attention. What I shall give here is useful, but only in the way of suggestion.

My method is this: first, study the motion to find the fundamental S. H. motion as in Arts. 306–8. Now make a skeleton drawing, tabulate the displacements for twenty-four equidistant positions of the crank and find the octave as in Art. 316. Alter the motion and see what its effect is upon the octave, and compare the result with the considerations of Art. 316. It would not, indeed, add greatly to the work to find in each case the terms in $\theta$, $2\theta$, and $3\theta$.

It is true that a person expert in dealing with trigonometrical expressions might be able to obtain the terms by making judicious approximations; unfortunately the very qualities that go with expertness in mathematics are usually those that prevent a man's being able to judge as to what terms he may, or may not, reject during the working out of a practical problem. I venture to offer the following as a suggestive method of dealing with links.

**326. Gooch Link Motion.**—*Open Eccentric Rods.*—Assume that the link $CC^1$, Fig. 308, is straight and that its middle point $F$ or $G$ has a horizontal motion in $OO^1$, $AA^1$ is the symmetrical position of the link; $AB$, $A^1B^1$ are the lines joining $A$ and $A^1$ with the two eccentric centres, when symmetrical, each making the angle $\beta$ with the line of centres $OO$. Let $CFO^1$ be $\psi$.

Find $y$ or $PQ$ the horizontal displacement of a block which keeps at the

distance $GQ=a$ from $OO^1$. Let the eccentricity of each eccentric be $r$ and length of link $AA^1=CC^1=2\lambda$.

**Approximation (1).** Assume that if the displacements $AC$ and $A^1C^1$ are projected on $NM$ and $N^1M^1$ we get $AB=x$ and $A^1B^1=x^1$, which are the simple

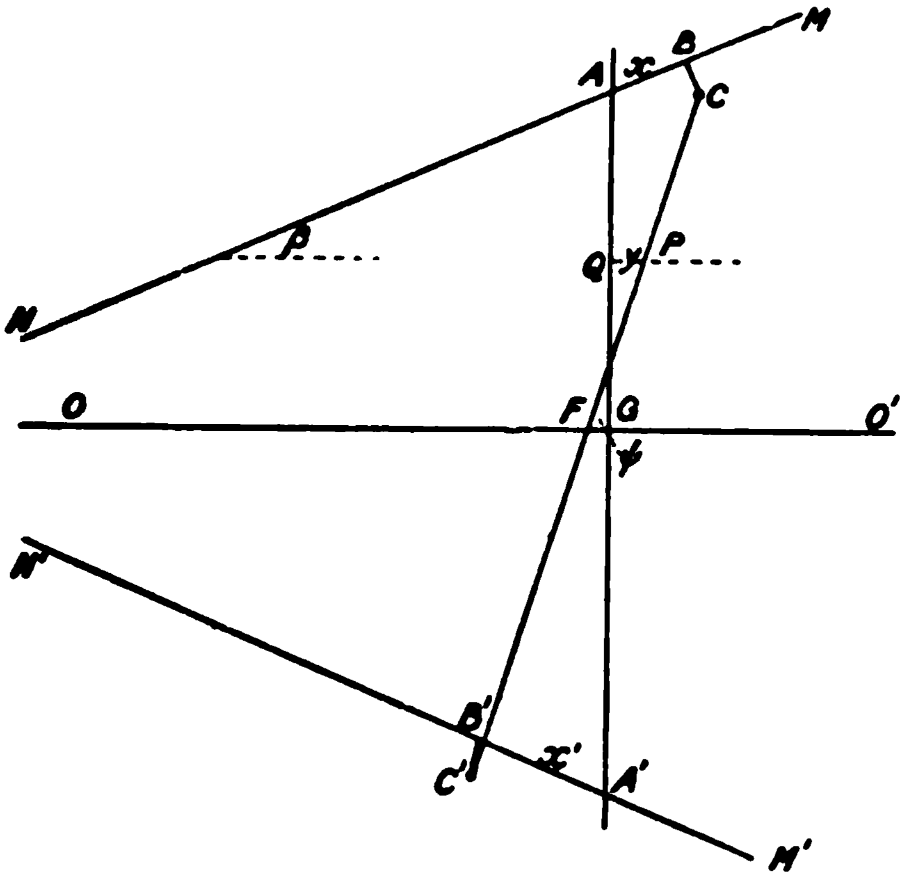



FIG. 308.

harmonic displacements which would occur along these lines, the eccentric rods being assumed to be infinitely long. In fact, if $a$ is the advance of either eccentric and $\theta$ is the angle which the main crank makes with its dead point position remote from the link in the direction of motion of the hands of a watch, and neglecting the octaves which are very small

$$x=r\{\sin(a+\beta+\theta)-\sin(a+\beta)\}$$

$$x^1=r\{\sin(a+\beta)-\sin(a+\beta-\theta)\}$$

Notice that $x^1$ is to the left and $x$ to the right.

By projecting horizontally and vertically,[1] or by simple geometry, making $CC^1$ the hypotheneuse of a right angled triangle with horizontal and vertical sides, joining $F$ with the right angle and projecting the horizontal base upon $N^1M^1$; or in other ways; it is easy to show that

$$\cos\psi=\frac{x+x^1}{2\lambda\cos\beta},\quad FG=\frac{\lambda\{\cos(\psi-\beta)-\sin\beta\}-x}{\cos\beta}$$

so that $y=a\cot\psi-FG$.

---

[1] By projection we get

$$\lambda\sin\psi=\lambda+x\sin\beta-CB\cos\beta$$
$$\lambda\cos\psi=FG+x\cos\beta+CB\sin\beta$$
$$\lambda\sin\psi=\lambda-x\sin\beta+C^1B^1\cos\beta$$
$$\lambda\cos\psi=-FG+x^1\cos\beta+C^1B^1\sin\beta$$

Eliminate $CB$ and $C^1B^1$

$$\lambda\cos(\psi-\beta)=\lambda\sin\beta+FG\cos\beta+x$$
$$\lambda\cos(\psi+\beta)=-\lambda\sin\beta-FG\cos\beta+x^1$$

$$2\lambda\cos\psi\cos\beta=x+x^1$$

Approximation (2). Let $\cot\psi = \dfrac{\cos\psi}{\sin\psi} = \dfrac{\cos\psi}{\sqrt{1-\cos^2\psi}} = \cos\psi(1+\frac{1}{2}\cos^2\psi)$

because $\cos\psi$ is a small quantity; then if we let

$$\frac{x}{\cos\beta} \text{ and } \frac{x^1}{\cos\beta} \text{ be called } X \text{ and } X^1$$

$$\cos\psi = \frac{1}{2\lambda}(X+X^1)(1+\tfrac{1}{2}\cos^2\psi)$$

$$FG = -X + \tfrac{1}{2}(X+X^1) + \lambda\,\frac{\sin\psi\sin\beta - \sin\beta}{\cos\beta}$$

Approximation (3), $-\dfrac{\lambda}{\cos\beta}\sin\beta(\sin\psi-1) = \frac{1}{2}\lambda\tan\beta\cos^2\psi$

and hence

$$y = \frac{a}{2\lambda}(X+X^1)(1+\tfrac{1}{2}\cos^2\psi) + X - \tfrac{1}{2}(X+X^1) + \tfrac{1}{2}\lambda\tan\beta\cos^2\psi$$

Now on the ordinary rough theory of Art. 306 the value of $y$ is

$$\frac{\lambda+a}{2\lambda}X - \frac{\lambda-a}{2\lambda}$$

Hence if I use $y^1$ to mean our new $y$ − old roughly approximate $y$;

$$y^1 = \left\{\frac{a}{4\lambda}(X+X^1) + \tfrac{1}{2}\lambda\tan\beta\right\}\left(\frac{X+X^1}{2\lambda}\right)^2$$

It is to be noted that $\alpha+\beta$ is what may be called the *true* advance of the ends of the link. If we let $X+X^1$, which is $2r\sin\theta\,\dfrac{\cos(\alpha+\beta)}{\cos\beta}$ be called $\mu\sin\theta$, we have

$$y^1 = \frac{a\mu^3}{16\lambda^3}\sin^3\theta + \tfrac{1}{8}\frac{\mu^2}{\lambda}\tan\beta\,.\,\sin^2\theta$$

Now $\sin^2\theta$ is $\frac{1}{2}-\frac{1}{2}\cos 2\theta$, and $\sin^3\theta$ is $\frac{3}{4}\sin\theta - \frac{1}{4}\sin^2\theta$

Hence neglecting the constant term $+\dfrac{1}{16}\dfrac{\mu^2}{\lambda}\tan\beta$

$$y^1 = \frac{3}{64}\frac{a\mu^3}{\lambda^3}\sin\theta - \frac{a\mu^3}{64\lambda^3}\sin 3\theta - \frac{1}{16}\frac{\mu^2}{\lambda}\tan\beta\,.\,\cos 2\theta$$

$$\text{or } y^1 = \frac{3}{64}\frac{a\mu^3}{\lambda^2}\sin\theta - \frac{1}{16}\frac{\mu^2}{\lambda}\tan\beta\cos 2\theta - \frac{a\mu^3}{64\lambda^3}\sin 3\theta$$

Taking some usual numbers

$r = 3$ inches, $\alpha = 30°$, $2\lambda = 15$ inches, ecc. rods 24 inches.

$$\text{Sin }\beta = \frac{7{\cdot}5}{3+24} = {\cdot}278 \text{ nearly so that } \beta = 16°{\cdot}12,\ \tan\beta = {\cdot}2890$$

$$\mu = 2r\frac{\cos(\alpha+\beta)}{\cos\beta} = 6\frac{\cos 46°{\cdot}12}{\cos 16{\cdot}12} = 4{\cdot}33,\ \mu^2 = 18{\cdot}75,\ \mu^3 = 80{\cdot}14$$

$$y^1 = {\cdot}009a\sin\theta - 0{\cdot}0450\cos 2\theta - {\cdot}003a\sin 3\theta.$$

The terms in $\theta$ and $3\theta$ are really of no importance; they are symmetrical and produce the same effects for the two ends of the cylinder; they are small. The term in $2\theta$ is also small. What there is of it is just the reverse of what is shown in Fig. 296. But the longer admission on one side of the cylinder than the other is so little marked that we may almost take this gear to be completely represented by the rough theory of Art. 306. Notice that the octave ${\cdot}045\cos 2\theta$ is of the same amount for all grades of expansion, and is therefore most important where the fundamental motion is small, that is, at the high grades of expansion.

When the octave is so small as this, it is comparable with the [s]mall octaves in $x$ and $x^1$ which we neglected, and whose amounts are [k]nown to us from Art. 304, or from the considerations of Art. 320. [I]t is quite easy to calculate the addition, but I prefer to neglect it, [a]nd indeed, the whole octave is negligable, as we must not attempt [s]o much accuracy when the quantities are so small. The constant [t]erm in each case is very nearly the same, and if this were a real [v]alve gear I should calculate from $y^1$ the limits of motion of the [v]alve.

**327. Slipping of Block.** In any link motion it will be noticed that suspension by a reversing link means, that whatever slipping occurs has a frequency twice as great as the fundamental motion. If the amplitude of the slip is $s$ it evidently means that there is a part of the motion which is nearly

$$\frac{rs}{\lambda \cos \beta} \sin (2\theta + \gamma) \cos (a + \beta) \sin \theta,$$

where $a + \beta$ is the real angle of advance in full gear, and $\lambda$ is the half length of the link. This is because the effect is to be continually altering slightly the respective fractions of the end motions which any intermediate point possesses.

This is a small term which may be written as

$$\frac{rs}{2\lambda \cos \beta} \cos (a + \beta) \{\cos (\theta + \gamma) - \cos (3\theta + \gamma)\}.$$

As it involves $\theta$ and $3\theta$ and not $2\theta$ or $4\theta$, it is a symmetrical term which has no practical effect on the valve motion, slipping is only objectionable on account of the wear and tear that it produces. We see that it must greatly simplify our study of link motions if we can leave out of account all effects due to slipping of the block: specifying the motion of the valve as being practically the same as the horizontal motion of a point in the link, which is the *average* position of the block.

**328.** From considerations of the above kind it is easy to show that the octave is of no practical importance in any of the six kinds of link motion, if the middle of the link has truly a horizontal motion, and if the proportions are what they usually are in locomotives. It is only when the eccentrics have as great throws and lengths of link and short rods as I have only seldom seen them even in marine engines, that the octave is of practical importance if the middle of the link is guided to move nearly in a straight line. When indeed the paths of the points approach some of the shapes shown in Fig. 287, we always have important octaves. I should say, however, that the best way of obtaining an octave sufficiently large to be really useful would be to have short eccentric rods with large throws and a long link. It seems also that the construction for finding the octave for any position of the gear in any link motion is almost exactly the

same as that used in finding the fundamental (Art. 307, and I give such a graphical rule in the note)[1] if it were not for the considerations of Art. 305, which show that the method of suspension destroys the usefulness of any such rule.

**329.** The following results obtained by my students will show how important the usually neglected terms become when eccentric rods are short. In every case the forward and back eccentrics have throws of 3 inches, 30° advance, lengths of rods 12 inches, slot 10 inches long, radius of slot 12 inches.

The horizontal displacement $y$ of the block is measured from an arbitrary zero.

$$y = A + a \sin(\theta + \alpha) + b \sin(2\theta + \beta) + c \sin(3\theta + \gamma) + d \sin(4\theta + \delta).$$

The values of the constants are not given in the following table, when they are very small. The angles $\alpha$, $\beta$, $\gamma$, $\delta$ are given in degrees.

| | | | $A$ | $a$ | $\alpha$ | $b$ | $\beta$ | $c$ | $\gamma$ | $d$ | $\delta$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Gooch link. Open rods | Centre moving horizontally | ⅔ full gear | — | 3·22 | 57 | ·6 | 114 | | | | |
| | | Half ,, | — | 2·93 | 66 | ·4 | 106 | | | | |
| | | Mid ,, | — | 2·73 | 90 | ·3 | 90 | | | | |
| | Centre suspended from link 6⅝ in. long | ⅔ full gear | — | 3·09 | 57 | ·685 | 104 | | | | |
| | | Half ,, | — | 2·86 | 66 | ·525 | 120 | | | | |
| | | Mid ,, | — | 2·725 | 90 | ·325 | 90 | | | | |
| | Graphical rule, Art. | ⅔ full gear | — | 3·09 | 63 | | | | | | |
| | | Half ,, | — | 2·90 | 72 | | | | | | |
| | | Mid ,, | — | 2·74 | 90 | | | | | | |
| | In every position of the gear the centre has a horizontal motion | Full gear | ·065 | 3·09 | 26 | ·13 | 306 | ·025 | 90 | ·025 | [illegible] |
| | | Half ,, | ·09 | 2·83 | 58 | ·13 | 276 | ·025 | 90 | ·0175 | [illegible] |
| | | Mid ,, | ·14 | 2·72 | 90 | ·13 | 270 | ·015 | 90 | ·0075 | [illegible] |
| Stephenson link. Open rods | Centre of link hung from a reversing link 11″ long, itself supported from an 8″ arm | Full forward | ·125 | 3·175 | 21 | ·0765 | 316 | ·0955 | 78 | ·027 | [illegible] |
| | | Half ,, | ·085 | 2·86 | 54 | ·136 | 278 | ·071 | 116 | ·01 | [illegible] |
| | | Mid gear | ·148 | 2·716 | 88 | ·1335 | 273 | ·0535 | 164 | ·0075 | [illegible] |
| | | Half backward | ·095 | 2·781 | 60 | ·125 | 282 | ·041 | 324 | ·0185 | [illegible] |
| | | Full ,, | ·029 | 3·003 | 29 | ·1685 | 306 | ·04 | 246 | ·0315 | [illegible] |
| | Bottom of link hung from a reversing link 15″ long, itself suspended from an arm 8″ long | Full forward | ·0675 | 3·03 | 30 | ·202 | -27 | ·044 | 186 | | |
| | | Half ,, | ·1005 | 2·82 | 56 | ·0535 | -106 | ·067 | 99 | | |
| | | Mid gear | ·081 | 2·805 | 91 | ·255 | 58 | ·105 | -141 | | |
| | | Half backward | ·0775 | 2·95 | 55 | ·385 | -91 | ·0155 | 156 | | |
| | | Full ,, | ·436 | 3·39 | 17 | ·425 | -146 | ·0155 | 75 | | |
| | Graphical rule, Art | Full gear | | 3·00 | 30 | | | | | | |
| | | Half ,, | | 2·43 | 54½ | | | | | | |
| | | Mid ,, | | 2·225 | 90 | | | | | | |

[1] Graphical rule for the octave, assuming that $A$ and $B$ move in paths parallel to $E O F$, the line of centres of an engine. When the crank is at dead point $O D$, Fig. 308A, let the eccentrics be at $o a$ and $o b$ working the link $A B$. $G o a = \alpha = H o b$ the angles of advance. We have seen how to find the fundamental S.H.M. of $C$. Now to find its octave. Draw $G O H$ perpendicular to $D O E$.

As in Art. 303, make $O A' = C B'$ = length of eccentric rod. Let $A' O E$ be called

must confess that what I have put before students on this ct is not its complete study, but only suggestive of how it may udied. Mr. Harrison has now arranged for me a large model h may quickly become either a Stephenson, Gooch, or Allan motion, which automatically draws either its own oval diagram,

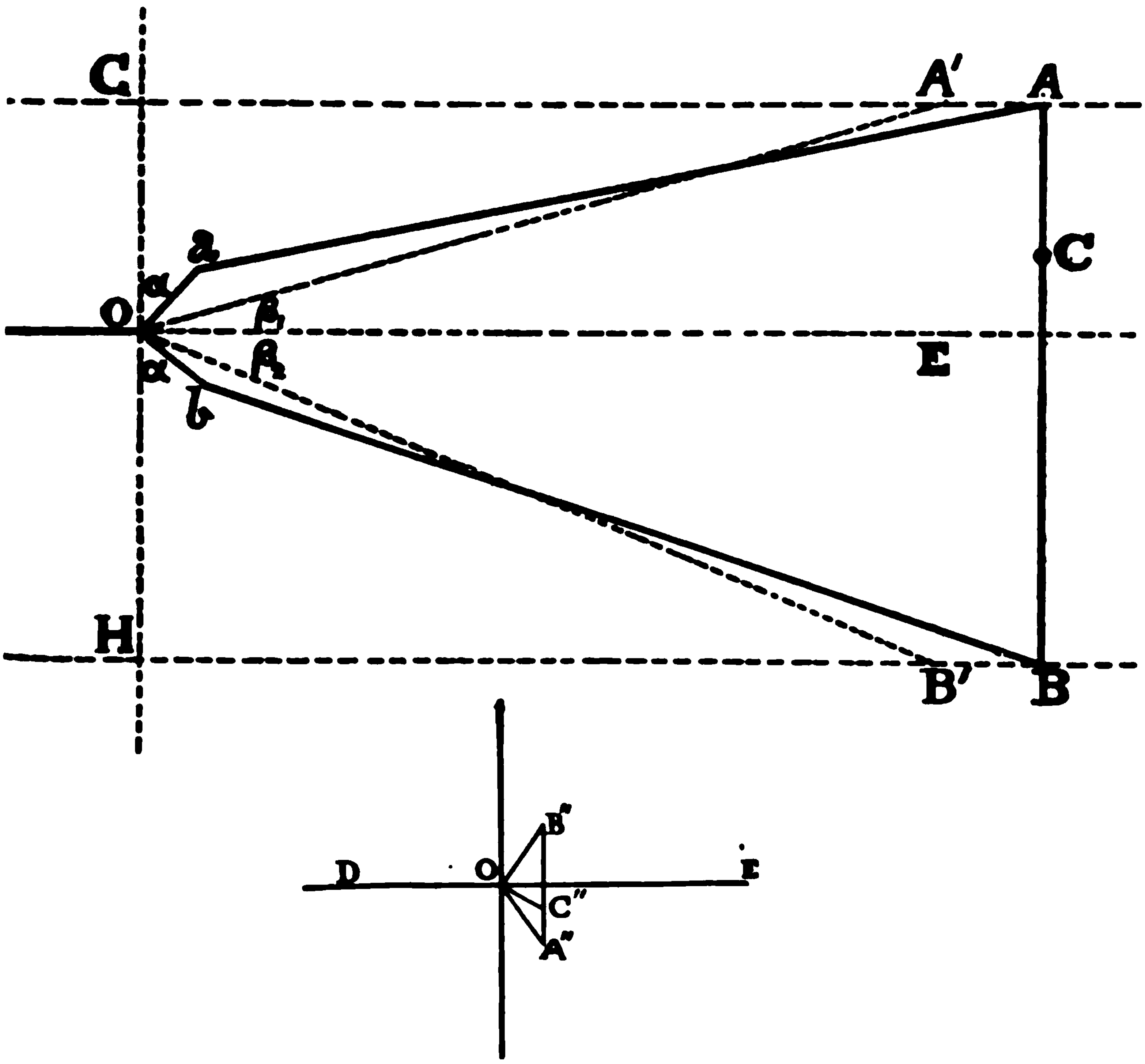



FIG. 309A.

curve showing $y$ and $\theta$. In a short time we hope to be in a tion to say with certainty exactly how the octave enters into the e motion with each of these gears with any method of suspension he link, but I am not sure that skeleton drawing may not be er, and I have shown how easy it is to get results by means of it.

nd let $B'OE$ be called $\phi_2$. Make $A''OE=2\ (\alpha+\phi_1)$, $B''OE = 2\ (\alpha+\phi_2)$. Let : $Ob = r$.

$$\text{Make } OA'' = r^2/4A'G,\ OB'' = r^2/4B'H.$$

oin $A''B''$ and divide in $C''$ in the proportion in which $C$ divides the link. Then lisplacement of $C$ from such a line as $GOH$ is a very nearly constant term, plus fundamental S.H. displacement found in Art. 303, together with

$$OC''\cos 2\ (\theta + EOC'').$$

# CHAPTER XXIX.

## INERTIA OF MOVING PARTS.

**330.** THERE are two kinds of problem worked by students.

1st. To find the forces acting at the cross head and crank pin in every position of the engine.

2nd. To find the forces acting between the earth and the frame of the engine, and to diminish them by balancing.

The first of these is very important if we consider the wear and tear of the engine. The changes in turning moment on the crank shaft are quite unimportant except in connection with torsional vibration.[1] The second has become very important, **because of the vibrations** set up in the ground or in a ship. In slow speed engines neither of them is of much importance.

**The general principle** of balancing may be put in this way Only for varying pressures in steam pipes, a very small matter, the resultant force in any direction on the frame work of the engine due to steam pressures is zero. The moment on the crank shaft does vary, see Art. 60; this may be reduced by the use of several cylinders. There is a moment acting, the nearly constant moment with which the machinery driven by the crank shaft resists motion, and this is balanced by a moment from the ground upon the frame. We need not now consider steady forces like this; we are concerned with forces due to relative motions of parts of the engine.

Now consider—if we had no friction, and no force of steam, and no external force—the engine revolving. Suppose its weight were exactly balanced; that it was free to move in any direction whatsoever. Then the frame will move in such a way that "the centre of gravity of the whole engine may not have any motion." This gives us one of the best points of view. For you will notice that in an actual engine we do not give to the frame the above freedom, and so we prevent its centre of gravity from keeping fixed. If we know the motion of the centre of gravity, we know from the simple law of motion what forces must be exerted on the frame by the

[1] There are critical speeds at which these greatly tend to produce fracture.

earth. Now what we aim at in balancing is this, that as an engine moves, its centre of gravity shall remain in the same point relatively to the frame of the engine. There is another condition also to be fulfilled—the moment of momentum of the engine about any axis must remain constant.

Another way of putting it is this:—

If any portion of mass $m$ has acclerations $\ddot{x}$, $\ddot{y}$, $\ddot{z}$ in three directions, regard $m\ddot{x}$, $m\ddot{y}$, $m\ddot{z}$ as forces in the three directions, the resultant of all such forces is the force exerted on the whole engine by outside bodies; or as the frame of the engine is itself fixed, it is the force with which the frame acts on the moving part. Or if we find the resultant for any part or parts of the engine this is the force with which outside things act on this part or parts.

For every moving part we have forces. A piston, piston rod, and cross head move together and may be considered together as giving rise to or requiring forces in one direction, and every sliding piece has to be considered in the same way. Rotating pieces are easily balanced by each other or with the help of pieces put on for balancing purposes. Pieces like connecting rods give most trouble, because of their curious angular motions.

**331. Balancing Rotating Parts.** Any portion of stuff of mass $m$, whose centre of gravity revolves at $v$ feet per second in a circle of radius $r$ feet, exerts a centrifugal force $mv^2/r$ pounds radially; and we know that an equal and opposite centripetal force of this amount must be acting upon the body. If the body has an angular velocity of $a$ radians per second, the force is $ma^2r$. If we apply this rule to every small portion of a rotating body, so as to get the loads due to centrifugal force, we can afterwards calculate the stresses produced. In this way we find the strengths of rotating objects such as fly wheels and coupling rods. Also we find the forces which must be exerted at the bearings to balance the centrifugal forces: we have easy problems in statics which may be worked graphically or arithmetically. If the axis of rotation passes through the centre of gravity of the whole of a body attached to a shaft with two bearings, the pressure on one bearing (due to centrifugal force) is at every instant equal and opposite to the pressure on the other, and by placing masses in proper positions the pressures on both bearings may be reduced to nothing. Thus, for example, if the centres of gravity of two masses are directly opposite to one another on a shaft, they may be made to balance. When not opposite they do not balance, but two masses may balance one, which is directly opposed to the resultant force of the two.

EXERCISE. Show that if there are masses $A$ and $B$, whose centres of gravity are at distances $OA$ and $OB$ from the axis $O$ in a plane at right angles to the axis, they produce the same effect as a mass 1 at $OC'$, if $OC'$ is the diagonal of the parallelogram of which $OAA^1$ and $OBB^1$ are the sides, where $OA^1 = A \cdot OA$, $OB^1 = B \cdot OB$, and that we may use a mass $C$ at $C$ in the line $OCC^1$ if $C \cdot OC = OC^1$.

EXERCISE. Show that a mass $A + B$, in the position of the centre of gravity of $A$ and $B$ will produce the same effect.

EXERCISE. Show that if there are masses $A$, $B$, $C$, $D$, &c., on a wheel, then a mass $A + B + C + D +$, &c., in the position of the centre of gravity of $A$, $B$, $C$, $D$, &c., will produce the same centrifugal force.

It is interesting to mount an axle to which a wheel is keyed, upon a not very rigid frame; fix a small mass on the wheel anywhere, and rotate rapidly. Even with small weights the effects of want of balance are very evident, and it is very easy by attaching other weights to the same wheel to show the principles of balancing, It does not at first come home to a student that the effect of centrifugal force in a badly balanced machine may be very great, and so he ought to work a few exercises like the following.

EXERCISE. What is the centrifugal force due to a body of 20 lbs. at 3 feet from an axis, revolving at 500 revolutions per minute?

*Answer.* $ma^2r$ becomes $wrn^2 \div 2{,}937$ if $w$ is weight in pounds and $n$ revolutions per minute. Hence we have a force of $20 \times 3 \times 25 \times 10^4 \div 2{,}937$ or 5,122 lbs. acting in every direction as the mass whirls round.

EXERCISE. A connecting rod 5 feet long, crank 1 foot. The connecting rod weighs 400 lbs., and its centre of gravity is $2\frac{1}{4}$ feet from the crank pin; we take it that in many inertia effects, it may be regarded as consisting of $\frac{2\frac{3}{4} \times 400}{5}$ or 220 lbs. situated at the crank pin. and $\frac{2\frac{1}{4} \times 400}{5}$ or 180 lbs. situated at the cross head. The crank (including the non-symmetrical part of the shaft near the crank) weighs 150 lbs., and its centre of gravity is 4 inches from the axis; this is equivalent in its centrifugal force to $150 \times \frac{4}{12}$ or 50 lbs. existing on the crank pin. Altogether, then, we have $220 + 50$ or 270 lbs. on the crank pin. What is the centrifugal force due to this when the speed is 250 revolutions per minute?

*Answer.* $270 \times 1 \times 250^2 \div 2{,}937 = 5{,}745$ lbs.

**332.** When a crank goes round uniformly, if the connecting rod

were infinitely long, **the motion of the sliding mass** would be simple harmonic. In this case the acceleration of the mass is always directed towards the middle of its path; it is proportional to distance from the middle, being greatest at the ends, and at the ends it is equal to the centripetal acceleration of the crank pin.

EXERCISE. If the piston and cross head weigh 460 lbs., and we include the above 180 lbs.; if the connecting rod were infinitely long; what are the forces due to the reciprocating motion at the end of the stroke?

*Answer.* Exactly equal to the centrifugal force of the same mass at a radius equal to that of the crank pin; or 13,620 lbs.

If we speak of the line of action of the engine as horizontal, note that the reciprocating forces are horizontal, and cannot be exactly balanced except by other reciprocating forces.

A mass $M$, with simple harmonic motion of amplitude $r$, may be exactly balanced just at the ends of the stroke. To do this we regard it as a mass $M$ on a crank pin $r$. But we have merely converted a horizontal action into an equal vertical action; all horizontal forces are balanced, but the vertical forces due to the balance weight are unbalanced. As the cross head of an engine has not a simple harmonic motion, we cannot balance even in this way all the horizontal forces. In a locomotive it is thought well to balance all the horizontal forces [a common English rule is to balance only two-thirds of the reciprocating forces in this way], and as this can be done approximately by rotating pieces, which, however, introduce vertical forces of their own, we put up with these as being less pernicious than horizontal forces. There can be no doubt that when this is done so that the horizontal forces alone are balanced, there is less of a tugging action, and consequently the coal bill is considerably diminished. One great objection to the method is that the pressure of the wheel on the rail varies greatly. For example, the highest speed of an English locomotive was attained in 1885; it was 85 miles per hour [same highest in America: greatest average speeds for over 100 miles were—English, 64·1; American, 64·9]. The driving wheel was 85 inches in diameter. EXERCISE: Show that, disregarding slip, the highest speed was 340 revolutions per minute; also, taking the above balance weight, the lifting force on each wheel was 10,630 lbs., or nearly 5 tons every revolution. Now this in itself would greatly produce slipping and make it exasperatingly difficult for a driver to get a greater speed, but the effect may be enormously magnified as the forced vibrations get to be more in time with the natural vibrations of the engine. The highest speeds can really only

$BA$ represents its velocity to such a scale that $GO$ represents $v$ or $qR$ or $\frac{2\pi n}{60}R$ feet per second; $BG^1$ represents the acceleration to

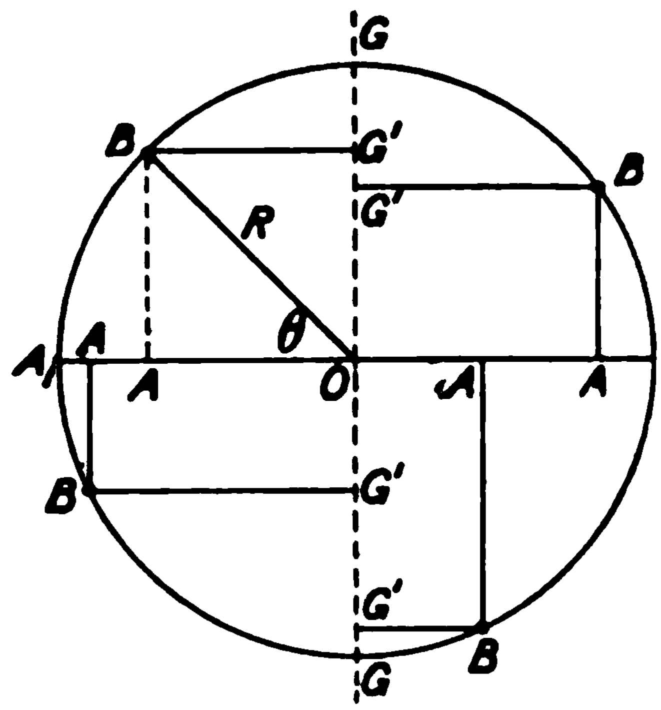



FIG. 309.

such a scale that $A_1O$ represents $v^2/R$ or $q^2R$ or $\frac{4\pi^2n^2}{3{,}600}R$ feet per second per second.

2. If the connecting rod is of length $l$. Let the slider be to the left hand of $A_1$. Draw an arc $HOH$ Fig. 309A with a radius equal to

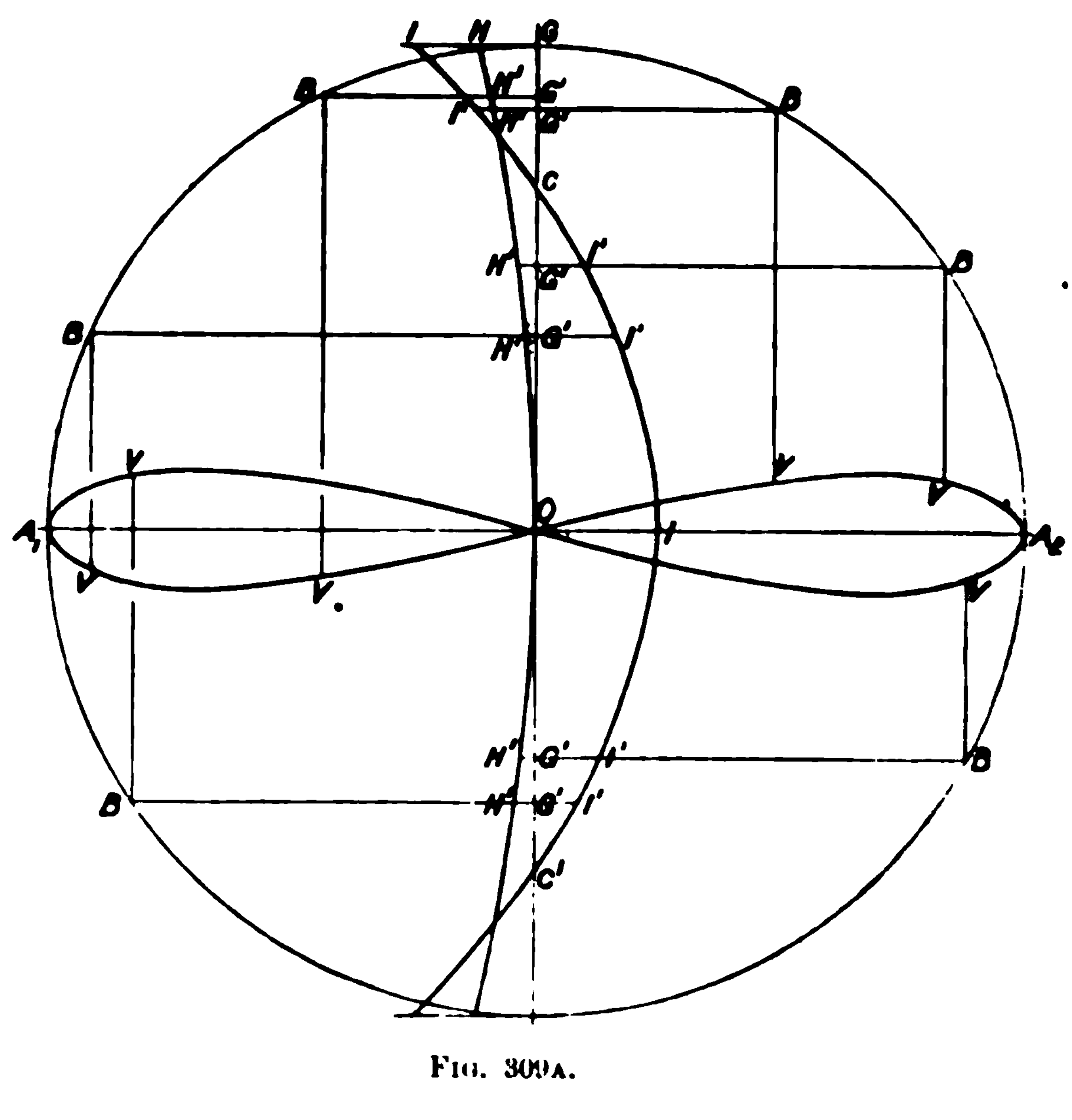


FIG. 309A.

the length of the connecting rod, its centre in $OA_1$ produced. Make $GI = OI = GI = 2GH$; also take it that there is no correction

needed when the crank is at 45° from its dead point. That is, find the points $C$ and $C^1$ such that $OC$ or $OC^1 = \cdot 707\, OG$. Now draw a curve $ICIC^1I$. If the connecting rod is five or more times the crank, an arc of a circle through $III$ will do very well. But, if the rod is of less length, the curve $ICIC^1I$ is not an arc of a circle, it is more nearly that of a parabola. Anyhow, it is easy to draw, since we have five points in it and know its symmetrical shape.

We may take it that for any position $B$ of the crank pin; $x$ the displacement of the slider from its mid position is not represented by $BG^1$, but by $BH^1$; and if the rod is not less than three and a half times the crank, or even if it is a little less, the acceleration is not represented by $BG^1$ but by $BI^1$. The construction for the velocity is not so simple. An approximate rule like this is only of importance during the study of this subject, as a useful way of putting one's ideas. It is hardly needed for practical purposes. The rule for the

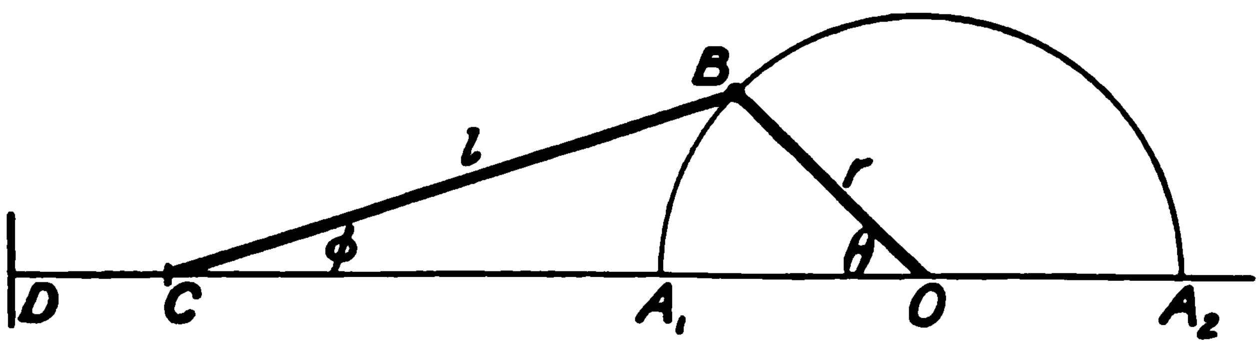



FIG. 310.

displacement is of course correct, and is well known. I have used a loop curve which takes the place of the line $A_1OA_2$ for velocity measurements, but these are not nearly so often required as the other two; besides, it is not so easily remembered. The distance $BV$ represents the velocity.

**337.** The following algebraic work will enable a student to look at the matter from another point of view, and ought to be used to test the above rule.

Let $DC = s$ (Fig. 310).

Projecting on the line of centres and at right angles to this we get

$$\left.\begin{array}{c} s + l\cos\phi + r\cos\theta = l + r \\ l\sin\phi = r\sin\theta \end{array}\right\} \quad . \quad . \quad . \quad . \quad (1)$$

As from the second of these

$$\sin\phi = \frac{r}{l}\sin\theta, \cos\phi = \sqrt{1 - \frac{r^2}{l^2}\sin^2\theta}$$

substituting in the first we eliminate $\phi$ and get

$$s = r(1 - \cos\theta) + l\left\{1 - \sqrt{1 - \frac{r^2}{l^2}\sin^2\theta}\right\}. \quad . \quad (2)$$

Or if $x$ is the distance to the left of the middle of the stroke, so that $r - s = x$, and if $\theta = qt$ where $q$ is the angular velocity in radians per second.

$$x = r\cos qt - l\left\{1 - \sqrt{1 - \frac{r^2}{l^2}\sin^2 qt}\right\} \quad . \quad . \quad . \quad (3)$$

If $l$ is not less than 5 times $r$ we may treat $\frac{r^2}{l^2}\sin^2 qt$ as so small a quantity that $\sqrt{1 - a} = 1 - \frac{1}{2}a$ . and

$$x = r\cos qt - \frac{r^2}{2l}\sin^2 qt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

But $2\sin^2 qt = 1 - \cos 2qt$, and hence

$$x = r\cos qt - \frac{r^2}{4l}(1 - \cos 2qt) \quad . \quad . \quad . \quad . \quad (5)$$

To the student of periodic motions in general, this form is very satisfactory. He sees that the motion of a slider worked from a uniformly rotating crank by a connecting rod is a simple harmonic motion of frequency $f = \frac{q}{2\pi}$ or of periodic time $\tau = \frac{2\pi}{q}$, together with an octave, as a musician might call it, a harmonic of twice the frequency and of smaller amplitude. The velocity $v$ and acceleration $a$ are then

$$v = \frac{dx}{dt} = -qr\left\{\sin qt + \frac{r}{2l}\sin 2qt\right\} \quad . \quad . \quad . \quad (6)$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -q^2r\left\{\cos qt + \frac{r}{l}\cos 2qt\right\} \quad . \quad . \quad (7)$$

Thus if $l = 5r$, we see that in the displacement $x$, the octave has an amplitude only one-twentieth of the fundamental; in the velocity the octave term is one-tenth of the fundamental. Whereas in the acceleration the octave term is as much as one-fifth of the fundamental. In fact, **any departure from simple harmonic motion** is very greatly accentuated in the acceleration; a matter of some importance to us in these days of high speeds of reciprocating machinery.

**338. Exercise for a Class** *of Students.*

Draw Fig. 309. When the main crank makes the angle $\theta$ with its dead point

$$x = r\left\{\cos\theta - \frac{r}{4l}(1 - \cos 2\theta)\right\} = BH, \text{ Fig. 311.}$$

$$\frac{dx}{dt} = v = -rq(\sin\theta + \frac{r}{2l}\sin 2\theta) = -BV$$

$$\frac{d^2x}{dt^2} = a = -rq^2(\cos\theta + \frac{r}{l}\cos 2\theta) = -BI.$$

Let $r = 1$, $q = 1$.

I. Let $l=5r$. Let a student calculate for various values of $\theta$, $BH$, $BI$, and $BV$, or rather let him plot the distances $HG^1$, $IG^1$, and $BV$. Take $\theta=0$, $10°$, $20°$, &c., right round to 360.

II. Let $l=4r$ and repeat.

III. Let $l=3r$ and repeat.

IV. Let $l=6r$ and repeat.

V. Let $l=10r$ and repeat.

In each case let him test with what accuracy the curve of Art. 336 represents his results, and to what extent he may depend upon correctness when an arc of a circle is used. Fig. 309A shows the result obtained by one of my students when $l/r=4$.

It is an excellent exercise for students to make a diagram, Fig. 312, in which the distances $BH^1$ are abscissæ and the distances $BI^1$ are ordinates, taking them from such a diagram as Fig. 309A. Or they may proceed as follows:—

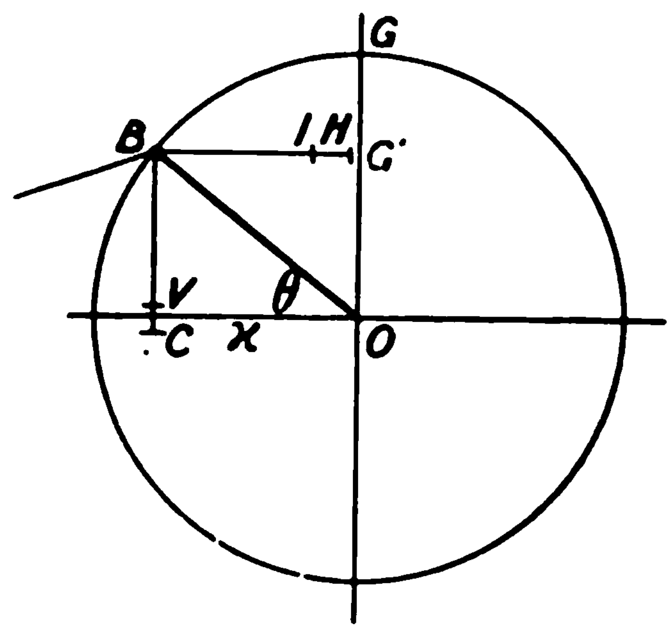



FIG. 311.

**339. *Accurate* Practical Rule.**—It will be found by the formula of Article 340 that the accelerations at the two ends of the stroke are, accurately,

$$\frac{v^2}{r}\left(1 \pm \frac{r}{l}\right)$$

being greater at the end remote from the crank; $v$ is the velocity of the crank pin; $r$ the length of the crank and $l$ the length of the connecting rod. Also, when $\theta$ is 90° (Fig. 310) the piston is the distance $l - \sqrt{l^2 - r^2}$ to the right of its mid-stroke, and its acceleration is then

$$r^2 \div \sqrt{l^2 - r^2}.$$

Thus in the case of Art. 58, if $x$ is distance to the *left* of the mid-stroke; if $r = 1{\cdot}25$ feet, $l = 6{\cdot}25$ feet, and the crank makes 120 revolutions per minute, we find

| $\theta$ | $x$ | accel. in feet per second per second. |
|---|---|---|
| 0 | 1·25 | -237 |
| 90 | -0·125 | 39·5 |
| 180 | -1·25 | 158 |

Many people merely recollect the accelerations at the ends, and assume that the acceleration is 0 when the crank and connecting rod

are at right angles. This rule is easy to remember, easy to apply and is wonderfully true except for very short connecting rods.

Sometimes the following exact rule is employed to get the intermediate point of no acceleration, but I am afraid that I am making too much of the matter, for in using these results I shall neglect friction and other things of much more importance than small errors here. $x$ is distance of piston to the left of mid-stroke when there is no acceleration, the crank being $r$ feet long; $l$ is the length of the connecting rod in feet.

| $l/r$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| $x/r$ | 0·153 | 0·132 | 0·109 | 0·091 | 0·078 | 0·068 | 0·060 | 0·054 | 0·050 |

To find these numbers from the formula of Art. 340 is an easy mathematical exercise.

If the connecting rod were infinitely long the acceleration in any position would be exactly proportional to distance from the

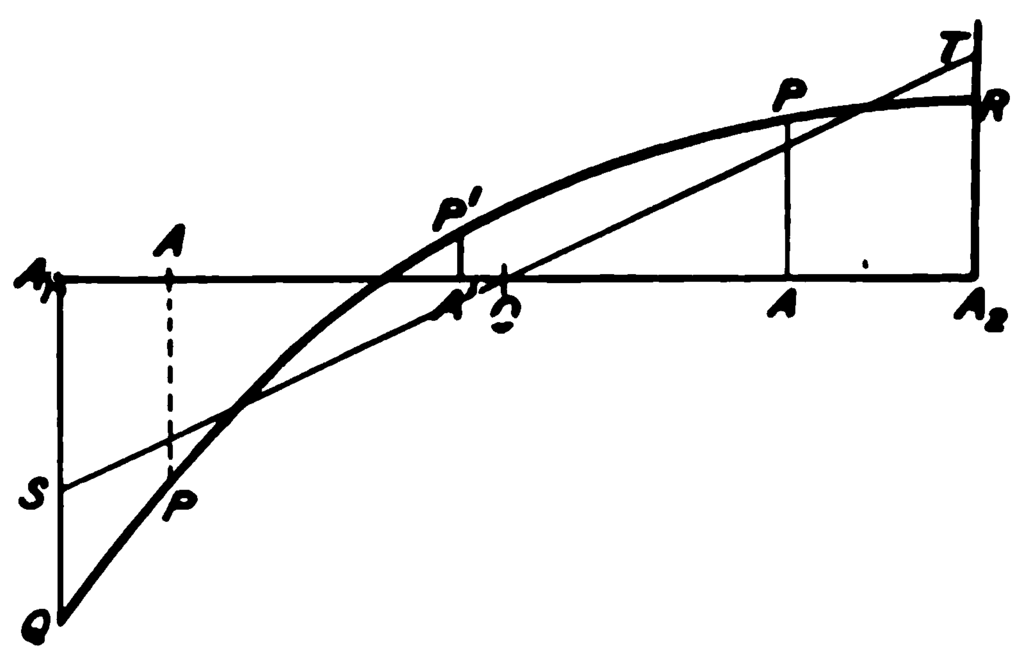



FIG. 312.

middle of the path, and would always be towards the middle; and the diagram of accelerations would be the straight line $ST$ (Fig. 312), $A_1S = A_2T$ representing 197·5 feet per second per second. If we take the case when $\frac{l}{r} = 5$ tabulated above, and $r = 1·25$ feet, the diagram of accelerations is the curve $QP'R$.

$A_1A_2$ represents the length of the piston stroke, $A_2$ being nearer the crank. I have made $A_1Q$ represent $-$ 237, $A_2R$ represents $+$ 158, and where $OA^1$ is 0·125 feet I have let $A^1P^1$ represent 39·5 and drawn by hand the curve $QP^1R$.

**340.** After equation (3) of Art. 337, we adopted an approximation

of great interest to students of periodic motions and mechanisms in general. But in the study of a particular mechanism, like the crank and connecting rod, it is sometimes thought well to do more than take a first approximation. For myself, I do not think it necessary to discuss small errors in this, regarding the many other things that we neglect, but for the sake of the weaker brethren I give the following construction:—

Starting with either (2) or (3), and differentiating twice and remembering as before that $\theta = qt$, we find the acceleration

$$a \text{ or } \frac{d^2x}{dt^2}\left(\text{or } -\frac{d^2s}{dt^2} \text{ as } x = r - s.\right)$$

$$\text{to be } -\frac{4\pi^2n^2r}{3600}\left\{\cos\theta + \frac{m\cos 2\theta + m^3\sin^4\theta}{(1 - m^2\sin^2\theta)^{3/2}}\right\} \quad . \quad . \quad . \quad . \quad (3),$$

when $m$ stands for $r/l$.

The student had better work out also

$$\frac{d^2\phi}{dt^2} = -\frac{4\pi^2n^2}{3600}\frac{m(1 - m^2)\sin\theta}{(1 - m^2\sin^2\theta)^{3/2}}.$$

The value of $\frac{d^2x}{dt^2}$ ought to be worked out for the following values of $\theta$. It is evidently the same for two values of $\theta$ equally distant from 0° or 180°. So that if we know it for $\theta = 40°$, it is the same for $\theta = -40$; if we know it for $\theta = 160°$, it is the same for $\theta = 200°$.

$$\theta = 0°, \text{ acceleration} = -\frac{4\pi^2n^2r}{3600}\left(1 + \frac{r}{l}\right)$$

$$\theta = 180°, \text{acceleration} = \frac{4\pi^2n^2r}{3600}\left(1 - \frac{r}{l}\right)$$

$$\theta = 45°, \text{acceleration} = -\frac{4\pi^2n^2r}{3600}\left\{1 + \left(\frac{2l^2}{r^2} - 1\right)^{-3/2}\right\}\frac{1}{\sqrt{2}}$$

$$\theta = 135°, \text{acceleration} = \frac{4\pi^2n^2r}{3600}\left\{1 - \left(\frac{2l^2}{r^2} - 1\right)^{-3/2}\right\}\frac{1}{\sqrt{2}}$$

$$\theta = 90°, \text{acceleration} = \frac{4\pi^2n^2r}{3600}\left(\frac{l^2}{r^2} - 1\right)^{-1/2}.$$

EXERCISE. Test the method of construction described in Art. 336. To do this, notice that the horizontal distance from the curve $III$ to $GOG'$ represents the term in the above expressions which differs from what there would be with an infinitely long connecting rod. Let $A_1O$, Fig. 309A, be called 1, and let $\frac{4\pi^2n^2r}{3600}$ be called 1; then it is easy to calculate that the horizontal distances from $III$ to $GOG$ are as follows:—distance from $I^1$ on the left to $G^1$ on the right being taken as positive.

| $\theta$ | $l/r=10$ | $l/r=6$ | $l/r=5$ | $l/r=4$ | $l/r=3$ |
|---|---|---|---|---|---|
| 0 | − ·100 | − ·167 | − ·200 | − ·250 | − ·333 |
| 45° | ·000 | − ·001 | − ·002 | − ·004 | − ·010 |
| 90° | + ·100 | ·169 | + ·204 | + ·258 | + ·353 |
| 135° | ·000 | − ·001 | − ·002 | − ·004 | − ·010 |
| 180° | − ·100 | ·167 | − ·200 | − ·250 | − ·333 |

If the student will try, he will see that the easy rule of Art. 336, is sufficiently accurate for all practical purposes until the connecting rod is less than four times the length of the crank. Even when it is so short as only three times the crank, the error is not great.

**341.** Many **geometrical constructions** have been given. I do not say that the following one is better than another. I am no good judge, because I never use any of them myself. Indeed, I do not like to see a student using any of them, as I consider the very simple method of Art. 338 not only accurate enough, but very much better, because it keeps important general principles before one's mind.

$AB$ is the connecting rod, $BO$ the crank, $OA$ the line of centres. Produce $AB$ to meet in $F$ the perpendicular $OF$. With $J$ the middle of the connecting

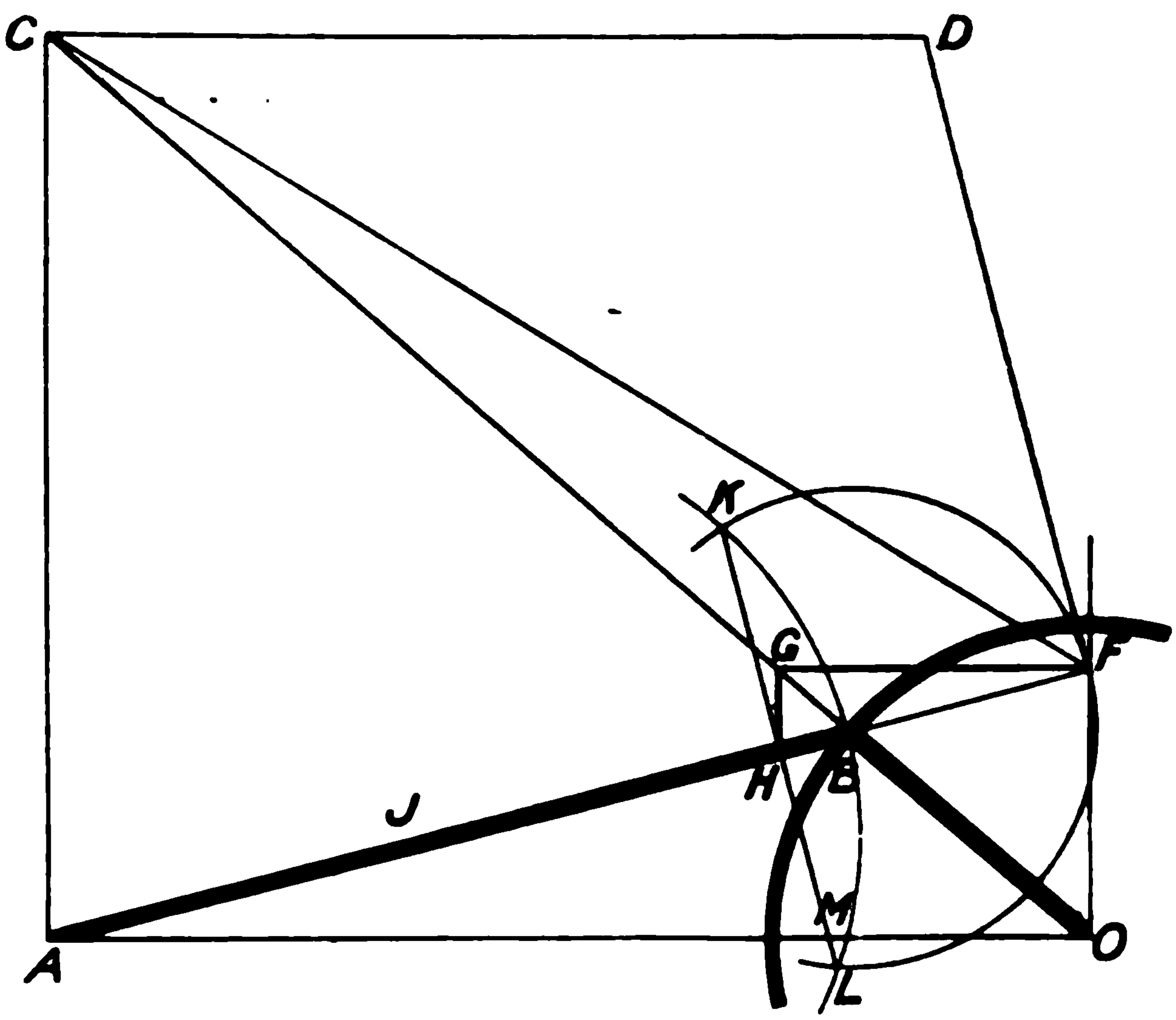



Fig. 313.

rod as centre, describe $KBL$ meeting the circle $KFL$ whose centre is $B$, in $K$ and $L$. Join $KL$, cutting $AO$ in $M$ and $AB$ in $H$.

To prove that

1. Velocity of piston in feet per second $= q \cdot FO$ if $FO$ is measured in feet.

2. Acceleration of piston in feet per second per second $= q^2 \cdot MO$ if $MO$ is measured in feet.

In fact if $BO$ represents the centripetal acceleration of the crank pin, $MO$ represents the acceleration of the piston on the same scale.

3. Angular acceleration of connecting rod, or $\frac{d^2\phi}{dt^2} = q^2 \cdot HM/AB$.

Draw $AC$ perpendicular to $AO$, and let it meet $OB$ in $C$. Draw $CD$ parallel to $AO$, and $FD$ at right angles to $ABF$.

From $H$ draw $HG$ parallel to $OF$, and join $GF$. Prove as a geometrical exercise that $GF$ is parallel to $AO$.

Note that the figures $FGHMOF$ and $AOFDCA$ are similar, and $OB$ in the one is similarly placed to $CB$ in the other, so that $\frac{OB}{CB} = \frac{OM}{CD}$ or $\frac{OB \cdot CD}{CB} = OM$.

1. The rod $AB$ has $C$ for its instantaneous centre, for $CA$ is at right angles to $A$'s motion, and $CBO$ is at right angles to $B$'s motion, so that

$$\frac{\text{velocity } v \text{ of } A}{\text{velocity } V \text{ of } B} = \frac{CA}{CB} = \text{and this is evidently } \frac{FO}{BO}.$$

Now $V=q \cdot OB$, and hence the proposition is proved.

2. Since $v=q \cdot OF$, the acceleration is $a=q \cdot \frac{d(OF)}{dt}$, or $q$ times the velocity of the point $F$ away from $O$. The velocity of $F$ away from $O$ may be studied in this way. $F$ is a point in the connecting rod (produced), and $C$ is the instantaneous centre. If $\alpha$ is the angular velocity of the rod, the velocity of $F$ or $\alpha \cdot CF$ resolved along $BF$ and $OF$ will evidently give $\alpha \cdot DF$ and $\alpha \cdot CD$, if $FD$ is drawn at right angles to $AF$ and $CD$ is parallel to $AO$ or at right angles to $OF$ (in fact $FCD$ is a triangle of velocities whose sides are at right angles to the three velocities).

Now $\alpha = \frac{V}{CB}$, and hence acceleration $a$ of the piston

$$= q \cdot \frac{V}{CB} \cdot CD = q^2 \cdot \frac{OB \cdot CD}{CB} = q^2 \cdot OM.$$

3. We see that $\frac{\alpha}{q} = \frac{OB}{CB} = \frac{BF}{AB}$, or $\alpha = \frac{q}{AB} \cdot BF$.

Hence $\frac{d\alpha}{dt} = \frac{q}{AB} \cdot \frac{d}{dt}(BF)$. But we have already shown that the velocity of $F$ in the direction $BF$ is $\alpha \cdot FD$.

Hence $\frac{d^2\phi}{dt^2}$ or $\frac{d\alpha}{dt} = q\alpha \frac{FD}{AB}$.

$$q^2 \cdot \frac{OB}{CB} \cdot \frac{FD}{AB} = q^2 \frac{OB}{AB} \cdot \frac{FD}{CB} = q^2 \cdot \frac{OB}{AB} \cdot \frac{HM}{OB} = q^2 \cdot HM/AB.$$

**342. Forces on the Frame of an Engine.**—If it were possible to imagine the effect of the mass of the connecting rod to be the same as that of two masses at its ends, it would be easy to balance engines; it would also be very easy to make all sorts of calculations which are difficult to make in the real case. Now it is important to know to what extent the easy method of working is wrong. The student ought here to read again Art. 330.

If $P$ is the resultant force from left to right on the piston, Fig. 314; if the distance of the piston or cross head to the right of the end of its stroke is $s$; if $M$ is the total mass of the piston, and what is rigidly attached to it, then

$$P - M\ddot{s} = F$$

where $\ddot{s}$ is Newton's way of writing $\frac{d^2s}{dt^2}$,

is the resultant force acting on the brasses of the connecting rod at the cross head.

In estimating $P$ we may assume a knowledge of friction as well as of the indicator diagrams. Or what is more usual, neglect the friction altogether.

Now if we dare imagine that the connecting rod acts as if its mass existed as its ends only, in portions $m_1$ at cross head and $m_2$ on crank pin, inversely proportional to the distances of the centre of gravity from these ends; we can readily imagine the $m_2$ part balanced like any other rotating mass by other

rotating masses, and the only part needing balance is $m_1$. In fact, in such a case we may say that, neglecting the forces of gravity :—

The turning moment on the crank shaft is $\{P - (M + m_1)\ddot{s}\}\,OQ$ where $OQ$ is shown in Fig. 315.[1]

I shall now speak of the forces with which the ground acts upon the frame. As $m_2$ is supposed to be balanced, we see that :—

1. There is no total vertical force on the frame.
2. The horizontal force $(M + m_1)\ddot{s}$ can only be balanced by one or more equal and opposite forces. Now imagine this balance effected by a similar piston, cross-head, &c., exactly opposite to the first, as shown, for example, in Fig. 316; or by two such systems. Notice that for such exact balance the balancing systems cannot be on the same side of $O$, as $\ddot{s}$ must be the same.
3. If the balance (2) is effected, the balancing is complete; there is no couple acting on the frame.

Now in the real case the effect of the motion of the connecting rod cannot be imagined to be exactly the same as that of the two detached masses $m_1$ and

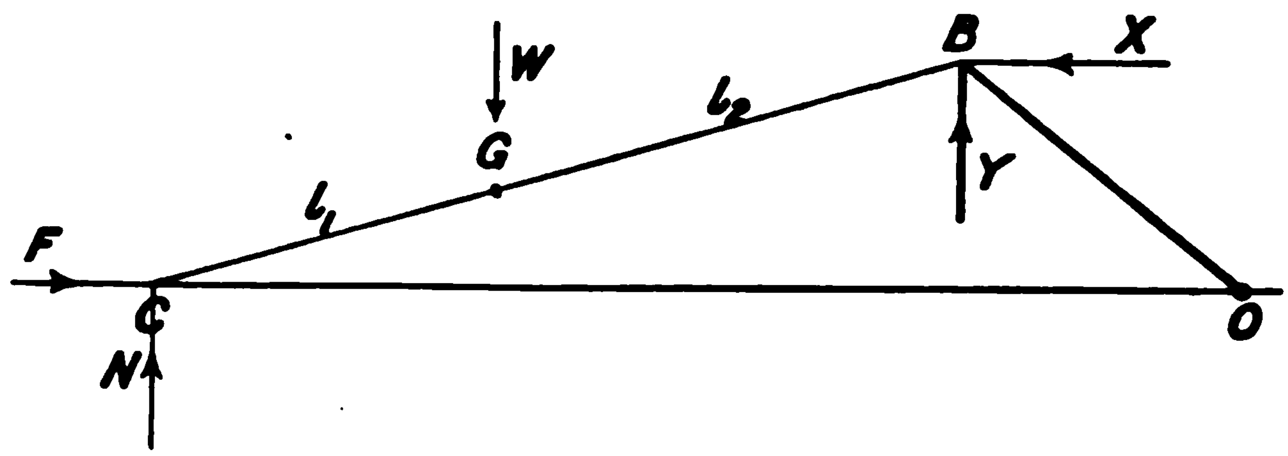



FIG. 314.

$m_2$, and this causes—*First*, an error in the above expression for the turning moment on the crank shaft : this error is not large ; in any case, fluctuations in the turning moment on the crank shaft are insignificant matters, except in very special cases. *Second*, an error which is only serious when we need very good balance : namely this, that in the real case, although the above statements 1 and 2 are correct, statement 3 is wrong. The student must see clearly what the amount of error in statement 3 is. I shall call it the **surging moment** on the frame. It is zero if the connecting rods are properly constructed.

**343. The Real Case.**—The figure (314) shows the connecting rod, $CB$, whose centre of gravity is at $G$; the resultant horizontal force $F$ acts at $C$; and $N$ is the normal component of the guiding force at $C$. Let us find $X$ and $Y$ the horizontal and vertical forces which must be exerted at $B$ to produce equilibrium.

I prefer always to use Newton's law (sometimes called three laws) of motion —a fundamental principle which cannot be forgotten if once learnt—whereas the many special rules which lead to quick working of exercises are readily forgotten. If the distance of $G$ horizontally to the right of some point is $x$, and if its vertical distance above the line of centres is $y$; the horizontal and vertical

[1] It may be worth while for the student to write out the exact mathematical expression for $\ddot{s}$. $OQ$ alone and to take a numerical example. Let him also work the following simple exercise :—Show that a mass $W$ lb. at the cross-head of a steam engine produces a turning moment of $-Wn^2r^2 \sin 2\theta / 5872$ pound feet if the rod is infinitely long and rotation uniform.

acceleration of $G$ may be written $\ddot{x}$ and $\ddot{y}$. Let $m$ be the mass of the connecting rod, or $W/g$ if $W$ is its weight.

$$X = F - m\ddot{x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$N + Y = m\ddot{y} + W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$Xl \sin\phi + Yl \cos\phi = I\ddot{\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

if $I$ is the moment of inertia of the rod about $C$, and $\phi$ is the angle $BCO$.

From (1) and (2) we see that the horizontal force $X$ and the *total* vertical force $N + Y$, depend only on the mass of the rod and the position of its centre of gravity, and therefore that in so far as these are concerned we may replace the rod with two masses, $m_1$ and $m_2$ at its ends, if $m_1 l_1 = m_2 l_2$.

From (3) and (1) $\quad Y = \dfrac{I\ddot{\phi} - l \sin\phi(F - m\ddot{x})}{l \cos\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$

If the actual calculations are made it is to be noted that

$$\ddot{x} = \frac{l_2}{l}\ddot{s} + \frac{l_1}{l} q^2 r \cos\theta, \quad \ddot{y} = -\frac{l_1}{l} r q^2 \sin\theta$$

$\ddot{s}$ being known from (3) of Art. 340 and $l = l_1 + l_2$.

It is easy to write out the expressions for the turning moment on the crank shaft and the centripetal force at $B$.

Now if $I^1$ is the moment of inertia with detached masses, $I^1 = m_2 l^2$. Whereas $I = m(k^2 + l_1^2)$. If $k$ is the radius of gyration about $G$.

Hence for perfect equivalence, since $m = m_1 + m_2$ and $m_1 l_1 = m_2 l_2$, it would be necessary to have $k^2 = l_1 l_2$.[1]

This cannot be effected unless the **connecting rod extends beyond** the cross head, or the crank pin, or in both ways, or if the mass of the rod be spread out laterally, as suggested by Mr. Harrison, a method of construction which might very well be used if the surging couple applied to the frame work and ground is to be done away with. [I find that Mr. Holroyd Smith has also made this suggestion.]

In any case it is only the $I\ddot{\phi}$ part of (4) which would be **different with the detached masses.** We know that if we have already obtained balance and calculated turning moment on the crank shaft, assuming detached masses, we have only now to consider that part of $Y$ which is represented by $(I - I^1)\ddot{\phi}/l \cos\phi$, or

$$\frac{m\ddot{\phi}}{l \cos\phi}(k^2 - l_1 l_2)$$

It is in the surging moment that the matter is really important, because there need be no surging moment with detached masses. The surging moment about any axis parallel to the crank shaft is

$$S = m(k^2 - l_1 l_2)\ddot{\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The extra value of $Y$ produces a turning moment on the crank shaft whose amount is

$$S \frac{r \cos\theta}{l \cos\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

---

[1] For $I = (m_1 + m_2)(k^2 + l_1^2) = m_2\left(\dfrac{l_2}{l_1} + 1\right)(k^2 + l_1^2) = m_2 l\left(\dfrac{k^2}{l_1} + l.\right)$

Hence $\quad \dfrac{k^2}{l_1} + l_1 = l = l_2 + l_1$, or $k^2 = l_1 l_2$.

**344. Example.** A crank is $r=1\cdot25$ feet long; the connecting rod 276 lbs. weight, $l=6\cdot25$ feet long, $l_1=3\frac{1}{3}$ feet, $l_2=2\frac{11}{12}$ feet, so that $l_1/l_2=\frac{8}{7}$; it has a radius of gyration about $G$ such that $k^2=l^2\div5^2=7\cdot8$ and $l_1l_2=9\cdot7$.

(5) Becomes $-\dfrac{276}{32\cdot2}(1\cdot9)\ddot{\phi}$, or $-16\cdot28\,\ddot{\phi}$

(6) Becomes $-3\cdot26\ddot{\phi}\dfrac{\cos\theta}{\cos\phi}$

If the speed is 120 revolutions per minute

$$\ddot{\phi}=-\frac{4\pi^2(120)^2}{3600}\,\frac{\frac{1}{5}(1-\frac{1}{25})\sin\theta}{(1-\frac{1}{25}\sin^2\theta)^{3/2}}$$

As we wish only to obtain a fairly correct notion of the effect we shall neglect the small terms and write

$$\ddot{\phi}=-31\cdot58\sin\theta,\text{ and }\cos\phi=1.$$

Hence (5) becomes $514\sin\theta$; (6) becomes $51\cdot6\sin2\theta$.

**345. Example.** Let the mass of the connecting rod of last example be replaced by two detached masses at its ends without alteration of its centre of gravity. There will be a mass of $\dfrac{276}{32\cdot2}\times\frac{7}{15}$ or $4\cdot00$ moving with the acceleration $\ddot{s}$ and a mass $\dfrac{276}{32\cdot2}\times\frac{8}{15}$ or $4\cdot56$ on the crank pin. Let the centrifugal force of this mass on the crank pin be balanced. The horizontal forces can only be balanced by other horizontal reciprocating masses. Let us study merely the turning moment on the crank. We must multiply the force at the cross head $4\cdot00\,\ddot{s}$ by $OQ$ in feet, Fig. 315. Or we may do the work numerically as follows:—

$$OQ=r\,\frac{\text{velocity of piston}}{\text{velocity of crank pin}}=\frac{rv\text{ of (6) Art.}}{rq}$$

$$OQ=1\cdot25\{\sin\theta+\tfrac{1}{10}\sin2\theta\}\text{ nearly}$$

also $$\ddot{s}=-197\cdot3\{\cos\theta+\tfrac{1}{5}\cos2\theta\}\text{ nearly}$$

| $\theta$ | $\ddot{s}$ acceleration of cross head. | $OQ$ in feet proportional to velocity of cross head | $4\cdot00\,\ddot{s}\cdot OQ$ turning moment on crank shaft. | Extra turning moment $51\cdot6\sin2\theta$. | Surging moment $514\sin.\theta$. |
|---|---|---|---|---|---|
| 0 | $-236\cdot8$ | 0 | 0 | 0 | 0 |
| 45° | $-139\cdot5$ | 1·01 | −564 | 51·6 | 363 |
| 90° | $+\ 39\cdot5$ | 1·25 | 197 | 0 | 514 |
| 135° | $+139\cdot5$ | 0·76 | 424 | · 51·6 | 363 |
| 180° | $+157\cdot8$ | 0 | 0 | 0 | 0 |

The figures in the last two columns show in what way the real case differs from the easily considered case of two detached masses. The extra turning moment on the crank shaft is of but little importance, but the surging moment is a most serious matter. To be sure in such an engine it is only about 1 per cent. of the greatest probable turning moment on crank shaft; but our speed was comparatively small and these effects increase as the square of the speed.

**346.** Students may be interested in the following interesting graphical construction for the **finding of a single force** which represents the resultant

of all the accelerating forces on a connecting rod. It is due to Mr. Harrison of the Royal College of Science. He uses first any of the well-known methods of expressing the acceleration of the cross head.

$AO$ line of centres ; $AB$ connecting rod ; $BO$ crank.

Produce $AB$ to $Q$ ($OQ$ is at right angles to $AO$) then $q.OQ$ is velocity of $A$.

Draw $QS$ parallel to $OA$, $SH$ parallel to $QO$, $Ha$ at right angles to $AB$.

Join $aB$. Then $OaB$ is a diagram of accelerations (see my "Applied Mechanics," Art. 476), that is, take any point $G$ in the rod, draw $Gg$ parallel to $AO$, then $g$ corresponds to $G$ in such a way that $gO$ represents in direction and magnitude the acceleration of $G$ to the same scale to which $BO$ represents the centripetal acceleration of the crank pin ; that is, if $gO$ is measured in feet the amount of the acceleration of $G$ is $q^2.gO$. If $G$ is the centre of gravity of the

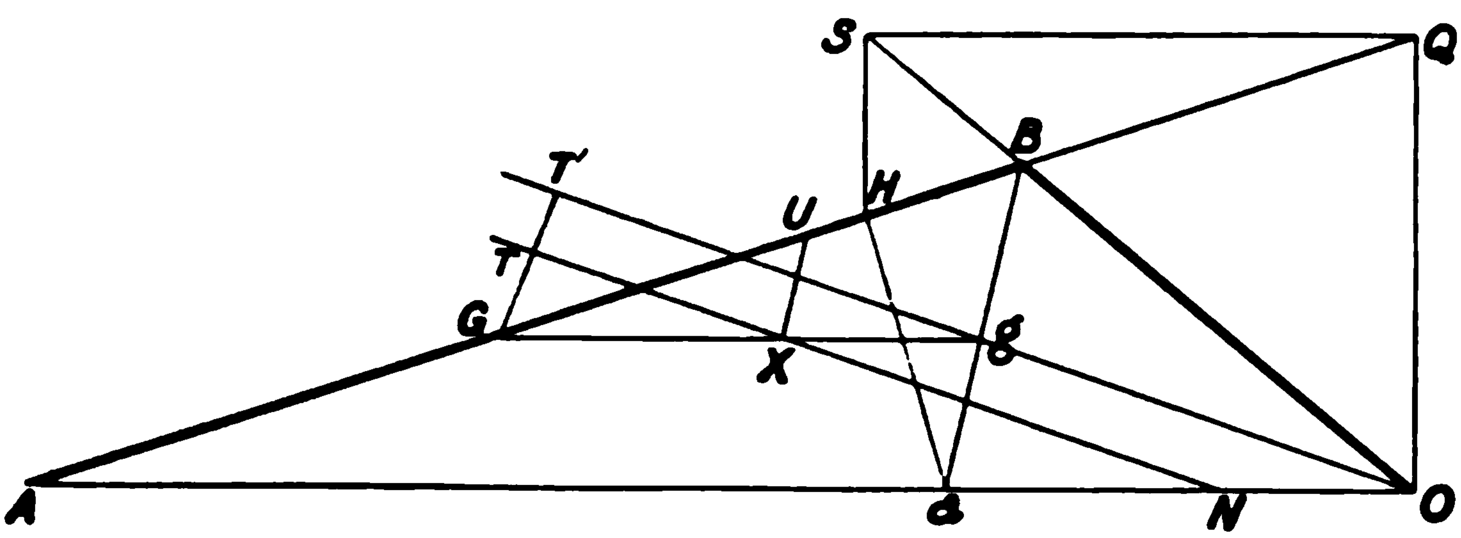



FIG. 315.

rod all the acceleration forces on the rigid rod are equivalent to a force through $G$ parallel to $gO$ and of the amount $m.q^2.gO$, together with a couple, $L = mk.^2q^2 \frac{Ha}{AB}$, where $k$ is the radius of gyration of the rod about $G$. These are really equivalent to a force $mq^2.gO$ parallel to $gO$ acting, say, along $TN$, such that the perpendicular $GT = \frac{\text{couple}}{\text{force}}$. It will be found that if we take $GU = k^2/AG$ and draw $UX$ parallel to $Ba$, meeting $Gg$ in $X$. Then :—

The resultant of all the acceleration forces in the rod is $mq^2.XN$, acting along $XN$ in the direction $X$ to $N$.

Of course if we could make $k^2 = l_1 l_2$ so that the total acceleration force passes through $O$ as is the case with detached masses, there would be balance in such a case as that of Fig. 316, where two cross heads and their cranks are exactly in line. Unless this condition is fulfilled (for example, as Mr. Harrison suggests, by prolonging the connecting rods) there is a surging couple acting on the frame of the engine and on the ground.[1]

[1] The proof of the above proposition is this :—

$$GT = mk^2q^2 \frac{Ha}{BA} \Big/ mq^2.g\theta = k^2 \frac{Ha}{g\theta.AB}$$

Now if $k^2 = l_1 l_2$ we have already seen that the connecting rod may be replaced y the mass $m_1$ at $A$ and the mass $m_2$ at $B$, and under these circumstances the total sultant force must act through $O$ and therefore must be like $TgO$. But $k^2$ is less an $l_1 l_2$, and it is evident that the real $T$ is such that $GT : GT^1 = k^2 : l_1 l_2 = GX : Gg$. ence we have proved that the real force passes through $X$ because we made $GU = /l_1$, or $GU : GB = k^2 : l_1 l_2 = GX : Gg$.

We can now find a vertical force at $A$ and some force at $B$ to equilibrate the [illegible] of the rod and resolve the $B$ force at right angles to and along the crank.

**347.** Using this construction one of my students, Mr. Rhind, has found the following answers. He took $l = 4\frac{1}{2}r$ and $k = \frac{4}{11}l$. Also $BG : GA = 11 : 16$; $F = 0$.

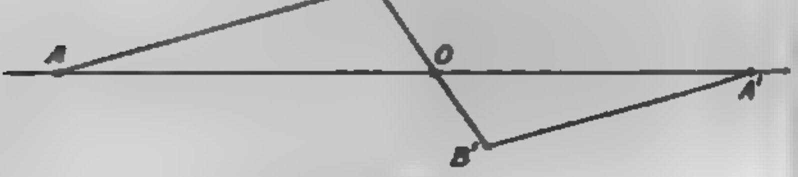



Fig. [illegible]

$R$ is the radial force at the crank pin: $R'$ is what it would be on the assumption of detached masses. $Q$ is the force at right angles to the crank, or the crank effort as it is sometimes called; $Q'$ is what it would be on the assumption of detached masses.

| | Inner dead point | 22½° | 45° | 67½° | 90° | 112½° | 135° | 157½° | Outer dead point |
|---|---|---|---|---|---|---|---|---|---|
| $Q$ | 0 | ·73 | ·96 | ·41 | – ·32 | – ·75 | – ·70 | ·42 | 0 |
| $Q'$ | 0 | ·74 | ·95 | ·32 | ·15 | ·68 | ·63 | – ·33 | 0 |
| $R$ | 3·12 | 2·92 | 2·10 | 1·49 | 1·48 | 1·85 | 2·30 | 2·63 | 2·75 |
| $R'$ | 3·3 | 2·97 | 2·11 | 1·53 | 1·36 | 1·97 | 2·36 | 2·63 | 2·74 |

If these numbers are multiplied by $\frac{Wq^2}{32 \cdot 2}$ we get the forces in pounds, $W$ being the weight of the rod in pounds, $q$ being angular velocity of the crank in radians per second.

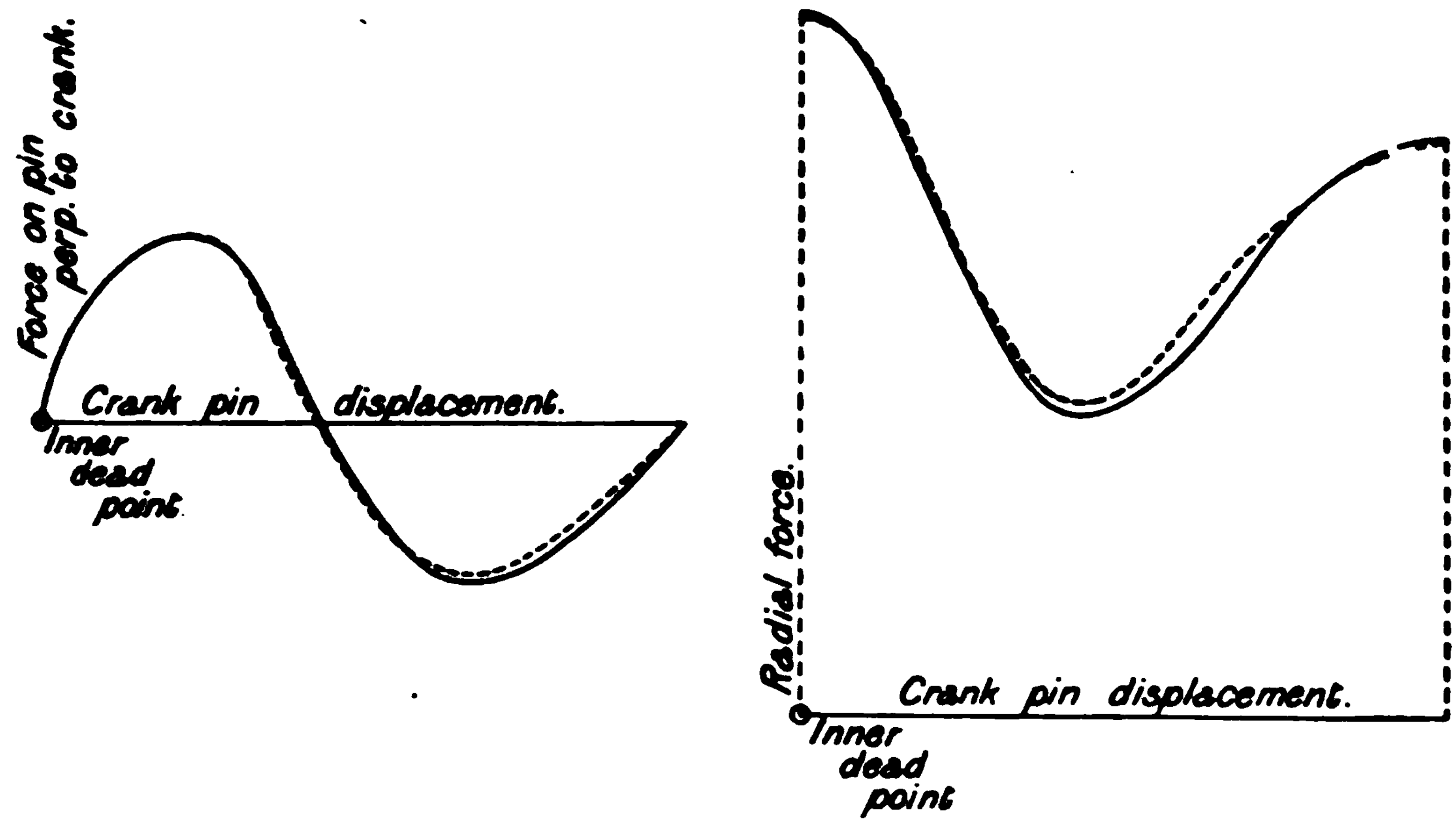
Figs. [illegible] and [illegible] show these results. The firm lines represent actual forces; the dotted lines show the result of our assumption of detached masses.

[illegible] such close agreement as this warrants our often [illegible] [illegible] when we know that in our most accurate calculations we [illegible] leaving out terms very much more important than any [illegible]

**348.** [illegible] I have given an example of how to deal with the indicator [illegible] engine. Among other things we found a diagram [illegible] the crank shaft. A student ought to take the [illegible] expansion engine, and combine the diagrams to see how the [illegible] when several pistons work the same shaft.

[illegible] to make elaborate investigations. But it is worth [illegible] important facts in regard to the **balancing of two and three cylinder engines.** We know the nature of the forces acting on [illegible] engine. If quite unbalanced we have a force $F$ in the line of [illegible] and a centrifugal force on the crank pin $mq^2r$ if [illegible]

is the mass of piston and rod, cross head and half (really the fraction $l_2/l$) of the connecting rod; $q$ the angular velocity in radians per second, and $r$ the length of the crank. We have no total force at right angles to the line of




FIGS. 317 AND 318.

centres except the component of the centrifugal force, but we have a surging couple whose amount is very nearly $b \sin \theta$.

Here $b$ stands for

$$-m(k^2 - l_1 l_2)\frac{4\pi^2 n^2}{3600}\,\frac{\frac{r}{l}\left(1 - \frac{r^2}{l^2}\right)}{\left(1 - \frac{r^2}{l^2}\sin^2\theta\right)^{3/2}}$$

and it is only very nearly a constant. $m$ is mass of the connecting rod.

The first force I shall denote by

$$-m_1\left(\cos\theta + \frac{r}{l}\cos 2\theta\right),$$

the second by $m_2$ acting in the direction $\theta$. Mere centrifugal force may be balanced, and it is possible to construct the connecting rod so as to destroy the surging couple.

### TWO LINE ENGINE UNBALANCED.

*Masses the same in both Lines.*

What forces acting at a point on the axis of the shaft mid way will balance the inertia forces, distance apart $2a$?

I. Cranks at right angles.

1. Resultant force in line

$$= -m_1\left(\cos\theta + \frac{r}{l}\cos 2\theta\right) + m_1\left(\sin\theta + \frac{r}{l}\cos 2\theta\right) = m_1\sqrt{2}\sin(\theta + 45^\circ)$$

2. Couple about vertical axis.

$$am_1\left\{-\left(\cos\theta + \frac{r}{l}\cos 2\theta\right) - \left(\sin\theta + \frac{r}{l}\cos 2\theta\right)\right\}$$

$$= -am_1\left\{\sqrt{2}\sin(\theta + 45^\circ) + \frac{2r}{l}\cos 2\theta\right\}$$

Resultant centrifugal force $m\sqrt{2}$ in the direction $\theta + 45$.

4. Couple due to centrifugal force $m_2 a\sqrt{2}$, about an axis which is at $\theta+45$ rotating.

5. Resultant surging couple $b\sqrt{2}\sin(\theta+45)$.

II. Cranks at 180°.

1. Resultant force in line

$$-m_1\left\{\cos\theta+\frac{r}{l}\cos 2\theta\right\}+m_1\left\{\cos\theta-\frac{r}{l}\cos 2\theta\right\}=-2m_1\frac{r}{l}\cos 2\theta.$$

2. Couple about vertical axis

$$-am_1\left(\cos\theta+\frac{r}{l}\cos 2\theta\right)+am_1\left(-\cos\theta+\frac{r}{l}\cos 2\theta\right)=-2am_1\cos\theta.$$

3. Resultant centrifugal force 0.

4. Couple due to centrifugal force, $2am_2$, about an axis which is at $\theta+90$ rotating.

5. Resultant surging couple 0.

### Three Line Engine Unbalanced.

I. Cranks 120° apart. Distance of lines apart $=a$. The forces at the point where the middle centre line meets the axis are

$$\text{crank at } \theta-120,\ m_1\left\{\frac{1}{2}\cos\theta-\frac{\sqrt{3}}{2}\sin\theta+\frac{r}{l}\left(\frac{1}{2}\cos 2\theta+\frac{\sqrt{3}}{2}\sin 2\theta\right)\right\}$$

$$\text{crank at } \theta+120,\ m_1\left\{\frac{1}{2}\cos\theta+\frac{\sqrt{3}}{2}\sin\theta+\frac{r}{l}\left(\frac{1}{2}\cos 2\theta-\frac{\sqrt{3}}{2}\sin 2\theta\right)\right\}$$

$$\text{crank at } \theta,\ -m_1\left(\cos\theta+\frac{r}{l}\cos 2\theta\right).$$

1. Resultant force in line

$$0+0+0+0,$$

that is, it is less than my approximations take account of.

2. Couple about vertical axis

$$m_1 a\left(\cos\theta+\frac{r}{l}\cos 2\theta\right)+m_1 a\left\{\frac{1}{2}\cos\theta-\frac{\sqrt{3}}{2}\sin\theta+\frac{r}{l}\left(\frac{1}{2}\cos 2\theta+\frac{\sqrt{3}}{2}\sin 2\theta\right)\right\}.$$

$$=m_1 a\sqrt{3}\left\{\cos(\theta+30)+\frac{r}{l}\sin 2(\theta+30)\right\}.$$

3. Resultant centrifugal force 0.

4. Couple due to centrifugal force. $\sqrt{3}\,m_2 a$ about an axis coinciding with the intermediate crank.

5. Resultant surging couple

$$b\left\{\sin\theta+\sin(\theta+120)+\sin(\theta-120)\right\}=0.$$

If we take the other possible arrangements of the cranks we shall find that the forces are exactly the same if the engine runs in the opposite direction.

[I have just seen a paper by Messrs. Robinson and Sankey (Inst. Nav. Arch. 1895) in which they point out that a perfect balance may be obtained by the use of two three line engines or a six line engine. This is evidently true as (2) may thus be balanced.—*October 31st*, 1898.]

# CHAPTER XXX.

## KINETIC THEORY OF GASES.

**349.** In mathematical calculations concerning stress and strain in solid and fluid bodies, we imagine the stuff to be continuous and homogeneous; we assume that stress is proportional to strain; in fluids we assume a law of internal friction; our mathematical results are of value because in many cases in which we can test them they agree with actual fact. When we leave mere **mechanics,** which is the name given to a particular kind of exercise in **mathematics;** when we consider chemistry or heat and other forms of energy, we are compelled to frame theories of the actual molecular constitution of matter. It is no longer continuous homogeneous stuff to us; we are compelled to study its coarsegrainedness.

The theory that a gas consists of molecules which are rushing about among each other with all sorts of velocities, is accepted by us because it and it alone agrees with all the facts that are known to us. At any instant nearly all the molecules are so far away from each other (compared with their own sizes) that there is practically no mutual attraction and they move in straight lines; when they do encounter, whether this is like the collision of a pair of billiard balls or other elastic bodies, or whether it is that each molecule goes round the other as a comet goes round the sun without actual contact, there is a communication of momentum which we call a collision. **The history of one collision** is probably very complicated. In all probability the analogy of a molecule with a solar system or star is fairly complete. We may imagine the millions of years which elapse before a star comes sufficiently near another for a collision to occur, and we may imagine a very complicated and tedious kind of collision between two stars, each with its planetary system. We must replace millions of years by the millionth of the millionth of a second to obtain the analogy. In a cubic millimetre of gas there are probably a million million million of molecules, and each of them meets with collision on an average 7,000

million times per second. There are probably all sorts of velocities from zero to some that are indefinitely large. In hydrogen at ordinary temperatures the average velocity (the square root of the mean square of the velocity) is greater than 1 mile per second. Each molecule of a gas consists of atoms tied together by a mutual attraction. If the gas is water stuff, each molecule consists of two atoms of hydrogen and one of oxygen; this is the simplest image of the molecule that we have which will suit the observed facts. Higher temperature means on the average more and more violent collisions, although there must be violent collisions at any temperature; greater density or a greater amount of stuff in a given volume means that each molecule has a shorter free path and that more collisions must happen per second. The theory allows us to imagine that when a collision is violent there may be a divorce (**dissociation** it is called) between the atoms of a molecule, and divorced hydrogen atoms may go roaming round very ready to combine with divorced oxygen atoms, but it is not until very high temperatures are reached that the average collision is so violent as to maintain a large proportion of the atoms in a state of dissociation. This idea of divorce and marriage is a very good working idea for the engineer to have who wants to know what occurs in the furnace of a boiler, or in a gas or oil engine cylinder. He had better remember that it is only a useful sort of notion. It is, however, a fact that if carbonic acid $CO_2$ is heated to a high temperature, its $p$, $v$ and $t$ do not obey the laws of perfect gases nearly so well as at lower temperatures; that is if $\frac{pv}{t}$ be called $R$, then $R$ increases in such a way that we are compelled to imagine a dissociation of $CO_2$ into carbonic oxide and oxygen (with disappearance of heat), just as at very high temperatures there is a dissociation of $H_2O$ into hydrogen and oxygen, and it is said that although in ordinary ways of cooling the dissociated CO and O becomes $CO_2$ (the heat reappearing), yet when cooling is effected very suddenly the stuff remains dissociated. Students will recollect the phenomenon of recalescence in iron and its hardening when suddenly cooled, as probably analogous with these dissociation phenomena.

At the time of an encounter the **internal motions** of the atoms in each molecule must be very complicated; but afterwards each molecule is left vibrating in some way or ways perfectly definite for this particular kind of molecule. We know the periodicities of some of these internal vibrations from spectrum analysis. Probably the internal energy of a molecule is of many kinds. One kind, mere potential and kinetic energy of the atoms, seems to re-arrange itself in amount at every collision. I do not

want here to go beyond the simplest dynamical notions, but the earnest student had better perhaps break off from mere dynamical notions for a while and try to understand an electro-magnetic molecular theory of matter. To us, just now, internal molecular energy that may not become heat or re-arrange itself, after, at all events, a few millions of collisions, that is, in less than the thousandth of a second, is beyond our consideration.

**350.** The **energy of the gas** which we consider, is first the kinetic energy of translation or flight. The average amount of this in any one direction is the same as in any other. We imagine the total energy of flight to be divided into three equal parts, one for each of the three degrees of freedom of a point. There must be many internal degrees of freedom in a molecule. It has been shown by Maxwell that the total kinetic energy divides itself equally among the degrees of freedom. Mere points have only three degrees of freedom. Perfectly smooth spheres would for our present purpose be regarded as having only three degrees, because, although each sphere has really three other degrees, being capable of rotation, it cannot suffer any change in such energy of rotation; whereas if the surfaces were rough there would be three other degrees. Smooth Ellipsoids of revolution might be regarded as having five degrees of freedom. Notions of this kind are, however, to be used with caution. We cannot imagine anything analogous to a molecule in a homogeneous sphere or ellipsoid.

We are groping towards a way of seeing how Maxwell's theorem may help us to understand from the kinetic theory how it can be true that the internal molecular energy in a perfect gas should keep proportional to the kinetic energy of flight. If I knew clearly what I might speak about, I should say that as in flight there are three degrees of freedom and if the whole energy of flight is $T$, then if in the molecule there are $f$ degrees of freedom, the total kinetic energy is

$$\frac{f}{3}T.$$

Unfortunately experimentally derived values of $\gamma$, the ratio of the specific heats, are such that this theory of Maxwell's leaves much to be explained, and therefore we shall put it, as Clausius did originally, that the average total energy of a molecule is $\beta$ times its energy of flight. I include in this not merely the internal kinetic energy of a molecule, but internal potential energy, or all the kinds of energy with which we deal in the thermodynamics of a gas.

On the kinetic theory the law for a perfect gas $pv/t = R$ a con-

stant, is true only if we neglect all attractions of molecules for one another and also the volumes of the molecules; that is, assume perfectly straight free paths of infinitely small particles. By taking account of possible attractions as I do in Art. 352, **Van der Waals** has arrived at his well-known equation

$$\left(p - \frac{m}{v^2}\right)\left(v - n\right) = Rt$$

where $m$, $n$ and $R$ are constants; which, however, is not found to be altogether in agreement with experiment for all substances.

**351.** It can be shown that in a perfect gas:—

1. If $m_1$ is the mass of each of one kind of molecule and $m_2$ is the mass of another kind of molecule when two gases are in the same vessel and $V_1$ and $V_2$ are their velocities. The average value of $m_1 V_1^2$ is the same as the average value of $m_2 V_2^2$.

2. If $v$ is the volume of unit mass of gas, so that $m$ being the mass of one molecule and there being $n$ molecules in unit volume, $mn = \frac{1}{v}$; then the pressure $p$ being really rate per second at which momentum is communicated through unit area of any interface in a normal direction by molecules flying one way is

$$p = \frac{1}{3} mn V^2$$

or

$$pv = \frac{1}{3} V^2$$

where $V^2$ is the mean square of all the velocities.

It is evident that this enables us to calculate $V$ for any of the permanent gases, and the student ought to make the calculation for hydrogen, oxygen, carbonic acid, and $H_2O$ gas.

3. Since by (2), $pv = \frac{1}{3} V^2$, and as $pv = Rt$, then $t$ stands for $V^2/3R$.

4. It is evident from (2) that as $\frac{1}{2} mn V^2$ is the kinetic energy of translation in unit volume, or $\frac{3}{2} p$, the kinetic energy of translation in unit mass is

$$\frac{3}{2} pv \text{ or } \frac{3}{2} Rt$$

And we take the total intrinsic energy to be $\beta$ times this,

or

$$E = \frac{3}{2} \beta pv \text{ or } \frac{3}{2} \beta Rt$$

If volume is kept constant, gain of $E$ when the temperature changes one degree, is the heat added, that is, it is the specific heat at constant volume,

$$k = \frac{3}{2}\beta R$$

But we know that $K = R + k$, and hence

$$K = R + \frac{3}{2}\beta R \text{ or } R\left(1 + \frac{3}{2}\beta\right)$$

and hence $$\gamma = \frac{K}{k} = \frac{2}{3\beta} + 1 \text{ or } \beta = \frac{2}{3}\frac{1}{\gamma - 1}$$

The best method of finding $\gamma$ is usually from experiments on sound. The following are known to be fairly accurately determined values of $\gamma$. The student is asked to calculate $\beta$ in each case. He may also be sufficiently curious to calculate $f$ (presumably degrees of freedom, in a crude application of Maxwell's theorem).

| | Atomicity of Molecule. | Observed $\gamma$ | Computed. $\beta$ | Computed. $f$ |
|---|---|---|---|---|
| Mercury (Hg) | 1 | 1·67 | 1 | 3 |
| Argon | 1 (?) | 1·65 | 1 | |
| Hydrogen ($H_2$) | — | 1·41 | | |
| Nitrogen ($N_2$) | — | 1·41 | | |
| Carbonic oxide (CO) | 2 | 1·40 | $\frac{5}{3}$ | 5·0 |
| Hydrochloric acid (HCl) | | 1·39 | | |
| Hydrobromic acid (HBr) | — | 1·42 | | |
| Hydroiodic acid (HI) | — | 1·40 | | |
| Chlorine ($Cl_2$) | | 1·32 | | |
| Bromine ($Br_2$) | 2 | 1·29 | | |
| Iodine ($I_2$) | — | 1·29 | $\frac{20}{9}$ | $6\frac{2}{3}$ |
| Iodine Chloride (ICl) | — | 1·31 | | |
| Carbonic acid ($CO_2$) | — | 1·308 | 2 | 6·5 |
| Nitrous oxide ($N_2O$) | 3 | 1·310 | | 6·5 |
| Sulphuretted hydrogen ($H_2S$) | | 1·340 | — | 5·9 |
| Carbon bisulphide ($CS_2$) | — | 1·239 | — | 8·4 |
| Ammonia ($NH_3$) | 4 | 1·30 | — | 6·7 |
| Methane ($CH_2$) | — | 1·313 | 2·13 | 6·4 |
| Methyl chloride ($CH_3Cl$) | — | 1·279 | 2·4 | 7·2 |
| Methyl bromide ($CH_3Br$) | 5 | 1·274 | 2·43 | 7·3 |
| Methyl iodide ($CH_3I$) | | 1·286 | 2·33 | 7·0 |
| Methylene chloride ($CH_2Cl_2$) | — | 1·219 | 3·07 | 9·2 |
| Chloroform ($CHCl_3$) | — | 1·154 | 4·33 | 13·0 |
| Carbon tetrachloride ($CCl_4$) | — | 1·130 | 5·13 | 15·4 |
| Silicon tetrachloride ($SiCl_4$) | — | 1·129 | 5·2 | 15·6 |

My reason for dwelling upon this matter and asking students to speculate on these results for themselves is this, that Mr. Macfarlane Gray asssumes that in a gas such as $H_2O$ gas we must have $R:k:K$ in the ratios $2:5:7$, and everybody who gives thought to steam engine theory must have a good reason for his action if he disagrees with Mr. Gray. Experimentally I find that this ratio holds only approximately in the case of some of the transparent diatomic gases, and it is certainly not the case in the coloured diatomic gases. As $H_2O$ is triatomic, we might expect the ratio of $K:k$ to be 1·31 to 1·34, or possibly as low as 1·239, but we have no *à prior* reason for thinking that it is 1·4. Indeed, the more we study the values of $\gamma$, or $\beta$, or $f$ (a more complete[1] table of gases is given in a paper by Dr. Stoney, *Phil. Mag.*, Oct. 1895, and it is from his paper that I have taken the above numbers) the more disinclined are we to assume that we know anything about either molecular degrees of freedom or the meaning of Maxwell's law in the kinetic theory of gases.

**352.** It is worth while here to say something of the kinetic theory when attractions are not neglected. If a particle of mass $m$ at $x, y, z$ is acted on by a force $X$, $Y$, $Z$, then $m\ddot{x} = X$. **Clausius transformed** this by using $\frac{d^2}{dt^2}(x^2) = 2\dot{x}^2 + 2x\ddot{x}$, so that we find

$$\tfrac{1}{2}m\dot{x}^2 = -\tfrac{1}{2}Xx + \tfrac{1}{4}m\frac{d^2}{dt^2}(x^2)$$

Integrating from 0 to $t$ and dividing by $t$ we get mean values. If the motion is periodic, the mean value of the last term on the right hand side is zero. Even if not strictly periodic, if $x$ and $\frac{dx}{dt}$ do not continually increase, the mean value of the last term gets to be smaller and smaller, and is negligible, and so we have, indicating averages by strokes,

$$\tfrac{1}{2}m\overline{\dot{x}^2} = -\tfrac{1}{2}\overline{Xx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Adding the three equations like (1) we get an expression for the kinetic energy of a particle. Adding the energies of all the particles we get total kinetic energy of the system

$$E = -\tfrac{1}{2}\Sigma\overline{(Xx + Yy + Zz)} \quad . \quad . \quad . \quad . \quad (2)$$

Now we may distinguish between forces externally applied and internal forces. For example, let the uniform pressure $p$, exerted by a confining vessel of volume $v$, be the only external force

$$\Sigma Xx = -p\iint x\cos\alpha \,.\, dS = -p\iint x \,.\, dy \,.\, dx = -pv,$$

if $dS$ is an element of area of the bounding surface, $\alpha$ the angle made by the normal there to the axis of $x$. Hence for the external forces the right hand expression of (2) becomes $\frac{3}{2}pv$.

Now as to internal forces. If $R$ is the attraction between two particles at $x, y, z$, and $x', y', z'$, whose distance apart is $r$,

$$Xx + X'x' = \frac{x'-x}{r}Rx + \frac{x-x'}{r}Rx' = -\frac{(x'-x)^2}{r}R.$$

[1] Modified from Mr. Capstick's table, *Science Progress*, June, 1895.

Adding such similar expressions we see that the right hand expression of (2) becomes for the internal forces $\Sigma\frac{1}{2}Rr$. This is called the **virial** (or sometimes merely the internal virial, the whole right hand expression of (2) being called the whole virial). Hence (2) becomes ;—Total kinetic energy of the system

$$E = \tfrac{3}{2}pv + \tfrac{1}{2}\Sigma Rr \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

When the virial is not indefinitely small, $pv$ is no longer constant when the temperature is constant if we assume that the temperature is proportional to the kinetic energy. Comparing (3) with experimental results we find that whereas in imperfect gases $R$ is an attraction, in liquids the virial becomes negative and $R$ is a repulsion. Further considerations of this kind led Van der Waals to his equation.

**353.** The study of the Van der Waals equation is recommended to students—in spite of its known incompleteness—because it does give fairly good notions of the behaviour of ordinary gases. It shows the way in which an actual gas differs in behaviour from what we call a perfect gas. We may imagine as the density gets greater how the free path gets shorter, and how collisions are more frequent until in the liquid state the molecules have no straight line paths, although they seem to be able to move about freely among each other, so that diffusion phenomena are explainable.

And yet how very crude these notions seem to be when we consider the enormous tensile forces which liquids are sometimes known to withstand, as if the repulsion which exists between molecules in the liquid state became a great attraction at slightly greater distances asunder. Again, in the solid state the roaming of molecules among each other seems to be quite given up; each seems to attract its neighbours with great cohesive forces. For what is known about molecular theory, the student must refer to advanced books on physics and chemistry. It must be of value in the study of heat engines. The student ought to have some molecular theory, simple or complex, which will enable him to imagine how things happen. To imagine how at the surface of a liquid some of the liquid molecules have sufficient velocity to jump out of the liquid, and how equilibrium is established when just as many of the vapour molecules get entangled in the liquid per second as there are others that jump out.

**354.** One of the most interesting things in connection with the kinetic theory of gases is its simple explanation of **viscosity**. We have only to imagine a great number of railway trucks moving near one another without friction, on many lines of rails at all sorts of speeds, crowded with men who are continually jumping from one truck to another; there is a continual communication of momentum, and momentum per second communicated is force—a force tending to the equalisation of the speeds of all trucks near one another, a

force of friction. The theory explains then, not only diffusion and conduction of heat in a gas, and viscosity, but how increase of temperature increases them all. In liquids, although at higher temperatures the diffusivity and probably the conductivity are greater, the viscosity is invariably less; and in all liquids except mercury, very much less. We believe that the study of the viscosity of liquids will lead us to better notions of molecular constitution.

**355. What is capillarity?** Of course the student knows the theory, probably in the form of the superficial tension analogy easily understood, as given, say, in Maxwell's book on Heat. But he must endeavour to have a private notion, however crude, of the cause of the phenomena, and how they are effected by electricity, for example. It is most important for us in connection with condensation and vaporisation when the liquid is in the shape of fine or large drops.

Water in a drop will evaporate more readily than if the surface were flat. Unless drops are so small that surface tension is altogether different in character from what it is in visible drops, if $p$ is the pressure necessary to prevent evaporation, and $p_0$ is the pressure of saturated vapour corresponding to the temperature; if $\tau$ is the surface tension, $r$ the radius of the drop, $\sigma$ the density of the vapour, $\rho$ that of the liquid; $p$ is greater than $p_0$ by the amount

$$\tau \frac{\sigma}{\rho - \sigma} \frac{2}{r}.$$

This explains why in dust free space the saturation pressure may be greatly exceeded without condensation. Moist air free from dust may be suddenly expanded so that the pressure is many times the saturation pressure without the formation of cloud. The presence of electrified zinc or the passage of Röntgen rays or ultraviolet light or light from Uranium glass causes cloud to form. It is evident from these and many other observations that we are very far from having an exact knowledge of what occurs inside a steam engine cylinder. So also for a bubble of steam to get larger in water, the saturated pressure $p_0$ of the steam corresponding to its temperature must be greater than that of the water $p$ by the above amount. One hardly sees how such a bubble could form were it not for 1, Dissolved gases. 2, Some action of the surface of the containing vessel or particles of foreign solid matter. Certainly there are cases known of drops of water existing surrounded by oil at atmospheric pressure, and 356° F., whereas the saturation pressure corresponding to this temperature is 10 atmospheres. Here we have evidence of great resistance to

nsile stress in water, and the phenomena of latent heat led Dupré think that the tensile stress called into play in changing water into pour is about 25,000 atmospheres. Water very free from air is w used in most boilers, and it is well to notice how in the Thorney-oft boiler the evaporation is assisted. It is this tendency to "boil th bumping" of air free water, that gives so much trouble in arting the fires of many marine boilers and makes artificial rculation so necessary.

**356.** So much for the behaviour of water and steam under **static onditions**; but we must expect that when sudden changes take lace, say under the conditions which hold inside a steam engine linder, the ordinary static law connecting pressure and temperature f saturated steam is not merely not a guide, but is actually mislead-ng. It seems almost impossible to study what goes on inside a team engine cylinder unless we are allowed to imagine that all the tuff, vapour and water, is at the same temperature at every instant. n truth, however, as the temperature changes rapidly, even if we magine the material of the cylinder to be itself non-conducting, here must be very curious differences of temperature in the fluid. Messrs. Callendar and Nicolson found that whilst the temperature hown by a thermometer in the body of the steam was nearly that f saturation corresponding to the pressure, the temperature shown y another thermometer inside the cylinder shows what we may gard as rapid superheating during cushioning and admission; hereas after a very rapid rise the temperature then fell rapidly. ll it was well below the saturation temperature, just before cut-off gan to take place. Professor Callendar has had so much experi-ice of the measurement of temperature that we must look upon his easurements as probably correct, however much they may seem to nflict with our other notions. He has himself given a good ex-anation of the fall after expansion begins; but I cannot accept his ew of the superheating, for it is practically impossible for me to iagine that the ordinary well-lagged cylinder using ordinary steam ever free from water. The explanation of the drop is this. Any ie who has worked a little with the $t\phi$ diagram knows that in the liabatic expansion of a pound of water stuff, containing $x$ lb. of eam and $1 - x$ lb. of water: if $x$ is nearly 1, condensation occurs iring expansion; if $x$ is nearly 0, evaporation occurs. That is, the iter tends to evaporate, and the steam tends to condense. Imagine, en, the struggle occurring at the surface of the water, and it becomes ident that if the expansion occurs rapidly there are really differ-ces of temperature between one portion of the fluid and another.

We may imagine the hotter portions of water being converted i steam and cooler portions of the steam becoming water. If : small, it is probable that on the whole the water is hotter than saturation temperature corresponding to the pressure, and if $x$ is la it is probable that on the whole the steam is cooler than the satu tion temperature corresponding to the pressure. Anyhow, we h no right to assume the saturation temperature and pressure to e throughout.

I have not here referred to the probable differences of temperat existing in a layer of water which gets thicker or thinner by c densation or evaporation. If the material of the cylinder w absolutely non-conducting, this layer is likely to be more unif in temperature during the evaporation process than the condensat a circumstance which tends slightly to diminish the amount of c densation in a steam engine cylinder.

## CHAPTER XXXI.

### THERMODYNAMICS.

**357.** WHEN we say that the state of a pound of stuff is defined by its $p$, $v$ and $t$, we understand that it is all at the same temperature, and that it is a fluid, or at all events, can only experience the sort of strain or stress which a fluid can experience. Our assumption is that there is no molecular structure in the fluid. It has only elasticity of bulk. If it was in the state $p$, $v$, and gets into the state $p + \delta p$, $v + \delta v$, then $- \delta v/v$ is called its *compressive* strain, accompanying the increase of stress $\delta p$. Any kind of elasticity is defined as a stress divided by the corresponding strain, and hence fluids can only have the elasticity,

$$e = \delta p \div ( - \delta v/v) \text{ or } - v \frac{dp}{dv} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The value of this may be $o$, if for example $\delta p = o$ and $\delta v$ has any value. Again, it may be $\infty$, if for example $\delta v = o$ and $\delta p$ has any value. There are two values of the elasticity which are considered more important than others, namely, the elasticity when temperature keeps constant, and this I shall call $e_t$; the elasticity when the stuff neither loses nor gains heat, and this I shall call $e_n$.

**358.** In all cases the state of a pound of stuff is completely known if we know two of the quantities, $v$, $p$ or $t$, if these are independent variables. It is supposed that physicists and chemists have provided for us this knowledge; given $p$ and $v$, or $t$ and $v$, we can calculate or find the other of the three. To give $p$ and $t$ during change of state will not define the state of the stuff as these are not then independent.

Any change of state is a change from $p$ to $p + \delta p$, a change of $v$ to $v + \delta v$, a change of $t$ to $t + \delta t$; any of these increments being positive or negative. If two of the changes are known, the third can be calculated because we are supposed to know the characteristic; therefore the change of state is completely defined if we

know $\delta v$ and $\delta t$, or $\delta v$ and $\delta p$, or (except in case of change of state from solid to liquid or liquid to gas) if we know $\delta t$ and $\delta p$.

In my calculus I have endeavoured to give in an easy way the idea underlying such a calculation as this:—Given $\delta t$ and $\delta v$, infinitely small changes, to find $\delta p$, $p$ being a function of $t$ and $v$,

$$\delta p = \left(\frac{dp}{dt}\right)\delta t + \left(\frac{dp}{dv}\right)\delta v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In the case of a perfect gas $\left(\frac{dp}{dt}\right) = \frac{R}{v}$, $\left(\frac{dp}{dv}\right) = -\frac{p}{v}$

and so

$$\delta p = \frac{R}{v}\,.\,\delta t - \frac{p}{v}\,.\,\delta v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

I gave examples: I took $t = 500$, $p = 2000$, $v = 14{\cdot}4$. Taking new values of $t$ and $v$ as follows, I could calculate the new $p$ in each case quite accurately from $pv = Rt$. I wanted to see with what accuracy (2) would give the same answer, knowing that (2) is more and more true as $\delta t$ and $\delta v$ are made less and less and is not absolutely true unless $\delta t$ and $\delta v$ are smaller and smaller without limit.

| $t$ | $v$ | true $p$ | assumed $\delta t$ | assumed $\delta v$ | true $\delta p$ | $\delta p$ calculated from (2) |
|---|---|---|---|---|---|---|
| 500 | 14·4 | 2000 | | | | |
| 501 | 14·5 | 1990·2 | 1 | 0·1 | -9·8 | -9·9 |
| 500·1 | 14·41 | 1999·2 | 0·1 | 0·01 | -1·0 | -0·99 |
| 500·01 | 14·401 | 1999·9 | 0·01 | 0·001 | -0·1 | -0·10 |

In the same way, for any substance

$$\delta v = \left(\frac{dv}{dt}\right)\delta t + \left(\frac{dv}{dp}\right)\delta p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$dt = \left(\frac{dt}{dv}\right)\delta v + \left(\frac{dt}{dp}\right)\delta p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Again, suppose that there is no change in $p$; put (1) = 0 and we have

$$\frac{dt}{dv} \text{ if } p \text{ does not alter} = -\left(\frac{dp}{dv}\right) / \left(\frac{dp}{dt}\right)$$

This is written as

$$\left(\frac{dt}{dv}\right) = -\left(\frac{dp}{dv}\right) / \left(\frac{dp}{dt}\right) \text{ or } \left(\frac{dp}{dv}\right)\left(\frac{dt}{dp}\right) \quad . \quad . \quad . \quad . \quad (5)$$

Similarly from (3),

$$\left(\frac{dt}{dp}\right) = -\left(\frac{dv}{dp}\right) / \left(\frac{dv}{dt}\right) \text{ or } -\left(\frac{dv}{dp}\right)\left(\frac{dt}{dv}\right) \quad . \quad . \quad . \quad (6)$$

Similarly from (4),

$$\left(\frac{dv}{dp}\right) = -\left(\frac{dt}{dp}\right) / \left(\frac{dt}{dv}\right) \text{ or } -\left(\frac{dt}{dp}\right)\left(\frac{dv}{dt}\right) \quad . \quad . \quad . \quad (7)$$

These statements are so new to some students that I advise them to illustrate what they mean by applying them all to the case of a perfect gas $pv = Rt$.

All the above merely follows from the mathematical fact that any two of $p$, $v$ and $t$ are independent, or that each of them is a function of the other two. In other words, there is some one law connecting $p$, $v$ and $t$ of one pound of any substance, although we may only know the law approximately for a limited range of states. This is also the same as "if a point in space represents by its three distances from the three standard planes the $p$, $v$ and $t$ of a pound of stuff, such points all lie in a surface." Students ought to practise the drawing of curves to express our knowledge of the behaviour of any stuff. They are supposed to have done this before beginning the study of steam engines. Thus, at any given temperature to draw a curve showing the $p$, $v$ diagram of 1 lb. of steam at constant $t$ from a superheated state until it is all liquid. The properties of carbonic acid are fairly well known to us, and its $p$, $v$ ($t$ constant), $p$, $t$ ($v$ constant), $v$, $t$ ($p$ constant) curves ought to be drawn. The drawing of the $v$, $t$ ($p$ constant) curve for water stuff from the ice to the superheated steam state at a few constant pressures is probably the most important exercise. My students have often drawn such curves (eking out the exact information of the books by guessing). They have then cut templates from one inch planks, 1 inch thickness representing 1 atmosphere; they have built these up with screws and glue carefully and chamfered off the edges, and so obtained a surface showing by its three co-ordinates the $p$, $v$, $t$ of water stuff. This is more easily done for a perfect gas (the templates for $p$, $t$ or $v$, $t$ are quite straight and the whole work takes only a few hours), and a student hitherto called stupid will sometimes begin to take an interest in more abstract mathematics after he has marked out the places for which $\phi$ is constant. To merely read about the doing of these things is surely a weariness to the flesh. When a student has actually done the work it is nearly impossible for him not to know that both $E$ and $\phi$ are the same if the state of the stuff is the same, and this means that he really knows the two laws of thermodynamics.

**359.** If we examine such a statement as (1) of Art. 191

$$\delta H = k\,.\,\delta t + l\,.\,\delta v$$

must remember that it is only true if the change of state is sidered to be smaller and smaller without limit. At the same e the two changes $\delta t$ and $\delta v$ are quite independent of one another; nay be $o$, or $\delta t$ may be $o$. It comes from the two assertions:— e heat given to the stuff in any small change of state is calcu- e," and "we know the whole change of state when we know the nge in $t$, and the change in $v$." As $\delta v$ may be large or small pared with $\delta t$, let $\delta v = o$, then the heat is $k\,.\,\delta t$, so we see that $t$ means "the heat given to the body during the change of tempera- $\delta t$ when the volume does not alter." Similarly $l\,.\,\delta v$ means "the t given during the change $\delta v$, when temperature does not alter." ice $l$ is what may be called a latent heat of expansion, and $k$ is specific heat at constant volume. In the same way $K$ is called specific heat at constant pressure. The student must read such ements as (1), (2) and (3) of Art. 191 in several ways, trying to exactly what each term means.

Now the three statements must agree in giving the same answer, and if we put them equal to one another in pairs we get relations between the co-efficients.

Thus $k.dt + l.dv = K.dt + L.dp$.

Substituting $dp = \left(\frac{dp}{dt}\right)dt + \left(\frac{dp}{dv}\right)dv$ from (1) of Art. 358, we have

$$k.dt + l.dv = K.dt + L\left(\frac{dp}{dt}\right)dt + L\left(\frac{dp}{dv}\right)dv.$$

This is true when $dv = o$, and also when $dt = o$, so that

$$k = K. + L\left(\frac{dp}{dt}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$l = L\left(\frac{dp}{dv}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Again, putting $k.dt + l.dv = P.dp + V.dv$ and substituting

$$dp = \left(\frac{dp}{dt}\right)dt + \left(\frac{dp}{dv}\right)dv$$

we have $\quad k.dt + l.dv = P\left(\frac{dp}{dt}\right)dt + P\left(\frac{dp}{dv}\right)dv + V.dv$

so that $\quad k = P.\left(\frac{dp}{dt}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$

and $\quad l = P\left(\frac{dp}{dv}\right) + V \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$

The student ought to write out another equality of the expressions of Art. 191 and get two other relations; also he may use other substitutions than what I have adopted. In this way he will obtain other relations, but he must not hope to find them all independent. For example, he will find

$$L + l\left(\frac{dv}{dt}\right) = K \quad . \quad . \quad . \quad . \quad (5)$$

and $\quad l\left(\frac{dv}{dp}\right) = L \quad . \quad . \quad . \quad (6)$

and again $\quad K\left(\frac{dt}{dp}\right) + L = P \quad . \quad . \quad . \quad (7)$

$$K\left(\frac{dt}{dv}\right) = V \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

but he might have found these by proper combinations of those already found. It is to be noticed that for so far we have only

employed algebra; we have assumed the existence of some law connecting $p$, $v$ and $t$, and that $\delta H$ is calculable, but the above relations are true algebraically even if we do not call $H$, heat; or $v$, volume; or $t$, temperature.

**360.** Exercise. The elasticity at constant temperature is $e_t = -v\left(\frac{dp}{dv}\right)$. To write out $e_H$ it is necessary to find the relation between $\delta p$ and $\delta v$ when $\delta H$ is $o$. Now

$$\delta H = P.\delta p + V.\delta v$$

so that when $\delta H$ is $o$, $\frac{dp}{dv}$ is $-\frac{V}{P}$, and hence

$$e_H = v\frac{V}{P}$$

Thus $\frac{e_H}{e_t} = -\frac{V}{P} \div \left(\frac{dp}{dv}\right)$. Taking $V$ from (8) above and $P$ from (3)

$$\frac{e_H}{e_t} = -K\left(\frac{dt}{dv}\right)\left(\frac{dp}{dt}\right) \div k\left(\frac{dp}{dv}\right).$$

But, as we have already seen in (5) of Art. 358

$$\left(\frac{dp}{dv}\right) \div \left(\frac{dp}{dt}\right) = -\left(\frac{dt}{dv}\right)$$

and hence *for any substance*

$$\frac{e_H}{e_t} = \frac{K}{k} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

This ratio I always denote by the letter $\gamma$.

Again, note that we have an important general statement which depends only on our definition of elasticity, and is merely algebraic, for what we call elasticity may have no physical meaning. We give it a physical meaning when we say what $v$ and $p$ and $H$ mean.

**361.** We leave mere algebra when we say: if $\delta H - p.\delta v$ be called $\delta E$; then $\delta E$ is a complete differential, that is, the value of the $E$ of a body depends on the state of the stuff. Now it is shown in elementary calculus books that if

$$M.dx + N.dy$$

is a complete differential, $\left(\frac{dM}{dy}\right)_x = \left(\frac{dN}{dx}\right)_y$; and hence if we subtract $p.\delta v$ from any of the expressions of Art. 191, and apply this criterion, we have three expressions for the **first law of thermodynamics.**

Thus $dE = dH - p.dv = k.dt + (l - p)dv$. The first law is then

$$\left(\frac{dk}{dv}\right)_t = \left(\frac{dl}{dt}\right)_v - \left(\frac{dp}{dt}\right) \quad . \quad . \quad . \quad . \quad (10)$$

or using $dE = dH - p.dv = P.dp + (V - p)dv$

$$\left(\frac{dP}{dv}\right)_p = \left(\frac{dV}{dp}\right)_v - 1 \quad . \quad . \quad . \quad . \quad (11)$$

With not much more trouble we find

$$\left(\frac{dK}{dp}\right)_t - \left(\frac{dL}{dt}\right)_p = \left(\frac{dv}{dt}\right) \quad . \quad . \quad . \quad . \quad (12)$$

Any one of (10), (11) or (12) may be called the first law of thermodynamics, as it is common enough to see a partial statement called a general law. The real first law is, however, "$dE$ is a complete differential."

Now let us apply **the second law.** Divide each of (1), (2) and (3) of Art. 191 by $t$, and let $\frac{dH}{t}$ be called $d\phi$, and make the statement that $d\phi$ is a complete differential. Thus

$$d\phi = \frac{dH}{t} = \frac{k}{t}.dt + \frac{l}{t}dv$$

The criterion after multiplying by $t$ gives

$$\left(\frac{dk}{dv}\right)_t = \left(\frac{dl}{dt}\right)_v - \frac{l}{t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Combining (13) and (10) we have

$$\frac{l}{t} = \left(\frac{dp}{dt}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Let the student show also that when (14) is combined with the criterion and the earlier equations we have,

$$\left(\frac{dk}{dv}\right)_t = t\frac{d^2p}{dt^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$\left(\frac{dv}{dt}\right) = \left(\frac{dK}{dp}\right)_t - \left(\frac{dL}{dt}\right)_p = -\frac{L}{t} \quad . \quad . \quad . \quad (16)$$

and

$$\left(\frac{dK}{dp}\right)_t = -t\frac{d^2v}{dt^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Also

$$\left(\frac{dV}{dp}\right)_v - \left(\frac{dP}{dv}\right)_p = \frac{1}{t}\left\{V\left(\frac{dt}{dp}\right) - P\left(\frac{dt}{dv}\right)\right\} = 1 \quad (18)$$

It immediately follows from (15) and (17) that

$$k = \mathbf{k} + t\int\frac{d^2p}{dt^2}.dv \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$$K = \mathbf{K} - t\int\frac{d^2v}{dt^2}.dp \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where **k** and **K** are *functions of the temperature only.*

**362.** EXERCISE 1. Show what the above relations become when substance follows the law $pv = Rt$. Number the corresponding quations in the same way and keep the results for reference. Note hat neither $K$ nor $k$ is necessarily a constant, but if either is contant, the other is so also. If not constant they must be functions of emperature only. Our answers are:—

$$l = p, \ L = -v.$$

$K - k = R$, and the expressions (1), (2), (3), of Art. 191 may be written

$$dH = k\,.\,dt + p\,.\,dv = K\,.\,dt - v\,.\,dp = \frac{1}{\gamma - 1}d(pv) + p\,.\,dv$$

Also

$$\phi - \phi_0 = k \log t + R \log . v\,; \ E - E_0 = k(t - t_0)$$

$$H_{10} = \frac{1}{\gamma - 1}(p_1 v_1 - p_0 v_0) + \text{work done.}$$

If expansion occurs according to the law $pv^s$ constant,

$$\text{Heat given during expansion} = \frac{\gamma - s}{\gamma - 1} \times \text{work done.}$$

or

$$\frac{dH}{dv} = \frac{\gamma - s}{\gamma - 1}p = h \text{ say.}$$

Hence if $h$ is $o$, that is, if the expansion is adiabatic, $s$ is $\gamma$.

The student ought to express $\phi$ in terms of $p$ and $v$, and also of $v$ and $t$, and show that an adiabatic may be expressed in any of the ways $pv^\gamma$ or $tv^{\gamma - 1}$ or $pt^{\gamma/(1-\gamma)}$ constant. See Art. 201.

**363.** EXERCISE 2. There is only one way that I know of, in the bsence of fresh experiments, to determine the value of $K$ for $H_2O$ as as. It is to assume that saturated steam at low temperatures and ressures is in the gaseous state. It is easy to show that this assumpion must be very nearly correct, for the volume pressure and temerature of saturated steam satisfy the law for a perfect gas more nd more closely at lower temperatures. Taking the atomic weight f oxygen as 15·88, we have

$$\frac{pv}{t} = R = 153{\cdot}8.$$

This 15·88 I have taken under the advice of authoritative hemists. We have from the above equations for a perfect gas

$$\frac{d\phi}{dt} = \frac{K}{t} - \frac{v}{t}\frac{dp}{dt}$$

Now let us suppose that $\frac{dp}{dt}$ is according to saturation, so that if $\lambda$ is the usual latent heat, $v\frac{dp}{dt} = \frac{\lambda}{t}$, see Exercise 5, and hence

$$\frac{d\phi}{dt} = \frac{K}{t} - \frac{\lambda}{t^2} \text{ or } = \frac{K}{t} - \frac{797}{t^2} + \frac{\cdot 695}{t} \quad . \quad . \quad (1)$$

using Regnault's formula. But the entropy of a pound of saturated steam is

$$\phi = \log\frac{t}{t_0} + \frac{\lambda}{t}$$

so that

$$\frac{d\phi}{dt} = \frac{1}{t} - \frac{797}{t^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Writing (1) and (2) equal, we find $K = \cdot 305$, and as $\frac{R}{J} = \cdot 11$ we have $k = \cdot 195$. I take these to be the specific heats of superheated steam about 0° C., and their ratio is 1·55. If we take Griffiths instead of Regnault, $K$ is ·399, $k$ is ·289, and their ratio is 1·38.

**364.** Exercise 3. Show what the above relations become when a substance follows the law

$$p = bt + a$$

where $a$ and $b$ are functions of *volume* only.

It will be seen that the equation of Van der Waals, Art. 350, is of this form, because we can write it as

$$p = \frac{Rt}{v - n} + \frac{m}{v^2}$$

In the first place, since $\left(\frac{dp}{dt}\right) = b$ and $\frac{d^2p}{dt^2} = 0$, we see from (19) Art. 361, that $k$ is a function of $t$ only. Also from (14), $l = tb$ or $l = p - a$, and therefore

$$dH = k\,.\,dt + tb\,.\,dv$$

gives
$$d\phi = \frac{dH}{t} = \frac{k}{t}\,.\,dt + b\,.\,dv$$

Or $\phi$ = a function of $t$ only + a function of $v$ only.

Also since $dE = dH - p\,.\,dv = k\,.\,dt + (tb - p)dv = k\,.\,dt - a\,.\,dv$, we have, $E$ = a function of $t$ only + a function of $v$ only.

Also
$$K = k + tb\left(\frac{dv}{dt}\right) = k - tb^2\left(\frac{dp}{dv}\right)$$

**365.** Exercise 4. Ramsay states that $e_H$, the adiabatic elas-

icity, is a linear function of $t$ if the volume is constant, and that he law assumed in Exercise 3 is generally true of all substances. how that these statements are consistent with one another.

Since $$\frac{e_H}{e_t} = \frac{K}{k} \text{ or } e_H = -v\left(\frac{dp}{dv}\right)\frac{K}{k}$$

re find $$e_H = v\left\{\frac{tb^2}{k} - t\frac{db}{dv} - \frac{da}{dv}\right\} = a + \beta t,$$

ay, where $a$ and $\beta$ are functions of volume only.

It will be found that this requires

$$k = tb^2/\left\{\frac{a}{v} + \frac{da}{dv} + \left(\frac{\beta}{v} + \frac{db}{dv}\right)t\right\}$$

nd hence if

$$\frac{a}{v} + \frac{da}{dv} = Ab^2$$

$$\frac{\beta}{v} + \frac{db}{dv} = Bb^2$$

here $A$ and $B$ are constants, the two statements are consistent ith one another, and give

$$k = \frac{t}{A + Bt}$$

function of the temperature. On this assumption $k$ is nearly roportional to the temperature when the temperature is small, but t very high temperatures tends towards a limiting value $1/B$.

**366.** Exercise 5.—Apply our equations to the case of a pound f stuff in a lower state (solid or liquid) at volume $s_1$ changing to a igher state (liquid or vapour) at volume $s_2$ at constant temperature eceiving latent heat $\lambda$.

Since $$dH = k.dt + l.dv,$$

r by Art. 361, $$= k.dt + t\left(\frac{dp}{dt}\right)dv$$

s the stuff is present in both states, $\frac{dp}{dt}$ is the same whether $v$ is contant or not.

If $t$ is constant,

$$dH = t\left(\frac{dp}{dt}\right)dv$$

r $$\lambda = t\left(\frac{dp}{dt}\right)(s_2 - s_1) \quad . \quad . \quad . \quad . \quad . \quad (1$$

f $\lambda$ is the usual latent heat.

**367.** EXERCISE 6. Show from the last result that at the melting point of ice, as $s_1$ is greater than $s_2$, since $s_2 - s_1$ is negative and $\lambda$ and $t$ are positive, $\frac{dp}{dt}$ must be negative. When ice melts at $0°$ C., or $t = 274$, $s_1 = \cdot 01747$, $s_2 = \cdot 01602$, $p$ being atmospheric pressure, or 2,116 lbs. per square foot, and $\lambda = 79 \times 1393$. Show that

$$\frac{dp}{dt} = -277{,}300.$$

That is, pressure lowers the melting point of ice at the rate of 131 atmospheres for one degree Centigrade.

**368.** EXERCISE 7. The volume of 1 lb. of water at 374° F is 0·018 cubic feet, and Rankine found by the above calculation that the volume of 1 lb. of steam is 2·476 cubic feet. He took Joule's equivalent as 772 and $L = 849$ heat units (F.) What value must he have taken for $\frac{dp}{dt}$? *Answer* 319.

# CHAPTER XXXII.

## SUPERHEATED STEAM.

**369. Regnault's Total Heat.** In steam engine calculations depend upon Regnault's measurements for temperatures above )° C. There are no others. We may have doubt as to whether really used the unit of heat which he thought he used, but we st make the best of what he gives us. It is for this reason that I ploy the Regnault unit of heat throughout this book, and I have d frankly that my only knowledge of it is that 100·5 of Regnault's its are equivalent to 100 of those of Reynolds, whose Joule's iivalent (778 in the Fahrenheit scale) is 1,399 London foot-pounds. ence my Joule throughout is 1,393 London foot-pounds (or 774 for e Fahrenheit unit). My trouble begins when I consider Regnault's sults below 100° C., because there have been other experiments d the results are said to conflict.

Our knowledge of $H$ from 63° C. to 88° C. is based on twenty-ree observations of Regnault. Assuming, as he did, that there is inear law connecting $H$ and $\theta$, I find $\frac{dH}{d\theta} = 0{\cdot}379$, the mean tem-rature being 77°·55 and the mean $H$ being 627·9. In fact, these enty-three measurements give me

$$H = 598{\cdot}61 + {\cdot}379\,\theta.$$

Regnault's thirty-eight measurements from 99°·27 C. to 100°·37 C. e a mean temperature 99°·88 C. and a mean value of $H$, 636·67.

**Mr. Griffiths** has found the latent heats of steam 572·60 at ·15 C. and 578·70 at 30° C., and I infer from his paper that his ues of $H$ are 612·75 and 608·70. It is not easy to say how ought to compare his unit (the heat for a degree on the nitrogen rmometer at 15° C.) with what I take to be Regnault's; but I d that the above numbers agree with a formula which he gives er, in which he uses as the value of Joule's equivalent $J =$ 99 × 10⁷. Now the $J$ which I use for Regnault's units corresponds 4·165 × $10^7$ Ergs (per gramme degree Centigrade). Hence I

take it that the values of $H$ given by Mr. Griffiths need to be multiplied by 4199/4165 if they are to be compared with Regnault's. They are therefore 617·74 at 40°·15 and 613·65 at 30° C. mean 615·7 at 35°·075 C.

Between Griffiths' mean and Regnault's 627·9 at 77°·55

$$\frac{dH}{d\theta} = 0{\cdot}287.$$

Between Griffiths' mean and Regnault's 636·67 at 99°·27 we find $\frac{dH}{d\theta} = 0{\cdot}327$. The mean of these values is 0·307.

Now I am aware that there is more certainty as to the measurements made by Mr. Griffiths, and that the units of the measurement made by Regnault below 100° C. are not well known, and his method of measurement between $-2°$ C. and 16° C. is open to criticism; but it will be seen by the above that **if we are to keep to Regnault's units,** we cannot do better for the present than to keep to his formula in which $\frac{dH}{d\theta}$ is the same for all kinds of steam, namely, ·305, not merely above 100° C., where we have nothing but Regnault, but also below 100° C., where Regnault cannot be regarded as altogether inconsistent with what Mr. Griffiths himself gives. I may at the same time say that although I cannot use Mr. Griffiths' formulæ between 0° and 100° C.,

$$\text{total heat } H = 596{\cdot}73 + {\cdot}3990\,\theta$$
$$\text{latent heat } l = 596{\cdot}73 - 0{\cdot}6010\,\theta,$$

his suggestion of their use causes me to feel that there is in the repetition of Regnault's work an excellent investigation waiting to be done by some National Physical Laboratory. Such an investigation ought to include the determination of the characteristic law for superheated steam and its specific heat.

**370. Superheated Steam.**—It will be seen in Art. 207 and elsewhere that it is very important for us to know with some accuracy what is the specific heat of superheated steam. Regnault's experiments give conflicting results. Thus when at atmospheric pressure he cooled steam at 225° C. to 125° C., the average specific heat seemed to be 0·48, a figure which is very generally taken to be nearly correct under all circumstances and of which great use is made; whereas when he cooled it from 124° C. to 100° C., Mr. Macfarlane Gray has shown that the average result was 0·378, although everybody uses 0·48. I cannot admit with Mr. Gray that we have any right to assume that we can calculate the specific heat of any gas from the atomic weight. See Art. 351.

We always assume that a superheated vapour, that is, a vapour . higher temperature than that which corresponds to its pressure the saturated or mixed condition, gets to be more and more rly like a perfect gas in its $p, v, t$ relations, the greater its tem- ature and its volume. I should be satisfied with this vague sort knowledge about superheated steam, if it were not that a more ct knowledge of the $p, v, t$ law is necessary before we can make ct statements about the specific heat.

On certain assumptions which have no experimental foundation, n, Zeuner, Ritter and others have obtained formulæ which have n made to fit **Hirn's experimental numbers.**

To find the density of superheated steam Hirn weighed a vessel taining the steam. His results are given in Table IV. at the of his "Exposition analytique et expérimentale de la Théorie anique de la Chaleur." I know of no other measurements on erheated steam except those of Regnault, who measured the cific heat at atmospheric pressure. I give in Art. 371, Hirn's ılts, converting $u$ into cubic feet per pound.

On any assumption, if we could obtain a simple empirical formula sfying our facts it would be valuable, and some of the formulæ en are supposed to agree with Hirn's results fairly well. It ns to have been forgotten by the framers of these formulæ and those who calmly accept such formulæ, that the formula of a fect gas must always agree fairly well with the results, and that test of a more correct formula is not as to whether it is nearly it for $u$ the volume, but for $v - u$, where $v$ is the volume of a nd of $H_2O$ as gas at the same pressure and temperature. Sub- ed to this test, I find that all the formulæ are as much as 16 per t. wrong; and it is easy to obtain many other simple formulæ ch are just as correct, but it would be very foolish to assume t any one of them is of value from the point of view of thermo- amic theory.

From the point of view of thermodynamic theory we should be d to obtain either

$v$ as a linear function of $t$, when $p$ is constant . . . (1)

$p$ as a linear function of $t$, when $v$ is constant . . (2),

ause it is evident from (19) and (20), Art. 361, that (1) means

$$K \text{ a function of } t \text{ only,}$$

(2) means

$$k \text{ a function of } t \text{ only.}$$

Dr. Ramsay asserts that for all vapours (2) is true; and we have

seen in Art. 365 that this is not inconsistent with his assertion that the adiabatic elasticity is a linear function of $t$ when $v$ is constant.

(2) is certainly not in accordance with Hirn's experimental results on superheated steam. To test it I took one of Hirn's volumes, 27·87 when $t = 392{\cdot}2$ and $p = 2116$; for the same volume saturated steam has $t = 372{\cdot}2$ and $p = 2010$. If (2) is true or

$$p = bt + a,$$

where $b$ and $a$ are functions of volume only; then for $u = 27{\cdot}87$, $a = 37$, $b = 5{\cdot}30$. Doing this for many of Hirn's observations, I found that $a$ could not be consistently regarded as a function of the volume. Thus I obtained

| vol. $u$ | 29·63 | 27·87 | 11·16 | 9·215 | 8·362 | 7·723 | 6·631 | 6·019 |
|---|---|---|---|---|---|---|---|---|
| $a$ | 5 | 37 | -335 | -1424 | -962 | -2390 | -209 | -1786 |
| $b$ | 5·09 | 5·30 | 14·11 | 19·42 | 19·90 | 24·75 | 22·54 | 28·51 |

so that $a$ is evidently no function of the volume.

It may be imagined that if the general rule (2) fails so seriously, the special form of it invented by Van der Waals will fail more seriously still, and this I have found to be so. When we try (1) above, we obtain what seems to me a better consistency. In trying (2) we had only two observations from which to determine each $a$ and $b$, whereas in testing (1) we have in some cases many observations: and it may be that it is on this account that (1) seems more consistent than (2).

Taking

$$u = bt + a$$

where $b$ and $a$ are functions of $p$ only, we obtain the following results:—

| $p$ in atmos. | 1 | 2½ | 3 | 3½ | 4 | 5 |
|---|---|---|---|---|---|---|
| $a$ | 1·29 | 0·10 | 1·23 | 1·70 | -0·70 | 0·45 |
| $b$ | 0·068 | 0·0309 | 0·0259 | ·0239 | 0·0189 | 0·0148 |

In carrying out this work it became evident that the discrepancies in $a$ were largely due to Hirn's errors of measurement; and certainly there is no disproof of the law (1), although, indeed, we cannot say that there is proof of it. I am disposed to think that neither (1) nor (2) is true, but that (1) is so much more nearly true than (2) that we may assume it true until we get further evidence.

**371.** It is on the whole better to test the **discrepance from the gaseous law.** The following Table shows the values of $\frac{Rt}{p} - u$ both for Hirn's results and for saturated steam at the same pressure. For each pressure I have plotted $v - u$ (calling $\frac{Rt}{p}$, $v$) and temperature, and it at once becomes evident that we can deduce no law from Hirn's results.

HIRN'S EXPERIMENTS.

| $p$ atmos. | $\theta$° C. | Hirn's. $u$ | $p$ lb. per sq. ft. | $t$ absol. | $u$ cub. ft. per lb. | $Rt/p - u$. |
|---|---|---|---|---|---|---|
| 1 | 118·5 | 1·74 | 2116 | 392·2 | 27·87 | 0·62 |
| ,, | 141 | 1·85 | 2116 | 414·7 | 29·64 | 0·49 |
| ,, | 148·5 | 1·87 | 2116 | 422·2 | 29·95 | 0·73 |
| ,, | 162 | 1·93 | 2116 | 435·7 | 30·91 | 0·70 |
| ,, | 200 | 2·08 | 2116 | 473·7 | 33·32 | 1·11 |
| ,, | 205 | 2·14 | 2116 | 478·7 | 34·28 | 0·50 |
| ,, | 246·5 | 2·289 | 2116 | 520·2 | 36·66 | 1·15 |
| 2·25 | 200 | 0·92 | 4762 | 473·7 | 14·73 | ·57 |
| 3 | 200 | 0·697 | 6349 | 473·7 | 11·165 | 0·315 |
| 3·5 | 196 | 0·591 | 7407 | 469·7 | 9·466 | ·206 |
| ,, | 201 | 0·6035 | 7407 | 474·7 | 9·668 | ·104 |
| ,, | 225 | 0·636 | 7407 | 498·7 | 10·188 | 0·162 |
| ,, | 246·5 | 0·6574 | 7407 | 520·2 | 10·531 | 0·269 |
| 4 | 165 | 0·4822 | 8465 | 438·7 | 7·724 | 0·245 |
| ,, | 200 | 0·522 | 8465 | 473·7 | 8·362 | 0·242 |
| ,, | 225 | 0·539 | 8465 | 498·7 | 8·634 | ·423 |
| ,, | 246·5 | ·5752 | 8465 | 519·7 | 9·214 | 0·227 |
| 5 | 160 | ·3758 | 10582 | 433·7 | 6·020 | ·282 |
| ,, | 200 | ·4095 | 10582 | 473·7 | 6·558 | 324 |
| ,, | 205 | ·414 | 10582 | 478·7 | 6·632 | ·323 |

The following values are for the saturated condition, multiplying Rankine's $u$ by 774/772:—

| $p$ atmos. | $\theta$° C. | $p$ lb. per sq. ft. | $t$ absol. | $u$ cub. ft. per lb. | $Rt/p - u$. |
|---|---|---|---|---|---|
| 1 | 100 | 2116 | 373·7 | 26·43 | 0·73 |
| $2\frac{1}{4}$ | 124·35 | 4762 | 398·1 | 12·33 | 0·52 |
| 3 | 133·6 | 6349 | 407·3 | 9·43 | 0·435 |
| $3\frac{1}{2}$ | 139·23 | 7407 | 412·9 | 8·15 | 0·422 |
| 4 | 144 | 8465 | 417·7 | 7·19 | 0·396 |
| 5 | 152·3 | 10582 | 426 | 5·83 | 0·36 |

In calculating $v$ which is $Rt/p$, I use $t = \theta + 273·7$, $p$ in pounds per square foot and $R = 153·8$ to suit an atomic weight of oxygen of 15·88. Hence, $v$, $p$ and $t$ are the volume pressure and temperature of $H_2O$ as a perfect gas. I had been in hopes that $v - u$ might be expressed as some simple function of pressure and temperature, or perhaps that $\frac{v - u}{v}$ might be so expressed.

There is no ground for any such assumption—a very much

simpler one is all that is warranted by the experiments. Take 1 atmosphere

| $\theta°$ C. | 100°C. | 118°·5 | 141° | 148°·5 | 162° | 200° | 205° | 246°·5 |
|---|---|---|---|---|---|---|---|---|
| $v-u$ | ·73 | ·62 | ·49 | ·73 | ·70 | 1·11 | 0·50 | 1·15 |

The mean of these eight observations 0·75, is almost exactly the value of $v-u$ at saturation.

We have only single observations of superheated steam at 2·25 and at 3 atmospheres. These conflict with one another when we compare them with $v-u$ at saturation, one showing an increase of $v-u$ with temperature, the other a diminution. But in view of the great number of results for one atmosphere which are so obviously unreliable, I feel quite sure that we cannot build upon any of these results of Hirn.

At 3·5 atmospheres Hirn's own results show no law, only inconsistency. But they are all lower than $v-u$ for saturation. It is only on the assumption, therefore, of the saturation value being correct that we get grounds for any assumption except constancy in $v-u$.

At 4 atmospheres we have the following values:—

| $\theta°$ C. | 144 | 165 | 200 | 225 | 246·5 |
|---|---|---|---|---|---|
| $v-u$ | ·396 | ·245 | ·242 | ·423 | ·227 |

Nothing but taking a mean can here satisfy us. But we always like to give greater weight to the saturation value, and by assuming it exactly right we may if we please suppose some fall in $v-u$ as $\theta$ increases. It is, however, unfair to draw any conclusion of consequence.

At 5 atmospheres we have more constancy:—

| $\theta$ C. | 152·3 | 160 | 200 | 205 |
|---|---|---|---|---|
| $v-u$ | ·36 | ·282 | ·324 | ·323 |

On the whole, therefore, I am disposed to say that **the only conclusion deducible** from Hirn's inconsistent and unreliable experiments is that there may be such a law as

$$v = \frac{Rt}{p} - f(p) \quad . \quad . \quad . \quad . \quad (1)$$

g the form and value of $f(p)$ from the more accurately known ers of Rankine for saturated steam. I find then that the ıation at our command at present gives for steam, whether in turated or superheated condition,

$$u = \frac{Rt}{p} - \left(0{\cdot}118 + \frac{3200}{p + 3350}\right) \quad . \quad . \quad . \quad . \quad (2)$$

e $R = 153{\cdot}8$, $t = \theta°$ C. $+ 273{\cdot}7$, $p$ is pressure in pounds per square

will now calculate $u$ for a few cases. Note that in the super-l cases I have chosen those which are most likely to be most in eement with observation.

SATURATED STEAM.

| $\theta$ | $p$ | Accepted $u$ | Now calculated $u$ | Percentage difference |
|---|---|---|---|---|
| 0°C. | 12·27 | 3398 | 3434 | +1·9 |
| 10 | 24·92 | 1736 | 1751 | − ·8 |
| 40 | 152·6 | 313·6 | 315·4 | − ·6 |
| 60 | 414·3 | 122·3 | 123·0 | − ·6 |
| 80 | 987·6 | 54·06 | 54·27 | − ·4 |
| 100 | 2116·4 | 26·43 | 26·48 | − ·2 |
| 120 | 4152 | 14·04 | 14·06 | − ·2 |
| 140 | 7563 | 7·995 | 8·007 | − ·7 |
| 160 | 12940 | 4·827 | 4·843 | − ·3 |
| 180 | 20990 | 3·065 | 3·078 | − ·4 |
| 200 | 32520 | 2·031 | 2·035 | − ·2 |

SUPERHEATED STEAM.

| | | Hirn's $u$ | | |
|---|---|---|---|---|
| 141 | 2116 | 29·64 | 29·46 | +1½ |
| 200 | 2116 | 33·32 | 33·74 | −1·3 |
| 205 | 2116 | 34·28 | 34·10 | + ·6 |
| 246·5 | 2116 | 36·66 | 37·08 | −1·6 |
| 200 | 4762 | 14·73 | 14·80 | − ·5 |
| 200 | 6349 | 11·165 | 11·03 | +1·2 |
| 201 | 7407 | 9·668 | 9·44 | +2·4 |
| 246·5 | 8465 | 9·214 | 9·06 | +1·7 |
| 160 | 10582 | 6·020 | 5·96 | +1·0 |

is easy to find a better formula than the above, if one wishes only to express perties of *saturated* steam. If $v = \frac{Rt}{p}$ as before, and if we plot log. $\left(\frac{v - u}{v}\right)$ . $p$ on squared paper, the points lie very closely in a straight line from es of one-fifteenth of an atmosphere to fourteen atmospheres, and my students duced the law

$$u = \frac{Rt}{p} - {\cdot}0113\, tp^{-0{\cdot}63}$$

**372. Our anxiety on this subject** is not so much to obtain an empirical formula, as to find the specific heat $K$ of superheated steam. One use to which we put $K$ is described in Art. 207, and it evidently is of considerable pecuniary importance. Thus, when we specify for plant for an electric lighting station, we commonly say "there must be one electrical unit produced (about $1\frac{1}{3}$ horse-power hours) for 21 lbs. of steam of 165 lbs. pressure." Now everything depends upon the wetness of the steam supplied, for the engine builder does not usually supply the boiler, and the easy way of finding the wetness depends upon our being able to use a value of $K$ in our calculation. See the other methods described in Art. 207.

The engine builder tests the engine at his own works by taking the steam through a reducing valve from a boiler at 215 lbs. pressure (if the cylinder is to be fed at 165 or less), so that he may be pretty certain of its dryness. Indeed, he is not likely to suffer because of having wet steam, for he knows how to object to what he considers an unfair test. I do not know if anybody ever tests whether the steam taken through such a reducing valve is not too dry, superheated in fact, a test excessively easy to apply.

**The Specific Heat of Superheated Steam.** As a matter of fact, we may say that Hirn's numbers do not help us to any better assumption than the use of $u = \dfrac{Rt}{p}$ in calculating $\left(\dfrac{du}{dt}\right)_p$ and $\left(\dfrac{dp}{dt}\right)_u$, or the assumptions that both $k$ and $K$ are functions of temperature only.

In Art. 363 I have found 0·305 to be the specific heat of superheated steam at 0° C., assuming Regnault's results to be correct. In the same sort of way we can find what is probably its specific heat in other conditions.

If $\phi$ is the entropy of a pound of dry saturated steam it will be found sufficiently accurate for almost all purposes to take

$$\phi = \log_e \frac{t}{273 \cdot 7} + \frac{797}{t} - 0 \cdot 695$$

This is deduced from taking Regnault's total heat as

$$H = 606 \cdot 5 + \cdot 305\theta\text{; or}$$

$$H = 523 \quad + \cdot 305t \text{ if } t = \theta^\circ \text{ C.} + 273 \cdot 7$$

and assuming that the latent heat is

$$L = H - \theta$$

so that we assume the specific heat of water to be constant.

But if we wish to be more accurate; if $\sigma$ is the specific heat of rater, $L = H - \int_{273\cdot 7}^{t} \sigma \, . \, dt$

Now I prefer to use the formula of Rankine for $\sigma$; it most robably agrees with Regnault's units, where $J = 1393$ or $774$. Converting into Centigrade

$$\sigma = 1 + 10^{-6}\theta^2 \text{ (or more exactly } 1 + 10^{-6}(\theta - 4)^2\text{)}$$

From this I find $\phi$, and also

$$\frac{d\phi}{dt} = \frac{1}{t} - \frac{797}{t^2} + \frac{10^{-6}}{3} \frac{(t - 278)^3}{t^2} \quad . \quad . \quad . \quad (1)$$

for steam which just keeps saturated.

But in superheated steam whose characteristic is

$$u = \frac{Rt}{p} - f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $f$ is some function of pressure and temperature, we see from Art. 361 that

$$d\phi = \frac{dH}{t} = \frac{K}{t} dt - \left(\frac{du}{dt}\right)_p dp$$

for any change of state. Now let us suppose the superheated steam to keep infinitely near to saturation, so that $\frac{dp}{dt}$ is defined. Then

$$\frac{d\phi}{dt} = \frac{K}{t} - \left(\frac{du}{dt}\right)_p \left(\frac{dp}{dt}\right)_{sat} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Putting (3) and (1) equal to one another, and neglecting the small term, we find

$$K = 1 - \frac{797}{t} + t\left(\frac{du}{dt}\right)_p \left(\frac{dp}{dt}\right)_{sat} . \quad . \quad . \quad . \quad (4)$$

Taking $u = \frac{Rt}{p} - f$ where $f$ is a function of $p$ only, $\left(\frac{du}{dt}\right)_p = \frac{R}{p}$ and hence

$$K = 1 - \frac{797}{t} + \frac{1}{J} \frac{Rt}{p} \left(\frac{dp}{dt}\right)_{sat} \quad . \quad . \quad . \quad . \quad (5)$$

Thus calculating we have the following results.

The value of $K$ for 0° C. may no doubt be obtained from this

formula if we can find $\frac{dp}{dt}$, but I prefer to take it as worked out in Art. 363.

| $\theta°$ C. | $t$ | $p$ | $\left(\frac{dp}{dt}\right)_{sat}$ | $K$ |
|---|---|---|---|---|
| 0 C. | — | — | — | ·305 |
| 40 | 314 | 152·6 | 8·176 | ·317 |
| 70 | 344 | 649·4 | 28·03 | ·322 |
| 100 | 374 | 2116·4 | 75·52 | ·341 |
| 130 | 404 | 5652 | 169·6 | ·366 |
| 160 | 434 | 12940 | 330·3 | ·385 |
| 190 | 464 | 26270 | 573·8 | ·400 |
| 210 | 484 | 39870 | 830 | ·464 |

There can be no doubt that these values for $K$ must be very nearly **correct near saturation** if Regnault's results are right. If we were only sure that (1) of Art. 370 were true, that is, that $K$ is a function of temperature only, we might use the tabulated values just given for $K$, not merely near saturation, but under all circumstances. I have already pointed out that Hirn's numbers are too incorrect to enable us to make any more exact assumption than (2) where $f$ sometimes increased with temperature and sometimes diminished, but in such an erratic way that we might assume $f$ a function of pressure only, just as fairly as anything else.

It seems to me now for the following reason that this must really be the case; that $f$ is, with some truth, merely a function of the pressure. Regnault's usually accepted value of $K$ or 0·48 is the average value between 224° C. and 125° C. at atmospheric pressure; yet it is not very different from the average value between the same temperatures of the table, although the tabulated values are for steam near saturation and at very great pressures.

Again, Mr. Macfarlane Gray obtained 0·38 as the average—from Regnault's experiments—between 100° C. and 125° C., and this is not very different from the tabulated value.

**Until some fresh experimental evidence** is before us, I am therefore disposed to accept the three numbers deduced from Regnault as being fairly correct, and to assume that $K$ is a function of the temperature. According to this notion the specific heat of superheated steam is

$$K = 0{\cdot}305 + 5{\cdot}75 \times 10^{-6}\theta^2$$

where $\theta$ is the temperature Centigrade. See Note, Art 187.

*Mathematical Exercise for Students.*—Assume that $K = \dfrac{a + b\theta^2}{1 + \beta\theta^2}$ Find $a$, $b$ and $\beta$, so that $K = {\cdot}305$ when $\theta = 0$, and that the average value of $K$ between $\theta = 100$ and $\theta = 125$ is 0·38, and the average value of $K$ between 125 and 225 is 0·48.

**373.** Exercise. If superheated steam follows the law

$$u = \frac{Rt}{p} - 2{\cdot}1\, p^{-0{\cdot}44}$$

and if $K$ is a known function of the temperature, what are the $t$, $\phi$ curves for constant pressure?

*Answer.* $d\phi = \dfrac{K}{t}\, dt$ or $\phi = \displaystyle\int \frac{K}{t} dt.$

Thus if we take $K = {\cdot}305 + 5{\cdot}75 \times 10^{-6}(t - 274)^2$

$$\phi = {\cdot}737 \log.\frac{t}{274} + 2{\cdot}875 \times 10^{-6}(t^2 - 1096\, t) + 0{\cdot}648.$$

It is to be noticed that in Art. 205 I assumed that superheated steam is a perfect gas, and furthermore that its specific heat $K$ is always 0·48. To draw the real $t\phi$ curves for constant pressure and volume would be more troublesome, and even when one has studied most carefully all the existing information, one has no great inclination to draw the curves even on the fresh information which I have given. In Art. 214 it will be seen that we greatly need correct figures to determine the weight of superheated steam used per hour per horse-power.

**374.** Exercise. Assume that for superheated steam

$$p = \frac{Rt}{u} - F,$$

where $F$ is a function of $u$ the volume only: as $k$ is a function of temperature only, see (19) of Art. 361, find it for various temperatures.

It is easy to show, as in the other case, that under nearly saturated conditions

$$k = 1 - \frac{797}{t} - t\left(\frac{dp}{dt}\right)_u \left(\frac{du}{dt}\right)_{sat} \quad . \quad . \quad . \quad . \quad (1)$$

$$k = 1 - \frac{797}{t} - \frac{Rt}{Ju}\left(\frac{du}{dt}\right)_{sat}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The following table of the values of $\dfrac{du}{dt}$ at saturation has been

calculated by Mr. D. Baxandall. The values of $u$ given, are Rankine's $\times \frac{774}{772}$, and $t$ is $\theta°$ C. + 274, $R$ is 153·8.

| $\theta°$ C. | $t$ | $u$ | $\left(\frac{du}{dt}\right)_{sat}$ | $k$ |
|---|---|---|---|---|
| 20 | 294 | 937 | - 55·25 | ·201 |
| 60 | 334 | 122·3 | - 5·351 | ·226 |
| 100 | 374 | 26·43 | - 0·89 | ·258 |
| 140 | 414 | 7·995 | - 0·2133 | ·293 |
| 180 | 454 | 3·065 | - 0·0663 | ·327 |
| 210 | 484 | 1·676 | - 0·0314 | ·353 |

My attention has recently been drawn to the values of $v$, $p$ and $t$ given by Professors Ramsay and Young in the *Phil. Trans.* 1892. These are evidently much more consistent with one another than those obtained by Hirn, but I find that they would not modify my conclusions because of their discontinuity with the calculated saturated volumes obtained by calculation, and it is close to saturation that I desire to know the properties of superheated steam. Also these gentlemen have not published the actual numbers obtained by them in experiment, but only those numbers "smoothed" on some system which is unknown to me. Smoothed on another system they would probably be different.

## CHAPTER XXXIII.

### HOW FLUIDS GIVE UP HEAT AND MOMENTUM.

**375.** THERE are two phenomena which we can understand only through our knowledge of diffusion in fluids:—

I. When a portion of a fluid has a greater temperature than the rest; how the **temperature** is equalised, that is, how it shares the average kinetic energy of its molecules with other portions of the fluid, and how it gives it up to the molecules of a solid body which it touches.

II. When a portion of fluid has, besides its molecular motion of agitation which is the same in all directions, a motion of its centre of gravity relatively to the rest of the fluid; that is, it has on the whole momentum in a particular direction; how it shares this **momentum** with the rest of the fluid, and especially how it gives up momentum to the molecules of a solid body which it touches.

I take it that these cases are identical, that the equalisation is by diffusion; that it proceeds very slowly indeed, at a rate which may be judged of in the following way:—

*Its rate in water.*—Get a large glass vessel of clear water; at a point $A$ let there be the end of a fixed tube, by means of which, with very little commotion, we may colour a small region in the water. If the water has no currents in it, at what rate will the coloured particles diffuse from $A$ through the whole mass? To be quite sure of freedom from currents, it is best to use a long, thin, vertical tube as our vessel. The diffusion is exceedingly slow compared with such equalisations of colour density as we are accustomed to in masses of water.

The molecular theory tells us what this rate is in gases. It is really slow, but it is practically impossible to test the theory by experimentally colouring some of the gas in a vessel.

But when there is such a motion that a portion of fluid $A$ gets

sandwiched out between other portions $B$, so that parts of $A$ are everywhere close to parts of $B$, there is no change in the actual rate of molecular diffusion anywhere, but the diffusion is between the neighbouring parts of $A$ and parts of $B$, and of course there is a rapid equalisation of properties.

If the peculiar property of $A$ is *colour*, colour is equalised.

If the property of $A$ is higher *temperature*, then temperature is equalised.

But in any case the mixing would be exceedingly slow except for the agency of unstable stream line motion which causes portions moving with very different velocities to become sandwiched, that is to come near together, so that diffusion may produce large effects. Now the rapid mixing cannot occur unless there are sandwiched streams, and diffusion is constantly equalising the velocities of the stream lines, and this is what we call friction; hence these three things seem to go together:—

1. Actual mixing of portions of the fluid.
2. Equalisation of temperature.
3. Friction.

**376.** In my 1873 edition I pointed out the importance of artificial obstructions in **the flues of boilers,** and when speaking to students I have persistently dwelt since then upon the apparent fact that anything which increases the friction of flue gases against the metal surface increases the rate of transmission of heat, but I must confess that I had no exact notions on the subject until, in 1897, Mr. Stanton, a pupil of Professor Osborne Reynolds, read the abstract of a paper before the Royal Society, published later in full in the *Philosophical Transactions.*

He forced water at two different temperatures through two concentric pipes, one surrounding the other, and showed that at quite different speeds the change of temperature produced in a given length of pipe was pretty much the same: that is, that twice as much heat passed when the speeds were twice as great. I at once put the matter in the following shape. My theory is very incomplete, but it is not at all bad to think about, and I think that it cannot differ greatly from that of Reynolds, which, to my great astonishment, I read a little about to-day [April, 1898] for the first time, in a short but most suggestive paper published in 1874 before the Manchester Literary and Philosophical Society. I found it referred to in the paper of Mr. Stanton, and I think that I may have heard of it before but mixed it up with a much more

laborate mathematical paper by Professor Reynolds (*Phil. Trans.*, 894).[1]

I feel that to publish this old and neglected paper here will be loing a service to all students, and I have asked for permission to ublish it as an appendix. At the same time I think that the ollowing rough theory, which I worked out after hearing Mr. Stanton's paper, will be welcome.

**377. When fluid is in motion filling a pipe** we know that here is a thin film or layer of fluid entangled among the molecules of the solid surface which is at rest, that is, it has no verage velocity relatively to the solid. Let us consider how heat gets into this film from the moving fluid. It is difficult to say whether one ought or ought not to take entrance of heat to this ayer of motionless fluid as entrance to the metal itself. There is qualisation of the momentum, and there may be equalisation of the emperature.

Now suppose $n$ molecules per second to enter this layer and the ame number to leave it; each of them enters with an average kinetic nergy proportional to $t$ the average temperature (absolute) in the ipe, and leaves with $t^1$ the temperature of the layer, and an average nomentum in the axial direction proportional to $v$ if $v$ is the average xial velocity in the pipe. There is a want of exactness in my lefinition of these averages, which is, I think, the only weakness in his investigation. Now axially directed momentum given to the ayer per second is what we mean by force of friction $F$.

$$\text{So that per unit area } F \propto nv \quad \dots\dots \quad (1)$$

And the heat $H$ or kinetic energy per second per unit area

$$H \propto n(t-t^1). \quad \dots\dots \quad (2)$$

Hence

$$H \propto F(t-t^1)/v \quad \dots\dots \quad (3)$$

Of course when $v$ is 0 we cannot use (1) in (2) to find (3), but we hall only use our equations in cases where $v$ has some value.

In the standard books on friction of fluids in pipes, the law is given

$$F \propto wv^2 \quad \dots\dots \quad (4)$$

where $w$ is the weight of the fluid per unit volume, and $v$ is the

[1] I believe that when I study the 1894 paper and other papers of Professor Reynolds, I shall write on this and many other subjects with certainty and clearness, but I have not yet found the necessary time. I know that the Manchester students have clear and correct notions on many subjects about which other students are ignorant.

average axial velocity in the channel. I am informed by Prof. O. Reynolds that the results of his 1883 paper in the *Philosophical Transactions* are applicable to gases, and taking his index there as 2 we have the same formulæ as (4); (3) and (4) give us

$$H = c'wv(t - t^1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where $c'$ is a constant. If instead of the conductivity of the layer or film at the surface being infinitely great, one side of the film is at $t''$ and the other at $t'$, and if $b$ is its thickness and $k$ its conductivity for heat, we get the equivalent of (5) from

$$(t' - t'')\frac{k}{b} = c'wv(t - t')$$

this gives

$$H \propto \frac{c'wv(t - t'')}{1 + c'wv\frac{b}{k}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

We see therefore that in using, as we shall do, (5) instead of the more correct (6) we shall be assuming $c'$ a constant, whereas it really depends upon the value of $wv\frac{b}{k}$. It is possible that $b$ diminishes as $v$ increases, so that $vb$ may be nearly constant; but $w$ is inversely as the absolute temperature and $k$ is probably proportional to the square of the absolute temperature. We shall proceed, therefore, using (5) and assuming $c'$ to be constant, but in the applications of our results we shall remember that $c'$ increases as the temperature increases.

We shall say then that the **heat resistance** per unit area between a fluid and a metal plate is inversely proportional to $wv$.

**378.** Let us consider two channels conveying fluids to be separated by a metal plate and to be conveying $W$ and $W_3$ lb. of fluid per second, of weights $w$ and $w_3$ lb. per unit volume; the cross sections being $A$ and $A_3$; the lengths of the perimeters of these cross sections which are in the metal plate are $P$ and $P_3$. The velocities being $v$ and $v_3$ and the average temperatures $t$ and $T$, $wv = W/A$, $w_3v_3 = W_3/A_3$.

I shall therefore use as the three resistances per unit area,

$$\frac{r}{wv}, \; R_2 \text{ and } \frac{r_3}{w_3v_3}$$

where $r$, $R_2$ and $r_3$ are constants, $R_2$ being the thickness divided by the conductivity of the metal, or

$$r\frac{A}{W}, \; R_2 \text{ and } r_3\frac{A_3}{W_3}$$

Hence, per unit area, the heat per second passing through is

$$H = \frac{t - T}{r\frac{A}{W} + R_2 + r_3\frac{A_3}{W_3}} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or

$$H = \frac{\frac{W}{A}(t - T)}{r + R_2\frac{W}{A} + r_3\frac{A_3}{W_3}\frac{W}{A}} \quad . \quad . \quad (2)$$

I shall take it that the two fluids move in opposite directions and as if in concentric pipes, and write (1) in the short form

$$H = c(t - T) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

At a distance $x$ from one end of the wall in the direction of motion of the hotter fluid if we take $P$ as the common touched perimeter, the one fluid gives up and the other receives at the elementary area $P.\delta x$ the heat $P.\delta x.c(t-T)$ per second, and in the same time we have the loss $-\ WK.dt$ and the gain $-\ W_3K_3.dT$.

$$-\ WKdt = P.dx.c(t - T) = -\ W_3.K_3.dT \quad . \quad (2)$$

Hence
$$T = \frac{WK}{W_3K_3}t + \text{constant}$$

and
$$t - T = \left(1 - \frac{WK}{W_3K_3}\right)t - C$$

When
$$x = o,\ t = t_1,\ T = T_1$$
$$x = l,\ t = t_2,\ T = T_2$$
$$C = T_2 - \frac{WK}{W_3K_3}t_2$$

$$t - T = \left(1 - \frac{WK}{W_3K_3}\right)t - \left(T_2 - \frac{WK}{W_3K_3}t_2\right) \quad . \quad . \quad (3)$$

Now (2) tells us that
$$\frac{dt}{dx} = -\ \frac{Pc}{WK}(t - T)$$

Let
$$\frac{Pc}{WK}\left(1 - \frac{WK}{W_3K_3}\right) \text{ be called } a,$$

Let
$$\frac{T_2 - \frac{WK}{W_3K_3}t_2}{1 - \frac{WK}{W_3K}} \text{ or } \frac{W_3K_3T_2 - WKt_2}{W_3K_3 - WK} \text{ be called } b$$

$$\text{then} \quad \frac{dt}{dx} = -a(t-b)$$

$$\log.\,(t-b) = -ax + \text{const.}$$

Now $t = t_1$ when $x = o$ so that

$$t = b + (t_1 - b)e^{-ax} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Let **the efficiency** $E$ be defined as $\dfrac{t_1 - t_2}{t_1 - T_2}$

$$\text{then } E = \frac{t_1 - b}{t_1 - T_2}\,(1 - e^{-al})$$

**379. Approximation.** Let us assume that $T_1 = T_2 = T$. This is nearly true in boilers. Then as

$$-\frac{dt}{dx} = \frac{Pc}{WK}(t - T)$$

$$\log.\,(t - T) = -\frac{Pc}{WK}x$$

$$t - T = (t_1 - T)\,e^{-\frac{Pc}{WK}x}$$

$$E = \frac{t_1 - t_2}{t_1 - T} = 1 - e^{-\frac{Pc}{WK}l}$$

The efficiency depends therefore upon the value of

$$\frac{Pcl}{WK} \text{ or } \frac{Pl/KA}{r + r_2\dfrac{W}{A} + r_3\dfrac{A_3}{W_3}\dfrac{W}{A}}$$

or neglecting the term in $r_2$ and leaving out $K$, the efficiency depends upon

$$l \div \left(r\frac{A}{P} + r_3\frac{A_3}{W_3}W\frac{1}{P}\right)$$

If $\dfrac{A}{P}$ and $\dfrac{A_3}{P}$ be called $m$ and $m_3$, the hydraulic mean depths, we see that the efficiency depends upon

$$l \div \left(rm + r_3 m_3 \frac{W}{W_3}\right)$$

The term $r_3 m_3 \dfrac{W}{W_3}$ gets less as there is better and better circulation of water: it is, in fact, inversely proportional to the velocity of the water close to the metal. It is to be noticed that this rapid circulation of the water is as necessary for efficiency as the rapid

circulation of the gases in the flue. Neglecting this term, or assuming that the water circulates so fast as to keep the metal practically at the temperature of the water, the efficiency would depend only upon $l/m$, the length of the flue divided by its hydraulic mean depth, and would be practically independent of the quantity of gases flowing, being $E = 1 - \epsilon^{-cl/m}$ where $c$ is a constant. The following expression will be found to be practically the same as this, and it is easier to deal with;

$$E = 1/(1 + cm/l)$$

where $c$ is a constant. Indeed, in a well constructed boiler the **mere area of the heating surface** ought to be of but slight importance. If in any class of boiler the efficiency depends upon mere area of heating surface, we have a proof of bad circulation of the water; a proof that the carrying off of heat by the water from the metal has not been attended to. It seems to me that when a **good scrubbing action** is established on both sides of the metal, there ought to be at least ten times, and may be more than 100 times as rapid an evaporation per square foot of heating surface as has yet been obtained in any boiler. In existing boilers the resistance of the metal itself is insignificant, as the following exercises will show. As better and better circulation is provided on both sides of the metal, it seems to me that the total resistance must approximate more and more to that of the metal itself.[1]

**380. Heat Resistance of a Metal Plate.**—The following exercises illustrate the insignificance of the metal plate resistance to the passage of heat in existing boiler flues and surface condensers.

[1] The above investigation shows that the **following simple way** of putting the whole matter is legitimate within certain limits of velocity, &c.

Assume the temperature $T$ of the water to be constant. Let $t - T$ be called $\theta$, $t$ being the absolute temperature of the gases at the distance $x$ from the furnace end of a tube of total length $l$ and diameter $d$. Let $W$ lb. of gases flow through the tube per second, the specific heat being constant.

Take our law as given in (5), Art. 377, to be that the loss of heat per second per unit area of tube is proportional to

$$v\theta/d \text{ or } W\theta/d^2$$

where $v$ is the velocity, for $v$ is proportional to $W/d^2$. Hence in the short length $\delta x$

$$-W.\delta\theta = cW\theta.\delta x/d$$

where $c$ is a constant. Solving this and taking $\theta = \theta_1$ at the furnace end and $\theta = \theta_2$ at the smoke box end, we have

$$\theta = \theta_1\epsilon^{-cx/d} \text{ and } \theta_2 = \theta_1\epsilon^{-cl/d}$$

and the **efficiency** is

$$E = 1 - \epsilon^{-cl/d}$$

In *CGS* units; the heat (in gramme of water degree units) $Q$ which flows in $t$ seconds between two parallel surfaces, $A$ square cm. in area $x$ cm. apart, the temperature difference being $\theta$ is

$$Q = \frac{A}{x} kt\theta$$

Where $k$ is the conductivity of the material. $k$ is probably proportional to the absolute temperature of metals. In the following table I give the Conductivities which are assumed to be correct in academic problems. Only the iron and trachyte are probably nearly correct, $k^1$ is $k \div 454$.

| Substance. | Conductivity. | $k^1$ of formula. |
|---|---|---|
| Steel | ·06 to ·1 | $1{\cdot}4 \times 10^{-4}$ to $2{\cdot}2 \times 10^{-4}$ |
| Iron | ·160 to ·2 | $3{\cdot}5 \times 10^{-4}$ to $4{\cdot}4 + 10^{-4}$ |
| Copper | 1·108 to 0·7 | $24 \times 10^{-4}$ to $15 \times 10^{-4}$ |
| Brass | 0·2 to ·3 | $4{\cdot}4 \times 10^{-4}$ to $6{\cdot}6 \times 10^{-4}$ |
| Trachyte | ·006 | $1{\cdot}3 \times 10^{-5}$ |
| Fire-brick | ·0017 | $3{\cdot}7 \times 10^{-6}$ |
| Plate glass | ·002 | $4{\cdot}4 \times 10^{-6}$ |
| Oak | ·0006 | $1{\cdot}3 \times 10^{-6}$ |

Mr. Callendar's latest numbers are $k = {\cdot}11$ for iron, ·12 for steel.

Now our unit of heat is in pound of water degree units and therefore if $Q^1$ is in these units; if $A$ is area in square cm., $x$ thickness in cm., and $k^1$ is the number in the table; then we have

$$Q^1 = \frac{A}{x} k^1 t\theta$$

If $\theta$ is in Fahrenheit or Centigrade degrees, $Q^1$ is in Fahrenheit or Centigrade pound of water units.

I often call $x \div Ak^1$ the heat *resistance* $R$ of the metal and then

$$Q^1 \text{ per second} = \theta \div R$$

EXERCISE. Find the resistance in our units of a copper plate 1 foot square $\frac{1}{2}$ inch thick.

*Answer.* As $x$ is $\frac{1}{2}$ inch or 1·27 cm., and as $A$ is 1 square foot or 929 square cm., if we take $k^1$ from the table as $24 \times 10^{-4}$ for copper $R = 1{\cdot}27 \div (929 \times 24 \times 10^{-4}) = 0{\cdot}57$.

EXERCISE. If 1·54 centigrade pound units of heat pass through the above square foot of copper per second, what must be the actual temperature difference $\theta$ in the metal?

Heat per second $= \dfrac{\theta}{R}$ so that $\theta = 1{\cdot}54 \times 0{\cdot}57 = 0{\cdot}88$ Centigrade degrees.

XERCISE. Probably no boiler produces more than 9 lbs. of steam our per square foot of metal. Take a plate of copper $\frac{1}{2}$ inch . Take the average temperature difference between gases and · as 750 Centigrade degrees. Take the heat per lb. of steam as ınits, what is the total heat resistance per square foot?

*nswer*. 750 divided by $\frac{9 \times 620}{60 \times 60}$, or by 1·54 units per second, gives stance 487 in foot second pound of water units.

ow we saw that a plate of copper 1 square foot (929 square cm.) ea, $\frac{1}{2}$ inch (1·27 cm.) thick, has a resistance of only 0·57. Hence **total resistance** is nearly 1,000 times that of the plate itself; ct the plate resistance may be neglected in comparison with the resistance even in boilers in which the skin resistances are ›tionally small. The actual thickness of the metal is obviously ry small importance therefore in flues.

XERCISE. If we have 2 square feet of surface in a **surface enser** per indicated horse-power, and if this means the con- tion of 15·4 lbs. of steam per hour per indicated horse-power, mperature being 135° F., the temperature of the water being '.; what is the total heat resistance? The tubes are of brass of an inch thick; compare the resistance of the metal with the . Take it that 1 lb. of steam gives out 950 Fahr. pound units ıdensing.[1]

*nswer*. The heat per second per square foot is 15·4 × 950 ÷ 60 × 60, or 2·03 Fahr. units. Total resistance per square foot face $= \frac{135 - 70}{2 \cdot 03}$ or 32. The resistance of a square foot (929 e cm.), $\frac{1}{20}$th of an inch (·127 cm.) thick is, taking $k^1 = 5 \times$ from the table.

$$\cdot 127 \div (929 \times 5 \times 10^{-4}) \text{ or } 0 \cdot 273$$

ıt the whole resistance is 118 times that of the metal itself.

[1] r. Callendar has recently obtained condensation at the rate of 1·07 Fahr. heat per second per square foot of metal per degree difference of temperature n steam and metal, or, for an actual temperature difference of 22 degrees he had 89 lbs. per hour per square foot. This would mean that 1 square foot suffice for about 6 horse-power. I believe that with steam more and more m air he would have obtained better and better results. We have no right ıme that the rapidity is proportional to temperature difference between water am; but if we might do so we should find more than 22 times the above rate ısmission, and the whole heat resistance would only be 5 times that of the This gives one of the best illustrations of what **a great improvement** will ted **in condensers** when the water is driven through very fine tubes at great y.

EXERCISE. An oak board 1 inch thick touched a plate of iron all over one face; its other face was exposed to the atmosphere of a room in which the temperature was 80° F. There was steam at 300° F. underneath the iron plate. The heat coming through into the room per square foot of surface was found to be 300 Fahr. units per hour, compare the resistance of the oak itself with the whole resistance.

The heat is $300 \div (60 \times 60)$ or ·0835 units per second $\frac{300-80}{\cdot 0835}$ or 2640 is the resistance.

Now the resistance of 1 square foot of oak 1″ thick is $2{\cdot}54 \div (929 \times 1{\cdot}3 \times 10^{-6})$ or 2100, so that the oak resistance is 80 per cent. of the whole. I am afraid, however, that neither the number in the table nor the above number can be relied upon, and this exercise creates a quite wrong notion of the accuracy of $k$ as given in the table.

## APPENDIX.

*Reprinted from the Proceedings of the Literary and Philosophical Society of Manchester*, 1874.

"On the Extent and Action of the Heating Surface for Steam Boilers," by Professor Osborne Reynolds, M.A.

The rapidity with which heat will pass from one fluid to another through an intervening plate of metal is a matter of such practical importance that I need not apologise for introducing it here. Besides its practical value it also forms a subject of very great philosophical interest, being intimately connected with, if it does not form part of, molecular philosophy.

In addition to the great amount of empirical and practical knowledge which has been acquired from steam-boilers, the transmission of heat has been made the subject of direct inquiry by Newton, Dulong and Petit, Péclet, Joule, and Rankine, and considerable efforts have been made to reduce it to a system. But as yet the advance in this direction has not been very great; and the discrepancy in the results of the various experiments is such that one cannot avoid the conclusion that the circumstances of the problem have not been all taken into account.

Newton appears to have assumed that the rate at which heat is transmitted from a surface to a gas and *vice versâ* is *cæteris paribus* directly proportional to the difference in temperature between the surface and the gas, whereas Dulong and Petit, followed by Péclet, came to the conclusion from their experiments that it followed altogether a different law.[1]

These philosophers do not seem to have advanced any theoretical reasons for the law which they have taken, but have deduced it entirely from their experiments, "à chercher par tâtonnement la loi que suivent ces résultats."[2]

[1] *Traité de la Chaleur*, Péclet, Vol. I., p. 365.
[2] *Ib.* p. 363.

In reducing these results, however, so many things had to be taken into account and so many assumptions have been made that it can hardly be a matter of surprise if they have been misled. And there is one assumption which upon the face of it seems to be contrary to general experience, this is, that the quantity of heat imparted by a given extent of surface to the adjacent fluid is independent of the motion of that fluid or of the nature of the surface ;[1] whereas the cooling effect of a wind compared with still air is so evident that it must cast doubt upon the truth of any hypothesis which does not take it into account.

In this paper I approach the problem in another manner from that in which it has been approached before. Starting with the laws recently discovered of the internal diffusion of fluids I have endeavoured to deduce from theoretical considerations the laws for the transmission of heat, and then verify these laws by experiment. In the latter respect I can only offer a few preliminary results ; which, however, seem to agree so well with general experience, as to warrant a further investigation of the subject, to promote which is my object in bringing it forward in the present incomplete form.

The heat carried off by air or any fluid from a surface, apart from the effect of radiation, is proportional to the internal diffusion of the fluid at and near the surface, *i.e.*, is proportional to the rate at which particles or molecules pass backwards and forwards from the surface to any given depth within the fluid, thus, if $AB$ be the surface and $ab$ an ideal line in the fluid parallel to $AB$, then the heat carried off from the surface in a given time will be proportional to the number of molecules which in that time pass from $ab$ to $AB$—that is for a given difference of temperature between the fluid and the surface.

This assumption is fundamental to what I have to say, and is based on the molecular theory of fluids.

Now the rate of this diffusion has been shown from various considerations to depend on two things :—

1. The natural internal diffusion of the fluid when at rest.

2. The eddies caused by visible motion which mixes the fluid up and continually brings fresh particles into contact with the surface.

The first of these causes is independent of the velocity of the fluid, if it be a gas is independent of its density, so that it may be said to depend only on the nature of the fluid.[2]

The second cause, the effect of eddies, arises entirely from the motion of the fluid, and is proportional both to the density of the fluid, if gas, and the velocity with which it flows past the surface.

The combined effect of these two causes may be expressed in a formula as follows :

$$H = At + B\rho vt, \qquad \text{(I)}$$

where $t$ is the difference of temperature between the surface and the fluid, $\rho$ is the density of the fluid, $v$ its velocity, and $A$ and $B$ constants depending on the nature of the fluid, $H$ being the heat transmitted per unit of surface of the surface in a unit of time.

If therefore a fluid were forced along a fixed length of pipe which was maintained at a uniform temperature greater or less than the initial temperature of the gas we should expect the following results.

1. Starting with a velocity zero, the gas would then acquire the same temperature as the tube. 2. As the velocity increased the temperature at which the gas would emerge would gradually diminish, rapidly at first, but in a

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[1] *Traité de la Chaleur*, Péclet, Vol. I., p. 383.

[2] Maxwell's *Theory of Heat*, chap. xix.

decreasing ratio until it would become sensibly constant and independent of the velocity. The velocity after which the temperature of the emerging gas would be sensibly constant can only be found for each particular gas by experiment; but it would seem reasonable to suppose that it would be the same as that at which the resistance offered by friction to the motion of the fluid would be sensibly proportional to the square of the velocity. It having been found both theoretically and by experiment that this resistance is connected with the diffusion of the gas by a formula:

$$R = A^1 v + B^1 \rho v^2 \qquad \text{(II)}$$

And various considerations lead to the supposition that $A$ and $B$ in (I) are proportional to $A^1$ and $B^1$ in (II). The value of $v$ which this gives is very small, and hence it follows that for considerable velocities the gas should emerge from the tube at a nearly constant temperature whatever may be its velocity.

This, as I am about to point out, is in accordance with what has been observed in tubular boilers as well as in more definite experiments.

In the locomotive the length of the boiler is limited by the length of tube necessary to cool the air from the fire down to a certain temperature say 500°. Now there does not seem to be any general rule in practice for determining this length, the length varying from 16 ft. to as little as 6, but whatever the proportions may be each engine furnishes a means of comparing the efficiency of the tubes for high and low velocities of the air through them. It has been a matter of surprise how completely the steam-producing power of a boiler appears to rise with the strength of blast or the work required from it. And as the boilers are as economical when working with a high blast as with a low, the air going up the chimney cannot have a much higher temperature in the one case than in the other. That it should be somewhat higher is strictly in accordance with the theory as stated above.

It must, however, be noticed that the foregoing conclusion is based on the assumption that the surface of the tube is kept at the same constant temperature, a condition which it is easy to see can hardly be fulfilled in practice.

The method by which this is usually attempted is by surrounding the tube on the outside with some fluid the temperature of which is kept constant by some natural means, such as boiling or freezing, for instance the tube is surrounded with boiling water. Now although it may be possible to keep the water at a constant temperature it does not at all follow that the tube will be kept at the same temperature; but on the other hand, since heat has to pass from the water to the tube there must be a difference of temperature between them, and this difference will be proportional to the quantity of heat which has to pass. And again the heat will have to pass through the material of the tube, and the rate at which it will do this will depend on the difference of the temperatures at its two surfaces. Hence if air be forced through a tube surrounded with boiling water, the temperature of the inner surface of the tube will not be constant but will diminish with the quantity of heat carried off by the air. It may be imagined that the difference will not be great: a variety of experiments lead me to suppose that it is much greater than is generally supposed. It is obvious that if the previous conclusions be correct this difference would be diminished by keeping the water in motion, and the more rapid the motion the less would be the difference. Taking these things into consideration the following experiments may, I think, be looked upon, if not as conclusive evidence of the truth of the above reasoning yet as bearing directly upon it.

One end of a brass tube was connected with a reservoir of compressed air,

the tube itself was immersed in boiling water, and the other end was connected with a small non-conducting chamber formed of concentric cylinders of paper with intervals between them in which was inserted the bulb of a thermometer. The air was then allowed to pass through the tube and paper chamber, the pressure in the reservoir being maintained by bellows and measured by a mercury gauge: the thermometer then indicated the temperature of the emerging air. One experiment gave the following results:—With the smallest possible pressure the thermometer rose to 96° F., and as the pressure increased fell, until with $\frac{1}{10}$ inch it was 87°, with $\frac{1}{2}$ inch it was 70°, with 1 inch it was 64°, with 2 inches 60°: beyond this point the bellows would not raise the pressure.

It appears, therefore, (1) that the temperature of the air never rose to 212°, the temperature of the tube, even when moving slowest; but this difference was clearly accounted for by the loss of heat in the chamber from radiation, the small quantity of air passing through it not being sufficient to maintain the full temperature, an effect which must obviously vanish as the velocity of the air increased; (2) as the velocity increased the temperature diminished, at first rapidly and then in a more steady manner. The first diminution might be expected from the fact that the velocity was not as yet equal to that at which the resistance of friction is sensibly equal to the square of the velocity as previously explained. The steady diminution which continued when the velocity was greater was due to the cooling of the tube. This was proved to be the case, for at any stage of the operation the temperature of the emerging air could be slightly raised by increasing the heat under the water so as to make it boil faster and produce greater agitation in the water surrounding the tube. This experiment was repeated with several tubes of different lengths and characters, some of copper and some of brass, with practically the same results. I have not, however, as yet been able to complete the investigation, and I hope to be able before long to bring forward another communication before the Society.

I may state that should these conclusions be established, and the constant $B$ for different fluids be determined, we should then be able to determine, as regards length and extent, the best proportion for the tubes and flues of boilers.

# CHAPTER XXXIV.

## JETS OF FLUID.

**381.** EVERY now and then during the last twenty years a student has asked for help in studying what will occur when **a jet of steam gives momentum** to a jet of water; his idea being to use the water in a turbine of some form, or, more directly still, in the propulsion of ships. This is a subject which is likely to become of great importance, and there is practically no help for the student in any of the books. Indeed, there is worse than no help. Mistakes are numerous in the best books on the flow of water; what must they be when the subject is the flow of a gas, and how absurd must the statements be on the flow of wet steam! I will not apologise for attempting to take up this subject in spite of the sense of my ignorance, because practical men feel the pressing need for some guidance, and there is what is much worse than no guidance in books at present. I shall assume that students know something about hydraulics. Not the misleading mixture of mathematical symbols and nonsense which is to be found in many books, but the common sense notions of the late **Professor James Thomson,** which really cover the whole ground of our knowledge. How do pressure and velocity alter along and across stream lines? the theory of the Thomson Jet Pump; what occurs near the frictional sides of a basin when water is flowing from it by a central hole at the bottom? the simple theory of the centrifugal pump and turbine. I have attempted in my book on Applied Mechanics, to give James Thomson's notions on these subjects. As to the way in which friction occurs in the passage of pumps, mathematical treatment of the subject is quite absurd in the present state of our knowledge; all we can do is to try to apply in a common sense fashion the general notions which the beautiful experiments of

Professor Osborne Reynolds have given us. I usually content myself with telling students how we get angles of vanes and velocities, so that fluid may leave one part of a contrivance and enter another moving with a different velocity, without shock; and how we ease off the sections of passages gradually so that there shall be small frictional loss of energy.

The rules for the steam turbine must, for the present, be the same as for the water turbine. The velocity of the rim of a wheel must be nearly equal to that which the fluid when flowing from one vessel to another would have at the orifice, if the pressure difference were half that between the supply and exhaust of the turbine; and hence it is that Mr. Parsons sends his steam through a series of such turbines, otherwise his velocities would be too great. See Art 389.

When a jet of fluid at very great velocity impinges on a jet of much greater mass, and they both go on together, there must be a great loss of energy. Fluids in passages are not altogether like colliding bodies in space, but the great general rule for such bodies must be borne in mind. When a moving body of mass $M_1$, and velocity $V_1$, strikes a body at rest of mass $M_2$, and they are found moving together afterwards, we know that the common velocity is $V = \dfrac{M_1 V_1}{M_1 + M_2}$ and

$$\frac{\text{lost energy}}{\text{remaining energy}} = \frac{M_2}{M_1},$$

so that the greater the stationary body the greater is the loss. Those inventors who wish to utilise a jet of steam in giving motion to water must bear this fact in mind. It does not necessarily mean that when we let a jet of steam give motion to water and allow the water to drive a turbine or exert propelling force, that the loss of energy will be exceedingly great compared with what occurs in a steam engine. Calculation and experiment may show that in spite of this loss of energy the efficiency of such a machine may compare favourably with that of a steam engine, and it may, besides, be more convenient in construction and application.

**382.** Until somebody makes a thorough experimental investigation, I do not see that we can make any accurate calculation except on the basis of the **following assumption.**

*A B C D* and *E F*, Fig. 319, are cross sections of a cylindric pipe. Normally to the portion *BC* of area $a_1$, there is a flow of fluid at the velocity $v_1$, the pressure there is $p_1$; normally to the rest (of area $a_2$)

of the section $AD$, there is a flow of fluid at the velocity $v_2$, and the pressure there is $p_2$.[1]

I shall neglect the action of gravity in the pipe, that is, difference of pressure due to difference of level. $EF$ is a cross section of area $a = a_1 + a_2$, through which the velocity $v$ is normal, the pressure being $p$. I assume that there is no friction at the boundaries of the

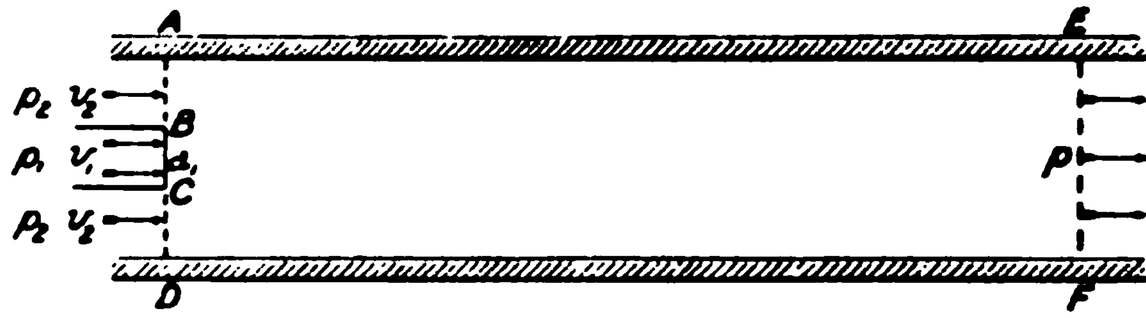


FIG. 319.

fluid, but there is sufficient friction in the fluid itself to cause the streams to get a common velocity at $EF$. Let $w_1$, $w_2$ and $w$, be the weight in pounds of a cubic foot of each fluid.

I. The quantity of fluid flowing in at $AD$ is equal to what flows out at $EF$,

$$a_1 v_1 w_1 + a_2 v_2 w_2 = a v w, \text{ and } a_1 + a_2 = a \quad . \quad . \quad . \quad . \quad (1)$$

II. The momentum per second communicated at $AD$, minus that going out at $EF$, is equilibriated by the pressure forces.

The weight of water per second through $a_1$ is $v_1 a_1 w_1$, and its momentum is $\frac{w_1}{g} a_1 v_1^2$, if $w$ is the weight of unit volume; so that

$$a_1 \left(\frac{w_1 v_1^2}{g} + p_1\right) + a_2 \left(\frac{w_2 v_2^2}{g} + p_2\right) = a \left(\frac{w v^2}{g} + p\right) \quad . \quad . \quad . \quad . \quad (2)$$

This is true because the pipe by assumption exerts no force in the axial direction, and there are no other forces acting on the whole mass from the outside than what I have considered. It is evident that we can calculate $v$ and $p$ from (1) and (2).

(1) May be written—

$$v = \boldsymbol{a}_1 v_1 + \boldsymbol{a}_2 v_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $\boldsymbol{a}_1$ stands for $a_1 w_1 / (a_1 + a_2) w$, and $\boldsymbol{a}_2$ for $a_2 w_2 / (a_1 + a_2) w$. Let us use $e$ to represent $\frac{v^2}{2g} + \frac{p}{w}$ and (2) may be written—

$$e = \boldsymbol{a}_1 e_1 + \boldsymbol{a}_2 e_2 - \frac{1}{2g}\left\{(\boldsymbol{a}_1 v_1 + \boldsymbol{a}_2 v_2)^2 - \boldsymbol{a}_1 v_1^2 - \boldsymbol{a}_2 v_2^2\right\} \quad . \quad . \quad (2)$$

[1] In a practical case, say of a jet pump, these velocities and pressures will be the average velocities and pressures.

The advantage of this way of putting the matter lies in this, hat, except for the usual friction which we meet with in pipes, $e$, after he mingling, will remain constant in the pipes, and we need no onger think that we can only make calculations at a section where $= a_1 + a_2$. Thus, if we want to consider to what height $h$ the ombined stream will rise against zero pressure, $e$ is $h$. If we want o consider $P$ the pressure in pounds per square foot in a vessel nto which the combined stream is o force its way through a pipe radually getting larger, $e$ is $\frac{P}{w}$. If ve want to consider the velocity $V$ with which the combined tream will enter the atmosphere, $e$ s $\frac{2116}{w} + \frac{V^2}{2g}$.

There is the further great advantage that from (2) we can easily ind the waste of energy due to he mingling of the streams. The ressures may be either absolute r measured from any zero.

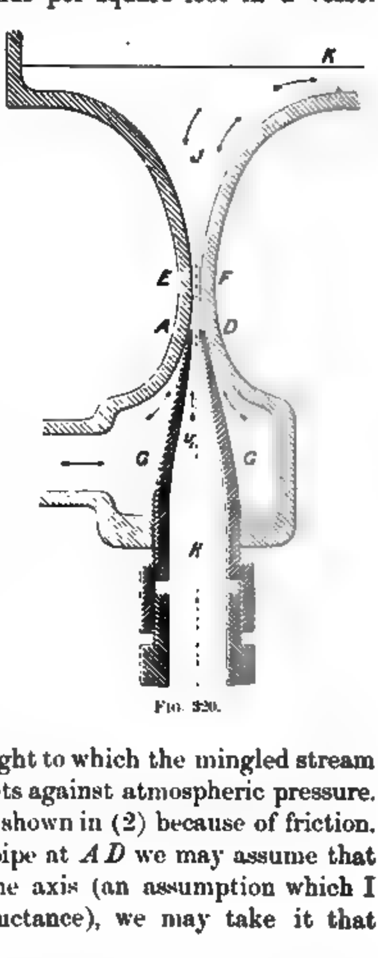


FIG. 320.

**383. In liquids** we take $w_1 = w_2 = w$ so that $a_1$ and $a_2$ mean $\frac{a_1}{a_1 + a_2}$ and $\frac{a_2}{a_1 + a_2}$ and $a_1 + a_2 = 1$. If the liquids have come rom tanks whose atmospheric still urfaces are $h_1$ and $h_2$ feet above he jets, we may take $h_1$ and $h_2$ as epresenting $e_1$ and $e_2$ and $e$ as the height to which the mingled stream ill lift itself above the level of the jets against atmospheric pressure. n all cases, of course $e$ is less than is shown in (2) because of friction.

If for a sensible length of the pipe at $AD$ we may assume that ne stream lines are all parallel to the axis (an assumption which I ever make except with great reluctance), we may take it that $p_1 = p_2$, and the work is simplified.

**384.** To illustrate the use of our formulæ let us consider a ase in which there are water streams only. In the **jet pump of rofessor James Thomson,** a rapidly flowing jet of water passes rough the nozzle $H$ (Fig. 320) and mingling at $AD$ with water hich comes into the chamber $G$, the two streams are discharged at $J$

Suppose that 1 cubic foot of water per second falls from a height of 60 feet, and passing through $H$ mingles with 6 cubic feet of water per second, which enters $G$ from a tank whose atmospheric still water surface level is the same as the level of the jet. To what height will the combined stream rise with atmospheric pressure above it, neglecting all frictional loss except what is absolutely necessary for the mingling?

Take the atmospheric pressure as zero and assume that $p_2$ is that of a partial vacuum such as we may obtain by a careful adjustment of the nozzles; say that $\frac{p_2}{w} = -25$ feet where $-34$ feet would represent a perfect vacuum.

$$\text{Hence} \qquad \frac{v_2^2}{2g} = 25, \ \frac{v_1^2}{2g} = 85, \ v_1 = 74, \ v_2 = 40$$

$$a_1 = \frac{1}{74} = \cdot 0135, \ a_2 = \frac{6}{40} = \cdot 1500, \ a = a_1 + a_2 = \cdot 1635$$

$$a_1 = \frac{\cdot 0135}{\cdot 1635} = \cdot 0826, \ a_2 = \cdot 9174.$$

Using (2) we find that

$$e = 11 \cdot 97 + 22 \cdot 94 - \frac{1832}{64 \cdot 4} = 6 \cdot 46$$

That is, the combined stream would rise to a height of 6·46 feet above $AD$ if there was no friction except what is necessary for the mingling of the streams.

It is to be noticed that before mingling we have the energy $1 \times 60$ as compared with $7 \times 6 \cdot 46$ or 45·22 after the mingling.

EXERCISE. With the figures of the above exercise, assume that the jet is 4 feet above one water level and 56 feet below the other. Here

$$\frac{v_1^2}{2g} = 56 + 25, \ \frac{v_2^2}{2g} = 25 - 4, \ v_1 = 72, \ v_2 = 36 \cdot 78$$

$$a_1 = \frac{1}{72} = \cdot 0139, \ a_2 = \frac{6}{36 \cdot 78} = \cdot 1631, \ a = \cdot 1770$$

$$a_1 = \cdot 0785, \ a_2 = \cdot 9215, \ e_1 = 56, \ e_2 = -4.$$

The student sees that I take motionless water on the level of the jet at atmospheric pressure as having $e = 0$.

Using (2) we find $e = 10 \cdot 75 + 15 \cdot 67 - 22 \cdot 62 = 3 \cdot 80$; or a lift of 3·80 feet above the jet; or a lift of 7·80 feet above the level of the lower tank.

We see therefore that the jet ought to be at as high a level as it

can be placed for best efficiency. In this case, by having the jet 4 feet above the water to be pumped, our energy before mingling being $1 \times 60$ becomes $7 \times 7{\cdot}80$ or $54{\cdot}6$, which is much better than the last case.

I have not yet worked long enough with the equation (2), nor worked enough numerical exercises to be able to put the matter more simply. The following exercise is less directly worked.

**385.** Suppose 1 cubic foot of water per second falls from a height of 60 feet, and passing through $H$, mingles with water which enters $G$ from a depth of 4 feet. Assuming that $\frac{1}{6}$th of the energy is wasted before the mingling (as if the height were only 50 feet), and that another 10 per cent. is wasted in ordinary pipe friction after the mingling and in the delivery of the water, How much is wasted in the mingling itself? Neglect the difference of level between $AD$ and $K$, the height to which the water is actually delivered.

We want to get some notion of the waste of energy, and we see at once that if $v_2 = v_1$, there is no waste. Here we have a very different state of things from that of the collision of two solid bodies. We can cause a jet with great energy per pound, but small quantity of stuff per second, to share its energy with another of great quantity, without loss of energy (in practice, without *much* loss of energy), if we take care that when the collision takes place we have produced, temporarily, an equal velocity in the jet of great quantity. Now in most practical cases the velocity $v_2$ will be limited. For example, in the jet pump the limiting value of $v_2$ will depend upon the height of $AD$ above the level of the water which is being pumped; even neglecting friction and with a zero lift we cannot have $v_2$ as much as 47 feet per second. The limiting value of $v_2$ cannot be so great as $\sqrt{2g(34 - h)}$ if $h$ is the lift to $AD$. In applying the method to the working of a turbine by water and steam we might have $v_2$ as great as $\sqrt{2g(34 + h)}$ if $h$ is the possible height in feet at which a tank might be kept for cooling the exhaust water from the turbine; but if the exhaust water might be cooled in coming from the turbine to the jet part by passing through tubes cooled by outside water, it seems as if it might be possible to get $v_2$ very great indeed.

Here $\dfrac{v_1^2}{2g} = 50 + 25$ or $v_1 = 69$ feet per second

$$\frac{v_2^2}{2g} = 25 - 4,\ v_2 = 37\ ;\ a_1 = \frac{1}{69} = {\cdot}0145 \text{ square feet.}$$

$a_2 = q_2/37$ if $q_2$ cubic feet of water are pumped per second. I find that I have worked this exercise from the first form of equation (2) in page 600, measuring pressure from absolute zero.

The whole energy of the water at $EF$ is to be the same as if it were motionless at atmospheric pressure plus 10 per cent. of the original energy per second of the jet water or 10 per cent. of $62{\cdot}3 \times 60$, or which is $373{\cdot}8$ or $6w$ if $w$ is $62{\cdot}3$.

$$av(p - 2116) + \frac{wav^3}{2g} - 6w = 0 \quad . \quad . \quad . \quad (1)$$

(4) becomes

$$p = 576 + \frac{w}{g}\left(\frac{69}{a} + 37^2\frac{a_2}{a} - v^2\right) \quad . \quad . \quad . \quad (4)$$

Our unknowns are $p, a_2, v, a$, and besides (1) and (4) we have

$$1 + 37a_2 = av \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$\cdot 0145 + a_2 = a \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Hence $$1 + 37a_2 = (\cdot 0145 + a_2)v.$$

**386.** Obviously the best way to find these unknowns is by trial. Now, if there were no loss of energy whatever, 1 cubic foot falling 44 feet (or 60 feet – the losses) could only lift 11 cubic feet 4 feet, and it is evident that we must look for a much smaller answer than this.

We first try therefore $q_2 = 6\frac{1}{6}$ or $a_2 = \frac{1}{6}$, $a$ is $\cdot 1812$, $p$ is 757 and (1) becomes 101 instead of 0. Trying other values of $a_2$ we at length find that $a_2 = \cdot 225$, $a = \cdot 24$, $v = 38{\cdot}93$, $p = 689$, $av = 9{\cdot}34$, so that the amount of water pumped is $9{\cdot}34 - 1$ or $8{\cdot}34$ cubic feet per second.

What vacuum is actually obtainable in jet pumps, I do not know; it does not seem to have been measured, but if it is less than that due to the 25 feet of water assumed above, the delivery will be less.

## 387. Flow of Steam from Orifices.

I shall assume that **the flow** to the orifice is **adiabatic.**

Let us consider what occurs at two cross sections at $A$ and $B$ of a stream tube in adiabatic flow, and neglect effects due to gravity.

A pound of stuff entering at $A$ brings with it its intrinsic energy $E$, and has work done upon it as it enters, $pV$ if $V$ is its volume: that is, the space gains the energy $E + pV$ with every pound of stuff that enters. Now, for every pound entering there is also a

pound leaving the space, and it carries away with it the value of $E + pV$ at $B$. Hence the values of $E + pV$ must be the same everywhere along a stream line if the flow is adiabatic.

Now, if at any place a pound of fluid consists of $x$ lb. of steam and $1 - x$ of water, and if $\lambda = l - pu$, $l$ being latent heat; if $u$ and $u'$ are the volumes of a pound of steam and a pound of water, and $v$ is the velocity, $h$ being the heat energy in a pound of water,

$$E = h + \lambda x + \frac{v^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$V = ux + u^1 (1 - x) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence along a stream line in adiabatic flow

$$h + \lambda x + \frac{v^2}{2g} + p \{ux + u^1 (1 - x)\} = \text{constant} \quad . \quad (3)$$

Thus, if steam flows from a boiler and $v = 0, x = 1$, where the pressure is $p_0$, and if at another place the pressure is $p$

$$h + x (\lambda + pu - pu^1) + pu^1 + \frac{v^2}{2g} = h_0 + \lambda_0 + p_0 u_0.$$

That is, the velocity is that due to the height

$$h_0 - h + \lambda_0 + p_0 u_0 - pu^1 - x \{\lambda + p (u - u^1)\} \quad . \quad (4)$$

of course $u^1$ is really negligible in this connection and $\lambda + pu = l$.

We may take $h_0 - h = J(\theta_0 - \theta)$

Hence

$$\frac{v^2}{2g} = J(\theta_0 - \theta) + l_0 - xl \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If we can state the amount of heat energy lost by every pound of steam because the flow is not truly adiabatic, this produces a lessening of $v^2/2g$.

Applying the second law of thermodynamics :—Since the flow is adiabatic, the entropy is constant, or

$$\frac{xl}{t} + \log. t \text{ is constant.}$$

In the above case

$$\frac{l_0}{t_0} + \log. t_0 = \frac{xl}{t} + \log. t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

or

$$x = \left(\frac{l_0}{t_0} + \log. \frac{t_0}{t}\right)\frac{t}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

**388. Graphically,** by means of the $\theta\phi$ diagram, $A_0B_0C_0$, Fig. 321, is the horizontal line drawn corresponding to the boiler temperature, and $ABC$ to any other temperature at any place in the stream; then $BP \div BC = x$.

If $AC$ is drawn corresponding to the lowest or terminal temperature where we want the greatest velocity $v$, $x$ is the dryness of the steam at the end of the operation, and the area $BB_0C_0PB$ represents the energy utilised, just as in a perfect engine on the Rankine cycle, Art. 214, only here the energy is stored up as kinetic energy.

Now, it is obvious that this adiabatic condition cannot hold close up to the water when steam and water jets collide; the whole of the steam becomes condensed because of the abstraction of heat

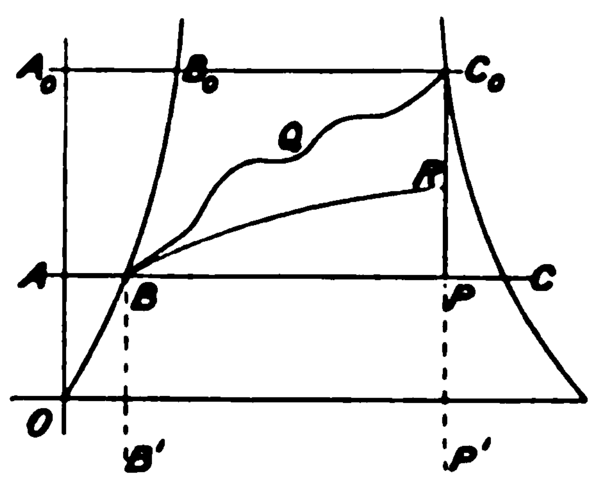



FIG. 321.

and if we know its rate of abstraction so that we can draw $C_0QB$ (the area between $C_0QB$ and the absolute zero line represents the total heat lost) we see that we must take the area $BB_0C_0QB$ as $\frac{v^2}{2g}$, instead of the whole area $BB_0C_0PB$. In fact, the whole gain of kinetic energy is to be calculated in every case as if the work of steam expanding from $p_0$ to $p$ were given to a piston. If the stuff gets rapidly cooled just at the end, we may in Fig. 321 assume adiabatic expansion, say to $R$, and then the curve of constant volume $RB$, as if the stuff were released from an ordinary cylinder without further expansion or doing of work. The area of an ordinary $pv$ indicator diagram, will illustrate this very well. Some such loss as 25 to 50 per cent. of the whole energy must be assumed in practical cases where steam jets collide with water-jets I think.

In academic exercises, like the following, I assume that real adiabatic expansion takes place.

*Exercises to be worked Graphically.*

Dry steam at the following boiler pressures and temperatures flows adiabatically, reaching the following lower temperatures with the velocities $v$.

| Boiler steam (dry). | | At the following lower temperatures the velocities are as given in feet per second. | | | |
|---|---|---|---|---|---|
| temp. Cent. | press. lb. per sq. in. | 120° C. | 80° C. | 40° C. | 20° C. |
| 0° C. | | | | | |
| 100 | 14·7 | — | 1630 | 2860 | 3340 |
| 110 | 20·8 | — | 1950 | 3045 | 3490 |
| 120 | 28·83 | — | 2225 | 3200 | 3570 |
| 130 | 39·25 | 1070 | 2450 | 3355 | 3750 |
| 140 | 52·52 | 1470 | 2625 | 3430 | 3860 |
| 160 | 89·86 | 1610 | 2635 | 3425 | 4060 |
| 200 | 225·9 | 2730 | 3450 | 4090 | 4390 |

**389.** It will presently be seen that the pressure in the jet is never less than ·578 of the higher pressure, and hence all the velocities of the above table, except two, are misleading, if we think of the steam flowing into an atmosphere. It may however be that at the nozzles of injectors these very great velocities do occur.

In calculating the flow of steam through an orifice, if $A$ is the area of the jet where the stream lines are most nearly parallel, $Av$ is the volume flowing per second, and divided by $ux$ (neglecting the volume of the water) it is the weight in pounds per second, or $Av/ux$.

Of course, if the flow is into the atmosphere or a vessel at lower pressure, the kinetic energy is changed into heat after passing through the orifice, and the wetness is lessened, or the steam becomes dry or superheated. But the steam will be wet near the orifice.

We may put the above result algebraically. When any fluid, water, or wet steam, or dry steam, or superheated steam, or air, or any other gas flows adiabatically from a vessel at pressure $p_0$ where its velocity is 0 to a place where its pressure is $p_1$; we find the work which it would do if admitted to a cylinder with no clearance, when expanding adiabatically to $p_1$, and we know that this work is the gain of its kinetic energy or $\frac{v^2}{2g}$. Thus for **air or any other gas** this will be found to give

$$v^2 = \frac{2g}{w_0}\frac{\gamma}{\gamma-1}\left(p_0{}^{1-\frac{1}{\gamma}} - p_1{}^{1-\frac{1}{\gamma}}\right)p_0{}^{1/\gamma} \quad . \quad . \quad . \quad (1)$$

**if $p_0$ is** the initial and $p_1$ the final **pressure, if** $w_0$ **is the weight of**

unit volume at $p_0$: $\gamma$ is 1·41 for air, 1·3 (doubtful) for superheated steam.

It will be found to answer also very nearly for **dry or wet steam** if we take as $\gamma$ the value given in the table, page 362.

| | |
|---|---|
| leaving the boiler dry, | $\gamma = 1{\cdot}130$ |
| leaving boiler with 25 per cent. water | $\gamma = 1{\cdot}113$ |
| „ „ „ 50 per cent. water | $\gamma = 1{\cdot}054$ |
| „ „ „ 75 per cent. water | $\gamma = 0{\cdot}959$ |

Thus, for example, taking dry saturated steam at 130° C. flowing to a place at 120° C. this method gives 1,074 feet per second, whereas the true answer in the above table is 1,070. Again, dry steam flowing from 200° C. to 20° C. gets a velocity of 4,400 feet per second, whereas the correct answer according to the table is 4,390. [It will presently be seen that both these answers are misleading.]

It will be found on trial that if $p_1$ is very little less than $p_0$, the above formula is **approximately** the same as

$$v^2 = \frac{2g}{w_0}(p_0 - p_1) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

EXERCISE. In a Thomson water turbine the velocity of the rim of the wheel is the velocity due to half the total available pressure; so in an air or steam turbine when there is no great difference of pressure, the velocity of the rim of the wheel is the velocity due to half the pressure difference. Thus if $p_0$ of the supply is 7,000 lbs. per square foot, and if $p_1$ of the exhaust is 6,800 lbs. per square foot, and if we take $w_0 = 0{\cdot}28$ lb. per cubic foot (as if it were air, or rather wet steam), then halving the pressure difference and using the above formula on 100 lbs. per square foot, we find

$$v = \sqrt{2g \times 100 \div {\cdot}28} = 151 \text{ feet per second.}$$

**390.** It is evident that as $p_1$ is made less and less, $v$ the velocity increases more and more, and so does $Q$ the cubic feet per second. But a large $Q$ does not necessarily mean a large quantity of fluid. It is worth while taking as an exercise $p_0 = 2$ atmospheres, and studying the result when $p_1$ is made less and less. Find $v$, using (1) in each case, and $W$, the weight in pounds flowing per second through an orifice one square foot in area. It will be found that $W$ is a maximum when $p_1$ is somewhat more than 1 atmosphere.

Such a numerical example suggests to us the general question what is the **maximum weight flow of a gas** through a throat.

Returning to (1) Art. 389, neglecting friction, if there is an orifice of area $A$ near which the flow is guided, so that the streams of air are parallel: $Q$ the volume in cubic feet flowing per second is $Q = vA$; the weight of stuff flowing per second is $W = vAw_1$ or $vAw_0\left(\frac{p_1}{p_0}\right)^{\frac{1}{\gamma}}$.

Hence, using $a$ for $p_1/p_0$ we have

$$W = A\sqrt{\frac{2g\gamma}{\gamma - 1}w_0 p_0\left(a^{2/\gamma} - a^{1+1/\gamma}\right)} \quad . \quad . \quad . \quad . \quad (1)$$

It is an easy exercise in the calculus to find what value of $a$ will cause $W$ to be a maximum. Statement (2) which follows this expresses the answer. It really comes to this, that there is maximum flow when $p_1$ is somewhat greater than half $p_0$.

**391.** If there is no loss or gain of energy by friction, &c., the above rules for the velocity are absolutely true. But mistakes may be, and are, often made in regard to the value of the pressure $p_1$.

When a jet of water is visible passing through the atmosphere, all round it there is atmospheric pressure, but what is the pressure inside? We guess at this. If the stream lines are evidently nearly parallel at a place, it is probable that there is the same pressure from inside to outside. Correct guessing is easy in the case of visible water. But in the case of gases the guessing may not be easy; and, indeed, it was found by **Napier's experiments** on steam and subsequent ones on air, that when $p_0$ is greater than twice $p_1$, the shape of the jet and the shapes of the stream lines near the orifice are so utterly different from those of water (we always base our notions on the behaviour of water jets which we have seen), that we rely upon experiment only, there being no theory to guide us. Whereas when $p_0$ is less than twice $p_1$, the theory is found to be as correct as with the flow of water. In fact, it is found that the pressure in the jet $p_1$ **never gets to be less** than the pressure corresponding to maximum flow, however low may be the pressure in the vessel into which the jet issues.

EXERCISE. Prove the following statements:—

1. When $p_0$ is less than $\frac{5}{3}$ of the external pressure, we may take as roughly correct the **flow of steam** in pounds per second through the area $A$ square feet to be

$$W = \frac{Ap_1}{34}\sqrt{\frac{p_0 - p_1}{p_1}},$$

the pressures being in lb. per square foot, and that this is right if $A$ is in square inches and the pressures are in lb. per square inch.

2. For $W$ in Art. 390 to be a maximum, we must have

$$a = \left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma}{1-\gamma}}, \text{ or } p = p_0 \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}}.$$

3. In the **case of air,** $\gamma = 1{\cdot}41$, so that if there is maximum flow $p_1 = {\cdot}527\, p_0$.

4. In the case of **superheated steam,** $\gamma = 1{\cdot}3$, possibly, so that if there is maximum flow $p_1 = {\cdot}546\, p_0$.

5. In the case of **dry saturated steam,** $\gamma = 1{\cdot}130$, so that if there is maximum flow $p_1 = {\cdot}578\, p_0$.

6. In the case of steam leaving the boiler with 25 per cent. of water $\gamma = 1{\cdot}113$, approximately, so that if there is maximum flow $p_1 = {\cdot}582\, p_0$.

7. If dry steam flows adiabatically from a boiler where the pressure is $p_0$ lb. per square foot to a place where the pressure is $p_1 = 0{\cdot}578\, p_0$, show that its weight, $w$ lb. per cubic foot, is $w_1 = 1{\cdot}762 \times 10^{-5} p_0^{0{\cdot}939}$.

To do this we may take $p_0 w_0^{-1{\cdot}13} = p_1 w_1^{-1{\cdot}13}$.

Also $p_0 w_0^{-1{\cdot}065} = 479 \times 144$. (See (9) Art. 180.)

EXERCISE. Calculate the values of $w_1$ for various values of $p_0$ given in the following table. It is evident from this that in rough calculations we may take it that $w_1 = 10^{-5}\, p_0$.

8. Show that the limiting velocity of a gas in (1) Art. 389 if the condition of maximum weight flow holds is

$$v_1 = \sqrt{\frac{2g}{w_0} \frac{\gamma}{\gamma + 1} p_0},$$

if $p_0$ is in lb. per square foot, $w_0$ being in lb. per cubic foot.

In the case of dry steam, taking $\gamma = 1{\cdot}13$, this becomes

$$v_1 = 5{\cdot}845 \sqrt{p_0 \div w_0}.$$

EXERCISE. Find the limiting velocity $v_1$ with which steam will rush into an atmosphere at a pressure less than ·57 of its initial pressure, if the initial pressure is as given in the table.

| $p_0 \div 144$. | $p_0$. | limiting $v_1$. | Value of $n$ if $w_1 = np_0$. | Values of $m$ in $W = mp_0A$. |
|---|---|---|---|---|
| 300 | 43200 | 1512 | $\cdot 9185 \times 10^{-5}$ | ·0140 |
| 200 | 28800 | 1496 | $\cdot 9414 \times 10^{-5}$ | ·0142 |
| 100 | 14400 | 1464 | $\cdot 9822 \times 10^{-5}$ | ·0145 |
| 50 | 7200 | 1432 | $1{\cdot}024 \times 10^{-5}$ | ·0148 |
| 30 | 4320 | 1410 | $1{\cdot}057 \times 10^{-5}$ | ·0150 |

We see, then, that the limiting velocities do not greatly differ from one another, although in every case the efflux may be into the atmosphere or a condenser. The student ought as an exercise to prove that this is the velocity of sound in the gas in the state in which it exists in the throat.

[*Added, October*, 1901.—In Osborne Reynolds's *Collected Papers*, vol. ii., page 311, I find the explanation of the Napier and other results. Imagine (1) of Art. 390 to refer, not to the whole system, but to one stream tube of cross section $A$ at the place where $p$ (substituted for $p_1$ in the formula) is the pressure; $W$ is constant everywhere. Hence (1) enables us to calculate $A$ if we know $p$. It will be found that taking less and less values of $p$, $A$ reaches a minimum value for $p = p_0 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$, easily found by making $\frac{dA}{dp} = 0$.]

In practical calculations I often take it that the limiting velocity is always 1450 feet per second, if $p_0$ has any value between 150 and 60 lb. per square inch.

EXERCISE. Find the limiting weight $W$ lb. of steam per second which will flow through an area $A$ square feet, using the above values of $v_1$ and $w_1$.

*Answer.* $W = \cdot 0194\, p_0^{0\cdot 969} A$.

EXERCISE. If we assume that $W = mp_0A$, what is $m$ for the values of $p_0$ in the table? The answers are given.

We see that we may in rough calculations take the following rule:—

9. The greatest weight of steam in pounds per second flowing through a throat of area $A$ square feet is $v_1 w_1 A$, or roughly,

$$W = \frac{1}{70} p_0 A.$$

This is the result arrived at **experimentally** by Mr. Napier. This formula may be used if $p_0$ is in lb. per square inch and $A$ is in square inches.

**392. Theory of the Injector.**—Dry saturated steam $W_1$ lb. per second from the boiler, at pressure $p_0$ and temperature $\theta_0$° C. reaches $B$, Fig. 323, adiabatically, where it is at $p_1$ and $\theta_1$° C., and it condenses, meeting $W_2$ lb. of water at $\theta_2$° C. and pressure $p_2$, which has risen from the feed tank by the pipe $A$. The combined stream at $\theta_3$° C. passes into the feed pipe at $E$ and through the valve $G$ to the boiler by $H$.

1. Assume no steam to escape condensation and no water to slip between $D$ and $E$. Also that the whole of the heat of a pound of steam leaving the boiler is in the mixture at $D$ and $E$; that is

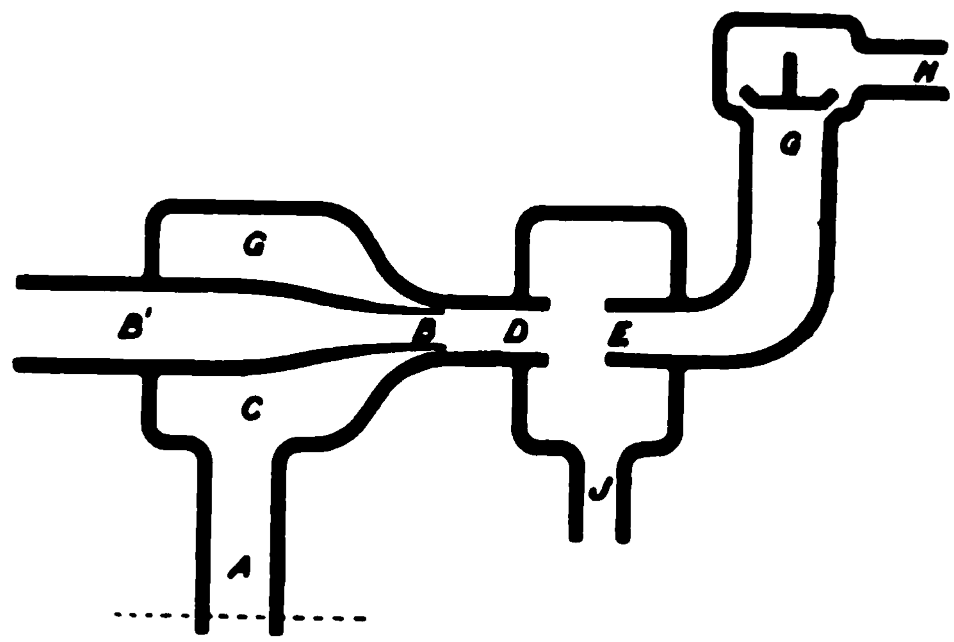



FIG. 323.

neglect the fact that a small fraction of the heat has been converted into kinetic energy. Then if $H_0$ is Regnault's total heat,

$$W_1(H_0 - \theta_3) = W_2(\theta_3 - \theta_2).$$

2. As the pressure in the overflow $J$ is atmospheric, assume that it is so at $E$, so that if $p_0$ is the boiler pressure in pounds per square inch, the velocity at $E$ must be

$$V = 12\sqrt{p_0 - 14{\cdot}7}$$

feet per second neglecting friction. The area $A$ at $E$ is

$$A = \frac{{\cdot}016\,(W_1 + W_2)}{V},$$

taking ·016 as the volume of 1 lb. of water.

3. I shall not attempt to give a theory of what happens when the streams of condensing steam and water meet, but we may take Fig. 324 as showing what may possibly occur at $ABCD$ and $EF$ of our old Fig. 320. Through the area $a_1$ square feet there is a flow of $W_1$ lb. of wet steam per second at the pressure $p_1$ and velocity $v_1$. Through the outer area $a_2$ we have $W_2$ lb. of water per second at the velocity $v_2$, which has come from a tank whose atmospheric still water level is $h_2$ feet *above* the jets. Through the area $a = a_1 + a_2$ or $EF$ we have $W_1 + W_2$ lb. of water flowing per second

ressure $p$, with velocity $v$, each pound of it possessing the total gy

$$\frac{v^2}{2g} + \frac{p}{w}.$$

If there were no friction except what is necessary for the gling, the total energy required if the water is to enter a er at pressure $P$ is $P/w$.

Until Napier's experiments the flow of steam from a er at $p_0$ into a place of low sure, no one dreamt that velocity was that corre- iding to the notion that e is a pressure 0·58 $p_0$ or $p_0$ in the throat. It would be very absurd for us our **exceedingly small wledge** to build a theory of the injector on the supposition $p_1$ is 0·6 $p_0$, and that $v_1$ is always about 1,450 feet per second, to use the formula of Art. 382. Should any one care to do nd if as before $v_2^2/2g$ is taken to be $h_2 + 25$ when the nozzles properly adjusted; if $W_1$ is taken to be $\frac{p_0 a_1}{70}$, and if $W_2/W_1$ is ed $y$, we arrive at the equations

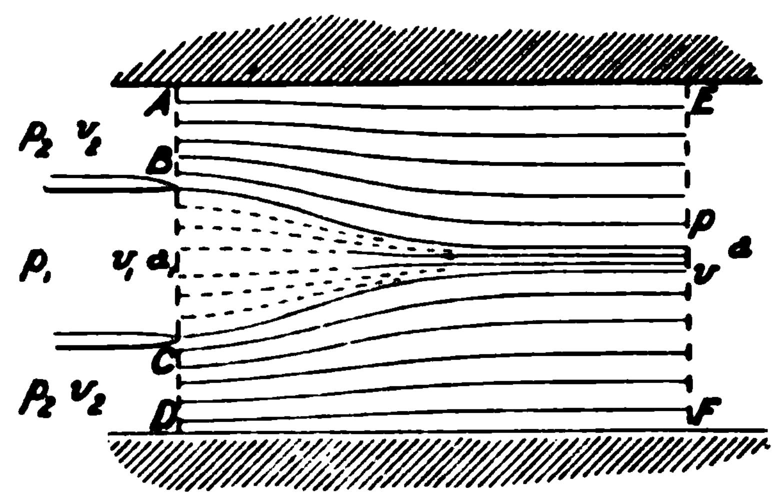



FIG. 324.

$$87 + \frac{2y}{v_2}(h_2 + 30) = (1 + y)\left(\frac{v}{g} + \frac{p}{vw}\right) \quad . \quad . \quad . \quad . \quad (2).$$

$$\frac{4360}{p_0} + \frac{y}{v_2} = \frac{1 + y}{v} \quad . \quad . \quad . \quad . \quad (1).$$

Given $y$ and $p_0$ we can find $v$ from (1); use it on (2) to find $p$ calculate

$$P = w\left(\frac{v^2}{2g} + \frac{p}{w}\right),$$

pressure in the vessel into which the combined jet may be ed.

But if the student uses this method he will find that although n $p_0 = 45 \times 144$ and $y = 9$, $P$ is sufficiently greater than $p_0$ to v that the injector would work; if he tries much higher values of nd $y$, the injector will not work. In fact his results do not agree experience, and therefore his theory is worthless. It is quite

possible indeed that $v_1$ reaches values very greatly exceeding th values found by Napier under such very different circumstances.

It may be worth while here to say that all the best writers us Napier's $v_1$ in the formula—

$$(1 + y)\left(V - \frac{gh_2}{V}\right) = v_1$$

to calculate $V$ the velocity with which the combined steam passes th space where it is at atmospheric pressure. When $h_2$ is 0 this is—

$$V = \frac{v_1}{1 + y}$$

as if we had two solid bodies of masses 1 and $y$ colliding in a vacuu with velocities $v^1$ and 0, $V$ being their common velocity after impac The formula is said to agree in a few cases with the actual expe mental results, but to greatly disagree in most cases. One thing know, it is always arrived at by what is called "a theory of the Injecto which is one of those pretences sometimes to be found in books applied science where weak mathematics hides the want of reason.

I hardly know if it is worth while here to say that if in my theo we assume $p_1$ and $p_2$ to be equal, and both equal to that due to th head $h_2$ and neglect the small term $h_2g/v_1$ and assume that $a_1$ much less than what it really is; in fact that $w_1 = w$, we get th commonly received formula. But I see no scientific reason for su assumptions.

**393.** I have never made accurate experiments with an inject I copy from Mr. Peabody's excellent "Thermodynamics of the Stea Engine," the following results of **experiments on a Selle injector** whose combining tube or water orifice is 6 mm. in diamet where smallest.

For each pressure of steam noted in column 1, the water w delivered by the injector into the boiler under approximately t same pressure. The delivery was measured by observing the indic tions of a water-meter.

The pressures in column 8 were obtained by throttling the stea supplied to the injector, and observing the pressure at which it ceas to work, each experiment being repeated several times with precis the same results.

The temperatures in column 9 were obtained by gradually heati the water supplied to the injector, and noting the temperature which it ceased to operate, each temperature recorded being check by several repetitions of the experiment.

**Experiments on a Sellers Injector.**

| Gauge pressure of Steam supplied to Injector, and pressure against which water is delivered, lbs. per square inch. | Delivery in cubic feet per hour. Maximum. | Delivery in cubic feet per hour. Minimum. | Delivery in cubic feet per hour. Ratio of minimum to maximum delivery. | Temperature, Fahrenheit degrees. Feed water. | Temperature, Fahrenheit degrees. Delivered water. At maximum delivery. | Temperature, Fahrenheit degrees. Delivered water. At minimum delivery. | Pressure of Steam required to deliver Water against pressure in column 1. | Highest temperature Fahr. admissible of feed-water. |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 75·3 | 63·6 | 0·845 | 66 | 100 | 94 | 3 | 132 |
| 20 | 82·4 | 61·2 | 0·743 | 66 | 108 | 104 | 9 | 134 |
| 30 | 94·2 | 56·5 | 0·600 | 66 | 114 | 116 | 16 | 134 |
| 40 | 100·1 | 60·0 | 0·599 | 66 | 120 | 123 | 22 | 132 |
| 50 | 108·3 | 64·7 | 0·597 | 66 | 124 | 125 | 27 | 131 |
| 60 | 116·5 | 63·6 | 0·546 | 66 | 127 | 133 | 34 | 130 |
| 70 | 124·8 | 63·6 | 0·510 | 67 | 130 | 142 | 40 | 130 |
| 80 | 133·0 | 67·1 | 0·505 | 66 | 134 | 144 | 46 | 131 |
| 90 | 141·3 | 69·5 | 0·492 | 67 | 136 | 148 | 52 | 132 |
| 100 | 147·2 | 64·7 | 0·456 | 66 | 140 | 159 | 58 | 132 |
| 110 | 153·0 | 67·1 | 0·439 | 67 | 144 | 162 | 63 | 132 |
| 120 | 156·6 | 73·0 | 0·466 | 67 | 148 | 162 | 69 | 134 |
| 130 | 161·2 | 74·2 | 0·460 | 66 | 150 | 165 | 75 | 130 |
| 140 | 166·0 | 78·9 | 0·476 | 66 | 153 | 166 | 81 | 126 |
| 150 | 170·7 | 70·6 | 0·414 | 66 | 157 | 167 | 18 | 121 |

Taking the case in which $p_0 = 150$ lb. per sq. in., $\theta_0 = 366°$ F., $\theta_3 = 157°$ F., $\theta_2 = 66°$ F., $H_0 = 1194$.

$$y = \frac{H_0 - \theta_3}{\theta_3 - \theta_2} = \frac{1037}{91} = 11·4.$$

$$v_1 = 5·845\sqrt{150 \times 144 \times 2·76} = 1430 \text{ feet per second}$$

$$v = \frac{1430}{1 + 11·4} = 115 \text{ feet per second.}$$

If $P$ lb. per sq. in. is the *gauge* pressure of delivery

$$115 = 12\sqrt{P}, \text{ or } P = 92,$$

that is, the pressure of the delivered water is only 0·62 of the boiler pressure in spite of our assumption of no friction. Hence the usually accepted formula has not only no scientific basis but it has not even the virtue of agreeing with experimental results. I think

that there can be no theory of the injector until some scientific man makes a complete experimental investigation of the subject.

**394.** EXERCISE. Taking the above case, that each pound of steam at 366° F. generated from feed water at 157° F. causes 11·4 lb. of water to enter the boiler. **Compare the performance** with the mechanical energy produced by a perfect non-condensing steam engine.

The work done per pound of steam is $\frac{11 \cdot 4}{62} \times 150 \times 144$ or 3972 foot pounds. A perfect non-condensing steam engine using steam at 366° F. would do (see Art. 214) 250,800 foot pounds per pound of steam. Of course it is to be remembered that the waste heat is utilized in heating the feed water.

**395.** This is not the place for other **speculations** such as I have made on injectors. It may, however, be worth while to mention that I anticipate greatly increased efficiency in the driving of water by steam jets by making the steam nozzle telescopic so that more and more steam enters as the water quickens in speed; not all entering at one place.

Fig. 138*a* shows one ordinary form of the single acting injector. To start it we open the steam valve a little, then the water supply valve, and as soon as water appears at the overflow we open the steam valve more and more until the overflow ceases. As air is drawn in to some extent and may be objectionable in condensing engines there is sometimes a non-return valve attached to the overflow, a weak spring pressing with a little more force than the weight of the valve.

Injectors ought **to be tested** for pressures of delivery 10 to 15 lb. above the boiler pressure, to allow for friction and the lifting of the valve. The **lift to the boiler** is seldom more than 20 feet. With a high lift there is sometimes difficulty on account of the non-condensation of the supply steam. The **feed tank temperature** ought to be as low as possible, else there may not be complete condensation of the steam. An injector whose nozzles are properly adjusted for a certain boiler pressure needs re-adjustment for other pressures, and there are **self-adjusting injectors** in the market. In **double action** injectors the water is first admitted to an intermediate space, and by means of a fresh jet of steam is injected into the boiler.

It will be seen by the above table that injectors will supply water at a greater pressure than that of the supply stream. Hence a jet of the **exhaust steam** from a non-condensing engine is sometimes used for feeding the boiler.

The injector is often used as a very inefficient pump, especially in chemical works. When the lift is small as in "**Ejectors**" and especially when the water enters through telescopic openings so that the water first set in motion by the steam is greatly added to later, it is said that there is a greatly increased efficiency. Ejectors are often arranged so that they act as a sort of combination of condenser and air-pump.

## CHAPTER XXXV.

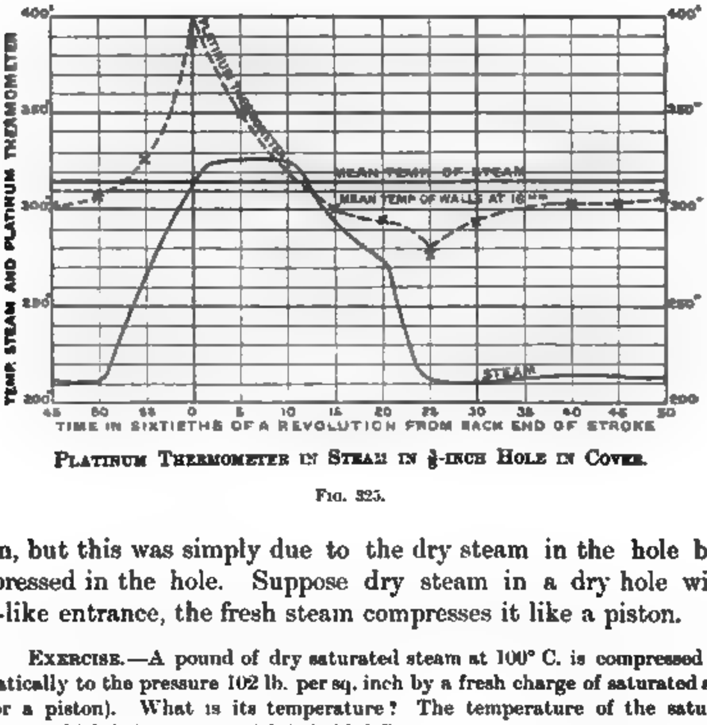
### CYLINDER CONDENSATION.

**396.** IN this chapter I consider the growth of water in the cylinder, using the answers to some mathematical problems in speculation. In our study we are apt to assume that the pressure of steam as given by the **indicator tells us the temperature** of steam and water everywhere in the cylinder. Indeed, this is the basis of our application of the $\theta\phi$ diagram to practical problems; an experienced man knows that the static laws are to be applied with caution and merely with the object of obtaining suggestions; the young student believes in the absolute truth of such a $\theta\phi$ diagram as my students draw for the whole of an indicator diagram.

I have already referred to the misleading notion conveyed by diagrams of cushioning. In further reference to this matter I will refer to Mr. Callendar's thermometer, which was fixed in a hole in the cylinder cover. The results are shown in Fig. 325. The temperatures corresponding to the pressure on the indicator diagram are shown in the full line curve; the temperatures given by the platinum thermometer are shown on the dotted curve, the temperature scale (Fahr.) being the same for both. **The superheating** shown at the end of the compression is very noticeable. During admission the temperature rapidly falls. Shortly after cut off the temperature is 2° or 3° Fahr. below that of the indicator diagram.

I hold that the thermometer in the end of the cylinder in Mr. Callendar's experiments measured something which was very different from the temperature anywhere else. No exact description has been given of the hole inside which the temperature was measured, but I take it that it was a **hole which might be dry** when other parts of the cylinder were wet, and that there probably was actual mechanical drainage from that hole of condensed water. Now all condensed

water draining away came into the hole as dry steam, and its latent heat is left behind, heating the steam about the thermometer and keeping it drier than other steam in the cylinder. He obtained higher temperatures there than the temperature of the incoming




PLATINUM THERMOMETER IN STEAM IN $\frac{3}{8}$-INCH HOLE IN COVER.

FIG. 325.

steam, but this was simply due to the dry steam in the hole being compressed in the hole. Suppose dry steam in a dry hole with a tube-like entrance, the fresh steam compresses it like a piston.

EXERCISE.—A pound of dry saturated steam at 100° C. is compressed adiabatically to the pressure 102 lb. per sq. inch by a fresh charge of saturated steam (or a piston). What is its temperature? The temperature of the saturated steam which is in contact with it is 165° C.

This is very easily worked out on the $\theta\phi$ diagram. $R$ Fig. 231 shows its state before compression at 100° C. $D$ at 165° C. is the end of the superheated steam curve $DE$ of 102 lb. pressure, and $RE$ is the line of adiabatic compression. I find on the diagram that $E$ means 328° C. I take here as roughly correct that the specific heat of superheated steam is 0·475.

Algebraically: The entropy of a pound of steam at $R$ is 1·744 ranks. This is also to be the entropy at $E$ of a pound of superheated steam at 102 lb. per sq. inch. Now the entropy of a pound of superheated steam at any temperature and pressure is given in (2) Art. 201 as 1·594 (the entropy at $D$) + $0{\cdot}475 \log \frac{t}{t_0}$ where $t_0$ is the absolute temperature at $D$, and $t$ is the absolute temperature at $E$. That is,

$$1{\cdot}744 = 1{\cdot}594 + 0{\cdot}475\,(\log_e t - \log_e 438{\cdot}7)$$

$$\log_e t = 2{\cdot}7795, \text{ so that } t = 602.$$

The temperature at $E$ is, then, 328° C.

Now, in the actual experiment, the superheating only reached 200° C., being followed by a sharp fall.

My figure, 328° C., could only be reached in a non-conducting hole and on the assumption of a very long narrow entrance. The paper says that it was a hole merely, and in all probability the thermometer was not far away from the fresh steam; as soon as the fresh steam had a little time to mix with the superheated steam there would be just such a fall of temperature as was noted. Some of Mr. Callendar's ingenious reasoning concerning dynamic effects as being different from static effects, with nearly all of which, however, I quite agree, are really based upon the temperature changes in a little well-drained pocket of steam and not the average steam in the cylinder, which is really that corresponding to the pressure.

I have referred to this matter at some length because I believe there is **always water** and the saturation temperature at all times of the stroke, **even in the driest cylinders,** unless a large amount of superheating is employed.

If the cylinder were for an instant quite dry, I do not believe that condensation would readily begin in the same stroke. It is to be remembered also that it is the existence of water round the piston and valves that enables leakage to be 50 times as great as if there was no water.

I have had the opportunity of watching smoke drawn into a glass cylinder with air after a piston for the purpose of noting whether or no the smoke and air kept separate. Any one who has seen, as I have seen, the immediate mixing that goes on in spite of all sorts of attempts to keep the stuffs separate, must know that it can only be in well-drained holes that any superheating can possibly take place. Mr. Callendar's other thermometer in the body of the steam showed a temperature corresponding to the pressure.

But although I think the temperature of the steam to be nearly the same everywhere, I do not think it possible that the water temperature is the same throughout. In much that follows I shall assume it to be the same throughout.

**397.** In the rough generalisation of Arts. 227–233 we have assumed that the resistance to the passage of heat between steam and metal skin is constant, and we have neglected the effect on $c$ of $w_3$ the water present before admission. It is my belief that neither of these assumptions is sound. **A more careful study of the whole question** seems to me necessary; a study of the growth and diminution of $w_3$ per cycle. It must not be imagined that I am looking merely for a simple formula. Indeed, it is quite obvious that there is **no simple formula** possible to express what

oes on in the cylinder of a steam engine. We have all notions bout what occurs; it is only when we **express these notions uantitatively** that their value can be judged of. It is of no mportance that we shall perhaps get no simple formula. Our nain business is to try to reason clearly about what occurs, with minimum of vagueness and "huggermugger."

The following problems are worked out exactly on certain assumptions. From the answers I shall endeavour to make reasonable peculations as to what goes on in the cylinder.

**398.** *Problem I.* A perfectly non-conducting vessel contains $w$ lb. of water, also dry saturated steam *at the same temperature* $\theta$. Let this temperature be supposed to alter, the steam being supposed to condense or vaporise just enough for the heating and cooling of the water, but to remain dry saturated steam. If $l$ is the latent heat of steam at the temperature $\theta$; if $\theta$ becomes $\theta + \delta\theta$, $w$ increases to $w + \delta w$ by the condensation of $\delta w$ of steam. Hence, $l \cdot \delta w = w \cdot \delta\theta$ or $dw/w = d\theta/l$, and as $l$ is a function of $\theta$, $w$ is a function of $\theta$. Hence, at the end of any cycle, as $\theta$ returns to its old value, $w$ returns to its old value.

Taking $l$ as $606\cdot5 - 0\cdot695\,\theta$ we see that $wl^\sigma$ remains constant throughout, $\sigma$ being $\frac{1}{\cdot695}$.

**399.** *Problem II.* A metal vessel of constant volume and internal area is filled with saturated steam at the temperature $\theta$° C. and this temperature follows a periodic law. There is of water $w$ lb. per unit area of the metal surface present at the time $t$. I assume that steam is condensed or water evaporated merely for the purpose of keeping the water at the temperature of the steam. $v$ is the temperature of the metal at a depth $x$ from the surface, and the metal is supposed to be so thick that time variations in temperature do not occur at its outer parts. The metal's thickness is $b$, and the outer surface is kept, by means of a steam jacket, at the temperature $v^1$ above the average temperature of the steam. We have in the metal, as before, in Art. 227,

$$\frac{d^2v}{dx^2} = \frac{s\rho}{k}\frac{dv}{dt} \quad \dots\dots\dots\dots \quad (1)$$

At the surface, between water and metal, if $e$ is the emissivity and $v_0$ is the surface temperature of the metal,

$$v_0 - \theta = \frac{k}{e}\frac{dv}{dx} \quad \dots\dots\dots \quad (2)$$

Also the condensation of water occurs according to the law

$$(v_0 - \theta)e + l\frac{dw}{dt} = w\frac{d\theta}{dt} \quad \dots\dots\dots \quad (3)$$

$l$ being the latent heat of steam at the temperature $\theta$. It will be noticed that I assume an instantaneous establishment of equilibrium of temperature between steam and water; I wish I could work to a more complex condition, but this will suffice for my present object.

$\theta$ and $v_0$ are periodic functions of the time. I have often, with my students, worked out the problem on the assumption of the most general shape of periodic function, but it will be found quite sufficient to take the simplest,

as in Art. 227. Let the skin temperature of the metal be

$$v_0 = a \sin qt + \theta_0 + kv^1/(eb + k) \quad \ldots \ldots \ldots \quad (4)$$

We know that

$$v = a\epsilon^{-ax} \sin(qt - ax) + \theta_0 + a\frac{v^1}{eb + k}(k + ex)$$

and

$$\theta = a\left(\frac{ka}{e} + 1\right)\sin qt + a\frac{ka}{e}\cos qt + \theta_0 \quad \ldots \ldots \quad (5)$$

where if $q = 2\pi n$, $a = \sqrt{\pi nsp/k}$
(3) is

$$\frac{dw}{dt} - \frac{w}{l}\frac{d\theta}{dt} = \frac{(\theta - v_0)e}{l}$$

Now

$$\int - \frac{1}{l}\frac{d\theta}{dt}dt = \int - \frac{d\theta}{l} = \int \frac{-d\theta}{606{\cdot}5 - {\cdot}695\theta} = \frac{1}{{\cdot}695}\log l = \sigma \log l$$

say, and hence we see that the solution of (3) is

$$w = l^{-\sigma}\left\{e\int l^{\sigma-1}(\theta - v_0)\,dt + c\right\} \quad \ldots \ldots \ldots \quad (6)$$

where $c$ is an arbitrary constant.

I am sorry to say that I can perform this integration only approximately. I am aware, from my experience in electrical work, how dangerous approximate calculations are likely to be in dealing with periodic functions, but I feel satisfied that my solution is sufficiently correct for my present purpose. Once I remember laboriously working out a second approximation, and the correction did not affect the conclusions which this first approximation leads to. The latent heat $l$ being $l_0 - {\cdot}695(\theta - \theta_0)$ where $l_0$ is the latent heat corresponding to $\theta_0$° C., my approximation consists in taking

$$l^{\sigma-1} = l_0^{\sigma-1}\left\{1 - \frac{{\cdot}695(\sigma - 1)(\theta - \theta_0)}{l_0}\right\} = l_0^{\sigma-1}\left\{1 - \frac{{\cdot}305}{l_0}(\theta - \theta_0)\right\}$$

I only want to know those terms in $w$ **which are not periodic**; terms which increase or diminish steadily with the time.

On writing out (6) there are many terms, each of which is easily integrated, and the answer is

$$w = \text{Periodic terms} + el^{-\sigma}l_0^{\sigma-1}\left[\text{Periodic terms} - \left\{\frac{kv^1}{eb + k} + {\cdot}305\,a^2k^2a^2\left(1 + \frac{e}{2ka}\right)\right\}t\right] \quad \ldots \ldots \ldots \quad (7)$$

The answer is, as I have said, approximate. In calculating its value numerically, to get ideas of its meaning, we may take $l_0$ instead of $l$.

I find, using $A$ for the amplitude of $\theta$ or $\frac{1}{2}(\theta_1 - \theta_3)$ and dividing the non-periodic terms by $n$, that the diminution of $w$ per cycle is

$$\frac{1}{n}\left\{\frac{kv^1}{l_0(b + k/e)} + \frac{{\cdot}305A^2}{l_0^2}\frac{e}{(1 + e/ka)^2 + 1}(1 + e/2ka)\right\} \quad \ldots \quad (8)$$

The steam jacket effect was, of course, obvious, and we need not have carried it through all the work as we have done. The other term was not by any means so obvious. We see that if $e$ is 0, so that it is as if the metal of the cylinder were non-conducting, there is no loss of $w$ per cycle, as we found in Art. 398.

If, as we may presume that it often is, $e$ is small, and we neglect the steam-jacket effect, we find the loss of $w$ per cycle to be proportional to $A^2e/n$ or to

$$e(\theta_1 - \theta_3)^2/n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If $e$ is large enough to make the amplitude of $v_0$ nearly the same as that of the steam, the lessening of $w$ per cycle is proportional to

$$(\theta_1 - \theta_3)^2/\sqrt{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

**A very striking result,** showing that the metal acts in an altogether different manner from that in which a quantity of water would act.

I will here indulge in a little speculation, and say that, just as in Art. 230, the departure from the sine function form of temperature change will be to cause us to use as the coefficient of $(\theta_1 - \theta_3)^2$ neither $\frac{1}{\sqrt{n}}$ nor $\frac{e}{n}$ but $\frac{\epsilon\left(g + \frac{h}{r}\right)}{\sqrt{n} + cn}$, where $\epsilon$ is proportional to the emissivity when small, but approaches a constant value when the emissivity is great, and where $g$ and $h$ and $c$ are constant.

**400.** I shall now try to approximate in a few problems which may be worked out mathematically to the departure in the actual condition of things from the above assumptions of Problems I. and II. There can be no doubt that Problem II. gives us pretty clear notions of the effect of the conductivity of the metal, our only trouble in this connection being our want of knowledge of $e$. In what follows I shall assume **a non-conducting cylinder.**

*Problem III. Admission—Expansion—Release.*—Non-conducting cylinder—Dry saturated steam supplied.—My reasoning will be mathematically correct on certain assumptions which ought to be criticised. I shall always neglect the volume of the water, and assume its specific heat to be constant.

### 1. Admission.

My assumption is that the steam is supplied at the pressure $p_1$. Whether there is wire-drawing or not does not matter, as I assume that valves and metal everywhere are non-conducting. The vessel is of volume $V$, containing $w_3$ lb. of water and $V/u_3$ lb. of steam. The intrinsic energy of 1 lb. of steam is $H - \frac{pu}{J}$ if Regnault's total heat $H$ is in heat units. $J$ is Joule's equivalent, $p$ the pressure in pounds per sq. foot, $u$ the volume of 1 lb. of steam in cubic feet.

The intrinsic energy of the stuff before admission is

$$\frac{V}{u_3}\left(H_3 - \frac{p_3u_3}{J}\right) + w_3\theta_3.$$

After admission we have $V/u_1$ lb. of steam and $w_1$ lb. of water. So that the quantity of stuff which enters is $V/u_1 + w_1 - (V/u_3 + w_3)$. The intrinsic energy is now $VH_1/u_1 - Vp_1/J + w_1\theta_1$.

Every pound of steam entering gives to the vessel the total energy $H_1$, because it brings with it its own intrinsic energy $H_1 - p_1u_1/J$, and also it has the work $p_1u_1$ done upon it by the steam which follows it up. Hence

$$VH_3/u_3 - Vp_3/J + w_3\theta_3 + H_1\{V/u_1 + w_1 - (V/u_3 + w_3)\} = H_1V/u_1 - Vp_1/J + w_1\theta_1.$$

Hence, if $w_3{}^1$ is $V/u_3$, the weight of the steam alone which is with the water before admission, we have

$$w_1 = w_3(H_1 - \theta_3)/l_1 + w_3{}^1\{H_1 - H_3 - (p_1 - p_3)u_3/J\}/l_1.$$

I am particularly anxious to know the effect of $w_3{}^1$ (or rather of the volume)

in diminishing the water, but I have also calculated other coefficients in th following cases. If it is recollected that $w_3^1$ is always a very small quantity, i will be seen that the drying effect due to volume is not great.

| $\theta_1$ | $\theta_3$ | | $\left(\frac{l_3}{l_1}\right)^e$ |
|---|---|---|---|
| 195° C. | 155° C. | $w_1 = 1{\cdot}092\, w_3 - 0{\cdot}123\, w_3^1$ | 1·092 |
| ,, | 100 | $w_1 = 1{\cdot}209\, w_3 - 1{\cdot}033\, w_3^1$ | 1·218 |
| ,, | 60 | $w_1 = 1{\cdot}295\, w_3 - 5{\cdot}301\, w_3^1$ | 1·311 |
| 180 | 145 | $w_1 = 1{\cdot}077\, w_3 - 0{\cdot}107\, w_3^1$ | 1·077 |
| | 100 | $w_1 = 1{\cdot}170\, w_3 - 0{\cdot}699\, w_3^1$ | 1·177 |
| | 60 | $w_1 = 1{\cdot}258\, w_3 - 3{\cdot}70\, w_3^1$ | 1·267 |
| 165 | 135 | $w_1 = 1{\cdot}061\, w_3 - 0{\cdot}091\, w_3^1$ | 1·064 |
| | 100 | $w_1 = 1{\cdot}135\, w_3 - 0{\cdot}425\, w_3^1$ | 1·140 |
| | 60 | $w_1 = 1{\cdot}218\, w_3 - 2{\cdot}50\, w_3^1$ | 1·227 |
| 145 | 100 | $w_1 = 1{\cdot}09\, w_3 - 0{\cdot}219\, w_3^1$ | 1·093 |
| | 60 | $w_1 = 1{\cdot}17\, w_3 - 1{\cdot}39\, w_3^1$ | 1·177 |

It will be evident from the magnitude of the other sources of growth a diminution of $w$ that we may neglect the effect of $w_3^1$ as a disturbing element our reasoning. I shall speak again of the relative magnitudes of the disturbi elements in Art. 403.

In my generalisation of Art. 398, I assume that

$$w_1 = w_3 \left(\frac{l_3}{l_1}\right)^e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the student will notice here the discrepance. Neglecting the evaporati effect of the clearance volume, we now find

$$w_1 = w_3 \frac{H_1 - \theta_3}{l_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

What is the reason that there is any difference? It is this:—To get (2 assumed that there was equilibrium of temperature everywhere only at the ginning and at the end; whereas to get (1) I assumed equilibrium of temperat at every instant during the change from $\theta_3$ to $\theta_1$. In (2) it is the same kind dry saturated steam which is condensing all the time; whereas in (1) the c densing steam gradually alters in character. A small amount of wetness in entering steam will cause the non-reversible operation to produce the same eff as the reversible one.

### 2. Adiabatic Expansion.

This is easily worked out on the $\theta\phi$ diagram. I shall here work it algebraically: $w_1$ lb. of water and $i$ lb. of indicated steam in a cylinder at expands adiabatically to $\theta_2$, becoming $w_2$ lb. water and $w_2^1$ lb. steam.

Let $\phi = \log \frac{t}{t_0}$ be the entropy of 1 lb. of water.

Let $\psi = \phi + \frac{l}{t}$ be the entropy of 1 lb. of steam.

The entropy at the beginning is equal to the entropy at the end, and therefore

$$w_1\phi_1 + i\psi_1 = w_2\phi_2 + w_2{}^1\psi_2$$

$$\text{Also } w_2{}^1 = w_1 + i - w_2$$

$$w_1\phi_1 + i\psi_1 = w_2\phi_2 + (w_1 + i - w_2)\psi_2$$

Hence

$$w_2 = w_1\left(1 - \frac{t_2}{l_2}\log.\frac{t_1}{t_2}\right) + i\left(1 - \frac{t_2}{t_1}\frac{l_1}{l_2} - \frac{t_2}{l_2}\log.\frac{t_1}{t_2}\right). \quad . \quad . \qquad (3)$$

In my generalisation I assume that the term created by $w_1$ is $w_1\left(\frac{l_1}{l}\right)$ The following examples show what sort of difference exists :—

| $r$ | $\theta_1$ | $\theta_2$ | $1 - \frac{t_2}{l_2}\log.\frac{t_1}{t_2}$ | $\left(\frac{l_1}{l_2}\right)^s$ |
|---|---|---|---|---|
| 2 | 195 | 165 | ·941 | ·936 |
| 5 | ,, | 130 | ·883 | ·871 |
| 13 | ,, | 102 | ·845 | ·824 |
| 2 | 175 | 142 | ·937 | ·930 |
| 5 | ,, | 117 | ·897 | ·888 |
| 13 | ,, | 90 | ·859 | ·842 |
| 2 | 155 | 130 | ·953 | ·951 |
| 5 | ,, | 104 | ·911 | ·903 |
| 2 | 135 | 113 | ·960 | ·958 |
| 5 | ,, | 87 | ·917 | ·911 |
| 2 | 115 | 95 | ·964 | ·962 |

The value of $p_1/p_2$ is roughly called $r$ merely for the purpose of giving some notion of the amount of expansion we are dealing with.

I take it, as I did in 1873 when I first wrote on this subject, following Rankine, that it is the term in $i$ that is the **most important wetting term** in the whole cycle. This is a term which is distinctly added on, and not contemplated in our generalisation over Problems I. and II. It may be written as above,

$$i\left(1 - \frac{t_2}{t_1}\frac{l_1}{l_2} - \frac{t_2}{l_2}\log.\frac{t_1}{t_2}\right)$$

Or in the handier form for calculation from the steam table

$$i\left(\frac{\psi_2 - \psi_1}{\psi_2 - \phi_2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

if $\psi$ is the entropy of a pound of steam and $\phi$ the entropy of a pound of water.

According to our generalisation, and, indeed, according to the next section, a quantity of water $w_2$ becomes after release $w_2\left(\frac{l_2}{l_3}\right)^s$ and consequently the addition of water per stroke on account of adiabatic expansion si

$$i\left(\frac{l_2}{l_3}\right)^s\left(\frac{\psi_2 - \psi_1}{\psi_2 - \phi_2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

We particularly want to know how the above coefficient of $i$, the wetting term, depends upon $r$, the ratio of cut-off, and I have calculated its value in many cases. The result is very interesting. Taking $r$ roughly as $p_1/p_2$ where $p_1$ is the initial pressure and $p_2$ the pressure at the end of the expansion. Taking $\theta_3$ as 60° C. in a condensing, and as 100° C. in a non-condensing engine I find :—

| Condensing engine. | | | Non-Condensing engine. | | |
|---|---|---|---|---|---|
| $r$ | $p_1$ | Wetting coefficient. | $r$ | $p_1$ | Wetting coefficient. |
| 2 | 203 | ·038 | 2 | 203 | ·041 |
| | 130 | ·043 | | 130 | ·047 |
| | 79 | ·037 | | 79 | ·040 |
| | 46 | ·035 | | | |
| | 25 | ·038 | | | |
| 5 | 203 | ·084 | 5 | 203 | ·093 |
| | 130 | ·083 | | 130 | ·090 |
| | 79 | ·084 | | 79 | ·090 |
| | 46 | ·085 | | | |
| 13 | 203 | ·129 | 13 | 203 | ·138 |
| | 130 | ·129 | | | |

On the whole, I am inclined to think that with great exactness we may say that **the wetting co-efficient** is independent of $p_1$, is nearly independent of $p_1$, and may be taken as being represented by $\frac{cr}{q + r}$ where $c$ and $q$ are constants in both condensing and non-condensing engines at all pressures. $c$ seems to be about ·25 in non-condensing and ·224 in condensing engines.

### 3. Exhaust.

The following investigation is put forward with some diffidence. The action is irreversible, and I have no doubt that my assumption will be objected to. I am not ashamed to say that I have worried over it a great deal, and in some years have had much correspondence about it with friends who are acknowledged authorities on thermodynamics. It seems at first an easy problem. It has been given up as insoluble or too troublesome by some of my friends, and I cannot accept the too easy solutions of the others.

$w_2$ lb. of water and $w_2^1$ lb. of steam, $W$ lb. altogether, in a non-conducting vessel of volume $v$, released to a condenser. Find the amount of water remaining, assuming no reverberatory back-flow. Neglect the volume of the water. At any instant let there be $w$ lb. of water, so that

$$W = w + \frac{v}{u}$$

Just previously $W$ was $W + \delta W$, $w$ was $w + \delta w$, and temperature was $\theta + \delta\theta$. The intrinsic energy of the stuff now present is what it was, except that the volume was $u.\delta W$ less than it now is. Imagine the escape to take place

through a small hole gradually. We have $W$ lb. of stuff, $w$ of water, and a volume $v - u \,.\, \delta W$ of steam, expanding to the volume $v$ doing the work $pu \,.\, \delta W$ in driving slowly the stuff $\delta W$ out of the hole (the hole being led to by a long fine tube, perhaps); therefore its intrinsic energy is now less by this amount.

$w + \delta w$ of water had the intrinsic energy $(\theta + \delta\theta)(w + \delta w) J$ and $\dfrac{v - u \,.\, \delta W}{u + \delta u}$ lb. of steam had the intrinsic energy,

$$\{H + \delta H - (p + \delta p)(u + \delta u)/J\} \frac{v - u \,.\, \delta W}{u + \delta u}$$

Subtracting $\dfrac{pu\delta W}{J}$ from the sum of these and equating to the intrinsic energy now, or

$$w\theta + \left(H - \frac{pu}{J}\right)(W - w)$$

we get an equation which reduces to

$$\frac{dw}{d\theta} - \frac{w}{l} = -v \frac{523 - \theta}{lut}$$

where $t = 273{\cdot}7 + \theta$. Letting $\dfrac{1}{{\cdot}695}$ be called $\sigma$ we find

$$w = l^{-\sigma}\left\{ -v \int \frac{523 - \theta}{ut} l^{\sigma - 1} . \, d\theta + C\right\}$$

where $C$ is an arbitrary constant.

Taking values of $\theta$ from 125° C. to 90° C. I have plotted the values of $\dfrac{523 - \theta}{ut} l^{\sigma - 1}$ and found that it might be expressed with great accuracy as a function of $\theta$.

$${\cdot}8829 - {\cdot}02309\,\theta + {\cdot}00021\,\theta^2,$$

which enables the integration to be performed easily.

Letting $w = w_2$ when $\theta = \theta_2$ it is easy to find $C$. Also let $r_2/u_2$ be called $w_2^1$ the weight of steam present before release; then the water $w_3$ present at the end of the release is

$$w_3 = w_2 \left(\frac{l_2}{l_3}\right)^{\sigma} + w_2^1 l_3^{-\sigma} u_2 \{{\cdot}8829(\theta_2 - \theta_3) - {\cdot}01154(\theta_2^2 - \theta_3^2) + {\cdot}00007(\theta_2^3 - \theta_3^3)\}. \quad . \quad . \quad (6)$$

To see the effect of the amount of steam present when water and steam are released, I have worked out the values of the co-efficients in the following cases.

**Comparison.**

Steam expands from $\theta_2$ to $\theta_3$ **adiabatically.**
Steam is **released** from $\theta_2$ to $\theta_3$.
Compare the amounts of water at the end of the two operations.
In the first we have the fraction

$$\frac{\psi_3 - \psi_2}{\psi_3 - \phi_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In the second we have the fraction

$$l_3^{-\sigma} u_2 \{{\cdot}8829(\theta_2 - \theta_3) - {\cdot}01154(\theta_2^2 - \theta_3^2) + {\cdot}00007(\theta_2^3 - \theta_3^3)\} \quad . \quad (8)$$

I do not see any easy method of comparison except by taking numerical examples:

| $\theta_2$ | $\theta_3$ | Fractional amount of water remaining. | |
|---|---|---|---|
| | | After adiabatic expansion. | After release. |
| 165 | 60 | ·1744 | ·0559 |
| 140 | 60 | ·1454 | ·0548 |
| 110 | 60 | ·1011 | ·0501 |
| 90 | 60 | ·0650 | ·0427 |
| 165 | 100 | ·1070 | ·0669 |
| 130 | 100 | ·0565 | ·0487 |
| 117 | 100 | ·0335 | ·0340 |

It seems, then, that when we release steam of even as high a pressure as 40 lbs. to the sq. inch, either to a condenser or to the atmosphere, if all that leaves the vessel is truly dry saturated steam, the water remaining is comparable with and may even approach in amount what would remain if the steam were adiabatically expanded to the lower temperature.

I have worked out the problem, because, although statements are often made in an off-hand manner concerning what happens on release, I believe that it has never been worked out before. And now that I have done it, I cannot make much present use of it, for, after all, the steam condensed in this way is not likely to remain in the cylinder. It is almost certainly carried off in the sudden rush of the uncondensed steam with which it is mechanically mixed, and I am going to neglect it altogether in the practical use to which I mean to put my results. Yet it must have an effect in cooling the valves and exhaust passage, and especially when the exhaust passage is also the steam passage will it tend to cause wetness. To make up for this neglect, I shall assume that the water due to the previous expansion remains in the cylinder. A more exact attempt to utilise my results would be to take both into account, multiplying each of them by a function of $n$ which gets less as $n$ gets greater. I have sometimes done so without great alteration to my general result, but with the feeling that there was a pretended exactness about the speculation much interfered with by my ignorance of the action at the valves.

It is to be noticed that the amount of water follows our old law, or $w_3 = w_2 \left(\frac{t_2}{t_3}\right)^s$ if we neglect the wetting effect of the steam which is present with the water.

As I wish to have no more vagueness than I can help, let me in conclusion ask students to check the answers to **the following exercises**:—$w_3$ lb. at $\theta_3$ increases to $w_1$ at $\theta_1$ by (2), and diminishes to $w_2$ at $\theta_2$ by adiabatic expansion, according to (3), putting $i = 0$; then further diminishes to $w_3{}^1$ at $\theta_3$ by (8). What percentage loss of $w$ occurs in the cycle? We know that this is the closest approximation we can make in a mathematical problem to what really occurs in a cylinder. If the cylinder were non-conducting, and there was thermal equilibrium just before and just after admission and during the expansion and release, and if we neglected the volume at admission and at release, and the volume of the steam at the beginning of the adiabatic operation

$$w_3{}^1 = w_3\left\{\frac{H_1 - \theta_3}{l_1}\right\}\left(1 - \frac{t_2}{l_2}\log.\frac{t_1}{t_2}\right)\left(\frac{t_2}{t_3}\right)^s \quad . \quad . \quad . \quad . \quad . \quad (9)$$

| $p_1$ | $r$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | Fractional evaporation. | Fractional condensation. |
|---|---|---|---|---|---|---|
| 203 | 2 | 195 | 165 | 60 | ·007 | — |
| | 5 | 195 | 130 | 60 | — | ·005 |
| | 13 | 195 | 102 | 60 | — | ·014 |
| | 2 | 195 | 165 | 100 | ·000 | — |
| | 5 | — | 130 | 100 | — | ·010 |
| | 13 | — | 102 | 100 | — | ·020 |
| 130 | 2 | 175 | 143 | 60 | ·007 | — |
| | 5 | — | 117 | 60 | — | ·001 |
| | 13 | — | 90 | 60 | — | ·012 |
| | 2 | 175 | 143 | 100 | — | ·000 |
| | 5 | — | 117 | 100 | — | ·009 |
| 79 | 2 | 155 | 130 | 60 | — | ·000 |
| | 5 | — | 102 | 60 | — | ·008 |
| | 2 | 155 | 130 | 100 | — | ·003 |
| | 5 | — | 102 | 100 | — | ·012 |
| 46 | 2 | 135 | 113 | 60 | ·002 | — |
| | 5 | — | 87 | 60 | — | ·003 |
| 25 | 2 | 115 | 95 | 60 | — | ·002 |

These examples show that the result of Problem I. applies fairly well to the more exact conditions studied in Problem III. If we must take into account such small tendencies to evaporation or condensation as we here observe (which seem to me, however, somewhat due to the inaccuracy of our knowledge of latent heat) we may take it that there is always a slight tendency of $w$ to increase or diminish at a rate proportional to its existing value.

**401.** I have worked out my problem exactly on certain assumptions. Other assumptions might be made and worked to, for in the irreversible operations of admission and exhaust there are various ways in which we may imagine the water to condense and vaporise. In release, for example, if the water is very thinly spread over a large surface, and especially if it is on a metal surface like the inside surface of a steam cylinder which has a steam jacket so that the metal is at slightly higher temperature than the water; the inner particles of water (touching the metal) may be warmer than the rest, and they may suddenly or explosively become steam, sending the other particles of water as water off into the outside space. There is reason to believe that this **explosive evaporation** does take place in some steam cylinders.

We might speculate on the case of the **water being in layers** of varying temperature as it is deposited and removed, but I have not yet been able to

frame an easy mathematical problem to illustrate such a state of things, and without such guidance I am afraid to speculate. It seems as if under such circumstances the water might have a drying action such as the metal has.

Any one who has worked in a heat laboratory must feel the impossibility of getting more than mere suggestion from one's general physical knowledge when dealing with this problem. We know a good deal about heat events that occur slowly, very little about those that **occur quickly.** Usually the surfaces of the metal are oily, but even in large modern engines in which oil is forbidden to be used in the cylinder we can see that **capillary actions** of a kind unknown to us must be acting to delay or accelerate evaporation and condensation. When it is almost impossible for us to realise the formation of particles of water in a dustless atmosphere; and we speak of this and other quite simple looking phenomena occurring with great rapidity in the cylinder, the surface of which is at quite different temperatures at different places, it is evident that what occurs inside the cylinder of a steam engine will not be well known to philosophers until long after cylinders of steam engines are only to be found in museums.

**402. The Practical Problem.** The above work gives me a little confidence in making the following assumptions. In future $w$ will mean the total water present at the end of the exhaust.

1. (9) of Art. 400 may be taken as showing how the gain of water $w$ per stroke depends on the value of $w$ itself. We cannot use it in a less vague way than what is suggested below.

2. Any source of steady supply of heat to the cylinder, not contemplated in Problem I., such as superheating or mechanical drainage of water, may be spoken of as if it were a steam jacket effect. A negative steam jacket effect will represent the cooling conditions of an unjacketed cylinder.

3. **The drying effect** due to conductivity of the metal and the steam jacket studied in Problem II. will account for all the drying effects if we assume that in well arranged cylinders, $e$, the surface emissivity is very small where there is no layer of water on the metal, and increases in proportion to the water present, but reaches a constant value if the water gets to be of considerable amount.

This applies only to the case in which the water $w$ **coats the metal in a thin layer,** and it is evident that when there is such a thin layer the drying tendency must be ever so much greater than when the water lies in pockets. **Water in pockets** seems to be altogether an evil. It takes in heat during rise of temperature and gives it out during the fall, but has very little tendency to diminution from one cycle to another as it does so. Water in globules caused by oil is nearly but not quite such a great evil. Whereas the metal with a surface of small resistance to the passage of heat (great $e$) although it acts evilly in much the same way, yet in doing so is always tending to make the cylinder dryer. A sort of equilibrium seems to be established by more of the metal getting a little wetter or dryer on its skin. I understand that a considerable amount of money has been spent in endeavouring to obtain a very non-conducting inside skin for cylinders; my investigation shows that such a skin would really increase the evils which it is intended to prevent.

**The wetting term** due to expansion is $ic'r/(9+r)$. In truth $c'$ is not a constant; it is supposed to diminish at higher speeds because the condensed water has less time to settle down and is hurried out in release with the steam. $c'$ is also less as the dimensions of the cylinder are greater, because of the greater average distance of the stuff from the walls in larger cylinders and passages. I shall call this term $i\,R$, and it is evident that without much alteration it will represent the wetting effect of any water which may cool the valves on release. For greater generality let us also include drying or wetting terms

such as are suggested by our problems. Thus we may take a small drying or wetting term proportional to $w$ say $\beta w$.

Also a wetting term $bi$ due to wet steam being supplied. $b$ is probably greater with low pressure steam because of its less density causing more priming in the boiler; also there is usually more surface exposed by the steam pipes per pound of steam supplied per hour. $b$ is negative if there is superheating.

The steam jacket term which may also be called a drainage term, and which may be negative for unjacketed cylinders, is $\frac{aSev'}{n}$, $e$ being the emissivity, and $v'$ being the excess of the jacket temperature above the mean steam temperature inside; $S$ is the average surface. The metal drying term is $\frac{Se(\theta_1 - \theta_2)^2(g+h/r)}{\sqrt{n} + cn}$. We might distinguish perhaps between what I call average surface for the steam jacket term and the other, but this is really not important. After a short run under steady conditions the drying and wetting balance one another so that

$$iR + ib = \beta w + \frac{ase}{n} v' + Se\phi^2 \frac{g + h/r}{\sqrt{n} + cn}$$

using $\phi$ for $\theta_1 - \theta_2$.

Now our notions about $e$ take the mathematical shape $e = \frac{a'w/s}{1 + mw/s}$ where $d$ and $m$ are constants. Students who delight in practical mathematics will find it interesting to take $e$ such a function of $w$ that it has a small constant value when $w = o$; that it increases proportionately to $w$ when $w$ is small; reaches a maximum for a certain value of $w$ and if $w$ is greater than this, $e$ slowly lessens again. I dare not venture here to give the answer which I obtain when I use this more complex function, and indeed in what follows I shall confine my attention to rather dry cylinders.

**403.** If a cylinder is fairly dry, the effect of $m$ is insignificant, and calling it $o$ we may take

$$i(R + b) = w\left\{\beta + \frac{aa^1v^1}{n} + a'\phi^2 \frac{g + h/r}{\sqrt{n} + cn}\right\}$$

Using the value of $w$ which this gives us in Art. 230 we find, taking $H_1 - \frac{1}{2}(\theta_1 + \theta_2$ as practically constant

$$y = \left(a^1 \frac{g + h/r}{\sqrt{n} + cn} + 1\right)(\theta_1 - \theta_2) \frac{c^1r/(g + r) + b}{\beta + \frac{aa^1v^1}{n} + \frac{a^1\phi^2 (g + h/r)}{\sqrt{n} + cn}} + A \frac{r}{nd}$$

The student must not imagine that I propose this as a working formula. There is no probability of our obtaining a cut and dried formula of general application. I have asked students to follow me in its working out because this sort of study will clear their ideas, and putting our notions down quantitatively on paper gives us a better idea of their real value. We can divide up this formula for a rather dry cylinder into

$$y = \frac{(F + 1)\phi}{\beta + J + F\phi^2} R + L.$$

$L$ is the leakage term, being proportional to $\frac{r}{nd}$; $R$ is also nearly proportional to $r$, but if the steam is supplied in a dry state or slightly superheated as $b$ may be negative, $R$ may also be regarded as proportional to $r$ – a constant. $RF$ is the predominant term in the numerator and this is a linear function of $r$, increasing as $r$ increases.

*S* gets less as the cylinder is larger, because *L* does so, and we saw that *R* also gets less as the cylinder gets larger. *L* is inversely proportional to *n*, and *F* the predominant term in the numerator also gets less as *n* increases, being inversely proportional to *n* in a dry cylinder and inversely proportional to $\sqrt{n}$ in a wet cylinder. As for the denominator of the first part of *y*, it consists of terms which indicate the three tendencies to drying. The Jacket term *J* is altogether good. The Water Film term *F*, we notice, does harm on the whole; we see it in the numerator where its harmful effect is shown; we see it in the denominator, and there its good effect is proportional to the square of the range of temperature, whereas its bad effect is only proportional to the temperature range.

If I am right, as soon as steam condenses it ought to be induced to drain away quickly from a cylinder. This serves two purposes: first there is a diminution of the altogether evil presence of heat-absorbing water; second there is the leaving behind it of its latent heat. The conditions inside a cylinder are critical. A little heat given by drainage or steam jacket or superheating may make all the difference between a wet cylinder with great loss and a dry cylinder with little loss. In my opinion, a metal surface dry at the end of the exhaust will take up but little heat and cause little loss, and the usual notion that we often have it has been invented by academic persons whose calculations (see Art. 209) are of no value unless this assumption is made.

# INDEX

# INDEX

*The References are to Pages*

D

T



U



V

W



Y



Z



THE END.

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52